On the Latent Space of Deep Generative Models

Honors Thesis Presentation

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Outline

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Generative Models

Problem

Given training data, generate samples from the same distribution as the training data (true distribution).

Deep Generative Models

Definition (Deep Generative Model)

A *deep generative model* is a (possibly randomized) function, denoted g and typically parameterized as a neural network, that accepts some vector $z \in \mathbb{R}^d$ for some d as input and outputs some image $X \in \mathcal{I}$, where $\mathcal{I}$ represents the space of images.

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1. Sample $z \sim \mathcal{N}(0, I_d)$ from *latent space*
2. Obtain $X = g(z)$
3. Hope that distribution of outputs is close to true distribution

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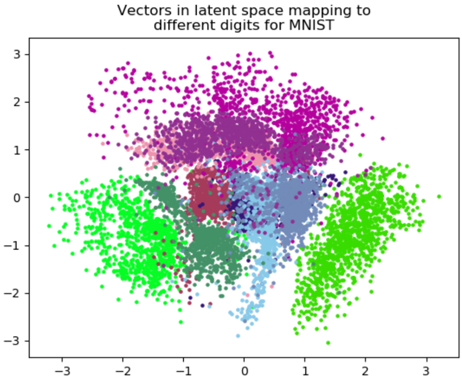
Examples


Vectors in latent space mapping to
different digits for MNIST
3
2
1
0
−1
−2
−3
−3
−2
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1
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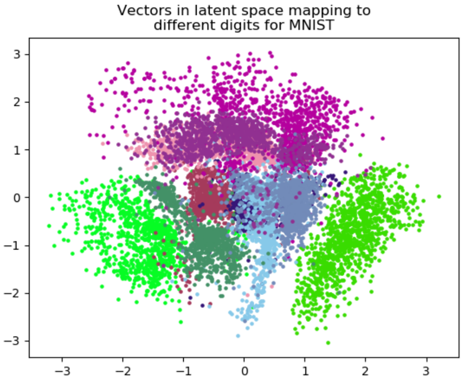

- Latent space of deep generative model mapping latent codes to handwritten digits

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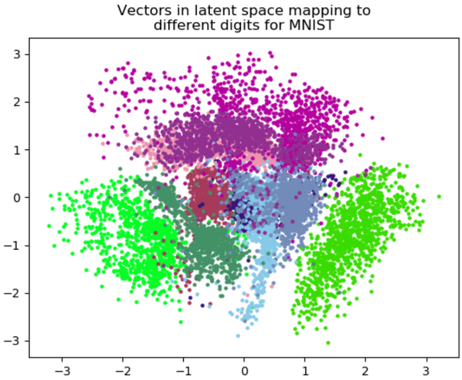

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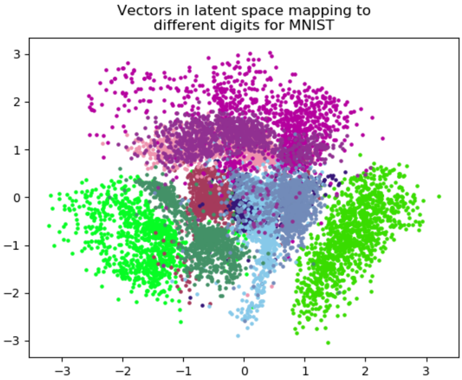

- Latent space of deep generative model mapping latent codes to handwritten digits
- Each colored region maps to a specific digit (class)
- Everything else: "holes"

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  - How far away am I from the closest point in a different class?

Assumptions

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Every latent vector maps to some image in the true distribution.

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Class Radius to Diversity

Theorem

Under the aforementioned assumptions, we have the following regarding the expected class radius of each image in the latent space of our generative model:

$$\mathop{\mathbb{E}}_{z\sim\mathcal{N}(0,I_d)} r(z) \leq \frac{\log\left(4\pi\log\left(1/\left\|p\right\|_2^2\right)\right)}{\sqrt{2\log\left(1/\left\|p\right\|_2^2\right)}}$$

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- How do we interpret $\log\left(1/\left\|p\right\|_2^2\right)$?

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$||p||_2^2$ as a Measure of Diversity

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- $\sum p_i^2 = \|p\|_2^2$

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Definition (α-order Rényi Entropy)

$$H_\alpha(p) = \frac{1}{1-\alpha} \log \left(\sum_{i=1}^{K} p_i^\alpha \right)$$

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Special Case: Collision Entropy

$$H_2(p) = -\log \left(\sum_{i=1}^{K} p_i^2 \right) = \log \left(\frac{1}{\|p\|_2^2} \right)$$

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Fact (Maximum Entropy Distribution)

For all α, the distribution maximizing the α-order Rényi Entropy is the uniform distribution, yielding:

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$$\mathop{\mathbb{E}}_{z \sim \mathcal{N}(0, I_d)} r(z) \leq \frac{\log(4\pi \log(K))}{\sqrt{2\log(K)}}$$

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   - Our contribution

Some Details

(Slightly rewritten) Result from Fawzi et al. (2018)

$$\mathbb{E}_{z\sim\mathcal{N}(0,I_d)} r(z) \leq \sum_{i=1}^{K} \Phi^{-1}(p_i) \cdot p_i + \frac{e^{-\Phi^{-1}(p_i)^2/2}}{\sqrt{2\pi}}$$

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Then Jensen:

$$\mathop{\mathbb{E}}_{z \sim \mathcal{N}(0, I_d)} r(z) \leq \gamma\left(\sum_{i=1}^{K} p_i \cdot p_i\right) = \gamma\left(\|p\|_2^2\right) = \boxed{\frac{\log\left(4\pi \log\left(1/\|p\|_2^2\right)\right)}{\sqrt{2\log\left(1/\|p\|_2^2\right)}}}$$

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Unfortunately, it is unclear that computing $\mathbb{E}\left[r(z)\right]$ is easier than computing $\|p\|_2^2$.

Adversarial Examples - A Tangential Connection

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- Under the probability decay assumption, above approaches 0 as the number of classes K grows arbitrarily large

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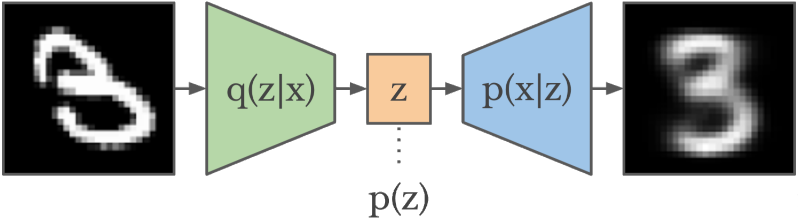
1 Introduction
- Deep Generative Models Overview
- Latent Space Overview
- Main Claim

2 Theory
- Notation and Assumptions
- Main Result
- Relationship to Diversity
- Proofs
- Potential Implications

3 Experiments
- Introduction to VAEs
- VAE Modification
- Experimental Setup
- Results

VAE Architecture



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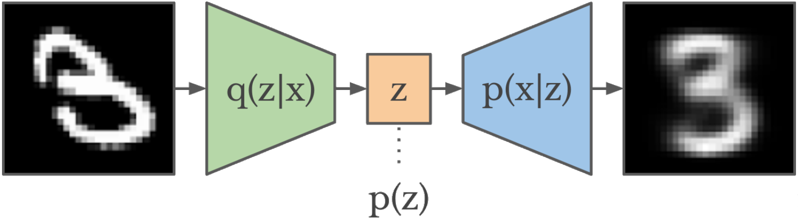

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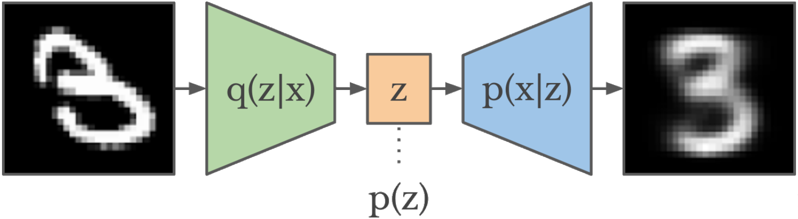

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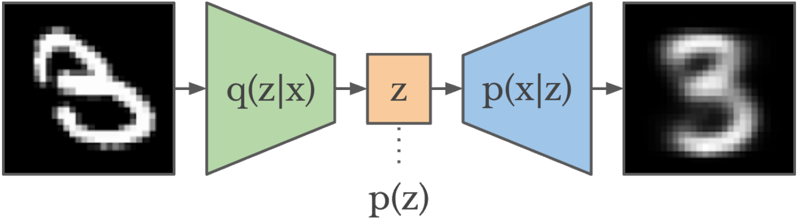

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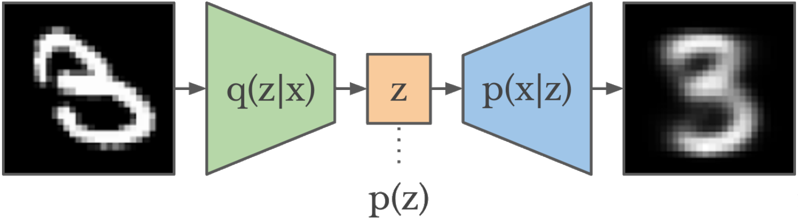

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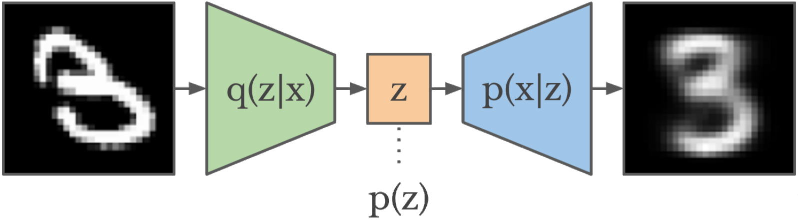

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- Objective: Minimize expected reconstruction loss

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Regularize with $\omega\left(\cdot, \cdot\right)$ *summed over many pairs of images.*

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Solution

Penalize distant encodings.

$$\omega\left(I_1, I_2\right) := \frac{\left\|f\left(I_1\right) - f\left(I_2\right)\right\|_2^2}{\left\|I_1 - I_2\right\|_2^2}$$

Regularize with $\omega\left(\cdot,\cdot\right)$ summed over many pairs of images. To force ω to be low, enforce Lipschitz constant via network weight clipping.

Outline

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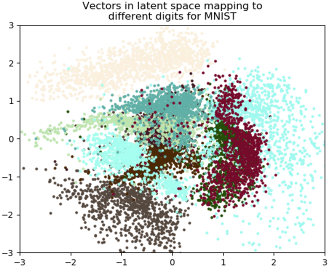
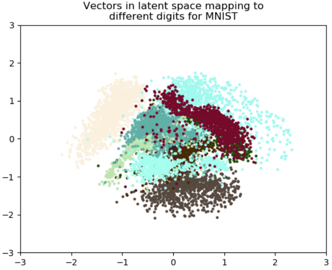
- Clip weights to $[-0.3, 0.3]$

Outline

Geometric Implications





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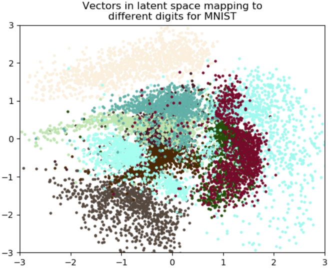

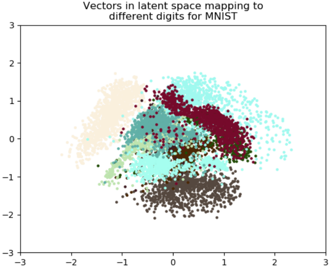

Conclusion: Modified VAE makes classes come closer to one another but decreases probability mass of space covered

Sample Quality

Sample Quality

Conclusion: Modified VAE does not change quality of generated samples noticeably

Interpretation of Results

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Problem

For future: Devise an optimization objective that favors both closer classes and better coverage of the latent space.

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Summary

We care about coverage of the latent space.

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- Test generative models for diversity
- Meta-Problem: Theoretically understand generative models better

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