Food Crisis: Leveraging mathematical modeling to develop effective food plans for K-12 schools

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Executive Summary

Nutritional management is an important concept that has rapidly risen in importance in the United States given the prominence of obesity and poor health.[1] Several programs have been initiated to mitigate the difficulties school systems are facing when attempting to satisfy the needs of students while maintaining inexpensive food production (Ibid.). It truly is difficult to appease the desires of students, administrators, and government officials, but mathematical modeling may hold the missing link to a viable solution to this problem. The United States Department of Agriculture has tasked us with developing mathematical models to predict the theoretical Calorie intake of students given a variety of individual characteristics, prepare a plan that will enable a subsidised school budget of $7 a week for lunch, and another plan that can sustain a $6 a week lunch plan.

The USDA first required us to determine the Calorie output of a certain individual given certain personal characteristics such as age, gender, and weight. To model this, we employed the Mifflin-St Jeor equation as a basis for determining the testing metabolic rate (RMR), which gives the amount of Calories necessary to maintain homeostasis. Based upon the RMR value, we derived a novel expression that utilizes age, weight, and height to determine Calorie output.

Next, we utilized predicted values for Caloric consumption at lunchtime for high schoolers aged 14-17 in the six different categories separated by gender and in order to find the value at which 95% of students would fulfill their Caloric requirements. Utilizing a normal distribution of Caloric consumption, we used the sample mean, sample standard deviation, and a t-distribution with 23 degrees of freedom, to find that a 750 Calorie meal would be adequate for 95% of students without severely exceeding Caloric limits for other students.

Finally, we define parameters for a meal plan and constructed it. The meal plan would follow the 750 Calorie requirement set in the previous step, and based on government subsidies and/or student payments, would cost $4.33 or less per meal per student on the $7 per student per week budget, and $4.13 or less per meal per student on the $6 per student per week budget. Based off a listing of nutritional requirements on a 2200 Calorie budget, we compiled a list of various menu plans that fulfilled the nutritional requirements of food groups while staying at around 731 Calories, giving us some flexibility. The entrees also came from a list of most-preferred entrees among students in 9th-12th grade. Our most expensive menu plan, complied from prices on a bulk foods website, was $4.09, which lies within the restrictions of both the $7 and $6 plan.

Introduction

Background

The issue of school nutrition has plagued countless high schools throughout the United States. Students have ardently denounced lackluster food quality through social media, boycotts, and parody videos.[2] The situation is not much better in middle schools- students have organized lunch strikes and candidly informed the media of their poor opinion of the food system (Ibid).

In contemporary consumerist culture, all parties involved in food nutrition desire the best of all worlds: students covet lunches that are filling and tasty, school districts want the lunches to be inexpensive and simple to produce, and the federal government desires lunches that are healthy and sustainable. Michelle Obama's "Healthy, Hunger-Free Kids Act of 2010" was one of the first effective actions undertaken to ensure that the American population would not succumb to obesity and unhealthy eating practices.[3] Her policy mandated that public schools were to follow revised nutritional plans which included items such as fruits, vegetables, and protein sources (Ibid.). To encourage school participation in the plan, she offered the incentive of extra federal funding for schools that partook in the proposal (Ibid.). Nevertheless, the plan was not received well by many students- the replacement of scrumptious foods such as ice cream and burgers with salads and fruits incited a multitude of complaints from students (Ibid.).Reconciling the wants of students, school districts, and the federal government remains one of the greatest challenges in the nutritional world. In this report we utilize mathematical models and computations to determine the Calorie requirements of students based on their personal attributes and develop a lunch proposal that is appealing to students and administrators alike.

Restatement of Problem

In this investigation, we were requested by the United States Department of Agriculture (USDA) to develop a mathematical model to approximate the optimal Calorie intake of a student at a school lunch based on individualized parameters. We were additionally asked to create a distribution of United State high school students within each of the parameters defined in the first section, and compute the percent of students who meet their Caloric requirements during school lunches. Finally we were requested to develop a school lunch plan based on food categories that could operate on just \$7 a week, and modify the plan to develop another plan that could operate on just \$6 a week.

Global Assumptions

1. We will start by making the assumption that a student consumes $27\frac{3}{11}\%$ of his or her daily Caloric intake during lunch. We obtained this number by noting that a student typically has a meal schedule in the order of breakfast, snack, lunch, snack, and dinner. If we let the percentage of the daily Caloric intake consumed during the snack be x and we assume that this percentage doubles during breakfast, triples during lunch, and quadruples during dinner, then, we have $2x + x + 3x + x + 4x = 11x = 100$, and $x = \frac{100}{11}$. Hence, $3x = \frac{300}{11} = 27\frac{3}{11}\%$. However, because such assumptions are typically rough in nature, we can round this up, making lunch approximately $\boxed{30\%}$ of the daily Caloric intake.

2. Furthermore, we will assume that none of the students whose data we are using to develop a mathematical model have pre-existing health conditions. In other words we will presume that no students have disorders such as Anorexia or extreme obesity that could influence the given data. Such an assumption ensures that the developed model will best represent optimal Calorie intake for a normal American population of school students.

3. Finally, we will assume that all students eat either all of the school lunch or none of it (i.e. they eat lunch from some source other than the school cafeteria). Thus there is no middle ground given for the students with respect to the school lunch; we will develop our model with the underlying facet that students cannot selectively eat parts of the school lunch. The Calories supplied by a given lunch will either be completely consumed or not consumed at all by a student.

Defining Variables

There are several factors that we have taken into account when generating the average number of Calories an average student should consume during a school lunch. These factors can be classified into independent variables (parameters the influence the dependent variables) an d dependent variables (parameters influenced by the independent variables). The independent variables include the student's age (in years) a, height (in inches) h, weight (in pounds) m, activity factor f, hours of sleep (in hours) s, and resting metabolic rate (in kcal/day) r. The dependent variables include the Caloric intake (in kcal) c and the hourly metabolic rate (in kcal/hr) m.

Analysis of the Problem and Model

You are What You Eat - Development of a Mathematical Formula for determining Calorie requirements

Assumptions

1. We will assume that there is no activity (denoted by a) generated when a person is sleeping. This assumption will be used when generating the overall model for the amount of Calories required for a individual based off of certain parameters.

Methodology

The greatest factor in calculating daily Calorie allotments is metabolic rate, which is the amount of Calories necessary to maintain homeostasis. It is the amount of Calories necessary to maintain functionality in ones body at rest. Basal metabolic rate (BMR) requires that the subject in a dark room after eight hours of sleep and twelve hours of fasting. Resting metabolic rate (RMR) is measured under less restrictive conditions. For the sake of our model, resting metabolic rate will be utilized, as it is more representative of the conditions of daily life for children.

RMR is definitively measured in gas analysis through indirect or direct Calorimetry, but estimates can be obtained through predictive equations using such attributes as height, weight, and age.[4] According to a 2005 study done by the J Am Diet Association, of the four RMR equations commonly used in clinical practice (Harris-Benedict, Mifflin-St Jeor, Owen, World Health Organization/Food and Agriculture Organization/United Nations University [WHO/FAO/UNU]), Mifflin-St Jeor produced the most accurate and reliable results, predicting RMR within 10% for every individual.[5]

The Mifflin-St Jeor equation accounts for the attributes of weight, height, age, and gender, and is as follows:

$$RMR = (\frac{10.0m}{1\text{kg}} + \frac{6.25h}{1\text{cm}} + \frac{5.0a}{1\text{year}} + s)\frac{\text{kcal}}{\text{day}}$$

where m is weight in kilograms, h is height in centimeters, a is age in years, and s is +5 for males and -161 for females (Ibid.).

The total Caloric intake of an individual is not solely his or her RMR, because humans are not always at rest. Multipliers must be included to account for differing lifestyles. The multiplier for a sedentary or slightly active lifestyle is 1.53, the multiplier for a lightly active or active lifestyle is 1.76, and the multiplier for a vigorous lifestyle is 2.25. The total Caloric allowance is thus calculated by multiplying an individual's estimated RMR from the Mifflin-St Jeor equation and the proper multiplier for their lifestyle.

Because a student's lunch is assumed to fulfill 30% of their daily intake, the total Caloric allowance is thus multiplied by .3 to determine the number of Calories that should be eaten at lunch.

Now, we will proceed to the derivation of the model. Previous attempts to model this have not included certain factors that could affect the daily Caloric consumption (and as a result, the lunch Caloric consumption is not accurate as well) such as the amount of sleep that a student gets nightly. The model we have developed takes this very important factor into account. In order to derive the equation, we will define a few variables.

We define the Sleep Metabolic Rate (SMR) to be the metabolic rate at sleep. We will call the ratio of the SMR to the RMR k. We will let the value of the RMR be r, and the activity factor (based on the amount of physical activity a particular student receives per day) be a. Additionally, in order to factor in the amount of sleep a particular student receives nightly, we will create a variable called h, which accomplishes this function. Then, we begin by calculating a "base" metabolic rate (different from the Basal Metabolic Rate) by simply multiplying the RMR by the activity factor. Hence, we begin by calculating the RMR. By the Mifflin-St. Jeor equation, we can calculate r. Then, we begin with the below:

$$ra$$

Then, we factor in sleep. The amount of Calories burned will come from the time spent sleeping, which is found by the below equation:

$$\frac{rkh}{24}$$

However, since we have double counted the RMR for the period spent sleeping, we need to subtract the below from our model:

$$\frac{arh}{24}$$

Hence, we obtain the following model:

$$ar + \frac{rkh}{24} - \frac{arh}{24}$$

And, when we simplify, we obtain a model for the amount of Calories consumed daily:

$$\frac{r}{24}(hk - ha + 24a)$$

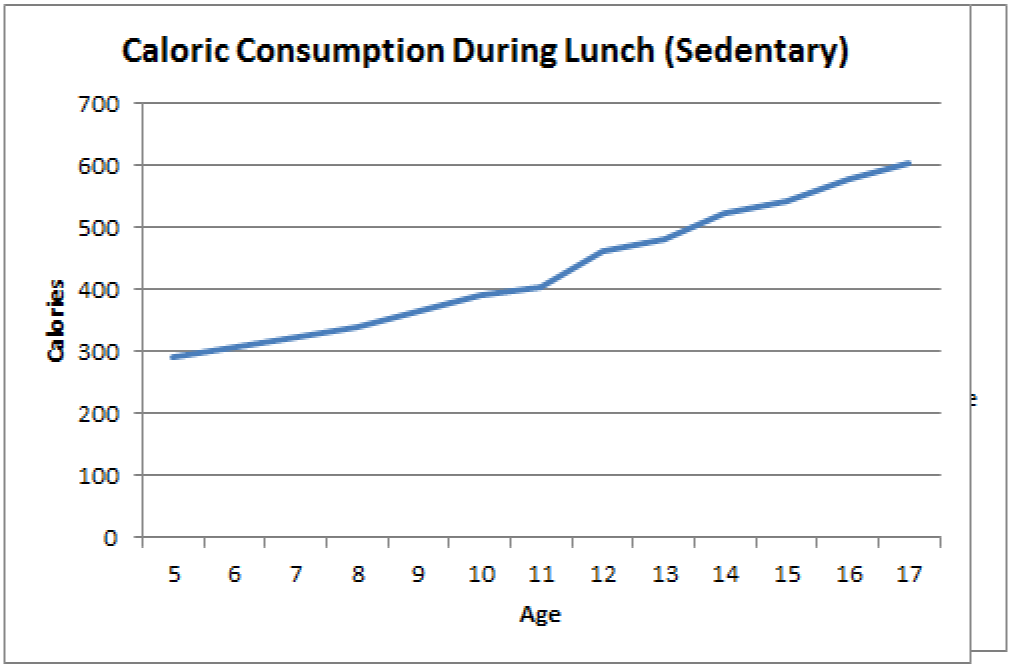
Multiplying this by .3 gives the final model for the amount of Calories consumed during lunch:

$$\boxed{\frac{r}{80}(hk - ha + 24a)}$$

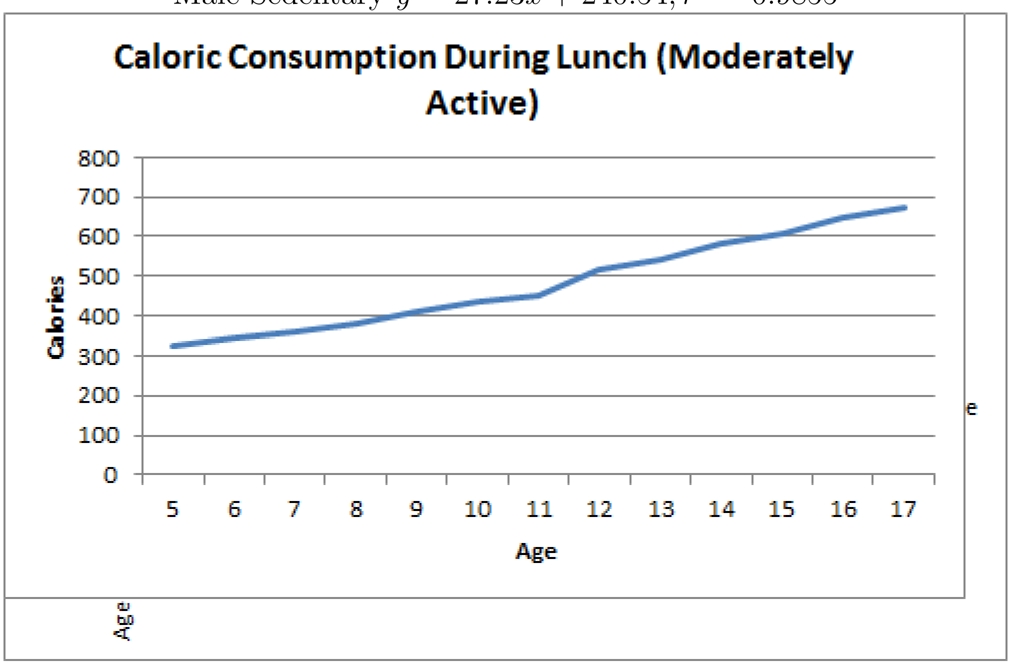
We then took this model and created data tables in Microsoft Excel. We gathered average data based on a height-weight-by-age chart found on www.disabled-world.com.[6] Using the formula tools in Excel, we were able to generate data tables which linked changes in independent variables to changes in the dependent variables (shown in bold). However, since there are so many ways to vary the independent variables, we chose to vary age, and we took the average. Between sheets, we varied gender and activity factors (based on how much physical activity a particular student engages in daily). For the sake of convenience, we have attached only one sample data table: that of moderately active males. However, we have attached all the graphs of the six sheets: three levels of activities for two different genders. Additionally, we computed linear regressions for these. In creating these graphs, we assumed that the height and weight of our sample was average given their ages, the entire sample had nine hours of sleep nightly, and the SMR factor was 0.65. The data is below. Note that $\hat{y}$ represents the number of Calories required and $\hat{x}$ represents the age of the student.

Weight, LBS	Weight, KG	Height, IN	Height, CM	Activity Factor (a)	Hours Sleep (h)	Age, YRS	**Resting Metabolic Rate (R)**	**Caloric Intake**	**Caloric Consumption During Lunch**	**Hourly Metabolic Rate**	SMR Factor (k)
41.8	18.96018361	40	101.6	1.76	9	5	804.6018361	1081.183717	324.3551	33.52508	0.65
46.2	20.95599242	42	106.68	1.76	9	6	851.3099242	1143.947711	343.1843	35.47125	
50.6	22.95180122	44	111.76	1.76	9	7	898.0180122	1206.711704	362.0135	37.41742	
57.2	25.94551442	45	114.3	1.76	9	8	938.8301442	1261.553006	378.4659	39.11792	
61.6	27.94132322	49	124.46	1.76	9	9	1017.288232	1366.981062	410.0943	42.38701	
70.4	31.93294082	51	129.54	1.76	9	10	1083.954408	1456.563736	436.9691	45.16477	
77	34.92665403	52	132.08	1.76	9	11	1124.76654	1511.405038	453.4215	46.86527	
88	39.91617603	59	149.86	1.76	9	12	1280.78676	1721.057209	516.3172	53.36612	
95	43.0913264	61	154.94	1.76	9	13	1339.288264	1799.668605	539.9006	55.80368	
110	49.89522004	64	162.56	1.76	9	14	1449.9522	1948.373269	584.512	60.41468	
120	54.43114913	65	165.1	1.76	9	15	1506.186491	2023.938098	607.1814	62.75777	
135	61.23504277	67	170.18	1.76	9	16	1600.975428	2151.310731	645.3932	66.70731	
145	65.77097187	69	175.26	1.76	9	17	1673.084719	2248.207591	674.4623	69.71186	

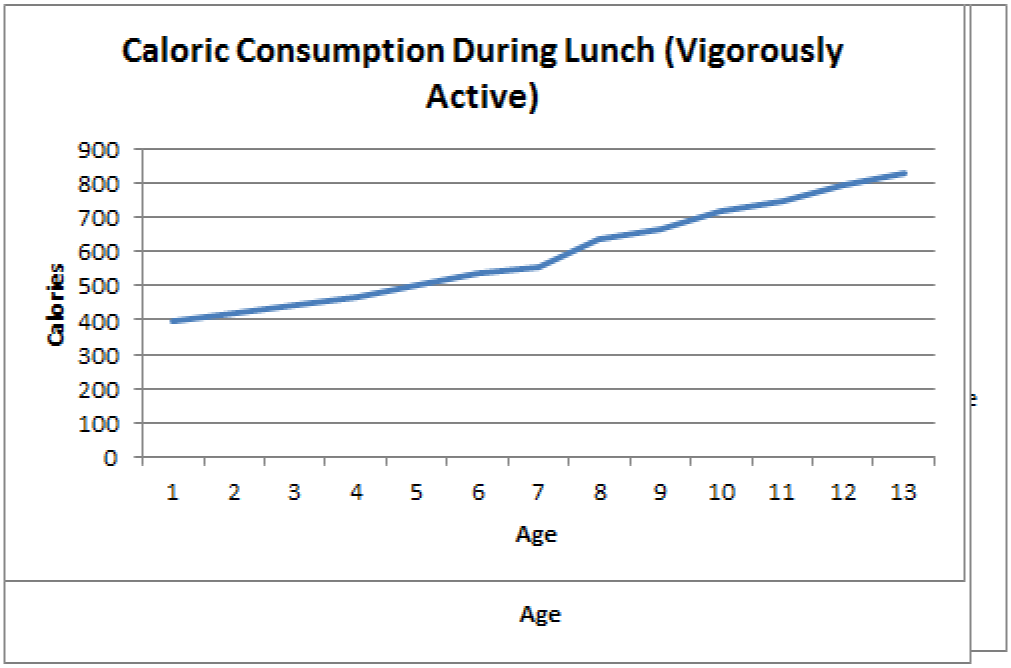
Data Table of Moderately Active Males



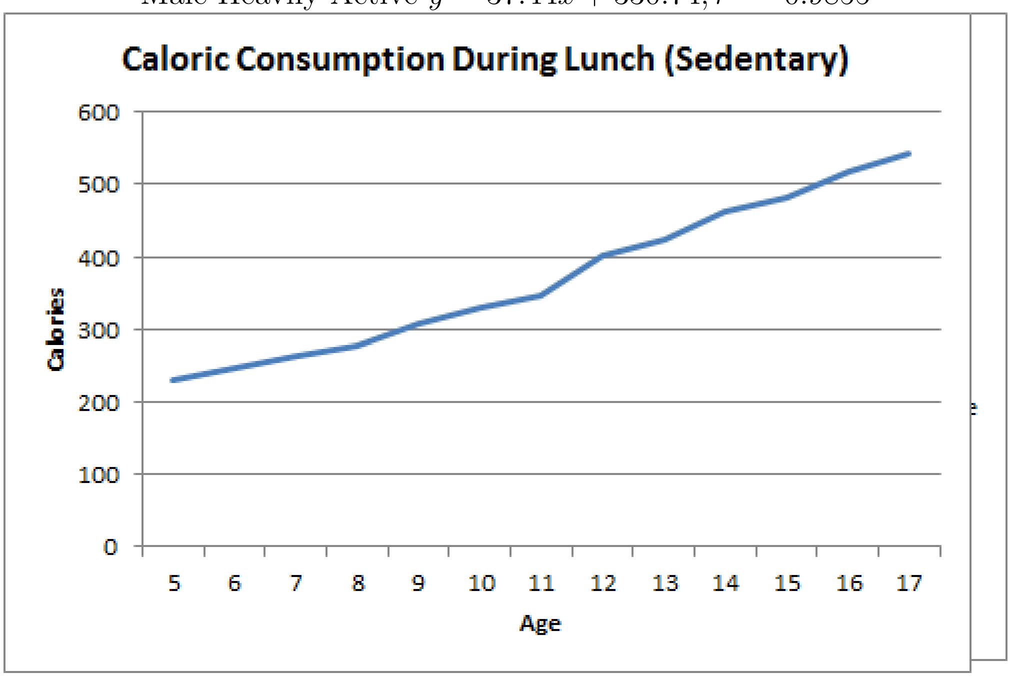
Male Sedentary $\hat{y} = 27.23\hat{x} + 240.54, r^2 = 0.9855$



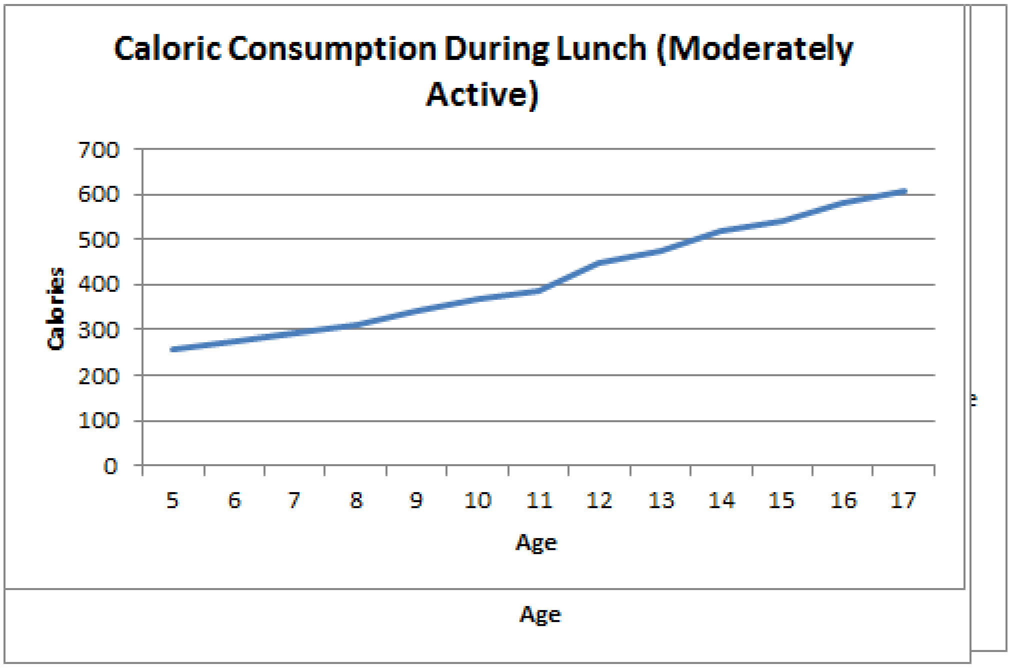
Male Moderately Active $\hat{y} = 30.49\hat{x} + 269.35, r^2 = 0.9855$



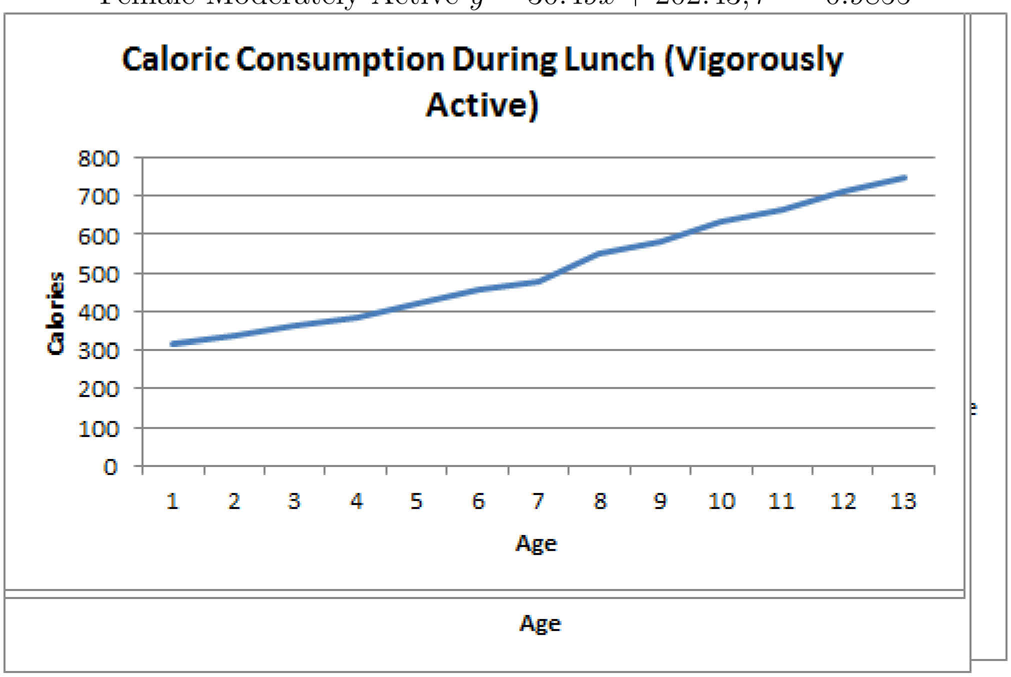
Male Heavily Active $\hat{y} = 37.44\hat{x} + 330.74, r^2 = 0.9855$



Female Sedentary $\hat{y} = 27.23\hat{x} + 180.78, r^2 = 0.9855$



Female Moderately Active $\hat{y} = 30.49\hat{x} + 202.43, r^2 = 0.9855$



Female Heavily Active $\hat{y} = 37.44\hat{x} + 248.57, r^2 = 0.9855$

One Size Doesn't Necessarily Fit All - Creation of a Calorie Consumption Distribution of U.S. High School Students

Assumptions

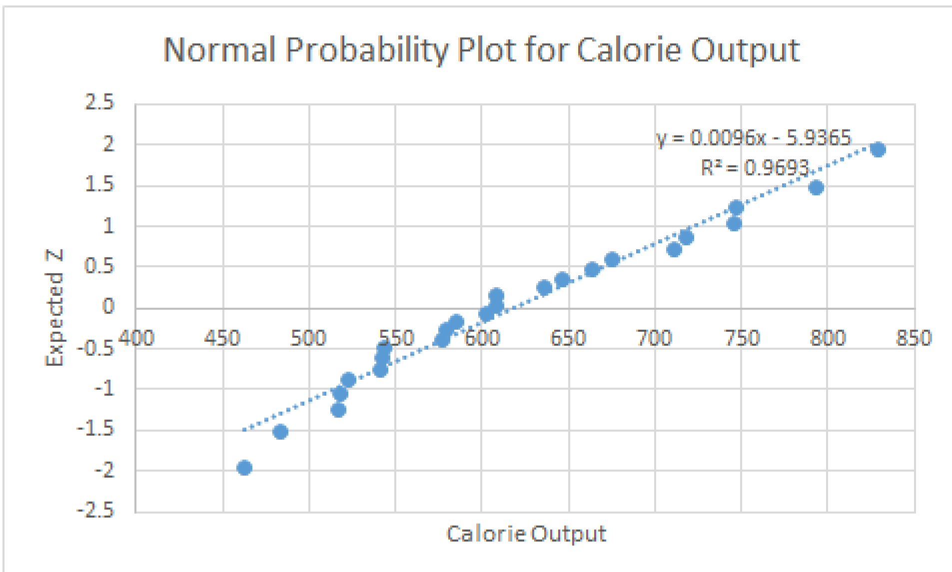
1. We assume there is an approximately equal number of male and female students. This means that we can incorporate both male and female predicted values without requiring adjustment or modification variables. This assumption is valid as the gender ratio of the entire human population

is approximately 1:1.

2. We assume high school students are between the ages of 14-17.

3. We assume that the average high school student sleeps for nine hours per night. Our model accounts for sleep time, but in order to find specific values to predict a threshold value, we needed to include one consistent sleep time value. Most teenagers require nine hours of sleep per night.

4. We assume the distribution of Caloric consumption during lunch of high school students is approximately normal, because the normal probability plot demonstrates a strong, positive, linear relationship with an correlation coefficient of 0.98.



Methodology

Using univariate statistics with the twenty-four values of Caloric consumption for 14-17 year old students in the six categories, we found that the mean Caloric consumption value for 14-17 year males and females in the three different activity levels was 618 Calories, with a sample standard deviation of 99 Calories. Using inverse t, with an area of .90 and degrees of freedom of 23, we found that the t* value would need to be 1.319. The upper limit of the .90 t-distribution will include approximately 95% of students, as t-distributions are centered around the mean and do not start from a lower bound value. Thus, the Caloric value at which 95% of students will receive their predicted lunch Caloric allotment is 618 + 1.319(99) which equals 749 Calories. The reason why 95% was chosen was to satisfy a large portion of students, but without going too high in exceeding Calorie counts for other students.

Conclusion

The standard school school lunch should supply approximately 750 Calories in order to fulfill the Caloric requirements for at least 95% of people without dangerously exceeding Caloric requirements for other students.

There is No Such Thing as a Free Lunch - Developing an Suitable Lunch Plan

Assumptions

1. We assume that in any given school week there are a maximum of 5 lunches per student.

2. We assume that the average dollar amount spent on each student per meal is $\$\frac{7}{5\text{days}} = \$1.40/\text{day}$; if we are to reduce the weekly budget by \$1.00 per student, then the amount spent per student per meal is $\$\frac{6}{5\text{days}} = \$1.20/\text{day}$

Methodology

In this problem, we were tasked with creating a lunch plan that adheres to several constraints. The plan must meet nutritional standards, be appealing to students, and cost no more than \$7/week to the district. Then we are asked to create an alternative plan for a more restricted meal plan of \$6/week.

To understand the requirements, we must first clarify a few points. First, the nutritional standards required are assumed to refer to the optimized Caloric intake calculated through modeling earlier in the problem. Second, it is stated that the district budget each week per student is \$7. This cost is not equivalent to the true accounting cost of each week of meals; the federal government's National School Lunch Program provides reimbursements for low-income and disadvantaged students, subsidizing the districts' school lunch costs[7]. In schools in which 60% of students or have free or reduced lunch, the government pays \$0.28 per lunch bought at full price, \$2.53 for each lunch bought at reduced price, and \$2.93 for each free lunch provided (Ibid.). In schools in which more than 60% of students have free or reduced lunch, the government pays an additional \$0.02 per meal, and in certain schools that are selected based on performance, the government pays an additional \$0.06 per meal on top of anything else already provided (Ibid.). For the purposes of this analysis, we will be assuming that the amount the school district has the option of spending per meal is equivalent to whatever portion of budget is designated for that meal addition to \$2.93 provided by any combination government subsidization and student payment; essentially we are assuming

that lunch costs \$2.65 at full price (subsidy for free lunch minus subsidy for paid lunch) and \$0.40 at reduced price (subsidy for free lunch minus subsidy for reduced-price lunch). That means that schools have the option to spend \$4.33 per meal, per student. These numbers are taken from a school with no achievement-based additional \$0.06 per meal or \$0.02 per meal for having more than 60% of students on free or reduced lunch, as we want all schools to be able to provide the selected meal plan for students; additional funding can go to reducing budget. As mentioned in problem 2, the approximate Caloric intake for the meal should be 750 Calories.
f Thus the parameters for constructing a meal plan for students are as follows: a healthy, nutritionally satisfying and filling meal that varies from day to day, appeals to students, 750 Calories, and costs \$4.33 per meal.

A daily meal plan of 2200 Calories, when divided by three, yields 731 Calories. This means dividing the food group servings of that daily meal plan into three should also yield 731 Calories, which allows some flexibility in maintaining a 750 Calorie limit. This translates to a lunch of 3 servings of grains, 1.3 servings of vegetables, 1 serving of fruit, one serving of dairy, and 2 servings of meat/protein.[8]

Conclusion

Based on the requirements for meals, sample meal plans fulfilling these food groups include:
1. A burger, either beef or chicken without cheese (protein, grains, vegetables), a fruit side such as a fruit cup (fruit), and milk (dairy).
2. A peanut butter and jelly sandwich (protein, grains, fruit), a vegetable side such as carrots (vegetables), and milk (dairy).
3. Chicken strips and toast (protein, grains), a vegetable side (vegetables), a fruit side (fruits), and milk (dairy).
4. Meat burrito, containing beef or chicken and cheese (protein, grains, dairy), a vegetable side or vegetable juice (vegetables) and a fruit side or fruit drink (fruits).

There are many variations of these plans that can be utilized, because different meats can be used in the burgers and the burritos, and different vegetable and fruit sides and juices can be implemented. All of the entrees on this list were top entree selections by 9th- 12th grade students.[9]

Item	Unit Price
Chicken Sandwich	2.15
Burger	1.33
PB&J	1.00
Chicken Strips	1.17
Chicken Burrito	1.07
Beef Burrito	1.17
Fruit Cups	0.61
Vegetable Side	0.19
Veggie Juice	1.32
Fruit Juice	0.81
Soy Milk	1.47
Milk	1.65

According to the price table aggregated from prices on bulk food websites[10] , the most expensive meal, composed of a chicken burger, fruit side, and milk, costs a total of $4.09. This is within both the price constraint of the $7 budget ($4.33 per meal) and the $6 budget ($4.13 per meal). If the budget were lowered to $6.00 per student, we would not have to make any changes; in fact, as it stands, the budget could be lowered to $6 per student to save money without any adverse effects.

Conclusion

In this analysis, we attempted to utilize mathematical modeling to determine the effect on certain individualized parameters on the Calorie intake of a student. We developed an innovative mathematical model that could yield the theoretical Calorie uptake based off of parameters such as age, height, and weight. This model utilizes the RMR and Mifflin-St. Jeor equation. Furthermore, we used a t-distribution of the approximately normally distributed data of Calorie consumption during lunch for 14-17 year old students to determine that 95% of students would have their Caloric requirements met during lunch with a Caloric intake of 750 Calories. Finally, we looked at the Caloric requirements from the previous question and monetary requirements based off of the designated budgets and government subsidies in order to define parameters for a meal plan as $4.33 per meal per student, 750 Calories, and 3 servings of grains, 1.3 servings of vegetables, 1 serving of fruit, one serving of dairy, and 2 servings of meat/protein, and developed a suggested lunch plan based off of a list of popular foods to fit within the set parameters.

Recommendations

To address the issue of students wanting taste and variety, administrators wanting cost efficiency, and parents wanting their kids to have healthy meals, we have devised the above sample meal plan.

By our previous calculations, we have identified that each student should eat a lunch containing 750 Calories. While we have provided a sample meal plan that a school could utilize in order to effectively satisfy the needs of all three parties, there are likely more plans that could satisfy these requirements. We would recommend that a school check with local suppliers to ensure that the prices of the food meet its students' needs.

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