# Checking Optimality Via Complementary Slackness

We consider LPs in standard form:

$$\begin{array}{lll} \mathcal{P}: & \text{maximize} & c^T x \\ & \text{subject to} & Ax \le b, \quad 0 \le x \end{array}$$

1. Does the vector $x = [1,1,1,1,1,1]^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \qquad c = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \qquad b = \begin{bmatrix} 10 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

2. Does the vector $x = [1.6, 3.6, 2.6, 0, 1.6, 0.6]^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \qquad c = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \qquad b = \begin{bmatrix} 10 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

3. Does the vector $x = [0,0,0,0,0,5,0]^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} 1 & -2 & 4 & 6 & 2 & 1 & 0 \\ -2 & -2 & 1 & 1 & 1 & 1 & 1 \\ 4 & 6 & 8 & 10 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 \end{bmatrix}, \qquad c = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 8 \\ 8 \\ -1 \end{bmatrix}, \qquad b = \begin{bmatrix} 10 \\ 10 \\ 20 \\ 10 \end{bmatrix}.$$

4. Does the vector $x = [0,0,0,0,0,10]^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -6 & -10 & 10 & 16 & -20 \\ 2 & 4 & 6 & 8 & 10 & 0 \\ 0 & 10 & 8 & 6 & 4 & 2 \\ 10 & -2 & 10 & -2 & 10 & -2 \end{bmatrix}, \qquad c = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 0 \\ 6 \\ 8 \end{bmatrix}, \qquad b = \begin{bmatrix} 10 \\ 10 \\ 20 \\ 20 \\ 10 \end{bmatrix}.$$

5. Does $x = (3, 1, 0)^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} -1 & 3 & -2 \\ 1 & -4 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \qquad c = \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}.$$

6. Does $x = (1, 2, 1, 0)^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} 3 & 1 & 4 & 2 \\ -3 & 2 & 2 & 1 \\ 1 & -2 & 3 & 0 \\ -3 & 2 & -1 & 4 \end{bmatrix}, \qquad c = \begin{bmatrix} -2 \\ 0 \\ 5 \\ 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 9 \\ 3 \\ 0 \\ 1 \end{bmatrix}.$$

7. Does $x = (0, 1, 1, 1)^T$ solve $\mathcal{P}$, where

$$A = \begin{bmatrix} -1 & 1 & 2 & 3 \\ 2 & 2 & -4 & 1 \\ -2 & -3 & 0 & 1 \\ 0 & 1 & 5 & 1 \\ -4 & 2 & 1 & 1 \\ -1 & 4 & 5 & 6 \end{bmatrix}, \qquad c = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 6 \end{bmatrix}, \qquad b = \begin{bmatrix} 7 \\ -1 \\ -1 \\ 7 \\ 5 \\ 16 \end{bmatrix}.$$

8. Change $b_6$ from 16 to 15 in problem 7 and check if $x = (0, 1, 1, 1)^T$ solves $\mathcal{P}$.

Answers: 1. No, $y$ does not exist. 2. Yes, $y = \frac{1}{5}(7, 4, 1, 0, 3, 13)^T$. 3. Yes, $y = (0, 0, 0, 4)^T$. 4. Yes (but tricky! Multiple dual solutions), $y_2 = y_3 = y_5 = 0$ and $y_1 + 2y_4 = 8$ with $2 \leq y_2 \leq 8$, 5. Yes, 6. No, 7. No, since no potential dual solution exists.