Homework 6

Maedeh Karkhane Yousefi

November 28, 2021

1. Exercise 7.1: SS MC and IS MC

The absolute answer is:

$$I = \int_0^2 e^{-x^2} = \sqrt{\pi}/2 \; erf(2)$$

ISMC:
We calculate the integral knowing the function $g(x) = e^{-x}$, and the integral is $\int_0^2 e^{-x^2}$. The exact answer is:

$$\int_0^2 e^{-x^2} = 1 - e^{-2}$$
$$y = -ln(1 - x(1 - e^{-2}))$$

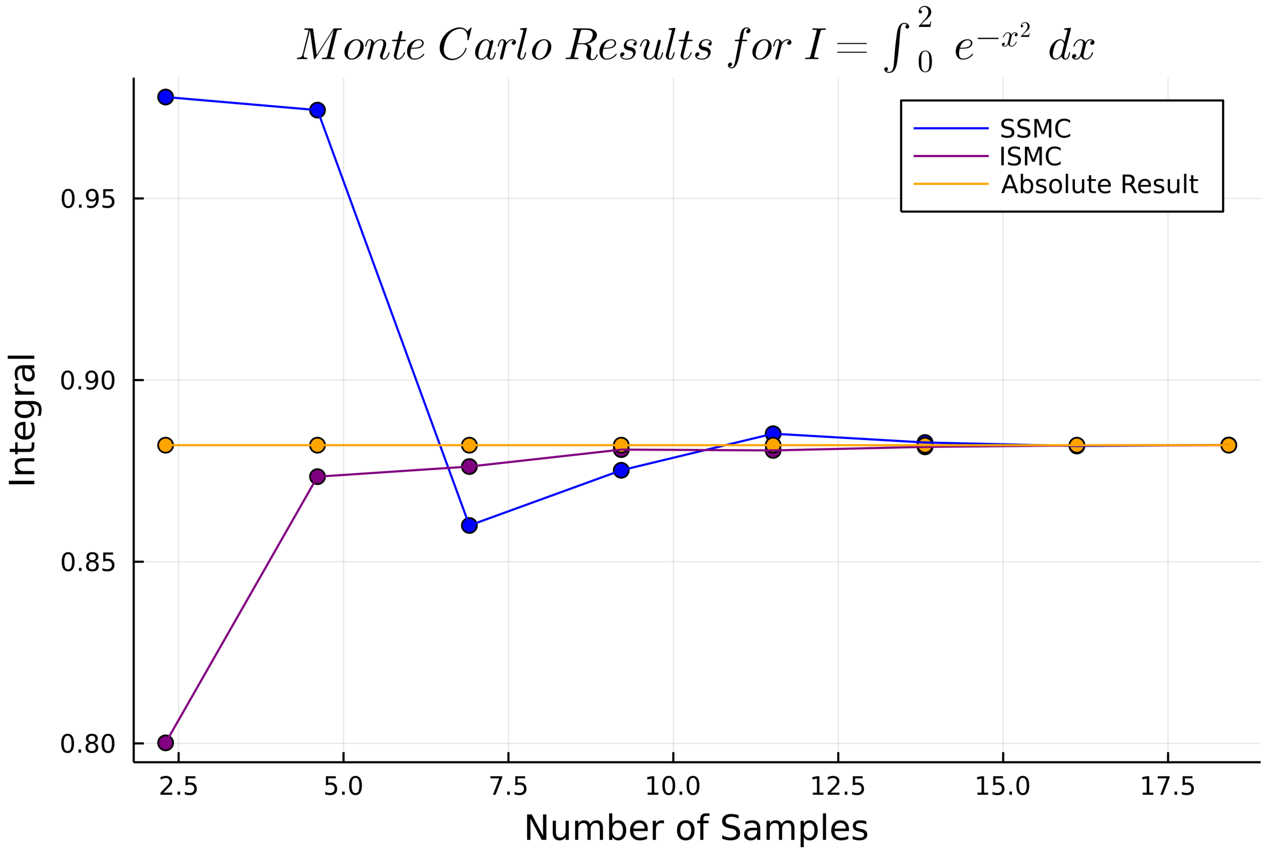


Figure 1: The Answers for 8 samples using SSMC and ISMC method. samples are between 10^1 to 10^8 multiplying by 10 each step.

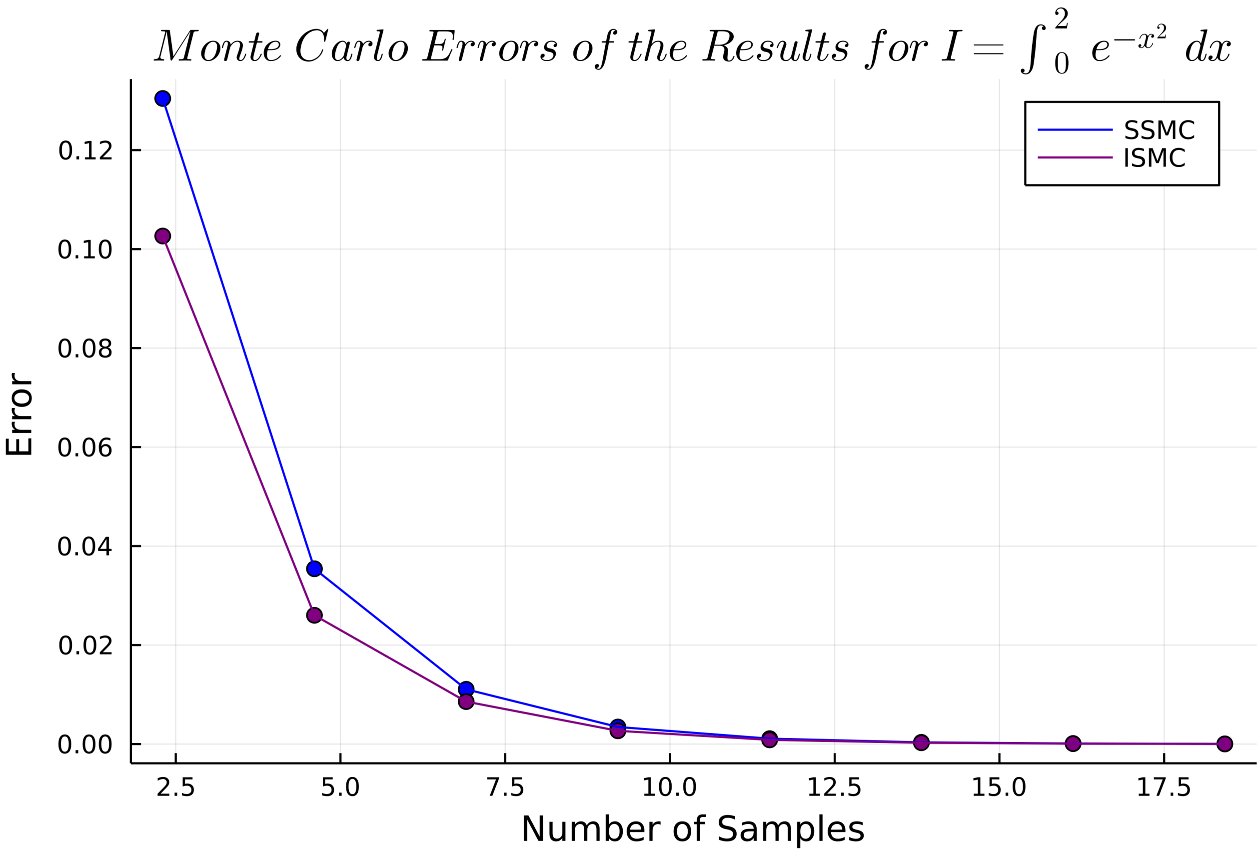


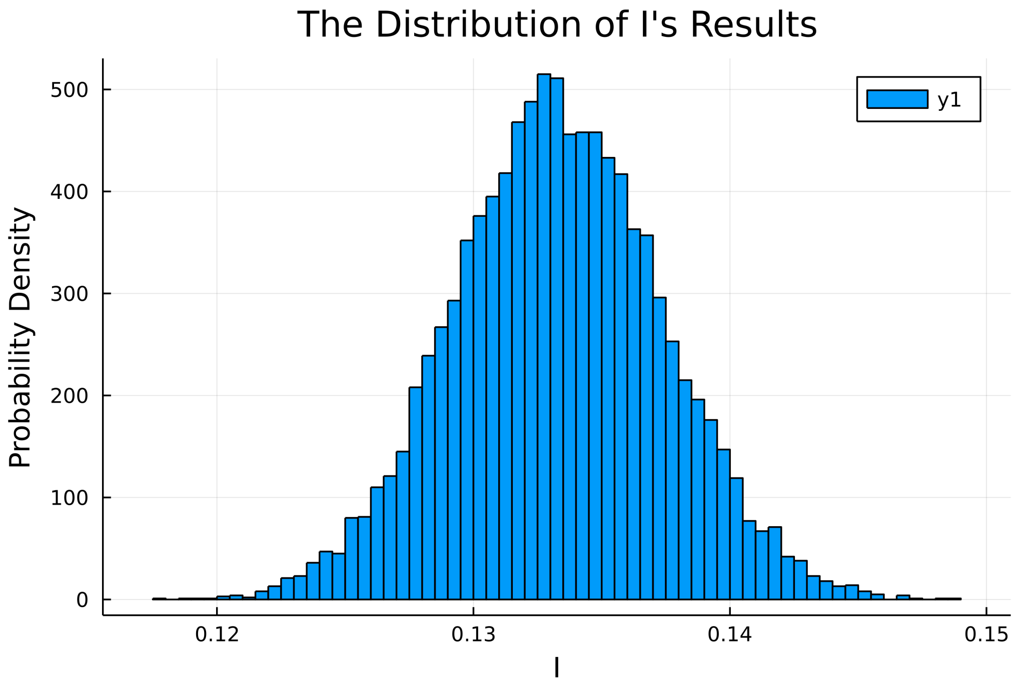
Figure 2: The errors of the answers for 8 samples using SSMC and ISMC method. samples are between 10^1 to 10^8 multiplying by 10 each step.

2. Exercise 7.2

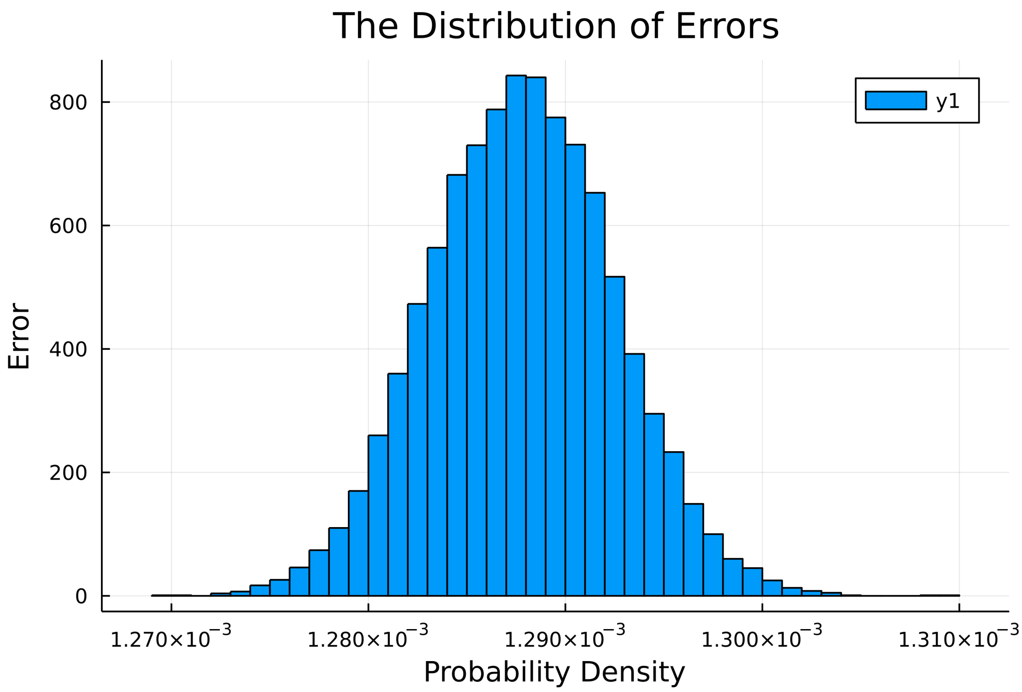
Using 2D Integration, I tried to find the center of mass of the sphere wanted in the textbook. knowing that:

$$R_{CoM} = \frac{I}{M}$$
$$R_{CoM} = \frac{\int_0^R \int_0^\pi (3+\frac{r}{R}\cos\theta) r^3 \sin\theta \cos\theta d\theta dr}{\int_0^R \int_0^\pi (3+\frac{r}{R}\cos\theta) r^2 \sin\theta d\theta dr}$$

The exact answer is: R/15 (assuming R=1, the answer is 1/15).



(a)

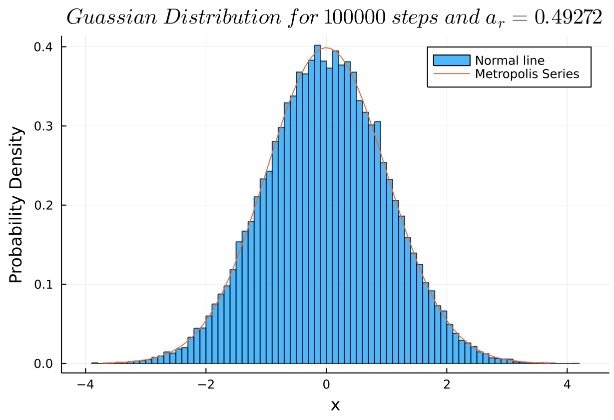


(b)

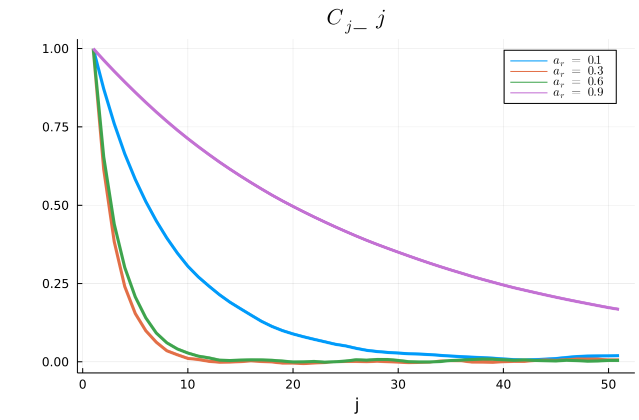
Figure 3: Plots given for Exercise 7.2. R=1. Number of Samples=10^4

3. Exercise 8.1: Metropolis

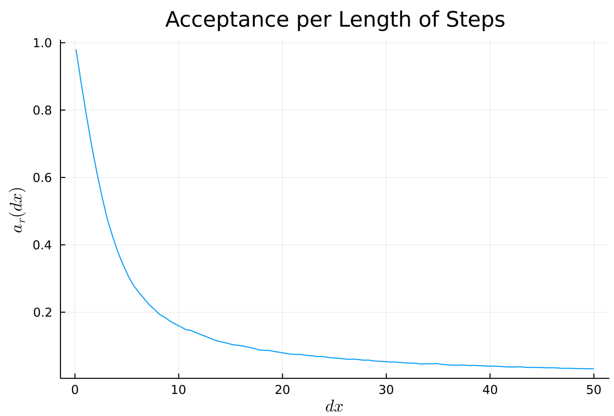
I used the Metropolis method in order to generate random numbers with a normal distribution. The relation between a_r and dx(unit length of steps) are shown and further we come up with the auto correlation and the correlation length.



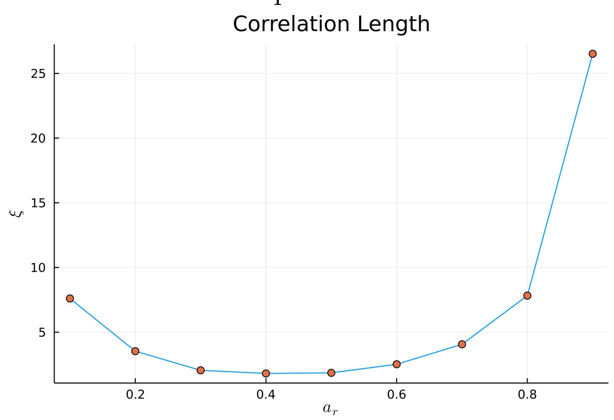
(a) Number of steps=10^5 , dx=3.



(b) dx are: 15.9, 7.95, 5.27, 3.88, 2.94, 2.2, 1.57, 1.03, 0.5. a_r=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. Number of Steps= 10^5.



(c) using the dx given in a list of steps from 0.1 to 50, a total number of 100 values. number of steps=10^5.



(d) Using the same data that were used in (b).

Figure 4: Plots given for example 8.1. $X_0 = 0$

The correlation Length is obtained by the equation given in the textbook.
refer to this link to check the saved data.