

Homework 6

Maedeh Karkhane Yousefi

November 27, 2021

1. Exercise 6.3: Central Limit Theorem

Knowing that the program's random generator gives a uniform distribution to us, we used this property to sum specific number of random numbers for 10000 samples.

Normalized Distribution of Samples

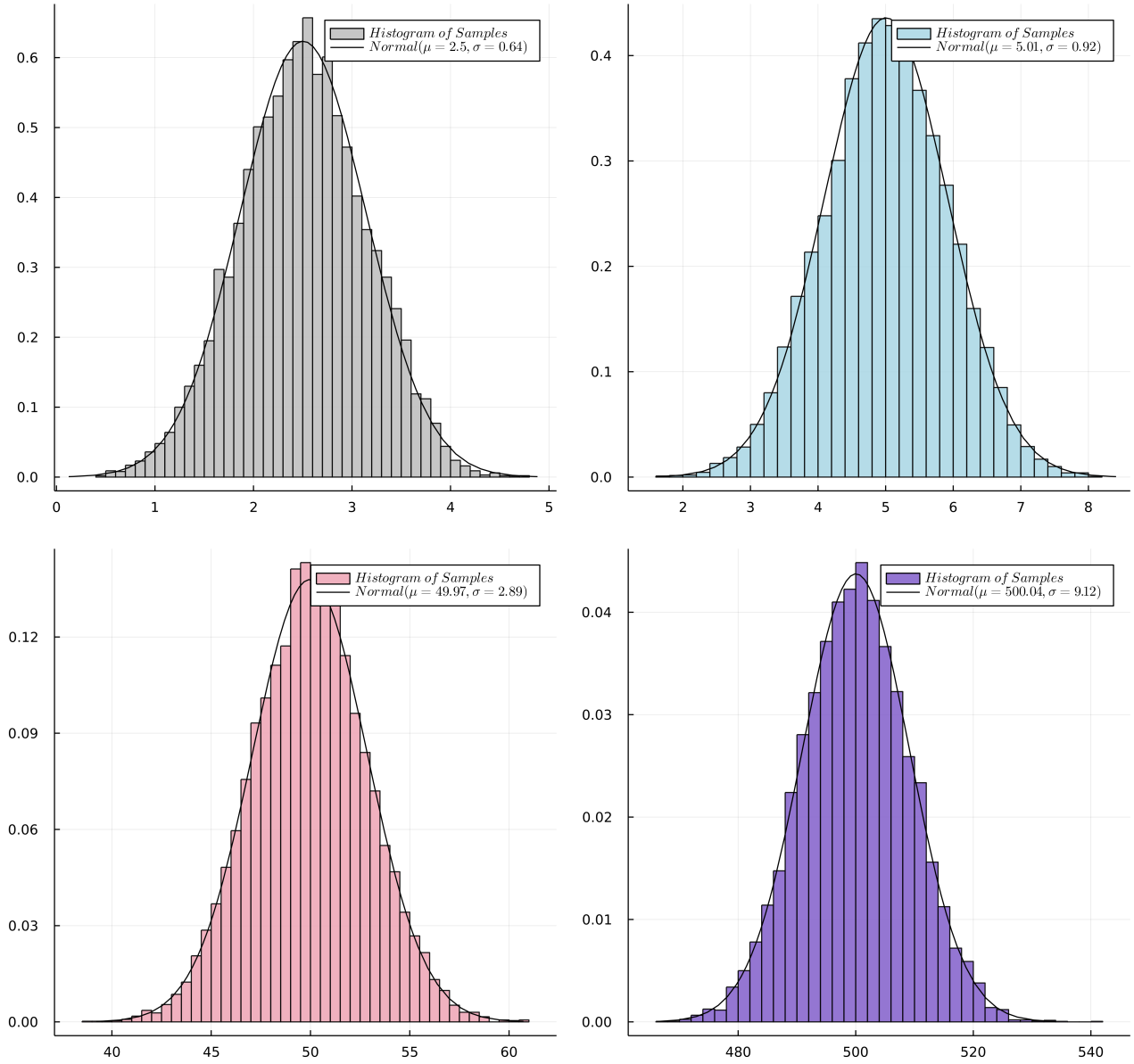


Figure 1: Central Limit Theorem for lengths: 5, 10, 100, 1000 for 10000 samples each.

2. Exercise 6.4: Transformation Matrix

$$\begin{aligned}
 f(\rho) &= \int_0^b \frac{1}{2\pi\sigma^2} \rho \exp\left(-\frac{\rho^2}{2\sigma^2}\right) d\rho d\theta \\
 f(\rho) &= \int_0^\rho \frac{1}{\sigma^2} \exp\left(-\frac{\rho'^2}{2\sigma^2}\right) d\rho' = x = 1 - \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \\
 \rho &= \sqrt{-2\sigma^2 \ln(1-x)} \\
 f(\theta) &= \int_0^\theta \frac{1}{2\pi} d\theta' = x' = \frac{\theta}{2\pi} \\
 \theta &= 2\pi x' \\
 y1 &= \rho \cos \theta \quad , \quad y2 = \rho \sin \theta
 \end{aligned}$$

With these results, I used an array of randomly generated numbers from the program's uniform distribution algorithm as input and considered N/2 of these elements as x1 and others as x2, and put into these equations.

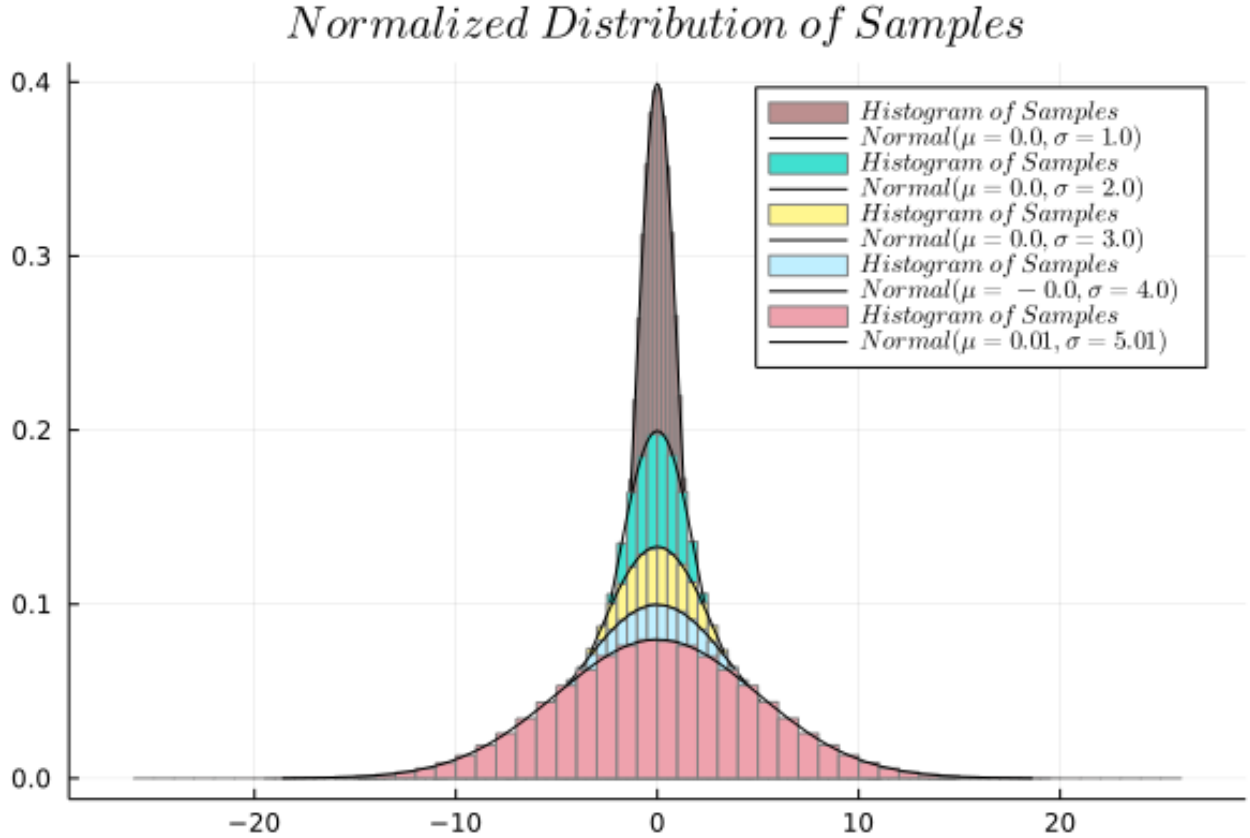


Figure 2: Random numbers' distribution using the Box-Muller transform getting use of the program's own uniform distribution random generator. $\sigma=[1, 2, 3, 4, 5]$, Sample Number=1000000

use [this link](#) to check the saved data.