G53FUZ Formulae - Craig Knott

Zadeh Sets

Discrete Set	$A = \mu_1/x_1 + ... + \mu_i/x_i$
Continuous Set	$A = \int_x \mu(x)/x$

Zadeh Definitions

α-cut	$A_\alpha = \{x \in X \mid \mu_A(x) > \alpha\}$
Strong α-cut	$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$
Fuzzy Complement	$\mu_{\bar{A}}(x) = 1 - \mu_A(x)\ \forall x \in X$
Fuzzy Convexity	iff $\mu_A(\lambda r + (1-\lambda)s)$ $> min(\mu_A(r), \mu_A(s))$ $\forall \lambda \in [0,1], \forall r, s \in \mathbb{R}^n$

Zadeh's Extension Principle

If $f = u \to v$, and $A = \mu_1/u_1 + ... + \mu_n/u_n$

$f(A) \quad \equiv f(\mu_1/u_1 + ... + \mu_n/u_n)$

$\qquad \equiv \mu_1 f(u_1) + ... + \mu_n f(u_n)$

Extended OR (IXI)	$\sum_{i,j}(\alpha_i \wedge \beta_j)/(v_i \vee w_i)$ (min of rhs)
Extended AND (IIX)	$\sum_{i,j}(\alpha_i \wedge \beta_j)/(v_i \wedge w_i)$ (max of rhs)

T-Norms

Definition	$[0,1] \times [0,1] \to [0,1]$
Min	$min(a,b)$
Product	ab
Bounded Difference	$max(0, a+b-1)$
Drastic Sum	a if $b = 1$; b if $a = 1$; else 0

T-Conorms

Definition	$[0,1] \times [0,1] \to [0,1]$
Max	$max(a,b)$
Probabilistic Sum	$a + b - ab$
Bounded Sum	$min(1, a+b)$
Drastic	a if $b = 0$; b if $a = 0$; else 1

Fuzzy Truth

Linguistic Truth	$\int_0^1 e^{\frac{-(1-v)^2}{0.2^2}}$
Definite Truth	$1/1$
Linguistic False	$\int_0^1 e^{\frac{-v^2}{0.2^2}}$
Definite False	$1/0$
Unknown	$? = \int_0^1 1/v$
Undefined	$\theta = \int_0^1 0/v$

Numeric Mamdani Defuzzification

Centroid	$x_c = \frac{\sum_{i=1}^N (\mu_i \cdot x_i)}{\sum_{i=1}^N \mu_i}$
Mean of Maxima	$x_m = x_i \mid \mu(x_i) = max_x(\mu(x))$
Largest of Maxima	$x_l = max_i(x_i) \mid \mu(x_i) = max_x(\mu(x))$
Smallest of Maxima	$x_s = min_i(x_i) \mid \mu(x_i) = max_x(\mu(x))$
Bisector	

Linguistic Mamdani Defuzzification

Euclidean Distance	$\delta^2 = \sum_{i=1}^N (\mu_i - \eta_i)^2$
Overlap	$\frac{A \cap B}{A \cup B}$

Mamdani Defuzzification Metrics

Membership grade at defuzzification point (μ_g)

Height of output set (μ_h)

Normalised Area	$\frac{\sum_{i=1}^N \mu_i}{N}$
Fuzzy Entropy	$S = \frac{\sum_{i=1}^N (-\mu_i ln(\mu_i) - (1-\mu_i) ln(1-\mu_i))}{N}$

ANFIS

First Layer	Fuzzify Inputs $O_{1,i} = \mu_{A_i}(x_1) \quad \forall i \in [1,2]$ $O_{1,i} = \mu_{B_{i-2}}(x_2)\ \forall i \in [3,4]$
Second Layer	Apply T-Norm to get weights $O_{2,i} = \mu_{A_i}(x_1)\mu_{B_i}(x_2)$
Third Layer	Normalise weights $O_{3,i} = \frac{w_i}{\sum_{j=1}^R w_j} \forall i \in [1,R]$
Fourth Layer	Apply weights to consequents $O_{4,i} = \overline{w_i}(p_i x_1 + q_i x_2 + r_i)$
Fifth Layer	Defuzzification $O_{5,i} = y = \frac{\sum_{i=1}^N w_i y_i}{\sum_{i=1}^N w_i}$

Back Propagation Update	$\Delta\alpha i = -\eta(\frac{\partial E}{\partial \alpha i})$
Learning Rate	$\eta = \frac{k}{\sqrt{\sum_{i=1}^N (\frac{\partial E}{\partial \alpha i})^2}}$
Back Prop. Err. Msr.	$E = \sqrt{\frac{\sum_{p=1}^P (T_p - Y_p)^2}{P}}$
Least Square Error	$\lVert AX - B \rVert^2$

Misc.

Modus Ponens	$((p \implies q) \wedge p) \perp q$
Modus Tollens	$((p \implies q) \wedge \neg q) \perp \neg p$
Grammar Definition	(N, Σ, P, S)
Agg. 0th order TSK	$\frac{\sum_{i=1}^N w_i k_i}{\sum_{i=1}^N w_i}$
Formula for SA[a]	$e^{-\Delta E/T}$
Accept move in SA	Low EC, High T: $e^{-0} \approx 1$
Reject move in SA	High EC, Low T: $e^{-\infty} \approx 0$
Formula for PID	$K_p et + K_i \int_0^t e\tau d\tau + K_d \frac{det}{dt}$

[a] Simulated Annealing