Simplex Algorithm for Problems in Standard Form and having Feasible Origin

Solve the following LPs using the simplex algorithm in simplex tableau form. At each stage of the simplex algorithm identify the BFS identified by the current tableau and give the associated objective vale z. All of the problems below are in standard form and have feasible origin.

1.
$$\begin{array}{lrcrcrcr} \text{maximize} & 4x & + & 3y & + & 2z & & \\ & & & & & & & \\ \text{subject to} & x & & & + & z & \le & 2 \\ & -x & - & y & + & z & \le & 1 \\ & x & + & y & + & z & \le & 3 \\ & 0 & \le & x, & y, & z & & \end{array}$$

Solution: $(2,1,0)$, optimal value $= 11$.

2.
$$\begin{array}{lrcrcrcr} \text{maximize} & 4x & + & 2y & + & 2z & & \\ & & & & & & & \\ \text{subject to} & x & + & 3y & - & 2z & \le & 3 \\ & 4x & + & 2y & & & \le & 4 \\ & x & + & y & + & z & \le & 2 \\ & 0 & \le & x, & y, & z & & \end{array}$$

Solution: $(1,0,1)$, optimal value $= 6$.

3.
$$\begin{array}{lrcrcrcr} \text{maximize} & -7x_1 & + & 9x_2 & + & 3x_3 & & \\ & & & & & & & \\ \text{subject to} & 5x_1 & - & 4x_2 & - & x_3 & \le & 10 \\ & x_1 & - & x_2 & & & \le & 4 \\ & -3x_1 & + & 4x_2 & + & x_3 & \le & 1 \\ & 0 & \le & x_1, & x_2, & x_3. & & \end{array}$$

Solution: $(4,0,13)$, optimal value $= 11$.

4.
$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & 7x_1 & + & 6x_2 & + & 5x_3 & - & 2x_4 & + & 3x_5 & & \\ & & & & & & & & & & & \\ \text{subject to} & x_1 & + & 3x_2 & + & 5x_3 & - & 2x_4 & + & 2x_5 & \le & 4 \\ & 4x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & + & x_5 & \le & 3 \\ & 2x_1 & + & 4x_2 & + & 4x_3 & - & 2x_4 & + & 5x_5 & \le & 5 \\ & 3x_1 & + & x_2 & + & 2x_3 & - & x_4 & + & 2x_5 & \le & 1 \\ & 0 & \le & x_1, & & x_2, & & x_3, & & x_4, & & x_5. \end{array}$$

Solution: $(0,4/3,2/3,5/3,0)$, optimal value $= 8$.