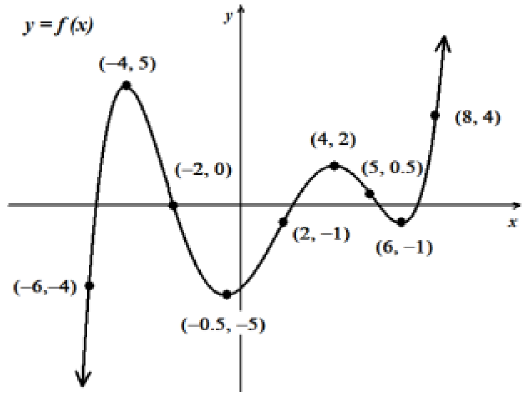
Name ______________________________

AP Precalculus
Midterm Review

Day	Date	Lesson	Homework
1	Monday 12/9	Review Unit 1A & 1B Tests	Unit 1A Midterm Review
2	Tuesday 12/10	Review Unit 2A Test	Unit 1B Midterm Review
3-4	Wed/Thurs 12/11-12	FRQ #1 & FRQ #2 Practice	Unit 2A Midterm Review
5	Friday 12/13	Review Unit 2B Test	Unit 2B Midterm Review
6	Monday 12/16	**Midterm Exam Free Response**	Study for Multiple Choice
7	Tuesday 12/17	**1st Period Exam** Questions and Work on Review (only 3rd, 5th, & 7th have class)	Study for Multiple Choice
8	Wednesday 12/18	**2nd & 6th Midterm**	Study for Multiple Choice
9	Thursday 12/19	**3rd & 5th Midterm**	Study for Multiple Choice
10	Friday 12/20	**4th & 7th Midterm**	None 😊

Unit 1A: Polynomials

1. Complete each blank for the graph of f at right.



Domain: ____________________ Range: ____________________

Interval(s) of increase: ____________________________________

Interval(s) of decrease: ____________________________________

Interval(s) where constant: __________________________________

Interval(s) where concave up: ________________________________

Interval(s) where concave down: ______________________________

Ordered pair(s) of inflection point(s): _________________________

Interval(s) where g is increasing and concave up: ___________________

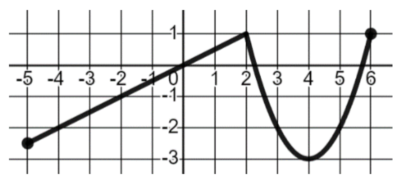
Interval(s) where g is decreasing and concave down: ___________________

Interval(s) where g is increasing at a decreasing rate: ___________________

Interval(s) where g is decreasing at an increasing rate: ___________________

Is $f(x)$ a function? ________ Justify.

2. A continuous function f is defined on the closed interval $-5 < x < 6$ and is shown on the graph below. For how many values of b, $-5 < b < 6$, is the average rate of change of f on the interval $[b, 5]$ equal to 0? Give a reason for your answer.



3. Selected values of continuous function $f(x)$ are given in the table below. Is $f(x)$ linear or quadratic? Justify your reasoning.

x	-3	-2	-1	0	1	2
$f(x)$	2.5	3	3.5	4	4.5	5

4. Selected values of continuous function $g(x)$ are given in the table below. Is $g(x)$ linear or quadratic? Justify your reasoning.

x	-4	-3	-2	-1	0	1
$g(x)$	-8	-14	-16	-14	-8	2

5. Find the degree and leading coefficient of the following polynomial functions.

(a) $f(x) = 5x^2 + 3x - 11$

Degree: ________

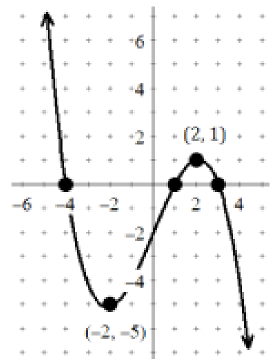
Leading Coefficient: ________

(b) $y = 2x^2(3 - x)(4x + 5)^2$

Degree: ________

Leading Coefficient: ________

6. Use the given graph to complete all characteristics asked for.



Zero(s): ________________________________

Interval(s) of Increase: ________________________

Interval(s) of Decrease: ________________________

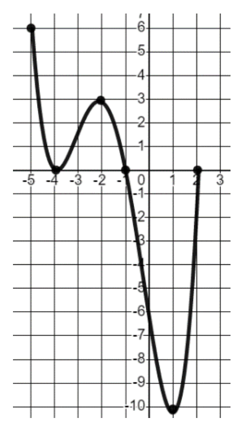
Relative Minima: ____________________________

Relative Maxima: ____________________________

Absolute Minimum: __________________________

Absolute Maximum: __________________________

7. Use the given graph to complete all characteristics asked for.



Zero(s): ________________________________

Interval(s) of Increase: ________________________

Interval(s) of Decrease: ________________________

Relative Minima: ____________________________

Relative Maxima: ____________________________

Absolute Minimum: __________________________

Absolute Maximum: __________________________

8. The table below give selected values of a polynomial function. Determine the degree of the polynomial. Justify your answer.

x	-3	-2	-1	0	1	2	3
$f(x)$	113	35	3	-1	5	3	-25

9. The table below give selected values of a polynomial function. Determine the degree of the polynomial. Justify your answer.

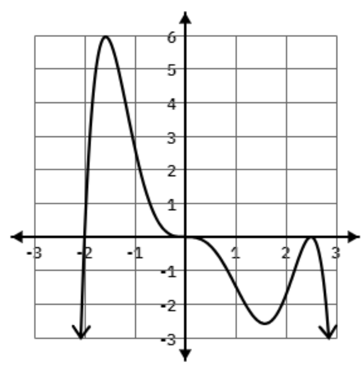
x	-2	-1	0	1	2	3	4
$g(x)$	95	5	-1	5	-1	5	95

10. Find the remaining zeros of the polynomial function f using the given information.

(a) Degree 4; Known zeros: $-5i, 2 + i\sqrt{7}$

(b) Degree 6; Known zeros: $0, -2, \sqrt{3}, -4 + i$

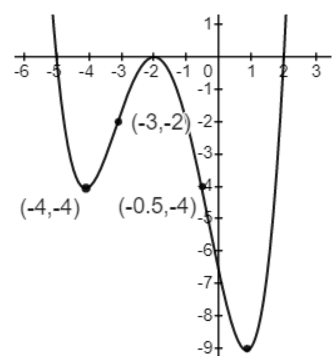
11. Compete the following subparts for the graph at right.



(a) Given the values of the zeros. Include the multiplicity of each zero.

(b) Write a possible equation for the function in the graph. Give your answer in factored form.

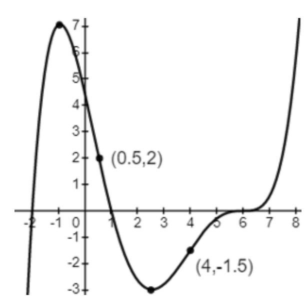
12. Compete the following subparts for the graph at right.



(a) Given the values of the zeros. Include the multiplicity of each zero.

(b) Write a possible equation for the function in the graph. Give your answer in factored form.

13. Compete the following subparts for the graph at right.



(a) Given the values of the zeros. Include the multiplicity of each zero.

(b) Write a possible equation for the function in the graph. Give your answer in factored form.

14. Complete the following subparts for the function $f(x) = 3x^2(x^2 - 9)(x^2 + 4)$

(a) Degree: __________

(b) List all real zeros. Include the multiplicity of each zero:

(c) Number of <u>distinct</u> real zeros: ______________

(d) Number of non-real zeros: ______________

15. Complete the following subparts for the function $g(x) = (x + 2)(x^2 - 3x - 10)$

(a) Degree: __________

(b) List all real zeros. Include the multiplicity of each zero:

(c) Number of <u>distinct</u> real zeros: ______________

(d) Number of non-real zeros: ______________

16. Determine the following for the polynomial function $a(x) = x(x - 8)^3(x + 3)^4$

(a) $a(x) > 0$

(c) $a(x) < 0$

(b) $a(x) \geq 0$

(d) $a(x) \leq 0$

17. Determine the following for the polynomial function $p(x) = -2x(x + 2)(x - 3)$

(a) $p(x) > 0$

(c) $p(x) < 0$

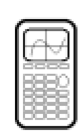
(b) $p(x) \geq 0$

(d) $p(x) \leq 0$

18. Determine the following if $f(x) = 2x^2 + 5x - 3$ and $g(x) = x^2 + 3x - 2$ (Calculator Allowed)

(a) $f(x) > g(x)$

(c) $f(x) < g(x)$



(b) $f(x) \geq g(x)$

(d) $f(x) \leq g(x)$

19. Match the equation to its end behavior. Each choice is only used once.

________ $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$

________ $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

________ $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$

________ $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$

A. $f(x) = 4x^3 + 2x^2 - 13x - 27$

B. $f(x) = -2x^3 + 5x^2 - 3x + 7$

C. $f(x) = 6x^4 - 7x^3 + 8x$

D. $f(x) = -5x^4 + 8x^2 - 2x + 6$

20. Given $f(x) = -2x^5 - 3x^4 + 2$, describe the end behavior as the input values decrease without bound. Write your answer as a limit statement.

21. Given $g(x) = -3x^4 + 2x^2 + 8x - 1$, describe the end behavior as the input values increase without bound. Write your answer as a limit statement.

22. Match the equation to a correct statement about its absolute extrema. Choices may be used more than once or not at all.

__________ $f(x) = 4x^3 + 2x^2 - 13x - 27$

__________ $f(x) = -2x^3 + 5x^2 - 3x + 7$

__________ $f(x) = 6x^4 - 7x^3 + 8x$

__________ $f(x) = -5x^4 + 8x^2 - 2x + 6$

A. Absolute maximum, but no absolute minimum

B. Absolute minimum, but no absolute maximum

C. Both an absolute maximum and an absolute minimum

D. Neither an absolute maximum nor an absolute minimum

23. Complete the table below if $f(x)$ is an <u>even</u> function.

x	-3	-2	-1	0	1	2	3
$f(x)$		-5		6	3		-2

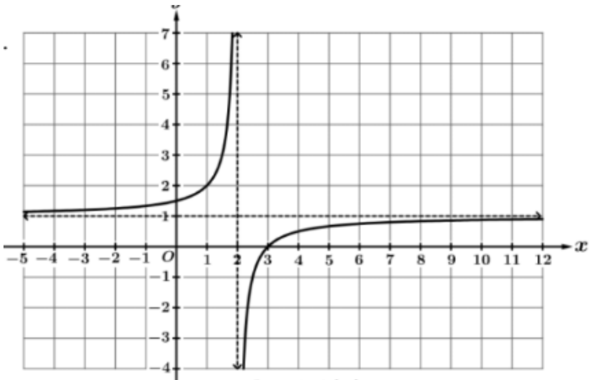
24. Complete the table below if $f(x)$ is an <u>odd</u> function.

x	-6	-4	-2	0	2	4	6
$f(x)$	3		-2	-1		7	

Unit 1B: Rationals

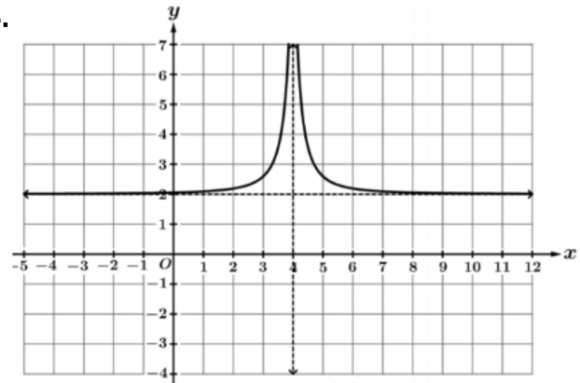
For #15-20; write limit statements to describe the left and right end behaviors.

25.



Graph of $f(x)$

26.



Graph of $g(x)$

27. $f(x) = \frac{2x^2-2x+1}{3x^2+5x+7}$

28. $h(x) = \frac{2x(x-3)}{(x+2)^2(x-1)}$

29. $f(x) = \frac{-2x^4+3x^2+x-1}{5x^2+2x+3}$

30. $h(x) = \frac{3(x-1)^2(x+5)}{(2x+3)^2}$

For #31-33: For each of the following rational functions, determine any values of x where the graph has a vertical asymptote, a hole, or a zero. Justify each answer.

31. $k(x) = \frac{(x+3)(x-2)}{(x+3)^2(x-2)}$

Vertical Asymptote with reason:

Hole with reason:

Zero with reason:

32. $p(x) = \frac{(x+7)(x+2)^3}{(x+1)(x+2)^2}$

Vertical Asymptote with reason:

Hole with reason:

Zero with reason:

33. $r(x) = \frac{x^3-x^2}{x^2+2x+1}$

Vertical Asymptote with reason:

Hole with reason:

Zero with reason:

34. Expand completely. $(3x-1)^4$

35. The domain of a function h is $-4 \le x \le 7$ and the range of h is $-6 \le y \le 0$. Find the domain and range of g, where $g(x) = 3h(x-2)$.

36. The domain of a function k is $2 \le x \le 14$ and the range of k is $-3 \le y \le 2$. Find the domain and range of r, where $r(x) = -2k(2x)$.

37. The domain of a function f is$-4 \le x \le 6$ and the range of f is $0 \le y \le 10$. Find the domain and range of g, where $g(x) = 2f(x-3) + 1$.

38. The domain of a function f is $-6 \le x \le 4$ and the range of f is $-10 \le y \le 3$. Find the domain and range of p, where $p(x) = 5 - 3f\big(2(x+1)\big)$.

39. The domain of a function f is$-4 \le x \le 6$ and the range of f is $0 \le y \le 10$. The graph of $y = k(x)$ is the result of the transformation $k(x) = 4f\left(\frac{x}{2}\right) + 1$. The point $(2, -3)$ on the graph of f transforms to which point on the graph of k?

40. The domain of a function f is$-4 \le x \le 6$ and the range of f is $0 \le y \le 10$. The graph of $y = p(x)$ is the result of the transformation $p(x) = -2f(x-3) + 4$. The point $(4, 1)$ on the graph of f transforms to which point on the graph of p?

41. (Multiple Choice) The function g is constructed by applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 4, a vertical dilation by a factor of 3, and a vertical translation by -7 units. Which of the following equations relating g and f is correct?

(A) $g(x) = 4f(3x) - 7$

(B) $g(x) = 3f(4x) - 7$

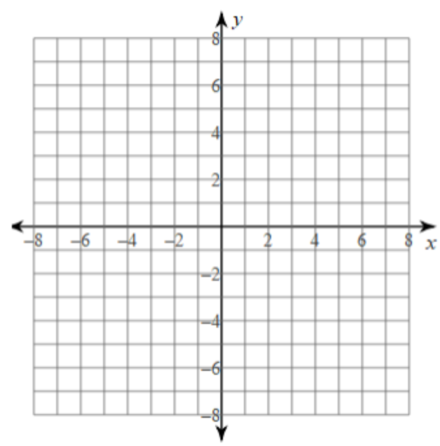
(C) $g(x) = 3f\left(\frac{x}{4}\right) - 7$

(D) $g(x) = 3f\left(\frac{x}{4}\right) + 7$

42. Write the equation of the slant asymptote for the following function.

$$f(x) = \frac{9x^3 - 12x^2 - 5x + 1}{3x^2 - 2x + 1}$$

43. Let $f(x) = \frac{1}{x+3} - 2$



(a) At which x value(s), if any, does f have a vertical asymptote?

$\lim_{x \to \infty} f(x) =$ ________

(b) Sketch a graph of f on the grid at right.

44. Given $f(x) = 4 - x^2$, find $g(x)$ if $g(x) = -f(3x) + 7$.

45. Let $h(x) = \begin{cases} \frac{x}{3} - 4 & x < -5 \\ 3 + 7x & -5 \le x < 6 \\ 5 - \frac{x}{8} & x \ge 6 \end{cases}$

(a) Evaluate $h(8)$ and $h(-5)$.

(b) Give the ordered pairs of all the open circles on the graph of h.

46. Tickets to an aquarium are \$25 per person. For parties greater than 4, the cost of each additional person is \$18. There is no tax.

(a) If a group of 3 buys tickets to the aquarium, what is the total cost of all their tickets?

(b) If a group of 7 buys tickets to the aquarium, what is the total cost of all their tickets?

(c) Let $C(n)$ be the total cost of buying n tickets to the aquarium. What values of n are reasonable in the domain? Explain.

(d) Write a piecewise equation for $C(n)$.

47. Selected values of function $f(x)$ are given in the table below.

x	14	18	28	33	35	38	41	48	53
$f(x)$	48	50	53	64	70	75	84	105	138

(a) A linear, quadratic, and cubic models are all fit to the data above. The residual plots for the linear and cubic models show a pattern. The residual plot for the quadratic model does not show a pattern. Which model is most appropriate for the data? Justify your reasoning.

(b) Find the equation of the regression curve that models $f(x)$.

(c) Use your equation to find the average rate of change from $x = 20$ to $x = 40$.

Unit 2A: Exponentials

For #48-49, find an equation that gives the n^{th} term of each sequence. Simplify your equation as much as possible. NOTE: You will have to determine if the sequence is arithmetic or geometric.

48. $17, 19, 21, 23, \ldots$

49. $\frac{2}{7}, 2, 14, 98, \ldots$

For #50-51, find an equation that gives the n^{th} term of each sequence. Instead of simplifying, use the k^{th} term of the sequence to write your equation, where k is given for each problem. NOTE: You will have to determine if the sequence is arithmetic or geometric.

50. $5, 8, 11, 14, \ldots \quad k = 1$

51. $32, 8, \ 2, \ \frac{1}{2} \ldots \quad k = 3$

For #52-53, a function has the following coordinate points. Could the function represent a linear function, exponential function, or neither? Justify your answer.

52.

x	3	5	7
$f(x)$	16	4	1

53.

x	21	22	23
$f(x)$	6	2	1

For #54-55, it is known that $f(x)$ is an exponential function that passes through the given points. Write an equation for this function.

54. (2, 40) and (5, 5)

55. (1, 3) and (3, 48) (NOTE: The base is positive)

For #56-59, answer the subparts given the function.

56. Given $f(x) = (0.9)^x$

(a) Is the function increasing or decreasing?

(b) Is the function concave up or concave down?

(c) Determine the end behavior. Write limit statements.

57. Given $f(x) = -24(2.3)^x$

(a) Is the function increasing or decreasing?

(b) Is the function concave up or concave down?

(c) Determine the end behavior. Write limit statements.

58. Given $f(x) = -5\left(\frac{1}{7}\right)^x$

(a) Is the function increasing or decreasing?

(b) Is the function concave up or concave down?

(c) Determine the end behavior. Write limit statements.

59. Given $f(x) = 2(6)^x$

(a) Is the function increasing or decreasing?

(b) Is the function concave up or concave down?

(c) Determine the end behavior. Write limit statements.

60. The function f is given by $f(x) = 3^x$, and the function g is given by $g(x) = \frac{f(x)}{81}$. Rewrite $g(x)$ so that it shows that a horizontal translation of $f(x)$.

61. The function h is given by $h(x) = 7 \cdot 4^{-\frac{x}{2}}$. What is the value for $h(3)$?

62. Black mold found as a result of water damage in buildings typically grows at a rate of 13% per week, depending on the weather. The basement of a particular building that has had water damage shows an initial amount of 2500 sp/m^3 (spores per cubic meters of air).

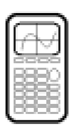
(a) Write an equation that gives the amount of black mold present in the basement t weeks after the original reading.

(b) Rewrite the equation so that t is the number of days after the original reading.

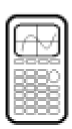
63. A new car's value decreases considerably over the years after purchasing. The following table shows the value of a new car for the given number of years after purchase.

Years After Purchase	0	1	3	5	6
Value ($)	$45,000	$36,000	$26,010	$18,792	$15,973

Using an exponential regression $y = ab^x$ to model this data, what is the car's predicted value to the nearest dollar 4 years after purchase?



64. The population for a small town in the land of Leibniz is shown below.



Time, yrs.	1995	2000	2005	2010	2015
Population, in thousands	8.21	8.63	8.49	8.84	8.92

(a) Find the quadratic regression model:______________________________

(b) Find the cubic regression model:______________________________

(c) Find the exponential regression model:______________________________

65. Use the given functions below to evaluate the following if possible.

$f(x) = 4x - 5$ $\qquad g(x) = x^2 - 2x + 4$ $\qquad h(x) = 3(2)^x$ $\qquad k(x) = 3 - 2x$

(a) $f(g(1)) =$

(b) $g(f(0)) =$

(c) $h(k(2)) =$

(d) $g(f(x)) =$

(e) $h(k(x)) =$

(f) $k(h(x)) =$

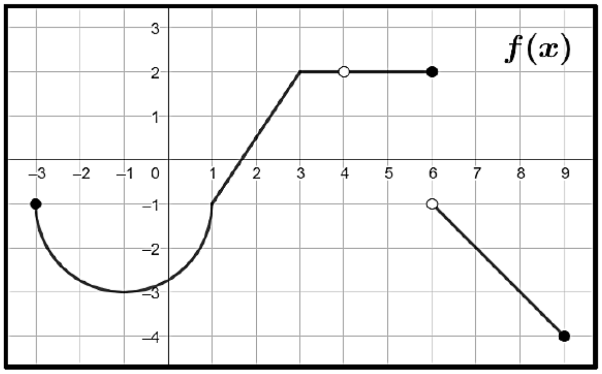
(g) (Multiple Choice) Which of the following expressions is NOT equivalent to $h(k(x))$?

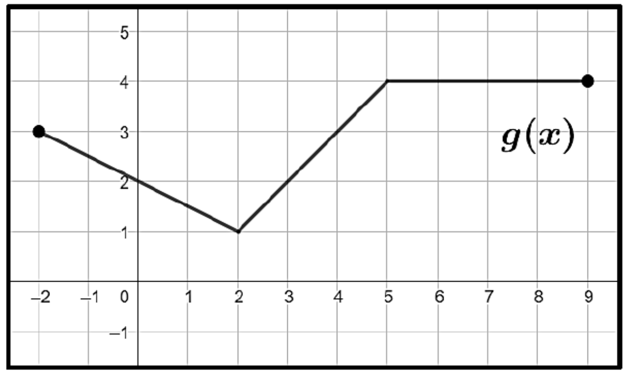
A. $24\left(\frac{1}{4}\right)^x$

B. $24(2)^{-2x}$

C. $24(4)^{-x}$

D. $27(2)^{-2x}$





x	-3	-1	2	6	9
$p(x)$	$f(6)$	e	-1	1	3

$$h(x) = \begin{cases} 8\left(\frac{1}{2}\right)^x, & x < 2 \\ 1 - x^2, & x = 2 \\ 4, & x > 3 \end{cases}$$

66. Use the given information above to evaluate the following, if possible.

(a) $f(g(4))$

(b) $(g \circ f)(6)$

(c) $(g \circ g)(-2)$

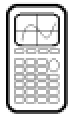
(d) $p(f(\pi))$

(e) $(f \circ g)(8)$

(f) $(g \circ h)(0)$

67. Find the inverse of the function $g(x) = \frac{3x-5}{-2x+7}$.

68. Given $f(x) = -3x^4 + 9$, find the domain of $f^{-1}(x)$.



69. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{2x-1}{x+2}$, find:

(a) $(f + g)(x)$

(b) $(f - g)(x)$

(c) $(f \cdot g)(x)$

(d) $(f \circ g)(x)$

(e) $(g \circ f)(x)$

(f) $g^{-1}(4)$

(g) Find the domain of $g^{-1}(x)$

Unit 2B: Logarithms

70. Expand as much as possible: $\log_7\left(\frac{a^2\sqrt[3]{b}}{c}\right)$

71. Condense as much as possible: $\frac{1}{3}\log a - 4\log b + 2\log c$

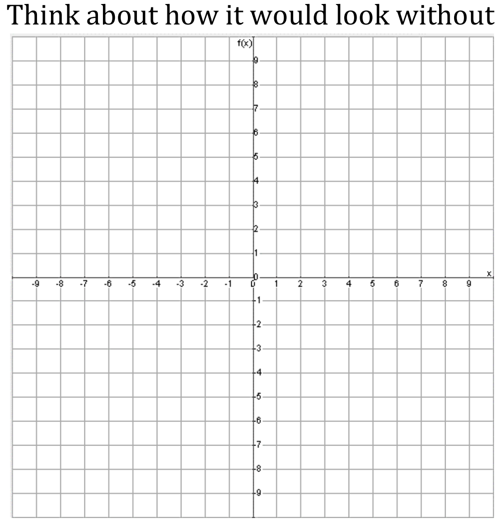
72. Find the inverses of the following functions. Use proper notation for the inverse. State any domain restrictions.

(a) $f(x) = -\frac{1}{2}\ln(3x-2) - 1$

(b) $f(x) = -2^{x+5} + 3$

73. Graph the following function and list the characteristics. (Note: Think about how it would look without the transformations, then shift it!)

$y = \left(\frac{2}{5}\right)^{x+1} + 2$



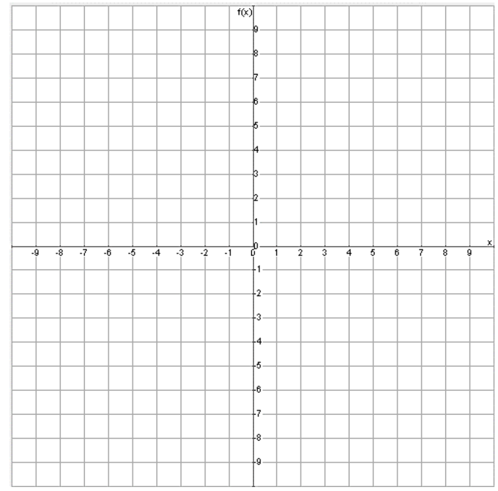
Domain ________________ Range ______________________

Any asymptotes? If so, give their equation.

End Behavior Limits:

74. Graph the following function and list the characteristics. (Note: Think about how it would look without the transformations, then shift it!)

$f(x) = \log_2(x - 3) + 4$



Domain ________________ Range ______________________

Any asymptotes? If so, give their equation.

End Behavior Limits:

75. Write an equivalent expression by condensing each expression to a single logarithm.

(a) $3 \ln x + 7 \ln y - 10 \ln z$

(b) $4 \log x - \frac{1}{2} \log y - \log z$

76. Write an equivalent expression by expanding each expression as much as possible.

(a) $\log_4 \left(\frac{x^6 y^5}{\sqrt[3]{z}} \right)$

(b) $\log_5 \left(\frac{125\sqrt{y}}{x^4} \right)$

77. Change to the base asked for.

(a) Change to base 10

$\log_5 14$

(b) Change to base 7

$\log_2 9$

(c) Change to base e

$\log_4 11$

78. For each problem below, $g(x)$ is a transformation of $f(x)$. Give TWO different (but equivalent) transformations for which the graph of g is the image of the graph of f.

(a) $f(x) = \log_2(x) \quad g(x) = \log_2(64x)$

(b) $f(x) = \log_5(x) \quad g(x) = \log_5(125x)$

79. Solve the following equations.

(a) $2^{3x-1} = 16$

(b) $125^{-3k-2} = 25^{-k}$

80. Solve the following equations. Solve without a calculator and give the EXACT answer. Then, use a calculator to find its decimal approximation rounded to three or more decimal places.

(a) $7^{x-8} + 1 = 11$

(b) $-3e^{x-1} + 4 = -20$

(c) $11^{4r} + 4 = 98.4$

81. Solve the following equations.

(a) $\log_5(x^2+33)=\log_5(-12x-3)$

(b) $\log_2(-4x)-\log_2(3)=\log_2(24)$

(c) $\log_6(2x)+9=7$

(d) $-6\ln(x-5)=0$

(e) $\log_5 x+\log_5(x-2)=\log_5 3$

(f) $\log_7(x+6)+\log_7 x=1$

82. Solve the inequality.

(a) $8^{x-1}\le\left(\frac{1}{2}\right)^{2x-1}$

(b) $\log_5(x-9)-\log_5 4>1$

(c) $\log_3(x-3)-\log_3 7<\log_3 23$

83. Find the inverse. State any domain restrictions.

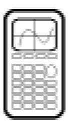
(a) $f(x) = 3 \cdot 2^{3-x} + 5$

(b) $g(x) = -4e^{3x-1} + 5$

(c) $h(\mathrm{x}) = \ln(4x + 1) - 2$

(d) $m(x) = 6 \log_3(x + 4) - 9$

84. Below is the data that represents $f(x) = a + b \ln x$.



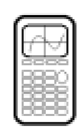
x	1	2	3	4
$f(x)$	2.1	4.8	6.4	7.5

(a) Determine the equation of the logarithmic regression function.

(b) Use the logarithmic function to find the value of the function when $x = 7$.

(c) Use the logarithmic equation to find where $f(x) = 6$.

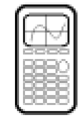
85. The temperature of a pizza, in degrees Fahrenheit, t minutes after it is removed from the oven can be modeled by the function F where $F(t) = 425(0.916)^t$.



(a) What is the temperature of the pizza after 20 minutes?

(b) How many minutes will it take for the temperature of the pizza to cool down to 95°F ?

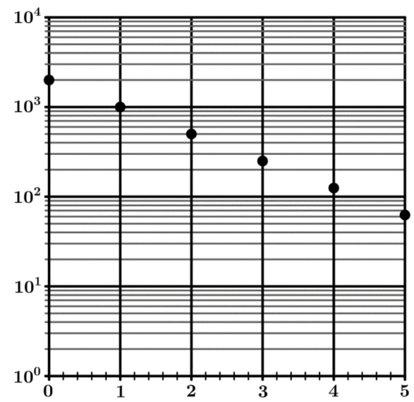
86. The Richter scale is used to measure earthquakes. The magnitude R of an earthquake is modeled by the equation $R = 0.67\log(0.37E) + 1.46$, where E is the energy in kilowatt-hours, released by the earthquake.



(a) Find the magnitude of an earthquake that releases 3.5×10^4 Kilowatt-hours of energy.

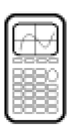
(b) How many Kilowatt-hours of energy is released in an earthquake that measures 2.5 on the Richter scale?

87. The function f is graphed on the semi-log plot below where the vertical axis has been logarithmically scaled. Write the equation of $f(x) = ab^x$.



88. A set of data points are graphed on a semi-log plot where the vertical axis has been logarithmically scaled. The points form a line on the semi-log plot. What does this tell you about the type of model the data follows?

89. Consider the function $f(x) = 200(1.3)^x$. The semi-log plot $y = \ln(f(x))$ is modeled by the linear equation of the form $\ln\big(f(x)\big) = bx + a$. Determine the equation for $y = \ln(f(x))$. Round a and b to three or more decimal places.



90. The semi-log plot $y = \log(f(x))$ is modeled by the equation $\log\big(f(x)\big) = 0.447158x + 1.07918$. Write the equation of $f(x) = ab^x$. Round a to the nearest whole number and b to the nearest tenth.