G53FUZ
Fuzzy Sets and Systems

Module Introduction

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Overview

- Module outline
  - module delivery
    - website
  - aims and objectives
  - weekly topics
  - assessment
  - resources
- Classical (Boolean) logic and set theory
  - refresher of basic concepts
  - deficiencies / weaknesses

Module Outline

Module Delivery

- Lectures
  - theoretical / conceptual issues
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- Labs
  - one 2 hour lab per week
  - practical issues
    - creating a fuzzy system in R
    - coursework assistance

Aims and Objectives

- Aim
  - present how fuzzy method can be used to represent knowledge and perform reasoning, in the presence of uncertainty, in a principled manner
- Objectives
  - to introduce the theory and principles of fuzzy logic, fuzzy sets and systems
  - to explain how fuzzy methods can be used to model uncertainty in real world examples
  - to convey the properties and concepts underlying fuzzy inference systems and their applications
  - to provide practical experience on the design and implementation of fuzzy logic systems

Note of Minor Late Change

- The previous module specification on SATURN
  - *Further, Fuzzy Logic Systems (FLSs) will be introduced and illustrated in conjunction to examples of real world applications in industrial control and other areas. As part of the practical work within the module, the design,* ***programming and deployment/testing of a fuzzy logic controller on a mobile robot*** *will provide a tangible real world example of some underlying concepts of FLSs.*
- Has been altered to now read
  - ***programming and deployment/testing of a fuzzy logic system***
  - not necessarily on a mobile robot

Weekly Topics

Week	Lecture	Lab
1	Module Introduction	MATLAB Fuzzy Toolbox
2	Fuzzy Concepts	Basic R
3	Linguistic Variables	Basic R
4	Fuzzy Inference and Defuzzification	Fuzzy Systems in R
5	TSK Inference: Practical Comparison	Fuzzy Systems in R
6	Applications: Control and Reasoning	Fuzzy Systems in R
7		CW Assistance
8	Learning using ANFIS	CW Assistance
9	Extensions to Type-1 Fuzzy Sets	CW Assistance
10	Non-Standard Inference Systems	CW Assistance

Module Assessment

- Coursework
  - 50%
  - implement a fuzzy inference system (in R)
  - write report describing your system
- Exam
  - 50%
  - 2 hours
  - one compulsory (30) and one from two/three (20)

Resources

- Fuzzy sets and systems
  - *Fuzzy Sets, Uncertainty and Information*, Klir and Folger
    - Prentice Hall PTR, 1988, ISBN 0133459845
  - *The Fuzzy Systems Handbook*, Cox
    - Academic Press, 1994, ISBN 0121942708
  - *Artificial Intelligence*, Negnevitsky
    - Addison Wesley, 2002, ISBN 0201711591
- MATLAB / R
  - MATLAB Fuzzy Logic Toolbox
  - R (www.r-project.org)
    - loads of online tutorials, docs, help, examples
    - R Fuzzy toolbox (`www.cs.nott.ac.uk/~jmg/fuzzy-v0_7.r`)

Classical (Boolean) Logic

Classical (Crisp) Logic

- Origins in Ancient Greece
  - Aristotle
  - Plato
- Two truth values
  - *true*, *false*
- Connectives
  - *not*, *and*, *or*

Propositional Logic

- All concepts (statements) are *true* or *false*
  - represented by symbols *T* and *F*
- Symbols used to represent connectives
  - *not* $\neg$ (negation / complement)
  - *and* $\wedge$ (conjunction / intersection)
  - *or* $\vee$ (disjunction / union)
  - *implies* $\Rightarrow$ (implication)
- Truth table defines meanings of connectives

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$
F	*F*	*T*	*F*	*F*	*T*
T	*F*	*F*	*F*	*T*	*F*
F	*T*	*T*	*F*	*T*	*T*
T	*T*	*F*	*T*	*T*	*T*

Boolean Algebra (Logic)

x	*y*	NOT *x*	NOT *y*	*x* AND *y*	*x* OR *y*
0	0	1	1	0	0
1	0	0	1	0	1
0	1	1	0	0	1
1	1	0	0	1	1

- Some properties
  - associativity $x \vee (y \vee z) \;=\; (x \vee y) \vee z$
  - commutativity $x \vee y = \; y \vee x$
  - distributivity $x \wedge (y \vee z) \;=\; (x \wedge y) \vee (x \wedge z)$
  - identity for $\vee$ $\quad x \vee 0 = \; x$
  - identity for $\wedge$ $\quad x \wedge 1 = \; x$
  - annihilator for $\wedge$ $\quad x \wedge 0 = \; 0$

Sets

- A set is a collection elements, from some *universe of discourse*
- The set of all elements in the universe of discourse is the *universal set*
  - denoted by X
- The set that contains no elements is the *empty set*
  - denoted by $\varnothing$
- Set theory and propositional logic are isomorphic
  - every theorem in one has a counterpart in the other

Complement

- The *complement* of a set A is the set of all members of the universal set, X, that are not in A
  - denoted by A^c or $\overline{A}$
- So, if

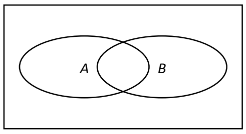
  $X = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$

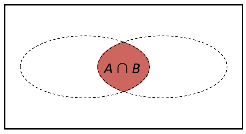
  $A = \{ 0, 2, 4, 6, 8 \}$
- Then

  $A^c = \{ 1, 3, 5, 7, 9 \}$
- Note that $X^c = \varnothing$, $\varnothing^c = X$, and $(A^c)^c = A$
- Complement is equivalent to logical *negation*

Intersection

- The *intersection* of sets A and B is the set containing all the elements that belong to both set A and set B
  - denoted by $A \cap B$
- Note that $A \cap X = A$, $A \cap \varnothing = \varnothing$, and $A \cap A^c = \varnothing$

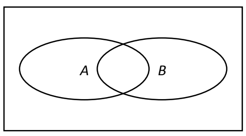


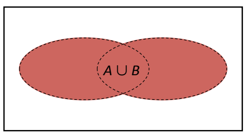


- Intersection is equivalent to logical *and*

Union

- The *union* of sets A and B is the set containing all the elements that belong to set A alone, to set B alone, or to both set A and set B
  - denoted by $A \cup B$
- Note that $A \cup X = X$, $A \cup \varnothing = A$, and $A \cup A^c = X$





- Union is equivalent to logical *or*

Deficiencies of Classical Logic

Aristotle, Zeno and Others

- Aristotle himself
  - future events: what is the truth of
    - it will snow tomorrow?
- Zeno
  - paradoxes (concerning infinities)
  - consider grains of sand
    - when have you got a 'heap'?

Cantor, Russell et al

- Let *R* be the set of all sets that are not members of themselves
  - if *R* qualifies as a member of itself, it would contradict its own definition as *a set containing all sets that are not members of themselves*
  - if *R* is not a member of itself, it would qualify as a member of itself by the same definition
- There are other such problems with classical set theory, including ones identified by Cantor

Exercise

- Who is in the set of short people?
- Who is in the set of tall people?

- Consider everyday concepts
  - children
  - middle-aged
  - old people

Fuzzy Logic – An Introduction

IEEE Computational Intelligence Society

- This first-prize-winning video was sponsored by the IEEE Computational Intelligence Society (CIS) in its 2011 Fuzzy Logic Video Competition
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Fuzzy Logic – Boiling Eggs

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Summary

- Classical two-valued logic has short-comings when representing the real-world
  - *classical*, *Boolean*, *two-valued*, ***crisp***
- Fuzzy logic relaxes the requirement to be either TRUE or FALSE
  - allows real-valued degrees of truth
- Logic and set theory are isomorphic
  - true/false statements can be mapped to set membership / non-membership