New algorithms for approximating massive datasets

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TTIC

2024 Feb 21

This talk – geometrically summarizing massive datasets

Guiding questions/motivations

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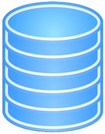
- quickly summarize a massive dataset in a geometrically meaningful way?



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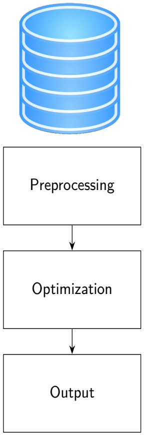
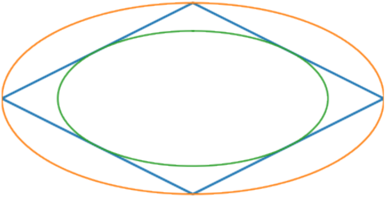
- use our summaries as optimization primitives or preprocessing routines?

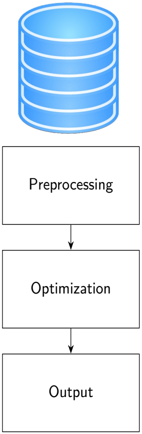


Guiding questions/motivations

Can we:

- quickly summarize a massive dataset in a geometrically meaningful way?
- use our summaries as optimization primitives or preprocessing routines?
- gain a better understanding of high-dimensional convex geometry?





Streaming algorithms for ellipsoidal approximation of convex polytopes – symmetric and asymmetric

https://arxiv.org/abs/2206.07250 (COLT 2022) and
https://arxiv.org/abs/2311.09460 (STOC 2024, to appear)

Yury Makarychev, *NSM*, Max Ovsiankin

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Ellipsoidal approximations

Basic problem

Given a convex body $X \subset \mathbb{R}^n$, find an ellipsoid $\mathcal{E}$ and a center $\boldsymbol{c} \in \mathbb{R}^n$ such that $\boldsymbol{c} + \mathcal{E}/\alpha \subseteq X \subseteq \boldsymbol{c} + \mathcal{E}$.

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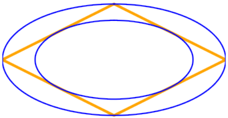

Figure: $\mathcal{E}/\sqrt{2} \subseteq X \subseteq \mathcal{E}$

Ellipsoidal approximations

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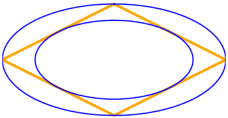

Figure: $\mathcal{E}/\sqrt{2} \subseteq X \subseteq \mathcal{E}$

Such an $\mathcal{E}$ allows us to succinctly represent a convex body (n^2 floats).

Offline solution

Ellipsoidal approximation – offline

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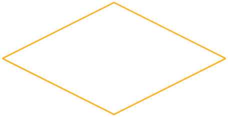

Figure: X

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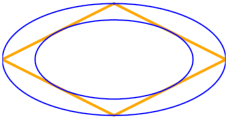

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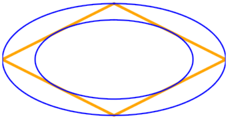

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If $\mathcal{E}$ is the minimum volume ellipsoid covering X, then we can always achieve $\alpha \leq n$.

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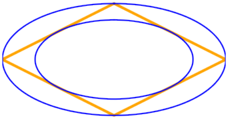

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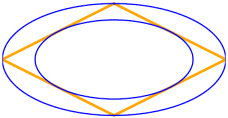

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There exists X for which *any* ellipsoidal approximation for X must achieve $\alpha = n$ (e.g. simplex). For symmetric polyhedrons with m linear constraints, John's Ellipsoid can be approximated in time $\widetilde{O}\left(mn^2\right)$ [CCLY19].

Streaming/online ellipsoidal approximations

Problem

Given a convex body $X = \text{conv}\,(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)$ as a stream of points, find a center $\boldsymbol{c} \in \mathbb{R}^n$ and ellipsoid $\mathcal{E}$ with $\boldsymbol{c} + {}^{\mathcal{E}}\!/_{\alpha} \subseteq X \subseteq \boldsymbol{c} + \mathcal{E}$ that minimizes α.

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$\boxed{\boldsymbol{x}_1}$

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Motivation – Suppose we want to summarize a dataset in a resource-constrained environment.

We want our algorithms to be efficient:

- Cannot store too many points at once;
- Update time in each iteration must be fast.

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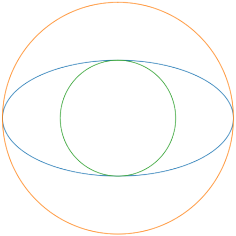

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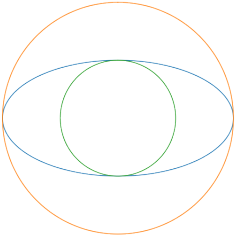

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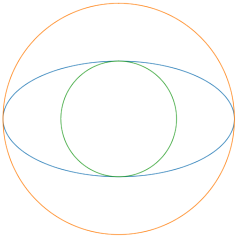

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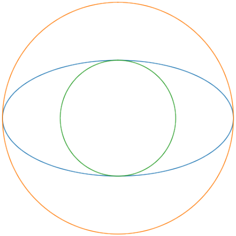

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Results for streaming ellipsoidal approximations

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- If we assume the points $\boldsymbol{x}_i$ have integer coordinates in $[-N,\dots,N]$, then we can replace $\log(R/r)$ with $\log n + \log N$.

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Greedy algorithm, symmetric case

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The algorithm is told a value of r_0 *such that* $r_0 \cdot B_2^n \subseteq X$.

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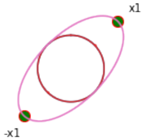

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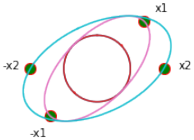

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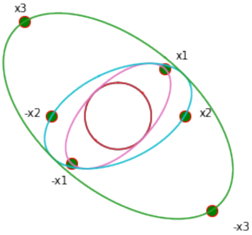

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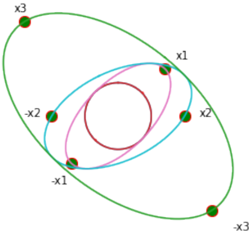

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Greedy algorithm, symmetric case

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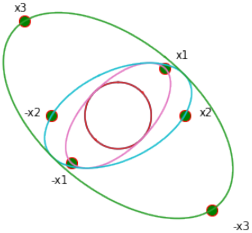

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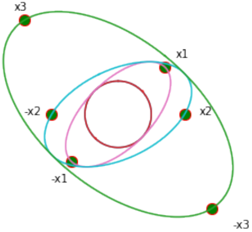

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Convex hull coresets

Desired approximation guarantee

Find a subset S of $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m$ such that there exists a center $\boldsymbol{c}_m \in \mathbb{R}^n$ for which

$$\begin{aligned} & \boldsymbol{c}_m + \operatorname{conv}(X_S) \\ \subseteq\ & \operatorname{conv}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m) \\ \subseteq\ & \boldsymbol{c}_m + \alpha_m \cdot (\operatorname{conv}(X_S) - \boldsymbol{c}_m) \end{aligned}$$

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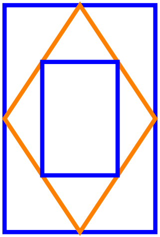

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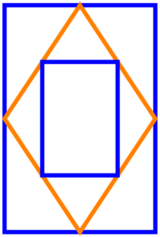

From ellipsoids to convex hull coresets (informal)

There exists an algorithm that calls the ellipsoidal approximation algorithm as a subroutine and chooses S in an online fashion such that:

$$\alpha_m \leq Cn \log\left(n\kappa^{\mathrm{OL}}(X)\right) \qquad \text{(asymmetric)}$$

$$\alpha_m \leq C\sqrt{n \log\left(n\kappa^{\mathrm{OL}}(X)\right)} \qquad \text{(symmetric)}$$

and $|S| \leq Cn \log\left(n\kappa^{\mathrm{OL}}(X)\right)$.

Conclusion – Streaming Algorithms for Ellipsoidal Approximation of Convex Polytopes

1. We gave **simple, nearly-optimal streaming algorithms** to calculate an ellipsoidal approximation for a convex polytope.

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4. Papers – https://arxiv.org/abs/2206.07250 (symmetric) and https://arxiv.org/abs/2311.09460 (asymmetric).

Approximating matrix block norms

https://arxiv.org/abs/2311.10013 (in progress)

NSM, Max Ovsiankin

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Matrix block norm sparsification

$$\|\mathbf{A}\boldsymbol{x}\|_{\mathcal{G}_p}^p := \sum_{i=1}^{m} \|\mathbf{A}_{S_i}\boldsymbol{x}\|_{p_i}^p .$$

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Applications – subroutine to speed up regression, dataset summarization, etc

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Our results

Regimes of p and $p_1, \ldots, p_m$:

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1. (Existence) If p and p_i belong to at least one of the regimes above, then there exists a weight vector $\beta \in \mathbb{R}^m_{\geq 0}$ such that

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Approximate leverage scores ($\boldsymbol{a}_i^T \left(\mathbf{A}^T\mathbf{D}\mathbf{A}\right)^{-1} \boldsymbol{a}_i$ for nonnegative diagonal $\mathbf{D}$) can be found in time $\widetilde{O}(\mathrm{nnz}(\mathbf{A}) + n^\omega)$.

Our results

Regimes of p and $p_1, \ldots, p_m$:

- $p > 0$ and $p_1, \ldots, p_m = p$;
- $p \geq 1$ and $p_1, \ldots, p_m \geq 2$.

Want a weighted subset of rows $\mathbf{A}' \in \mathbb{R}^{\widetilde{m} \times n}$ such that

$$\|\mathbf{A}\boldsymbol{x}\|_{\mathcal{G}_p}^p \in (1 \pm \varepsilon) \cdot \|\mathbf{A}'\boldsymbol{x}\|_{\mathcal{G}_p}^p$$

Theorem

1. (Existence) If p and p_i belong to at least one of the regimes above, then there exists a weight vector $\beta \in \mathbb{R}^m_{\geq 0}$ such that

$$\widetilde{m} := \|\beta\|_0 = C(p, p_1, \ldots, p_m) \cdot \frac{n^{\max(1,p/2)} \log\left(n/\varepsilon\right) (\log n)^2}{\varepsilon^2}.$$

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Dependence on n is essentially optimal (we need $\widetilde{m} \gtrsim n^{\max(1,p/2)}$ [LWW19]).

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Our task: Find $\mathcal{D}$ such that for a "small" $\widetilde{m}$, w.h.p, for all $\boldsymbol{x} \in \mathbb{R}^n$:

$$(1-\varepsilon) \cdot \frac{f(\boldsymbol{x})}{\widetilde{m}} \le \|\mathbf{A}\boldsymbol{x}\|_{\mathcal{G}_p}^p \le (1+\varepsilon) \cdot \frac{f(\boldsymbol{x})}{\widetilde{m}}.$$

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4. Paper – https://arxiv.org/abs/2311.10013.

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The input in a data science problem instance isn't always clean or accessible.

Untrustworthy and unexpected data

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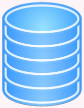

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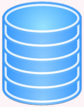

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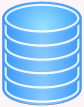

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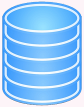

Questions?

Thank you!!

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