

# On the Latent Space of Deep Generative Models

## Honors Thesis Presentation

Naren Manoj

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# Outline

- 1 Introduction
  - Deep Generative Models Overview
  - Terms
  - Main Claim
- 2 Theory
  - Assumptions
  - Main Result
  - Relationship to Diversity
  - Proofs
  - Potential Implications
- 3 Experiments
  - VAE Modification
  - Experimental Setup
  - Results

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  - Use  $\ell_2$  distance between two images

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- Formulated as min-max game

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Generally, **theoretical understanding of deep generative models is still lacking.**

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# Deep Generative Model

## Definition (Deep Generative Model)

A *deep generative model* is a (possibly randomized) function, denoted  $g$ , that accepts some vector  $z \in \mathbb{R}^d$  for some  $d$  as input and outputs some image  $I \in \mathcal{I}$ , where  $\mathcal{I}$  represents the space of images.



# Encoder

## Definition (Encoder)

An *encoder* is a (possibly randomized) function, denoted  $f$ , that accepts some input image  $I \in \mathcal{I}$  (where, again  $\mathcal{I}$  is the space of images) and outputs some vector  $z \in \mathbb{R}^d$ .

# Class

## Definition (Class)

Suppose we are given a classifier into  $K$  classes  $c : \mathcal{I} \rightarrow [K]$  that accepts as input an image and outputs a class label which is an integer index in  $[K] := \{i\}_{i=1}^K$ . Then, the *class* of an image  $I$  under  $c$  is given by:

$$K_{c(I)} := \{z' \mid c(g(z')) = c(I)\}$$

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In words,  $K_i$  is the subset of the latent space that is classified into label  $i$  when a point  $z \in K_i$  is passed through  $g$  and subsequently classified by  $c$ .

# Latent Hole

## Definition (Latent Hole)

A *hole* is a point  $z \in \mathbb{R}^d$  such that there does not exist an image  $I$  in the data manifold for which  $f(I) = z$ .

# Class Radius

## Definition (Class Radius)

Consider the probability vector  $p \in \mathbb{R}^K$  such that:

$$p_i \propto \nu(K_i)$$

Then, the *class radius* is the expected distance from a random point  $z \sim p$  in some class to the closest point in a different class. Specifically, define  $r(z)$  below, and let  $c(g(z)) = i$ :

$$r(z) := \inf_{z' \in \bigcup_{j \neq i} K_j} \|z - z'\|$$

Then the class radius is simply:

$$\mathbb{E}_{z \sim p} [r(z)]$$

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# Assumptions

## Probability Decay

$$\|p\|_{\infty} \leq \min \left( \frac{1}{5}, h(K) \in \left[ \Omega \left( \frac{1}{K} \right), o(1) \right] \right)$$



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## Partition of Latent Space

$$\bigcup_{i=1}^K K_i = \mathbb{R}^d$$

- Every latent vector maps to some image in the data manifold



# Class Radius to Diversity

## Theorem

*Under the aforementioned assumptions, we have the following regarding the expected class radius of each image in the latent space of our generative model:*

$$\mathbb{E}[r(z)] \leq \frac{\log \left( 4\pi \log \left( 1 / \|p\|_2^2 \right) \right)}{\sqrt{2 \log \left( 1 / \|p\|_2^2 \right)}}$$

- Upper bound depends *only* on  $\|p\|_2^2$

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# $\|p\|_2^2$ as a Measure of Diversity

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*I have a bunch of socks in a drawer distributed according to  $p$ . If I draw two socks, what is the probability that they are of the same type?*

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- $\sum p_i^2 = \|p\|_2^2$

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## Definition ( $\alpha$ -order Rényi Entropy)

$$H_\alpha(p) = \frac{1}{1-\alpha} \log \left( \sum_{i=1}^K p_i^\alpha \right)$$

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$$\lim_{\alpha \rightarrow 1} H_\alpha(p) = - \sum_{i=1}^K p_i \log(p_i)$$

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## Special Case: Collision Entropy

$$H_2(p) = -\log \left( \sum p_i^2 \right) = \log \left( \frac{1}{\|p\|_2^2} \right)$$

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## Fact (Maximum Entropy Distribution)

*For all  $\alpha$ , the distribution maximizing the  $\alpha$ -order Rényi Entropy is the uniform distribution, yielding:*

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$$\mathbb{E}[r(z)] \leq \frac{\log(4\pi \log(K))}{\sqrt{2 \log(K)}}$$

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- 3 Massage a little
  - Our contribution

# Some Details

(Slightly rewritten) Result from Fawzi et al. (2018)

$$\mathbb{E}[r(z)] \leq \sum_{i=1}^K \Phi^{-1}(p_i) \cdot p_i + \frac{e^{-\Phi^{-1}(p_i)^2/2}}{\sqrt{2\pi}}$$

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Rewrite some:

$$\sum_{i=1}^K \Phi^{-1}(p_i) \cdot p_i + \frac{e^{-\Phi^{-1}(p_i)^2/2}}{\sqrt{2\pi}} \leq \sum_{i=1}^K p_i \left( \frac{\log(4\pi \log(1/p_i))}{\sqrt{2 \log(1/p_i)}} \right)$$



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Then Jensen:

$$\mathbb{E}[r(z)] \leq g\left(\sum_{i=1}^K p_i \cdot p_i\right) = g\left(\|p\|_2^2\right) = \frac{\log\left(4\pi \log\left(1/\|p\|_2^2\right)\right)}{\sqrt{2 \log\left(1/\|p\|_2^2\right)}}$$

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# Test for Diversity

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Unfortunately, computing  $\mathbb{E}[r(z)]$  might be more difficult than computing  $\|p\|_2^2$  unless  $K$  is extremely large

# Adversarial Examples

- Recall:

$$r(z) = \inf_{z' \in \bigcup_{j \neq i} K_j} \|z - z'\|$$



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$$L \cdot \frac{\log \left( 4\pi \log \left( 1 / \|p\|_2^2 \right) \right)}{\sqrt{2 \log \left( 1 / \|p\|_2^2 \right)}}$$

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- Under the probability decay assumption, above approaches 0 as the number of classes  $K$  grows arbitrarily large



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*Regularize with  $\omega(\cdot, \cdot)$  summed over many pairs of images. To force  $\omega$  to be low, enforce Lipschitz constant via network weight clipping.*



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  - DCGAN-like encoder and decoder
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- Clip weights to  $[-0.3, 0.3]$

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  - Terms
  - Main Claim
- 2 Theory
  - Assumptions
  - Main Result
  - Relationship to Diversity
  - Proofs
  - Potential Implications
- 3 Experiments
  - VAE Modification
  - Experimental Setup
  - Results



# Geometric Implications

- Talk about geometry of latent space before and after

# Sample Quality

- Talk about sample quality before and after

# Conclusion

## Summary

- Summarize **main results**
- Scheme achieves asymptotic throughput of **100%**

# Conclusion

## Summary

- Summarize **main results**
- Scheme achieves asymptotic throughput of **100%**

## Future Work

- What do I want to do but am too lazy to do or don't have time for

# References I

- Alhussein Fawzi, Hamza Fawzi, and Omar Fawzi. Adversarial vulnerability for any classifier. *arXiv preprint arXiv:1802.08686*, 2018.
- Alireza Makhzani, Jonathon Shlens, Navdeep Jaitly, Ian Goodfellow, and Brendan Frey. Adversarial autoencoders. *arXiv preprint arXiv:1511.05644*, 2015.