University of Tehran
Department of Electrical and Computer Engineering

Problem Description: From Continuous Signals to Discrete Ones

As the data in a computer is stored and processed digitally, the signal processing in MATLAB is also performed in this manner. The topic of signal sampling is the bridge between these two types of signals, makes it possible to translate analog concepts over discrete-time signals.

Suppose that we have an array in our MATLAB workspace such that it contains N samples which correspond to T seconds. In other words, we have sampled our data with a sample rate of $t_s = \frac{1}{f_s} = \frac{T}{N}$. So the time range corresponding to this array of samples would be $t = t_{start}: t_s: t_{end} - t_s$.

In order to transform this array of data from time domain to frequency domain, one can use MATLAB's `fft` command. The FFT is abbreviate for "fast Fourier transform" which is an algorithm for discrete Fourier transform.

Using the basic syntax of fft function would return another array consists of N numbers. As noted, these numbers correspond to a specific frequency value between $[0, 2\pi]$, which is called angular frequency ($\omega = 2\pi f$). As discussed in the class, the Fourier transform of discrete signals is periodic with a period of 2π. So if we swap the first half of fft output with its second half, the information content won't change, due to its inherent periodicity. The MATLAB function that performs this action is called fftshift. Using fftshift(fft(x)), you get the same frequency characteristics of signal, but this time each of these values correspond to an angular frequency between $-\pi$ and π.

Finally, let's convert angular frequency to "traditional" frequency. The maximum frequency contained in a signal sampled with f_s is $\frac{f_s}{2}$, as it is proved by Nyquist, so we can map the values between $(-\pi, \pi)$ to $(-\frac{f_s}{2}, \frac{f_s}{2})$ using a linear relationship, which means that frequency values corresponding to each value in the output of fftshift(fft(x)) are $-\frac{f_s}{2}:\frac{f_s}{N}:\frac{f_s}{2}-\frac{f_s}{N}$.

Conceptual Questions

1. Provide a brief summary of what has been instructed in problem description in Persian. The goal of this question is to check whether if you had understood the concept of fft.
2. Consider the concept of frequency resolution. It is the smallest step size that you can take in frequency domain. Show that frequency resolution is independent of sampling frequency and is conversely related to duration of data (T).

For each of the following signals, plot their spectrum (absolute value of fft) in decibels, followed by its phase in the same figure but different axis. Use meaningful labels for x and y axes of each axis such that your final figure looks like the output of MATLAB's `freqz` function. Then calculate the Fourier transform theoretically, and compare the theoretical result with the ones you plotted using fft. Normalize the spectrum graphs by their maximum value. No need to normalize phase graphs.

In example 4, manually reduce the phase of frequencies with small amplitude to 0 and display phase as a multiplier of π. You may use a code like this:

```
tol = 1e-6;
y(abs(y) < tol) = 0;

theta = angle(y);

plot(f, theta/pi)
xlabel('Frequency (Hz)')
ylabel('Phase / \pi')
```

Spectral Response

1. $x_1(t) = \cos(10\pi t)\,, t_{start} = -1, t_{end} = 1, fs = 50$
2. $x_2(t) = \Pi(t), t_{start} = -1, t_{end} = 1, fs = 50$
3. $x_3(t) = \mathrm{x}_1(\mathrm{t})\mathrm{x}_2(\mathrm{t}) = \cos(10\pi t)\,\Pi(t), t_{start} = 1, t_{end} = -1, f_s = 50$
4. $x_4(t) = \cos\left(30\pi t + \frac{\pi}{4}\right), t_{start} = 0, t_{end} = 1, fs = 100$
5. $x_5(t) = \sum_{k=-9}^{9} \Pi(t-2k),\ t_{start} = -19, t_{end} = 19, f_s = 50$[1]

[1] In the last example, you might encounter an impulse train in the spectrum. Report the distance between these impulses and validate that theoretically.

Now let's discuss about relationship between variations of signal in time domain and their spectral counterparts. Sharp transitions in time domain usually translates in high frequency features while an increase in slow transitions over time mean a rise low frequency spectrum.

What is the sharpest possible transition in time domain? Dirac's delta function. It is definitely the most discontinuity that you can see in a signal, so it's reasonable to assume that its spectrum should contain power in all of high-frequency band, even in $\pm\infty$. On the contrary, lowest variation would be no variation, or constant signal. Obviously, its Fourier transform consists of a single impulse at zero frequency.

Having this in mind, plot the spectrum graph as specified in previous problem and discuss your results.

Bridging Time domain to Frequency

1. $x_6(t) = \delta(t), t_{start} = -1, t_{end} = 1, f_S = 50$
2. $x_7(t) = 1, t_{start} = -1, t_{end} = 1, f_S = 50$

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Now that we have worked with MATLAB's fft, let's dive deeper into other methods to evaluate spectrum. First by means of MATLAB's symbolic toolbox, then by a writing a DFT code from scratch:

1. Using MATLAB's `syms`, `fourier` and `ezplot` evaluate and plot the spectrum of the following signal:

$$x_8(t) = \mathrm{e}^{-\frac{\mathrm{t}^2}{8}}$$

2. Evaluate the value of the following signal between 0 to 8 with $t_S = 0.01$, then evaluate the value of its spectrum at frequencies $-\frac{f_s}{2} : \frac{f_s}{N} : \frac{f_s}{2} - \frac{f_s}{N}$ using Fourier synthesis equation.

$$x_9(t) = \sin(100t)\, e^{-j\pi n}$$

   In general, this method is very inefficient comparing to FFT, but you can make it faster by performing array computations in MATLAB. Although this is not mandatory, but try to make your code as efficient as possible, using the possible number of loops in your code.

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General Notes about Homework!

- Ask your questions about this homework from me!
- Cheating is strongly prohibited in this course. You may cooperate with your fellow classmates but avoid duplicating each other's solutions.
- Refer to course's general rules for further details.