LAB 6 : Regression

Regression is generally used for curve fitting task. Here we will demonstrate regression task for the following :

1. **Fitting of a Line (One Variable and Two Variables)**
2. **Fitting of a Plane**
3. **Fitting of M-dimensional hyperplane**
4. **Practical Example of Regression task**

In [22]:

```
import numpy as np
import matplotlib.pyplot as plt
```

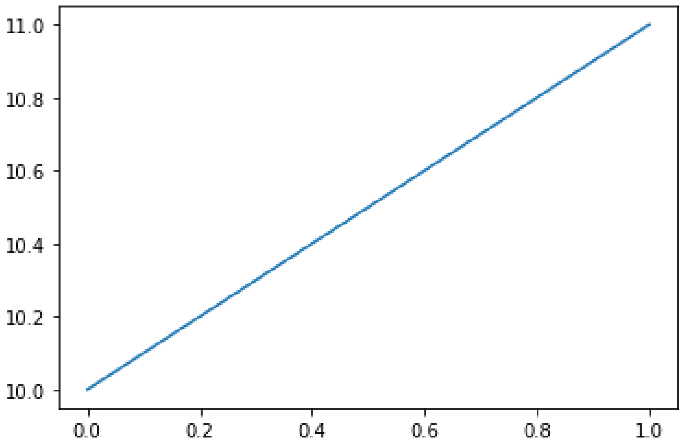
Fitting of a Line (One Variable)

Generation of line data ($y = w_1x$) $+ w_0$

1. **Generate x, 1000 points from 0-1**
2. **Take $w_0 = 10$ and $w_1 = 1$ and generate y**
3. **Plot (x,y)**

In [23]:

```
## Write your code here
w0,w1 = 10,1
x = np.linspace(0,1,1000)
y = w1*x + w0
plt.plot(x,y)
plt.show()
```



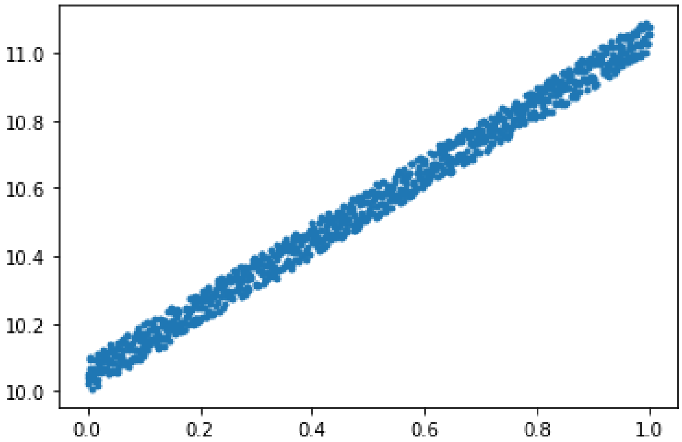
Corruption of data using uniformly sampled random noise

1. **Generate random numbers uniformly from (0-1) with same size as y**
2. **Corrupt y and generate y_{cor} by adding the generated random samples with a weight of 0.1.**
3. **Plot (x,y_{cor}) (use scatter plot)**

In [24]:

```
## Write your code here
```

```
rand_num = np.random.random(y.shape)
y_corr = y + 0.1*rand_num
plt.plot(x,y_corr,'.')
plt.show()
```



Heuristically predicting the curve (Generating the Error Curve)

1. **Keep $w_0 = 10$ as constant and find w_1**
2. **Create a search space from -5 to 7 for w_1, by generating 1000 numbers between that**
3. **Find y_{pred} using each value of w_1**
4. **The y_{pred} that provide least norm error with y, will be decided as best y_{pred}**

$$error = \frac{1}{m}\sum_{i=1}^{M}(y_i - y_{pred_i})^2$$

5. **Plot error vs search_ $w1$**
6. **First plot the scatter plot (x,y_{cor}) , over that plot (x,$y_{bestpred}$)**

In [25]:

```
a = np.array([1,6,7])
b = np.array([2,3,4])
np.outer(a,b)
```

Out[25]:

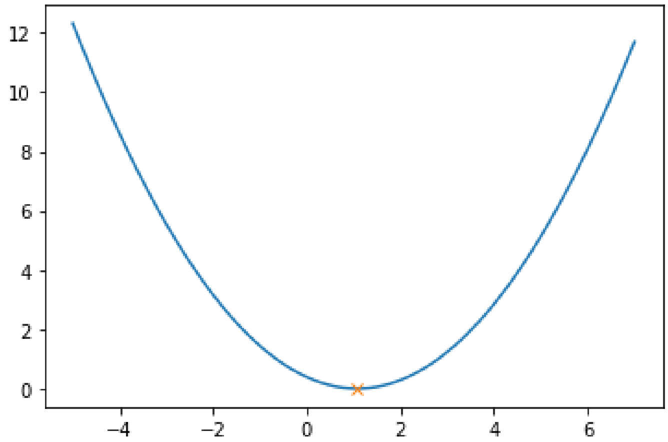
```
array([[ 2,  3,  4],
       [12, 18, 24],
       [14, 21, 28]])
```

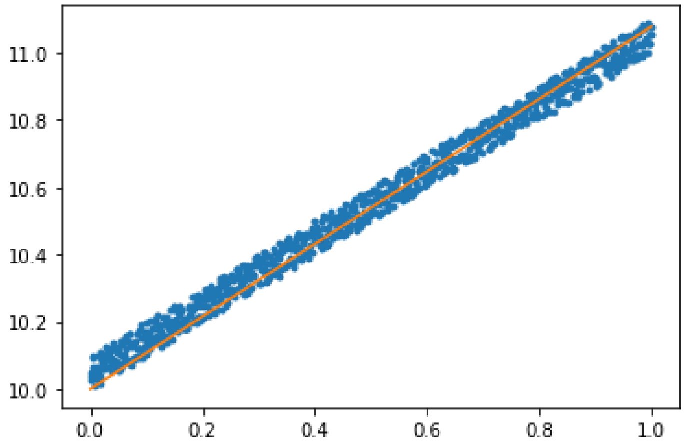
In [26]:

```
## Write your code here
def optimal_sol(x,y,w0,search_space):
    y_pred = np.outer(search_space,x) + w0
    error = np.sum((y_pred - y)**2,axis = 1)/y.shape[0]
    index = np.argmin(error)
    return error,index

w0 = 10
search_space = np.linspace(-5,7,1000)
error,min_index = optimal_sol(x,y_corr,w0,search_space)
y_opt = search_space[min_index]*x + w0
print(f'Optimal Value of w1 is : {search_space[min_index]}')
plt.plot(search_space,error)
plt.plot(search_space[min_index],error[min_index],'x')
plt.show()
```

```
plt.plot(x,y_corr,'.')
plt.plot(x,y_opt)
plt.show()
```

```
Optimal Value of w1 is : 1.0780780780780779
```





Using Gradient Descent to predict the curve

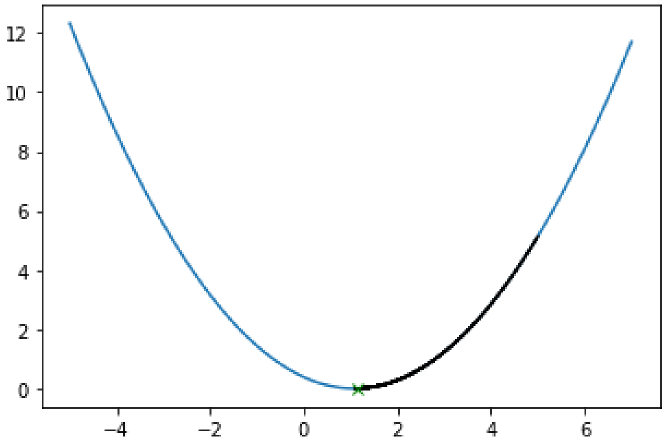
1. $Error = \frac{1}{m}\sum_{i=1}^{M}(y_i - y_{pred_i})^2 = \frac{1}{m}\sum_{i=1}^{M}(y_i - (w_0 + w_1 x_i))^2$
2. $\nabla Error|_{w1} = \frac{-2}{M}\sum_{i=1}^{M}(y_i - y_{pred_i}) \times x_i$
3. $w_1 |_{new} = w_1 |_{old} - \lambda \nabla Error|_{w1} = w_1|_{old} + \frac{2\lambda}{M}\sum_{i=1}^{M}(y_i - y_{pred_i}) \times x_i$

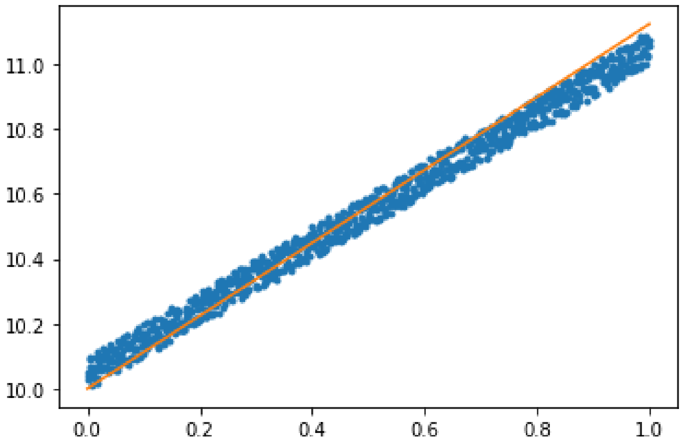
In [27]:

```
## Write your code here
def grad(x,y,y_pred):
    return (-2 * np.sum((y-y_pred) * x))/y.shape[0]

def loss(y,y_pred):
    return np.sum((y_pred - y)**2)/y.shape[0]

plt.figure()
plt.plot(search_space,error)
w1 = 5
w0 = 10
epsilon = 1e-5
lr = 0.01

diff_in_errors = 1000000
iter = 0
while iter < 1000 and diff_in_errors > epsilon:
    new_w1 = w1
```

```
    old_y_pred = w1 * x + w0
    # print(grad(x,y_corr,old_y_pred))
    new_w1 = new_w1 - lr * grad(x,y_corr,old_y_pred)
    new_y_pred = new_w1 * x + w0
    diff_in_errors = np.abs(loss(y_corr,new_y_pred) - loss(y_corr,old_y_pred))
    plt.plot([w1,new_w1],[loss(y_corr,old_y_pred),loss(y_corr,new_y_pred)],color = 'k')
    w1 = new_w1
    iter += 1
    # break

print(f'Optimal Value of w1 is : {w1}')
y_opt = w1 * x + w0
plt.plot(w1,loss(y_corr,y_opt),'x',color='g')
plt.show()
plt.figure()
plt.plot(x,y_corr,'.')
plt.plot(x,y_opt)
plt.show()
```

```
Optimal Value of w1 is : 1.1245149404210937
```





Fitting of a Line (Two Variables)

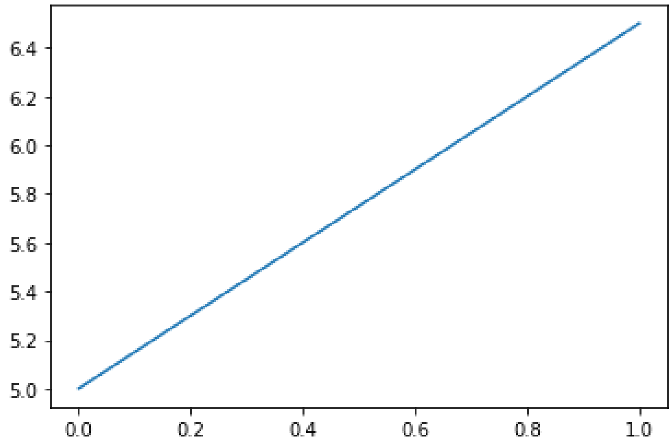
Generation of Line Data ($y = w_1 x$) $+ w_0$

1. Generate x, 1000 points from 0-1
2. Take $w_0 = 5$ and $w_1 = 1.5$ and generate y
3. Plot (x,y)

In [28]:

```
## Write your code here
x = np.linspace(0,1,1000)
```

```
w0,w1 = 5,1.5
y = w1 * x + w0
plt.plot(x,y)
plt.show()
```

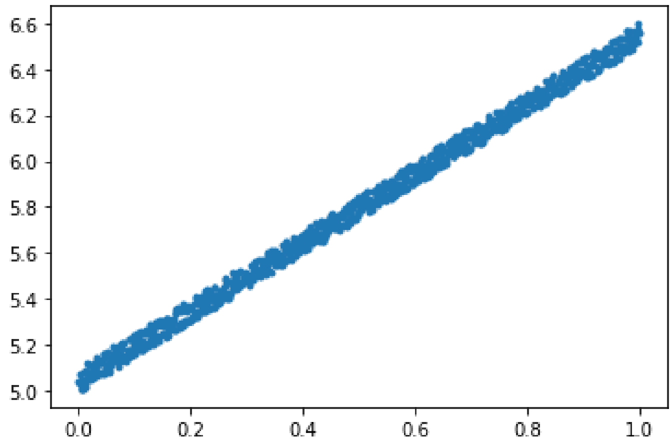


Corrupt the data using uniformly sampled random noise

1. **Generate random numbers uniformly from (0-1) with same size as y**
2. **Corrupt y and generate y_{cor} by adding the generated random samples with a weight of 0.1**
3. **Plot (x,y_{cor}) (use scatter plot)**

In [29]:

```
## Write your code here
rand_num = np.random.random(y.shape)
y_corr = y + 0.1 * rand_num
plt.plot(x,y_corr,'.')
plt.show()
```



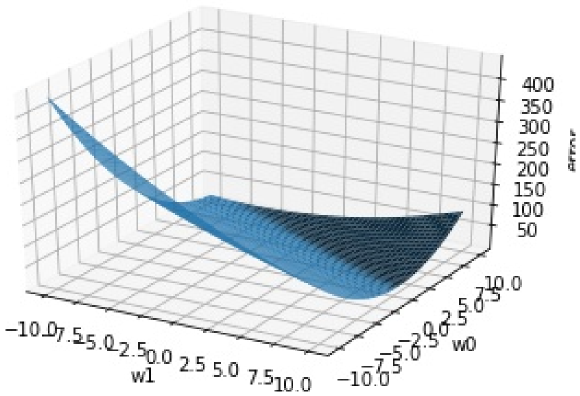
Plot the Error Surface

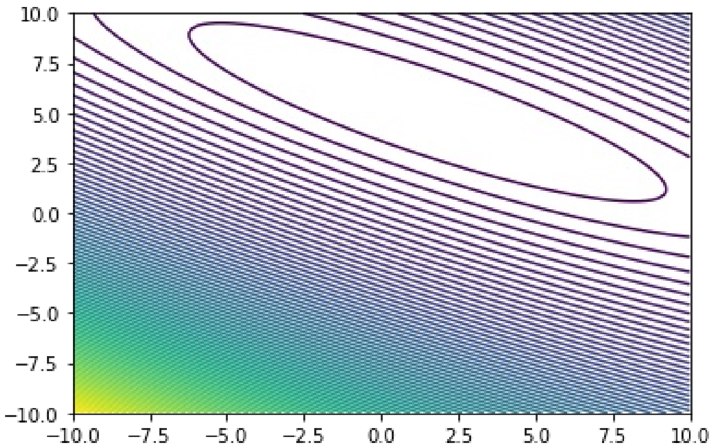
1. **we have all the data points available in y_{cor}, now we have to fit a line with it. (i.e from y_{cor} we have to predict the true value of w_1 and w_0)**
2. **Take w_1 and w_0 from -10 to 10, to get the error surface**

In [30]:

```
## Write your code here
w0 = np.linspace(-10,10,100)
w1 = np.linspace(-10,10,100)
mesh_w0,mesh_w1 = np.meshgrid(w0,w1)

def loss(w1,w0,x,y):
    error = np.zeros(w1.shape)
    for a,b in zip(x,y):
        y_pred = w1 * a + w0
        error += (y_pred - b)**2
```

```
    return error/x.shape[0]

error = loss(mesh_w1,mesh_w0,x,y_corr)

plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(mesh_w1, mesh_w0, error)
ax.set_xlabel('w1')
ax.set_ylabel('w0')
ax.set_zlabel('error');
plt.show()

plt.figure()
plt.contour(mesh_w1, mesh_w0, error,100)
plt.show()
```





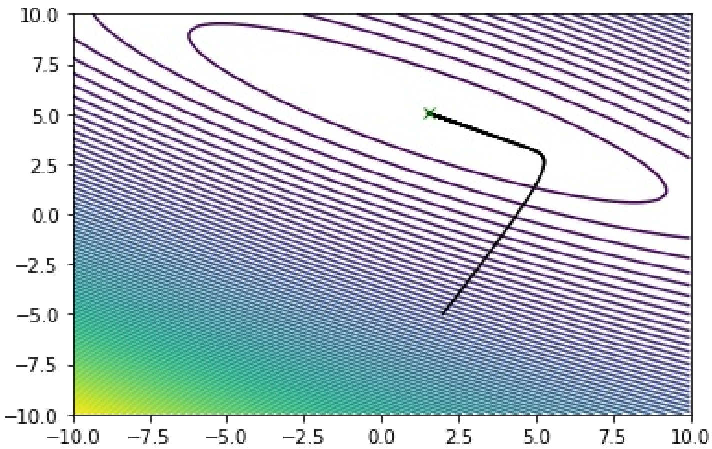
Gradient Descent to find optimal Values

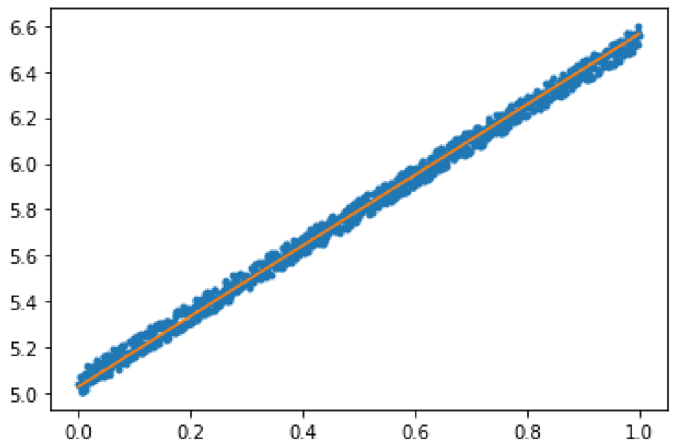
In [31]:

```
## Write your code here
w1 = 2
w0 = -5
lr = 0.1
epsilon = 1e-6

def grad_w1(x,y,y_pred):
    return (-2 * np.sum((y-y_pred) * x))/y.shape[0]

def grad_w0(x,y,y_pred):
    return (-2 * np.sum(y-y_pred))/y.shape[0]

def loss(y,y_pred):
    return np.sum((y_pred - y)**2)/y.shape[0]

plt.figure()
plt.contour(mesh_w1,mesh_w0, error,100)

diff_in_errors = 1000000
iter = 0
```

```python
while iter < 1000 and diff_in_errors > epsilon:
    new_w0 = w0
    new_w1 = w1
    old_y_pred = w1 * x + w0
    # print(grad(x,y_corr,old_y_pred))
    new_w1 = new_w1 - lr * grad_w1(x,y_corr,old_y_pred)
    new_w0 =  new_w0 - lr * grad_w0(x,y_corr,old_y_pred)
    new_y_pred = new_w1 * x + new_w0
    diff_in_errors = np.abs(loss(y,new_y_pred) - loss(y,old_y_pred))
    plt.plot([w1,new_w1],[w0,new_w0],color = 'k')
    w1 = new_w1
    w0 = new_w0
    iter += 1
    # break

print(f'Optimal Value of w0 and w1 is w0 = {w0} and w1 = {w1}')
plt.plot(w1,w0,'x',color='g')
y_opt = w1 * x + w0
plt.show()
# plt.plot(w1,loss(y_corr,y_opt),'x',color='g')
# plt.show()
plt.figure()
plt.plot(x,y_corr,'.')
plt.plot(x,y_opt)
plt.show()
```

```
Optimal Value of w0 and w1 is w0 = 5.029484136355913 and w1 = 1.5347003507873118
```





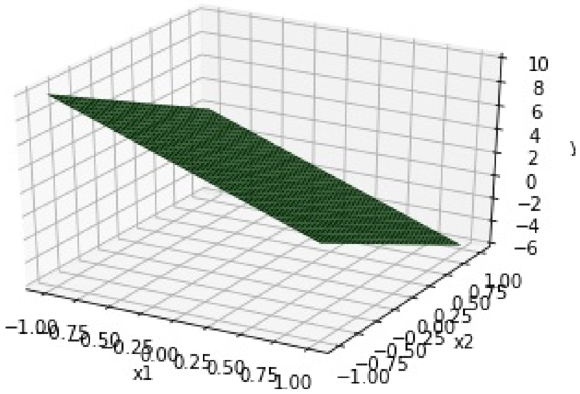
Fitting of a Plane

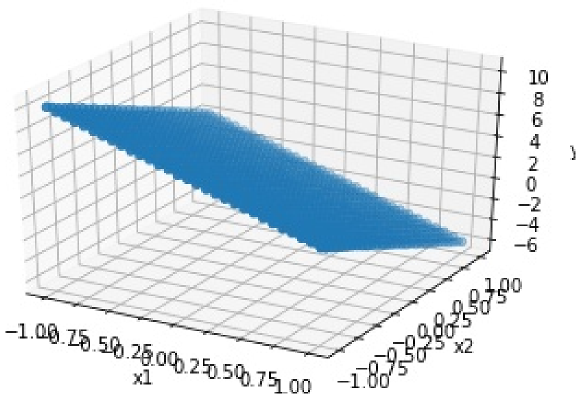
Generation of plane data

1. **Generate x_1 and x_2 from range -1 to 1, (30 samples)**
2. **Equation of plane $y = w_0 + w_1 x_1 + w_2 x_2$**
3. **Here we will fix w_0 and will learn w_1 and w_2**

In [32]:

```
## Write your code here
x1 = np.linspace(-1,1,30)
x2 = np.linspace(-1,1,30)

w0,w1,w2 = 2,-4,-4

y= w0 + w1 * x1 + w2 * x2

X1,X2 = np.meshgrid(x1,x2)
Y = w0 + w1 * X1 + w2 * X2
plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(X1, X2, Y, color = 'g')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');
plt.show()

# Adding a noisy plane
rand_noise_plane = np.random.uniform(0,1,Y.shape)
Y_corr = Y + 0.1 * rand_noise_plane

plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_corr,'.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');
```





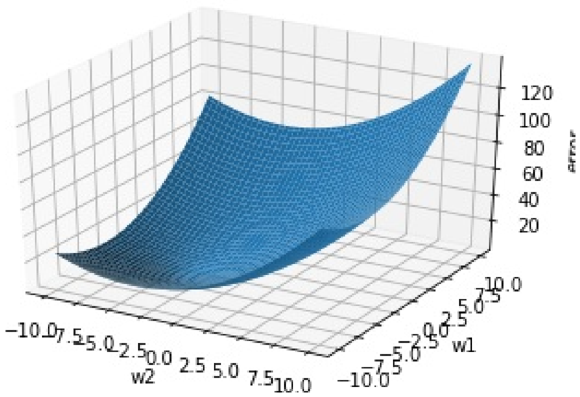
Generate the Error Surface

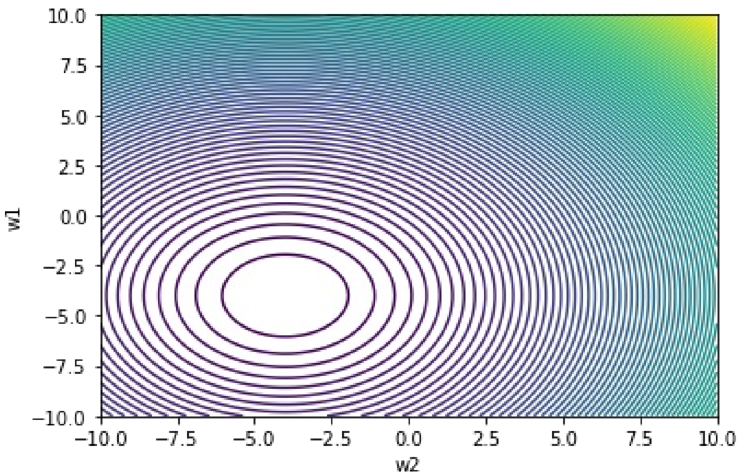
1. **Vary w_1 and w_2 and generate the error surface and find their optimal value**
2. **Also plot the Contour**

In [33]:

```
## Write your code here
x1 = X1.flatten()
```

```
x2 = X2.flatten()
y_corr = Y_corr.flatten()

search_w1 = np.linspace(-10,10,100)
search_w2 = np.linspace(-10,10,100)

W2,W1 = np.meshgrid(search_w2,search_w1)

def loss(w2,w1,w0,x1,x2,y):

    error = np.zeros(w1.shape)
    for a,b,c in zip(x1,x2,y):
        y_pred = w0 + w1 * a + w2 * b
        error += (y_pred - c)**2

    return error/x1.shape[0]

error = loss(W2,W1,w0,x1,x2,y_corr)

plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(W2, W1, error)
ax.set_xlabel('w2')
ax.set_ylabel('w1')
ax.set_zlabel('error');
plt.show()

plt.figure()
plt.contour(W2, W1, error,100)
plt.xlabel('w2')
plt.ylabel('w1')
plt.show()
```





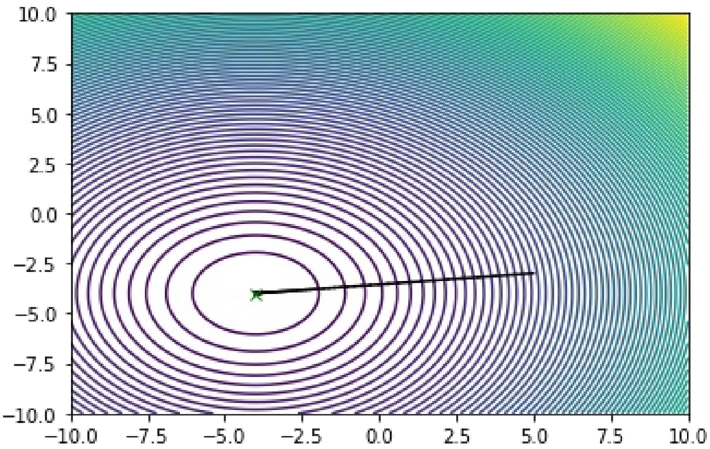
Prediction using Gradient Descent

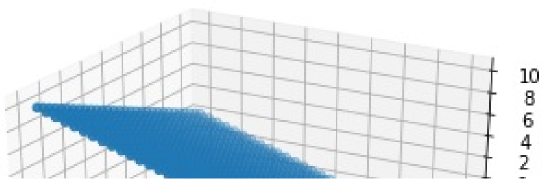
In [34]:

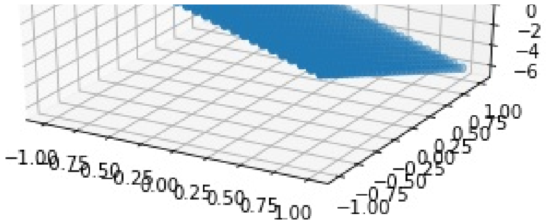
```
## Write your code here
w2 = 5
w1 = -3
```

```
lr = 0.1
epsilon = 1e-6
w0 = 2

def grad_w2(x2,y,y_pred):
    return (-2 * np.sum((y-y_pred) * x2))/y.shape[0]

def grad_w1(x1,y,y_pred):
    return (-2 * np.sum((y-y_pred) * x1))/y.shape[0]

def loss(y,y_pred):
    return np.sum((y_pred - y)**2)/y.shape[0]

plt.figure()
plt.contour(W2, W1, error,100)

diff_in_errors = 1000000
iter = 0
while iter < 10000 and diff_in_errors > epsilon:
    new_w1 = w1
    new_w2 = w2
    old_y_pred = w0 + w1 * x1 + w2 * x2
    new_w1 = new_w1 - lr * grad_w1(x1,y_corr,old_y_pred)
    new_w2 =  new_w2 - lr * grad_w2(x2,y_corr,old_y_pred)
    new_y_pred = w0 + new_w1 * x1 + new_w2 * x2
    diff_in_errors = np.abs(loss(y_corr,new_y_pred) - loss(y_corr,old_y_pred))
    plt.plot([w2,new_w2],[w1,new_w1],color = 'k')
    w1 = new_w1
    w2 = new_w2
    iter += 1

print(f'Optimal Value of w1 and w2 is w1 = {w1} and w2 = {w2}')
plt.plot(w2,w1,'x',color='g')
plt.show()

plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_corr,'.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

y_opt = w0 + w1*X1 + w2*X2

ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, y_opt,'.')
plt.show()
```

```
Optimal Value of w1 and w2 is w1 = -3.9997592580265104 and w2 = -3.9926522699841107
```







Fitting of M-dimentional hyperplane (M-dimention, both in matrix inversion and gradient descent)

Here we will vectorize the input and will use matrix method to solve the regression problem.

let we have M- dimensional hyperplane we have to fit using regression, the inputs are $x1, x2, x3, \ldots, x_M$ **. in vector form we can write** $[x1, x2, \ldots, x_M]^T$ **, and similarly the weights are** $w1, w2, \ldots w_M$ **can be written as a vector** $[w1, w2, \ldots w_M]^T$ **, Then the equation of the plane can be written as:**

$$y = w1x1 + w2x2 + \ldots + w_M x_M$$

$w1, w2, \ldots, wM$ **are the scalling parameters in M different direction, and we also need a offset parameter w0, to capture the offset variation while fitting.**

The final input vector (generally known as augmented feature vector) is represented as $[1, x1, x2, \ldots, x_M]^T$ **and the weight matrix is** $[w0, w1, w2, \ldots w_M]^T$ **, now the equation of the plane can be written as:**

$$y = w0 + w1x1 + w2x2 + \ldots + w_M x_M$$

In matrix notation: $y = x^T w$ **(for a single data point), but in general we are dealing with N- data points, so in matrix notation**

$$Y = X^T W$$

where Y is a $N \times 1$ **vector, X is a** $M \times N$ **matrix and W is a** $M \times 1$ **vector.**

$$Error = \frac{1}{N}||Y - X^T W||^2$$

it looks like a optimization problem, where we have to find W, which will give minimum error.

1. **By computation:**

$\nabla Error = 0$ **will give us** W_{opt}**, then** W_{opt} **can be written as:**

$$W_{opt} = (XX^T)^{-1} XY$$

1. **By gradient descent:**

$$W_{new} = W_{old} + \frac{2\lambda}{N} X(Y -$$

$- X^T W_{old})$

1. **Create a class named Regression**
2. **Inside the class, include constructor, and the following functions:**

   a. grad_update: Takes input as previous weight, learning rate, x, y and returns the updated weight.

   b. error: Takes input as weight, learning rate, x, y and returns the mean squared error.

   c. mat_inv: This returns the pseudo inverse of train data which is multiplied by labels.

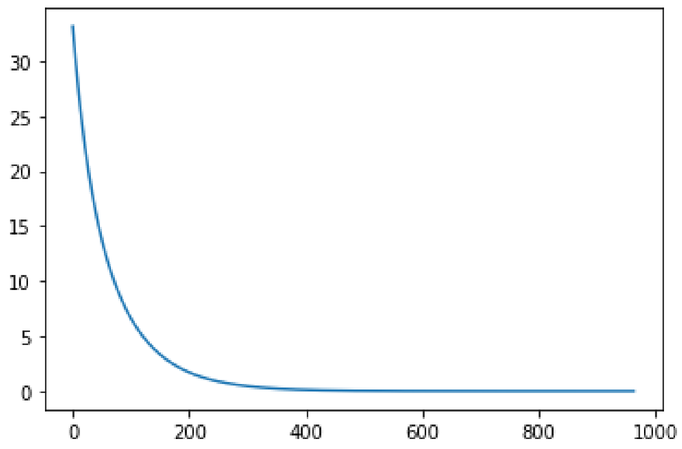
   d. Regression_grad_des: Here, inside the for loop, write a code to update the weights. Also calulate error after each update of weights and store them in a list. Next, calculate the deviation in error with new_weights and old_weights and break the loop, if it's below a threshold value mentioned the code.

In [35]:

```
class regression:
  # Constructor
  def __init__(self, name='reg'):
    self.name = name  # Create an instance variable

  def grad_update(self,w_old,lr,y,x):
    w = w_old + lr * (2/x.shape[1]) * (x @ (y - (x.T @ w_old)))
    return w

  def error(self,w,y,x):
    return np.mean((y-x.T @ w)**2)

  def mat_inv(self,y,x_aug):
    return np.linalg.pinv((x_aug @ x_aug.T)) @ x_aug @ y

  # By Gradien descent

  def Regression_grad_des(self,x,y,lr):
    epsilon = 1e-6
    error = []
    for i in range(1000):
      if i == 0:
        w = np.random.uniform(-1,1,(x_aug.shape[0],1))
        w_old = w
        w_pred = self.grad_update(w_old,lr,y,x_aug)
      else:
        w_old = w_pred
        w_pred = self.grad_update(w_old,lr,y,x_aug)

      error.append(self.error(w_pred,y,x_aug))
      dev = np.abs(self.error(w_pred,y,x_aug) - self.error(w_old,y,x_aug))

      if dev <= 1e-6:
        break

    return w_pred,error


############################################################################
# Generation of data

sim_dim=5
sim_no_data=1000
x=np.random.uniform(-1,1,(sim_dim,sim_no_data))
print(x.shape)

w = np.array([[4],[-3],[1],[2],[8],[0]])
print(w.shape)

## Augment the Input
x_aug = np.concatenate((np.ones((1,x.shape[1])), x),axis=0)
print(x_aug.shape)
```

```
y=x_aug.T @ w   # vector multiplication
print(y.shape)

## Corrupt the input by adding noise
noise=np.random.uniform(0,1,y.shape)
y=y+0.1*noise

### The data (x_aug and y) is generated ###

############################################################################

# By Computation (Normal Equation)
reg = regression()
w_opt=reg.mat_inv(y,x_aug)
print(w_opt)

# By Gradien descent
lr=0.01
w_pred,err=reg.Regression_grad_des(x_aug,y,lr)
print(w_pred)

plt.plot(err)
plt.show()
```

```
(5, 1000)
(6, 1)
(6, 1000)
(1000, 1)
[[ 4.05113343e+00]
 [-3.00006012e+00]
 [ 9.98736792e-01]
 [ 2.00357498e+00]
 [ 8.00147673e+00]
 [-1.40017087e-03]]
[[ 4.05080792e+00]
 [-2.99083852e+00]
 [ 9.99421245e-01]
 [ 2.00092608e+00]
 [ 7.98942134e+00]
 [-2.02359004e-03]]
```



Practical Example (Salary Prediction)

1. **Read data from csv file**
2. **Do train test split (90% and 10%)**
3. **Compute optimal weight values and predict the salary using the regression class created above (Use both the methods)**
4. **Find the mean square error in test.**
5. **Also find the optimal weight values using regression class from the Sci-kit learn library**

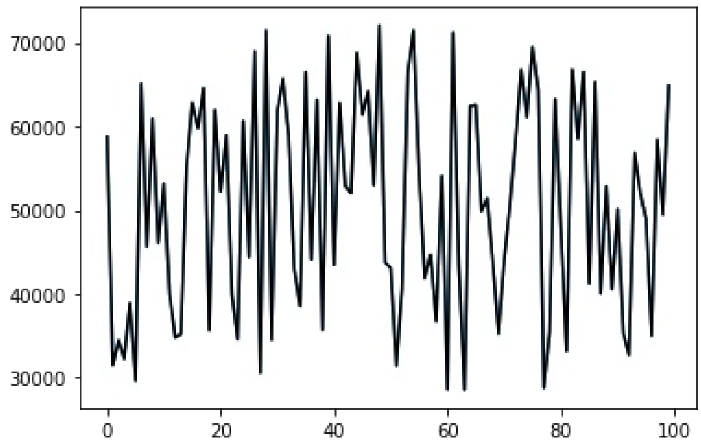
In [36]:

```
## Write your code here
import math
import pandas as pd
from sklearn.model_selection import train_test_split

data = pd.read_csv('salary_pred_data.csv')
X = data.drop(['Salary'],axis =1)
Y = data['Salary']
x_train,x_test,y_train,y_test = train_test_split(X,Y,test_size = 0.1,random_state=4)
x_train,x_test,y_train,y_test = x_train.T,x_test.T,y_train.T,y_test.T
# print(x_train.shape,x_test.shape,y_train.shape,y_test.shape)
x_train = np.asarray(x_train)
x_test = np.asarray(x_test)
x_train = np.concatenate((np.ones((1,x_train.shape[1])), x_train),axis=0)
x_test = np.concatenate((np.ones((1,x_test.shape[1])), x_test),axis=0)
y_train = np.asarray(y_train).T
y_test = np.asarray(y_test).T
reg=regression()
# print(x_train.shape,x_test.shape,y_train.shape,y_test.shape)
w_pred_matrix = reg.mat_inv(y_train,x_train)
error_train = reg.error(w_pred_matrix,y_train,x_train)/((np.max(y_train)-np.mean(y_train
))**2)
error_test = reg.error(w_pred_matrix,y_test,x_test)/((np.max(y_test)-np.mean(y_test))**2
)
y_pred = x_test.T @ w_pred_matrix
print('Normalized training error = ',error_train,'\n')
print('Normalized testing error = ',error_test,'\n')
print('predicted salary = ',y_pred[0:3],'\n')
print('actual salary = ',y_test[0:3])
print("\nWeights with Matrix Inversion")
```

```
Normalized training error =  2.785366539722834e-28

Normalized testing error =  3.4968574342306293e-28

predicted salary =  [58744. 31490. 34464.]

actual salary =  [58744 31490 34464]

Weights with Matrix Inversion
```

In [37]:

```
plt.plot(y_test)
plt.plot(y_pred,'k')
plt.show()
```



In [38]:

```
from sklearn.linear_model import LinearRegression

reg = LinearRegression()
reg.fit(x_train.T,y_train)

final_weights = reg.coef_
```

```
print(final_weights)

y_pred = reg.predict(x_test.T)

plt.plot(y_test)
plt.plot(y_pred,'k')
plt.show()
```

```
[0.e+00 2.e+03 1.e+02 2.e+00 3.e+02 5.e+03]
```