Math 407 Section A

SAMPLE PROBLEMS FOR THE FIRST QUIZ

1. Consider the system

$$
\begin{array}{rcrcrcl}
4x_1 & & & - & x_3 & = & 200 \\
9x_1 & + & x_2 & - & x_3 & = & 200 \\
7x_1 & - & x_2 & + & 2x_3 & = & 200 .
\end{array}
$$

   (a) Write the augmented matrix corresponding to this system.

   (b) Reduce the augmented system in part (a) to echelon form.

   (c) Describe the set of solutions to the given system.

2. Represent the linear span of the four vectors

$$
x_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad
x_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \quad
x_3 = \begin{bmatrix} 2 \\ 1 \\ 7 \\ 1 \end{bmatrix}, \quad
\text{and} \quad
x_4 = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 5 \end{bmatrix},
$$

   as the range space of some matrix, and give a basis for the null space of this matrix.

3. Compute a basis for $\mathrm{nul}\,(A^T)^{\perp}$ where A is given by

$$
A = \begin{bmatrix}
1 & -1 & 2 & 3 \\
0 & 1 & 1 & -2 \\
2 & 1 & 7 & 0 \\
1 & -2 & 1 & 5
\end{bmatrix} .
$$

4. Find the inverse of the matrix $B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -4 & 1 \\ 0 & 2 & 1 \end{pmatrix}$.

5. Compute a basis for the null space of the matrix

$$
\begin{bmatrix}
2 & 1 & 3 & 4 & 5 \\
1 & 3 & 2 & 7 & 8
\end{bmatrix} .
$$

6. Solve the following system of linear equations

$$
\begin{array}{rcrcrcl}
x_1 & + & 2x_2 & & & = & 1 \\
-x_1 & - & 4x_2 & + & x_3 & = & 2 \\
& & 2x_2 & + & x_3 & = & 0.
\end{array}
$$

7. Determine whether the following system of linear equations has a solution or not.

$$
\begin{pmatrix}
1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 1 & 0 \\
0 & -1 & -1 & 0 & 1 \\
0 & 0 & 0 & -1 & -1
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}
=
\begin{pmatrix} 1 \\ 2 \\ -2 \\ 0 \end{pmatrix} .
$$

8. Find a 2 by 2 square matrix B satisfying

$$A = B \cdot C,$$

where $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} -1 & -3 & 0 \\ 8 & 9 & 3 \end{pmatrix}$.

9. Suppose the matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is such that $A \in \mathbb{R}^{a \times 3}$, $B \in \mathbb{R}^{2 \times b}$, $C \in \mathbb{R}^{c \times d}$, and $D \in \mathbb{R}^{5 \times 4}$.

   (a) What are the values of $a, b, c,$ and d?

   (b) Suppose that the matrix multiplication MT is well defined. Further suppose that it can be done in block form where T has the structure

   $$T = \begin{bmatrix} U & V & W \\ Q & R & S \end{bmatrix}.$$

   What are the possible dimensions of the matrices $U, V, W, Q, R,$ and S?

10. Consider the matrix

$$\begin{bmatrix} 2 & 1 & 3 & 4 & 9 \\ 2 & -2 & -4 & 2 & 8 \\ 4 & -1 & 2 & 1 & 7 \\ 1 & 1 & 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}.$$

If $A \in \mathbb{R}^{a \times 2}$ and $D \in \mathbb{R}^{2 \times d}$, determine a and d then compute the matrix product CB.