Dynamics of Optical Solitons in Dual-Core Fibers via Two Integration Schemes

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Abstract

This article studies the dynamics of optical solitons in dual-core fibers with group velocity mismatch, group velocity dispersion and linear coupling coefficient under Kerr law nonlinearity via two integration schemes, namely, Q-function scheme and trial solution approach. The Q-function scheme extracts dark and singular 1-soliton solutions, along with the corresponding existence restriction. This scheme, however, fails to retrieve bright 1-soliton solution. Moreover, the trial solution approach extracts bright, dark and singular 1-soliton solutions. The constraint conditions, for the existence of the soliton solutions, are also listed. Additionally, a couple of other solutions known as singular periodic solutions, fall out as a by-product of this scheme. The obtained results have potential applications in the study of solitons based optical communication.

Keywords: Optical solitons, dual-core fibers, integrability.

1 Introduction

The dynamical systems of soliton propagation through optical fibers for trans-continental and trans-oceanic distances is one of the most fascinating and attractive areas of research, now a days. The most of these systems, in optical fibers, are usually expressed in time domain, and when fields at different frequencies propagate through the fiber the common practice is also to write a distance equation for each field component. Most of dynamical systems are represented in the form of nonlinear complex partial differential equations [1-12, 19-26].

In past two decades, there has been a considerable advancement in the field of fiber optical communication systems. Furthermore, introduction of fiber amplifier, optical solitons and nonlinear effects in optical fiber for transmitting data through lossy optical fibers hundreds of kilometers, even sub-marine, are the sign of extended improvement in this era. Now, the demand for the high capacity, high performance and ultra wide-band communication networks have faced scientist with new problems and emerging of new ideas as a consequence. Therefore, it is imperative to address the dynamics of soliton pulses from a mathematical perspective. This will lead to a deeper understanding of the engineering aspects of these solitons.

This paper will study these solitons in dual-core optical fibers with Kerr law nonlinearity from a purely mathematical standpoint. The focus of this paper therefore will be to extract exact 1-soliton solution for the governing model by two integration schemes. This model is described the coupled nonlinear Schrödingers equation [13, 14]. The Q-function scheme extracts dark and singular 1-soliton solutions, along with the corresponding existence restriction and while the trial solution approach extracts bright, dark and singular 1-soliton solutions. The constraint conditions, for the existence of the soliton solutions, are also listed. Additionally, a couple of other solutions known as singular periodic solutions will naturally emerge as a by-product of this integration tool.

The rest of the article is organized as follows, in section 2 the dual core model is described. In section 3, the Q-function scheme has been discussed along with mathematical analysis to celebrate soliton solutions. The trial solution approach is also described and then implemented to construct the solitons in next section 4. In last section 5, the conclusions have drawn.

2 The governing model

Pulse propagation in a decoupled two-core fibers has distinction from continuous wave propagation. In a conventional two core fiber, pulse propagation has been studied extensively by solving the coupled mode equations; where the light coupling between the two cores is characterized by a structure dependent parameter called the coupling coefficients. The model for decoupled NLSE read as

$$\begin{aligned} i\left(\frac{\partial\psi_1}{\partial x}+\alpha_1\frac{\partial\psi_2}{\partial t}\right)+\alpha_2\frac{\partial^2\psi_1}{\partial t^2}+\alpha_3\left|\psi_1{}^2\right|\psi_1+\alpha_4\psi_2=0,\\ i\left(\frac{\partial\psi_2}{\partial x}+\alpha_1\frac{\partial\psi_1}{\partial t}\right)+\alpha_2\frac{\partial^2\psi_2}{\partial t^2}+\alpha_3\left|\psi_2{}^2\right|\psi_2+\alpha_4\psi_1=0, \end{aligned} \tag{1}$$

where ψ_1 and ψ_1 are the field envelopes, while x is the $1/\alpha_1$, α_2 and α_4 are group velocity mismatch, group velocity dispersion and linear coupling coefficient, respectively. It may also be noted that propagation co-ordinate and α_3 is defined by $\alpha_3 = 2\pi n_2/\vartheta A_{eff}$ where n_2, ϑ and A_{eff} are nonlinear refractive index, the wavelength and effective mode area of each wavelength, respectively. These details are already known [13, 14].

In order to solve Eq. (1), we use the following complex wave transformation

$$\psi_l(x,t) = P_l(\xi)\, e^{i\phi(x,t)}, \quad l = 1, 2, \tag{2}$$

where represents the shape of the pulse and

$$\xi = k(x - vt), \tag{3}$$

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{4}$$

In Eq. (2), the function $\phi(x,t)$ is the phase component of the soliton. Then, in Eq. (4), κ is the soliton frequency, while ω is the wave number of the soliton and θ is the phase constant. Finally in Eq. (3), v is the velocity of the soliton. By replacing Eq. (2) into Eq. (1) and separating the real and imaginary parts of the result, we have

$$v = \frac{1}{2\alpha_2\omega - \alpha_1} \tag{5}$$

and

$$\alpha_2\, k^2 v^2 P_l'' + \alpha_3\, P_l^3 + (\alpha_4 - \alpha_1\omega)\, P_{\tilde{l}} + \left(\kappa - \alpha_2\, \omega^2\right) P_l = 0, \tag{6}$$

for $l = 1, 2$ and $\tilde{l} = 3 - l$.

3 The Q-function scheme

This section has been divided into two subsections, in first subsection the scheme has been described and while in next subsection it is implemented to celebrate optical solitons.

3.1 Description of the scheme

In this subsection we outline the main steps of the Q-function scheme [15, 16, 17] as following: let us consider the nonlinear evolution equation (NLEE):

$$F\left(u, u_t, u_x, u_{xx}, u_{xt}, ...\right) = 0, \tag{7}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its various partial derivatives u_t, u_x with respect to t, x respectively, in which the highest order derivatives and nonlinear terms are involved.

Using the traveling wave transformation

$$u(x,t) = u(\xi), \;\; \xi = k(x - ct), \tag{8}$$

where k, ω are constant to be determined later. Then Eq. (7) is reduced to a nonlinear ordinary differential equation (NLODE) of the form

$$P\left(u, u', u'', ...\right) = 0. \tag{9}$$

Step1. Suppose that Eq.(9) has the formal solution

$$u(\xi) = \sum_{j=0}^{n} a_j Q^j(\xi) \tag{10}$$

where $a_j (j = 0, 1, ..., n)$ are constants to be determined later, such that $a_n \neq 0$, while $Q(\xi)$ has the form:

$$Q(\xi) = \frac{1}{1 + Kexp(\xi)} \quad , \tag{11}$$

a solution of the Riccati equation

$$Q'(\xi) = Q^2(\xi) - Q(\xi), \tag{12}$$

wher K is an arbitrary constant.

Step2. We determine the positive integer n in (10) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (9).

Step3. We substitute Eq. (10) with Eq. (12) into Eq. (9), and calculate all the terms of the same powers of the function $Q(\xi)$ and equating them to zero, we obtain a system of algebraic equations which can be solved by Maple or Mathematica to get the unknown parameters $k, c, a_j (j = 0, 1, 2, ..., n)$ and Consequently, we obtain the exact solutions of Eq. (7).

3.2 Application to the governing model

The Q-function scheme is applied to obtain the optical solitons of the governing equation. Balancing P_l'' with P_l^3 with $l = 1, 2$ in Eq. (6), we get $m = n = 1$. This suggests the choice of P_l in Eq. (6) as

$$P_1 = a_0 + a_1 Q, \tag{13}$$

$$P_2 = b_0 + b_1 Q, \tag{14}$$

where a_0, a_1, b_0 and b_1 are constants to be determined such that $a_1, b_1 \neq 0$.

Substituting Eqs. (13) and (14) with Eq. (12) in Eq. (6) and then setting the coefficients of $Q(\xi)$ to zero, we obtain a set of algebraic equations as following:

$$\alpha_3 a_1^3 + 2\alpha_2 a_1 k^2 v^2 = 0, \tag{15}$$

$$\alpha_3 b_1^3 + 2\alpha_2 b_1 k^2 v^2 = 0, \tag{16}$$

$$3a_1 \left(a_0 a_1 \alpha_3 - \alpha_2 k^2 v^2\right) = 0, \tag{17}$$

$$3b_1 \left(\alpha_3 b_0 b_1 - \alpha_2 k^2 v^2\right) = 0, \tag{18}$$

$$a_1 \left(3\alpha_3 a_0^2 + \kappa + \alpha_2 \left(k^2 v^2 - \omega^2\right)\right) + b_1 \left(\alpha_4 - \alpha_1 \omega\right) = 0, \tag{19}$$

$$a_1 \left(\alpha_4 - \alpha_1 \omega\right) + b_1 \left(3\alpha_3 b_0^2 + \kappa + \alpha_2 \left(k^2 v^2 - \omega^2\right)\right) = 0, \tag{20}$$

$$a_0 \left(\kappa - \alpha_2 \omega^2\right) + \alpha_3 a_0^3 + b_0 \left(\alpha_4 - \alpha_1 \omega\right) = 0, \tag{21}$$

$$a_0 \left(\alpha_4 - \alpha_1 \omega\right) + b_0 \left(\kappa - \alpha_2 \omega^2\right) + \alpha_3 b_0^3 = 0. \tag{22}$$

Solving this system with the aid of Mathematica gives

$$\begin{aligned} &a_0 = \pm\sqrt{\frac{\alpha_2 \omega^2 - \alpha_1 \omega + \alpha_4 - \kappa}{\alpha_3}}, \quad a_1 = -2a_0, \quad b_0 = -a_0, \\ &b_1 = -a_1, \quad k = \pm\sqrt{\frac{2\left(-\alpha_2 \omega^2 + \alpha_1 \omega - \alpha_4 + \kappa\right)}{\alpha_2 v^2}}, \end{aligned} \tag{23}$$

where $\alpha_j (j = 1, 2, 3, 4), \omega$ and κ are arbitrary real constants.
The solution of Eq. (1) corresponding to (23) is

$$\psi_l(x,t) = \pm\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_3}} \left(1 - \frac{2}{1 + Kexp(\xi)}\right) e^{i(-\kappa x + \omega t + \theta)}, \tag{24}$$

If we set $K = \pm 1$ in Eq. (24), we obtain

$$\begin{aligned}\psi_l(x,t) = \pm &\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_3}} \tanh\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{2\alpha_2 v^2}}(x - vt)\right] \\ &\times e^{i(-\kappa x + \omega t + \theta)},\end{aligned} \tag{25}$$

and

$$\begin{aligned}\psi_l(x,t) = \pm &\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_3}} \coth\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{2\alpha_2 v^2}}(x - vt)\right] \\ &\times e^{i(-\kappa x + \omega t + \theta)}.\end{aligned} \tag{26}$$

These solutios are referred as dark and singular 1-soliton solutions and are valid for

$$\alpha_2 \left(-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa\right) > 0.$$

4 The trial solution approach

This section has also been divided into two subsections, in first subsection the scheme has been described and while in next subsection it is implemented to celebrate optical solitons.

4.1 Description of the method

In this subsection we outline the main steps of the trial equation method [18, 17] as following:
Step 1. Take the trial equation

$$(u')^2 = F(u) = \sum_{j=0}^{s} a_j u^j, \tag{27}$$

where $a_j, (j = 0, 1, ..., s)$ are constants to be determined. Substituting Eq. (27) and other derivative terms such as u'' or u''' and so on into Eq. (9) yields a polynomial $G(u)$ of u . According to the balance principle we can determine the value of s . Setting the coefficients of $G(u)$ to zero, we get a system of algebraic equations. Solving this system, we shall determine

c, k and values of $a_0, a_1, ..., a_s$.

Step 2. Rewrite Eq. (27) by the integral form

$$\pm(\xi - \xi_0) = \int \frac{1}{\sqrt{F(u)}} du. \tag{28}$$

According to the complete discrimination system of the polynomial, we classify the roots of $F(u)$, and solve the integral equation (18). Thus we obtain the exact solutions to Eq. (7).

4.1.1 Application to the governing model

The trial equation method is applied to obtain the exact solutions of the governing equation. Using the balancing principle leads to

$$P_{\tilde{l}} = -P_l.$$

Balancing P_l'' with P_l^3 in Eq. (6), then we get $s = 4$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$2a_4\alpha_2k^2v^2 + \alpha_3 = 0, \tag{29}$$

$$\frac{3}{2}a_3\alpha_2k^2v^2 = 0, \tag{30}$$

$$\alpha_2\left(a_2k^2v^2 - \omega^2\right) + \alpha_1\omega - \alpha_4 + \kappa = 0, \tag{31}$$

$$\frac{1}{2}a_1\alpha_2k^2v^2 = 0. \tag{32}$$

Solving the above system of algebraic equations, we obtain the following results:

$$a_1 = 0, \quad a_2 = \frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_2k^2v^2}, \quad a_3 = 0, \quad a_4 = -\frac{\alpha_3}{2\alpha_2k^2v^2}. \tag{33}$$

Substituting these results into Eq. (28), we get

$$\pm(\xi - \xi_0) = \int \frac{dP_l}{\sqrt{a_0 + \frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_2k^2v^2}P_l^2 - \frac{\alpha_3}{2\alpha_2k^2v^2}P_l^4}}. \tag{34}$$

where a_0 is an arbitrary real constant. Now, we discuss two cases as following:

Case1. If we set $a_0 = 0$ in Eq. (34) and integrating with respect to P_l, we get

$$\psi_l(x,t) = \pm\sqrt{\frac{2\left(\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa\right)}{\alpha_3}}\operatorname{sech}\left[\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_2k^2v^2}}(k(x - vt) - \xi_0)\right] \times e^{i(-\kappa x + \omega t + \theta)}, \tag{35}$$

or

$$\psi_l(x,t) = \pm\sqrt{-\frac{2\left(\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa\right)}{\alpha_3}}\operatorname{csch}\left[\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_2 k^2 v^2}}(k(x - vt) - \xi_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}. \tag{36}$$

These solutios are referred as bright and singular 1-soliton solutions and are valid for

$$\alpha_2\left(-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa\right) < 0.$$

$$\psi_l(x,t) = \pm\sqrt{\frac{2\left(\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa\right)}{\alpha_3}}\sec\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{\alpha_2 k^2 v^2}}(k(x - vt) - \xi_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{37}$$

or

$$\psi_l(x,t) = \mp\sqrt{\frac{2\left(\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa\right)}{\alpha_3}}\csc\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{\alpha_2 k^2 v^2}}(k(x - vt) - \xi_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}. \tag{38}$$

These solutios are referred as singular periodic solutions and are valid for

$$\alpha_2\left(-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa\right) > 0.$$

Case2. If we set $a_0 = -\frac{\left(\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa\right)^2}{2\alpha_2\alpha_3 k^2 v^2}$ in Eq. (34) and integrating with respect to P_l, we get

$$\psi_l(x,t) = \pm\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_3}}\tanh\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{2\alpha_2 k^2 v^2}}(k(x - vt) - \xi_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{39}$$

or

$$\psi_l(x,t) = \pm\sqrt{\frac{\alpha_2\omega^2 - \alpha_1\omega + \alpha_4 - \kappa}{\alpha_3}}\coth\left[\sqrt{\frac{-\alpha_2\omega^2 + \alpha_1\omega - \alpha_4 + \kappa}{2\alpha_2 k^2 v^2}}(k(x - vt) - \xi_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}. \tag{40}$$

These solutios are valid for $\alpha_2\left(-\alpha_2\omega^2+\alpha_1\omega-\alpha_4+\kappa\right)>0$.

$$\psi_l(x,t)=\pm\sqrt{-\frac{\alpha_2\omega^2-\alpha_1\omega+\alpha_4-\kappa}{\alpha_3}}\tan\left[\sqrt{\frac{\alpha_2\omega^2-\alpha_1\omega+\alpha_4-\kappa}{2\alpha_2k^2v^2}}(k(x-vt)-\xi_0)\right] \times e^{i(-\kappa x+\omega t+\theta)}, \tag{41}$$

or

$$\psi_l(x,t)=\mp\sqrt{-\frac{\alpha_2\omega^2-\alpha_1\omega+\alpha_4-\kappa}{\alpha_3}}\cot\left[\sqrt{\frac{\alpha_2\omega^2-\alpha_1\omega+\alpha_4-\kappa}{2\alpha_2k^2v^2}}(k(x-vt)-\xi_0)\right] \times e^{i(-\kappa x+\omega t+\theta)}. \tag{42}$$

These solutios are valid for $\alpha_2\left(-\alpha_2\omega^2+\alpha_1\omega-\alpha_4+\kappa\right)<0$.

5 Conclusions

This paper studied the dual-core optical fibers with Kerr law nonlinearity. The application of the Q-function and the trial solution algorithms revealed a different types of soliton solutions to this model. These are bright, dark and singular soliton solutions with certain constraint conditions. In the future these algorithms will be considered with other forms of nonlinearity.

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