HomeWork 9

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1. First Order ODE

RC circuit equation should be solved using the Euler algorithm for this part. The Euler algorithm:

$$v_{n+1} = v_n + a_n \Delta t$$
$$x_{n+1} = x_n + v_n \Delta t$$

The equation that should be solved is:

$$\dot{q} = \frac{V_0}{R} - \frac{q}{RC}$$

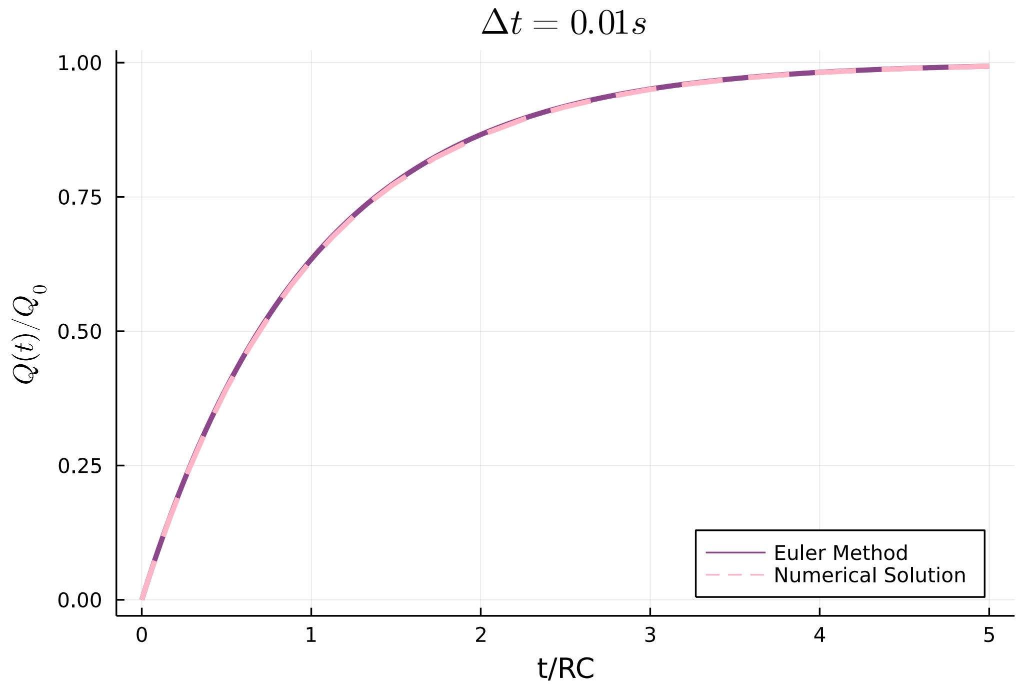
We put the following form for the function in code:

$$\dot{q} = q_0(1-q)/RC \ , \ q_0 \equiv CV_0$$

The increment of the global error with changing the step values is also shown for the Euler algorithm.

At last, I've used the following algorithm, which was mentioned in class, to solve the problem. The instability is obvious from the plot.

$$x_{n+1} = x_{n-1} + 2\Delta t \dot{x}(t_n, x_n)$$



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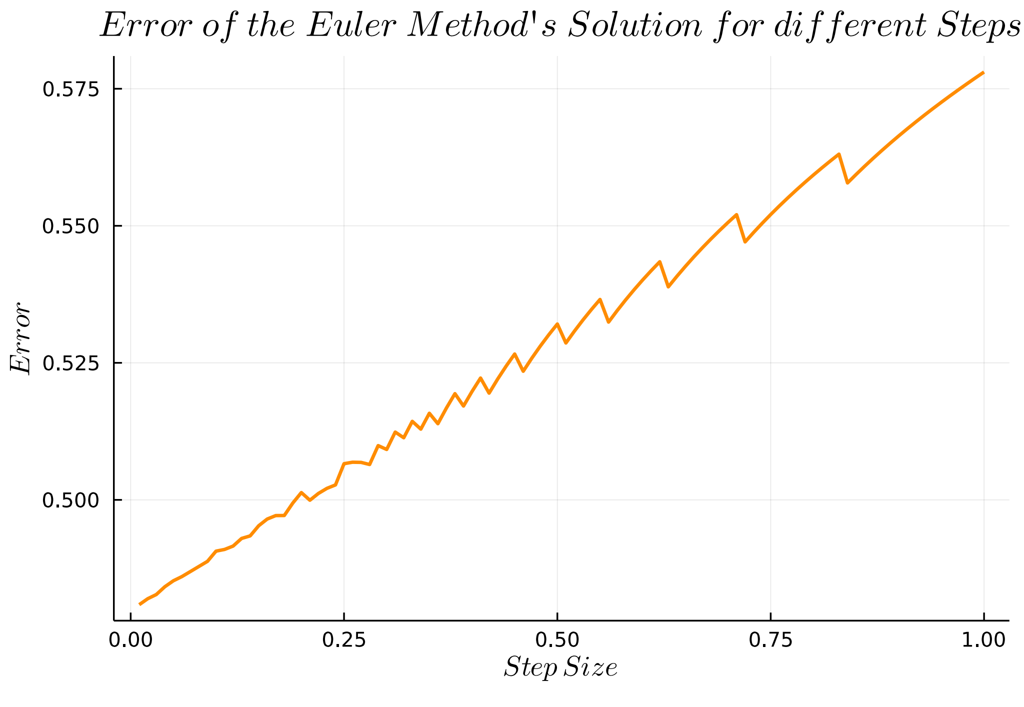

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Figure 1: Solving the RC circuit equation using the Euler algorithm. the time range is between 0.0 to 5.0 seconds. step equals to 0.01.

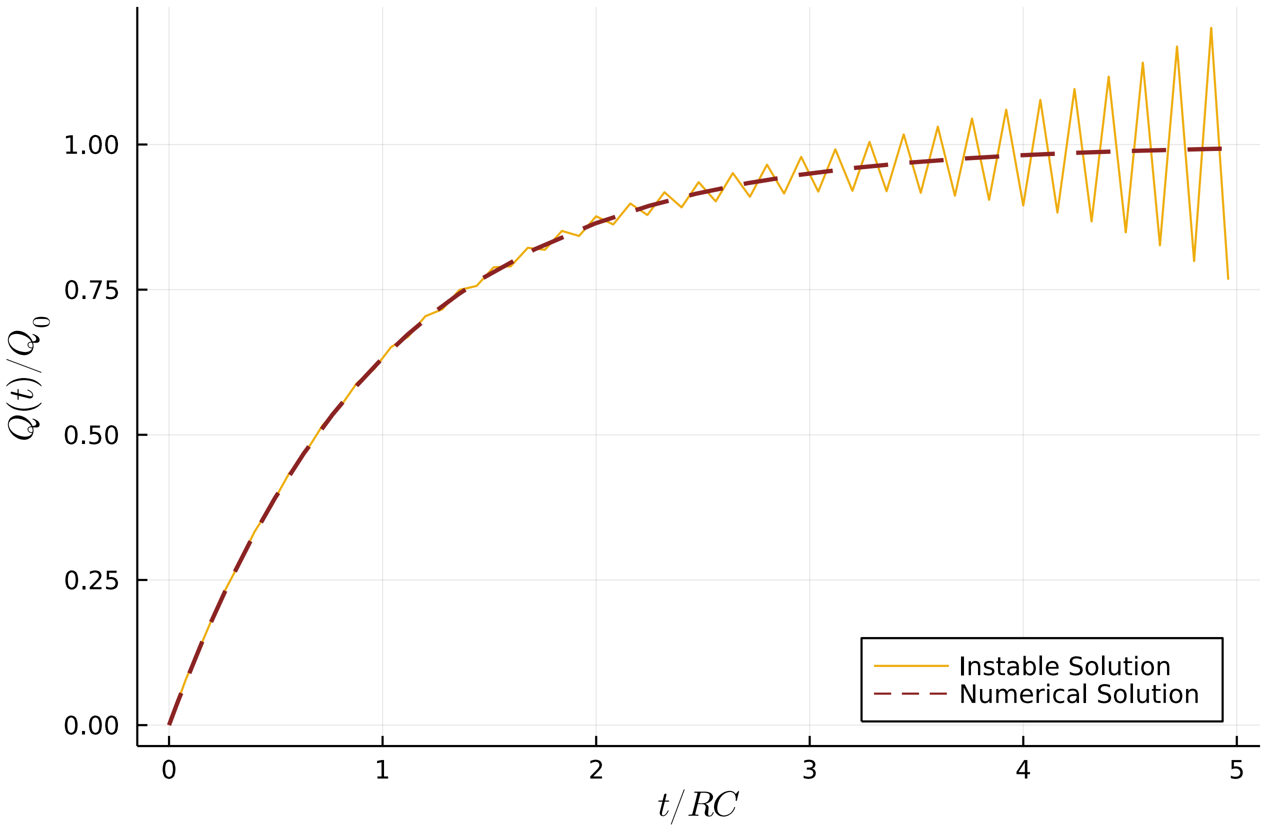


Figure 2: Instability plot of RC circuit equation for the given algorithm. Step size= 0.08. time ranging from 0.0 to 5.0 seconds.

2. Second Order ODE

Simple Harmonic Oscillator's equation has been solved with the Euler Method and the following algorithms and the comparison plots and the phase space plots are followed.

Euler-Cromer Algorithm :

$$v_{n+1} = v_n + a_n \Delta t$$
$$x_{n+1} = x_n + v_{n+1} \Delta t$$

Verlet Algorithm :

$$x_{n+1} = 2x_n - x_{n-1} + a_n (\Delta t)^2$$
$$v_{n+1} = \frac{x_{n+1} - x_n}{h}$$

Velocity Verlet Algorithm :

$$x_{n+1} = 2x_n + v_n \Delta t + \frac{1}{2} a_n (\Delta t)^2$$
$$v_{n+1} = v_n + \frac{1}{2}(a_{n+1} + a_n) \Delta t$$

SHM equation:

$$\ddot{x} = -\omega x$$

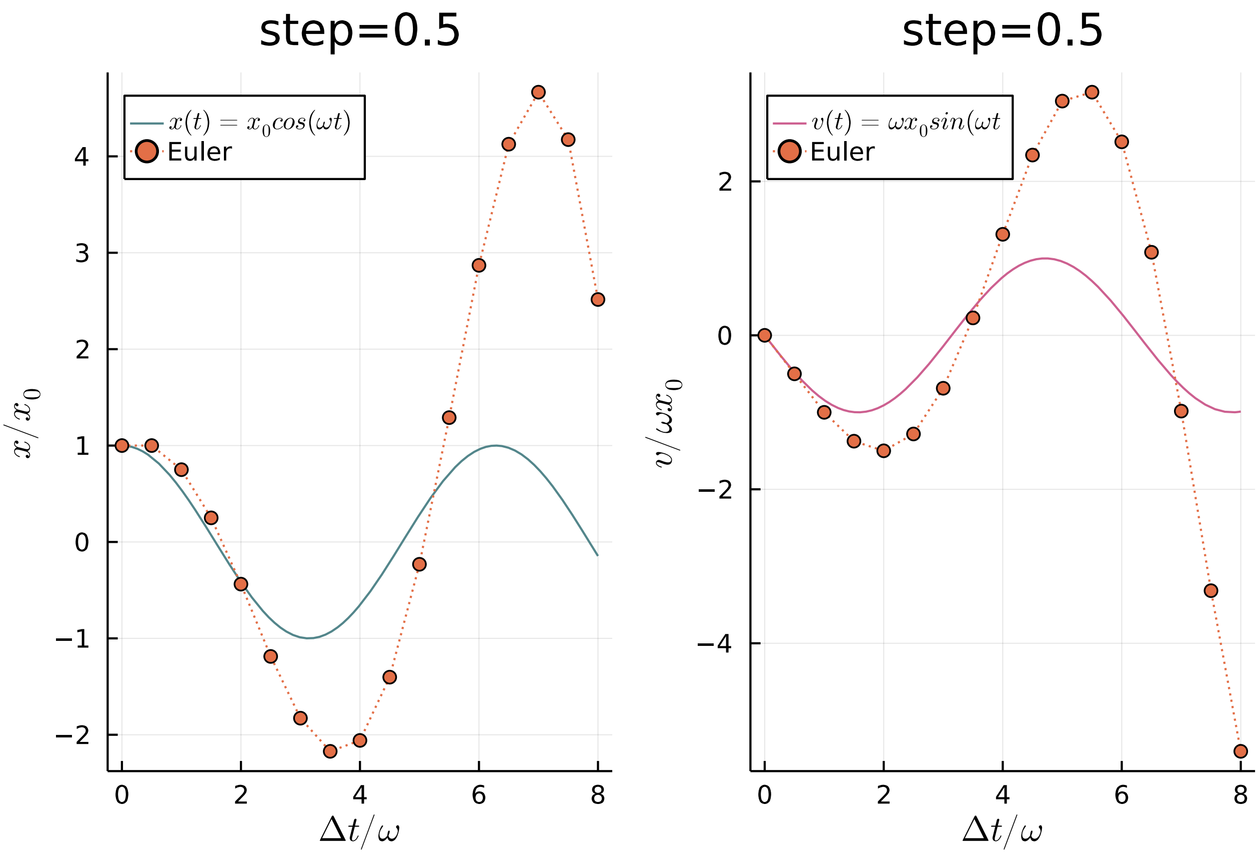
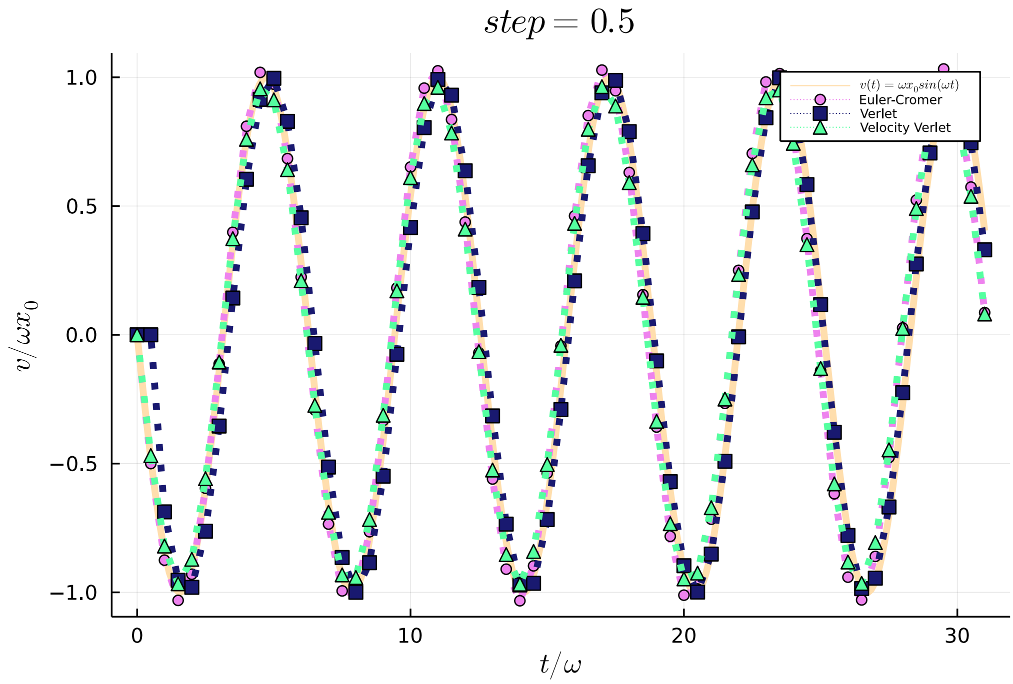


Figure 3: Place-Time and Velocity-Time Plots. Comparison between Euler method's and the analytic solution for SHM (n=5) and time range between 0.0 and 8.0 seconds. $x_{initial} = 1.0$ and $v_{initial} = 0.0$.



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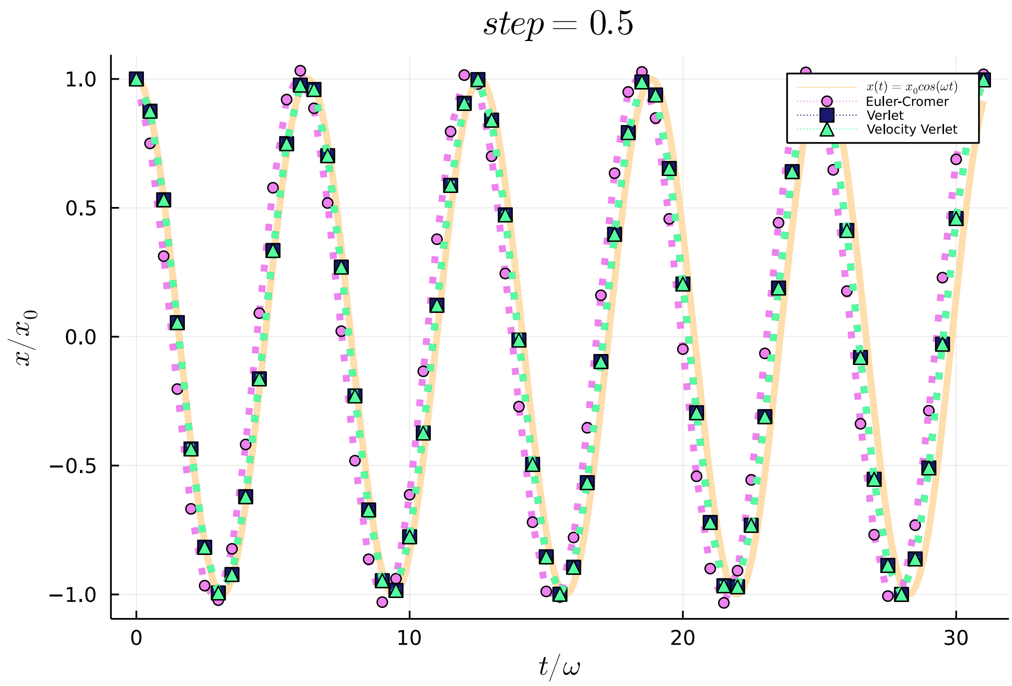

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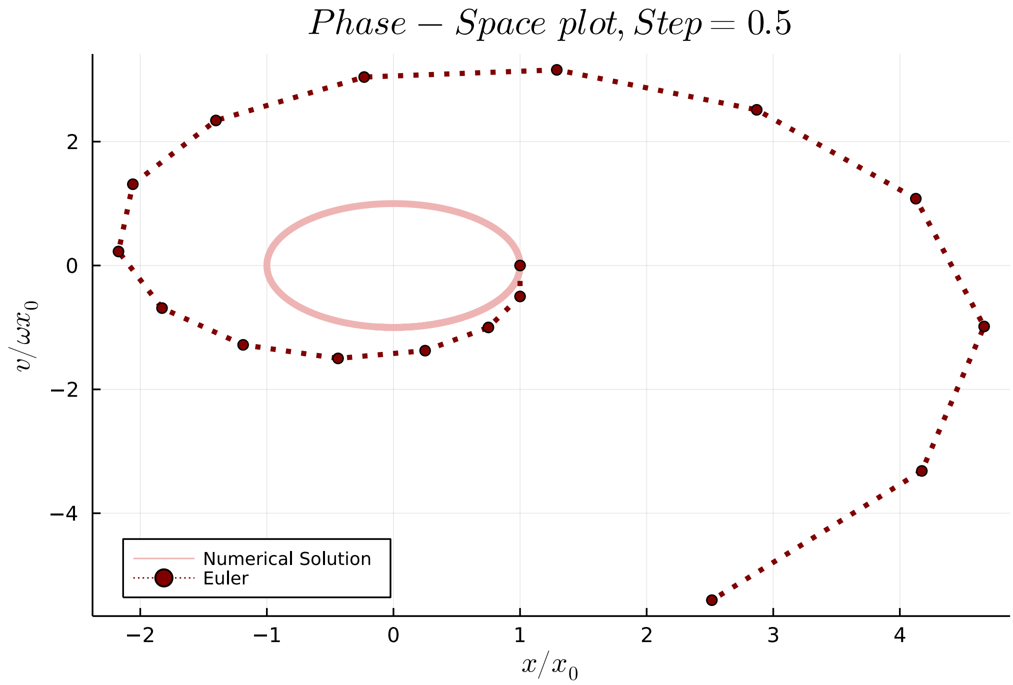
Figure 4: Place-Time and Velocity-Time Plots. Time range between 0.0 and 31.0 seconds, n=5, $x_{initial} = 1.0$ and $v_{initial} = 0.0$.



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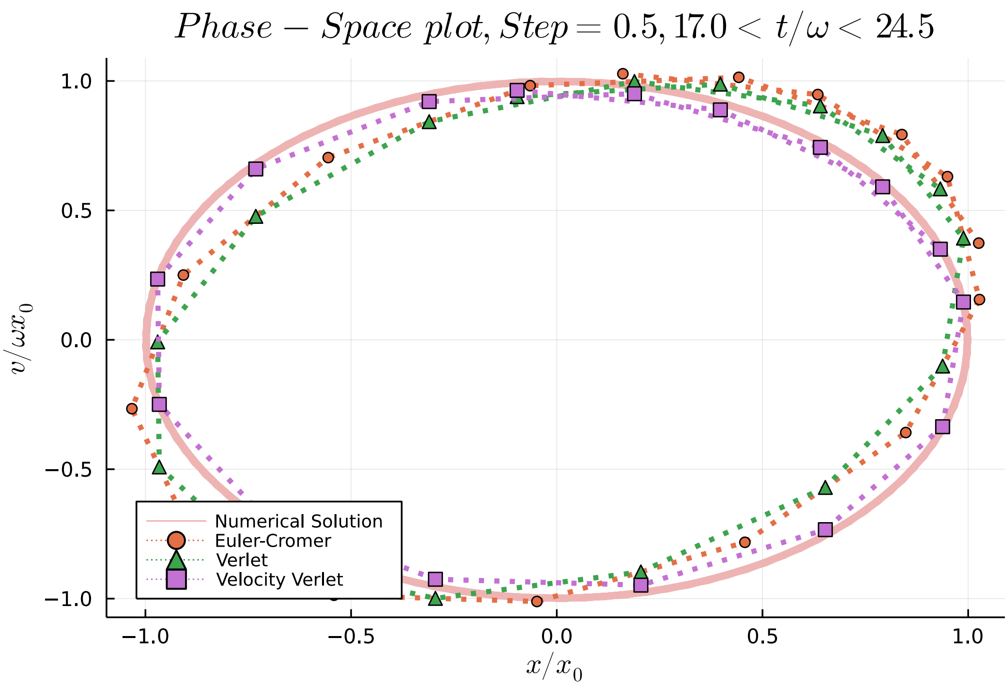

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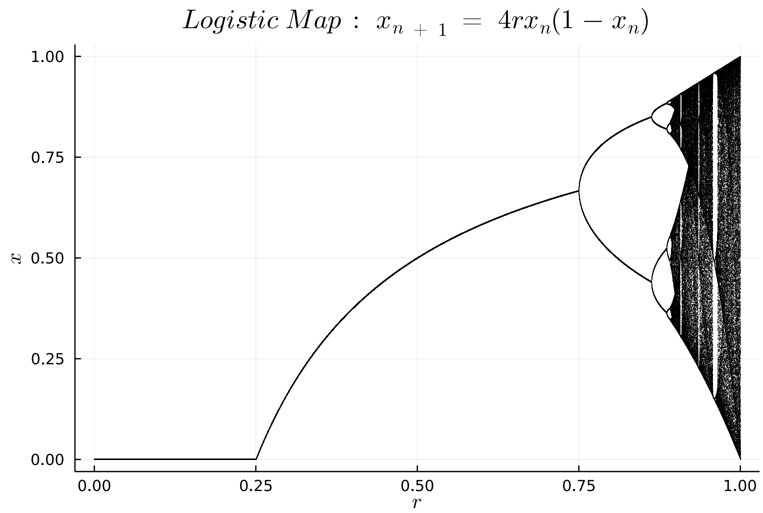
Figure 5: Phase space plots. n=5, $x_{initial} = 1.0$ and $v_{initial} = 0.0$.

3. Bifurcation

Mathematically the logistic map is written as:

$$x_{n+1} = 4rx_n(1-x)$$

By using this function, the bifurcation can be seen from the plot. I created a range of r values in between 0.0 and 1.0, then for each r, I generated an array of x_0 values in range (0.0, 1.0). These values are passed to the function, and the final x values will be returned after a repetition of 10^4.



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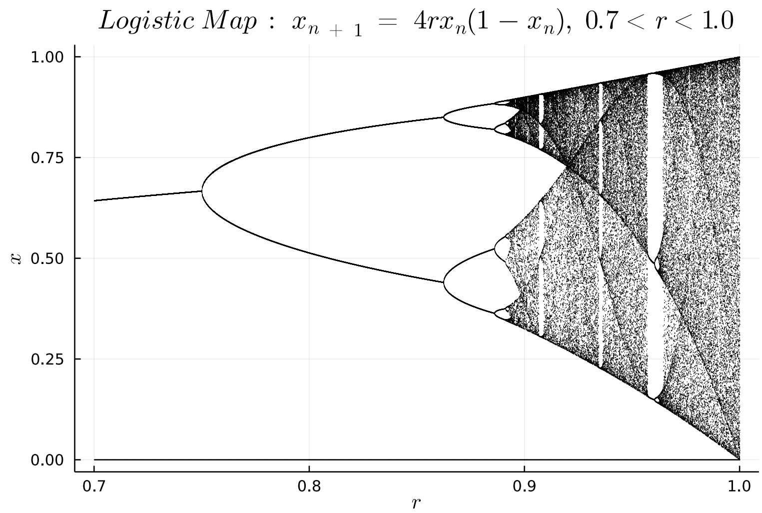

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Figure 6: Bifurcation plots. $\Delta r = 10^{-4}$, $0.0 < r < 1.0$, $n = 10^4$.

Feigenbaum constants

$$\sigma = \lim_{n\to\infty} \frac{r(d_n) - r(d_{n-1})}{r(d_{n+1}) - r(d_n)} = 4.6692$$

And I acquired the value $\sigma \sim 4.627451$, for step= 10^{-4} and n $= 10^4$.

$$\alpha = \lim_{i\to\infty} \frac{L_{i-1}}{L_i} = 2.502907$$

And I acquired the value $\alpha \sim 1.5294$, for step= 10^{-4} and n $= 10^4$.