G53FUZ
Fuzzy Sets and Systems

Revision Lecture

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1. Module Overview

- Six main topic areas
  - lectures 2-7
    - fuzzy sets and operations
    - linguistic variables and fuzzy logic
    - Mamdani inference and defuzzification
    - TSK inference and fuzzy control
    - fuzzy model identification and tuning
    - ANFIS
    - [non-standard fuzzy inference]

2. Fuzzy Sets and Operations

- Concept and formal definition of a fuzzy set
  - basic notation
- Properties
  - alpha-cuts, support, normality, convexity
  - meaning of membership values (not probabilities)
- Operations
  - basic complement, intersection, union
  - operator axioms and families

3. Linguistic Variables and Logic

- Concept and formal definition of linguistic variables
  - (quintuple) aspects of variables
  - terms and membership functions
- Membership function types
  - piecewise linear (triangular, trapezoidal), Gaussian, sigmoids
- Linguistic truth and 'full' fuzzy logic
  - logical and; logical or

4. Mamdani and Defuzzification

- Concept and basis of Mamdani inference
  - essential methodology
  - complete worked example
- Defuzzification
  - numeric and linguistic defuzzification
  - centre-of-gravity, mean-of-maxima, etc.
  - problems with defuzzification
    - additional metrics provide more information
  - linguistic defuzz. using similarity measures

5. TSK and Control

- Concept and basis of TSK inference
  - essential methodology
  - zeroth order, first order, worked examples
  - differerences between Mamdani and TSK
- Control examples
  - controlling a DC motor; PID control
  - fuzzy approach to control
  - example systems: 4 rules and 9 rules

6. Model Identification and Tuning

- Model identification
  - structure and parameter identification
  - performance metrics and model evaluation
- Tuning methods
  - exhaustive search; monte carlo
  - hill climbing; stochastic local search
  - simulated annealing; simplex method
- Case study
  - tuning umbilical acid-base analysis system

7. ANFIS

- Concept and methodology
  - description of the layers (1-5)
  - overview of tuning processes
- Case study
  - complete worked example on iris data
  - fully automated approach
  - hand-crafted approach
  - hybrid (semi-automated) approach

8. Beyond Type-1 [not examined]

- Type-2 fuzzy sets
  - essential concepts and operational overview
- Non-stationary fuzzy sets
  - essential concepts and operational overview
- Case studies
  - modelling variation in umbilical acid-base analysis
  - improving decision support in breast cancer

Exam

- Two from four questions
  - multi-part free response
  - each question carries equal weighting (25 marks)
  - 2 hours, so should be plenty of time
- Module is new, so no past papers
  - see G52ARB 2010 for an example question (3)
- Top tips
  - read all the questions carefully before starting
  - look carefully at the mark distribution

Example Questions

(a) Given three fuzzy sets, *A*, *B* and *C*, defined over a universe of discourse of discrete integers between zero and ten (inclusive), where:

$A = 0.8/0 + 0.9/1 + 1.0/2 + 0.8/3 + 0.5/4 + 0.1/5,$
$B = 0.4/2 + 0.8/3 + 0.5/4 + 0.5/5 + 0.6/6 + 0.1/7,$
$C = 0.5/5 + 1.0/6 + 0.5/7 + 0.0/8 + 0.6/9 + 0.1/10$

Define the ‘standard’ (Zadeh) fuzzy operators, and use them to calculate the following fuzzy sets:

(i) (NOT *A*) AND *B*
(ii) A OR (*B* AND *C*)

[9]

(b) Give the formal properties that fuzzy intersection and union operators must possess, and re-calculate the two sets above using the probabilistic family.

[12]

(c) Someone states that fuzzy membership values are exactly the same as basic probabilities. Discuss.

[4]

Example Questions

(a) Draw example fuzzy sets for the input variable *error*, featuring two fuzzy sets, and input variable *delta_error*, featuring three fuzzy sets, suitable for use in a fuzzy controller. Label your diagrams fully, and use them to explain what a *fuzzy set* is. [9]

(b) Provide a set of rules, representing the rules both in linguistic form and in corresponding rule table form, that may be used in a fuzzy controller featuring the above described input variables and an *output* variable featuring five fuzzy sets. [8]

(c) Explain what alterations to the fuzzy controller are necessary if the *error* variable is altered to have three fuzzy sets and the *output* variable to have seven fuzzy sets. [3]

(d) What is *defuzzification* and when is it necessary? [5]

(a) Describe in detail two mechanisms of fuzzy inference and hence explain the differences between them.

[12]

(b) Explain what is meant by *numeric defuzzification* and *linguistic defuzzification* and describe the advantages and disadvantages of each.

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(c) If two fuzzy sets have the same ‘centre of gravity’, describe some other numeric metrics that might be used to distinguish between them.

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