Problem set 1.

N 1.

$f(x) = 5x^2 + 3$

$\frac{d f(x)}{dx} = 2 \cdot 5x + 0 = 10x$

N 2.

$f(x) = 3e^{2x}$

$\frac{d f(x)}{dx} = 2 \cdot (3e^{2x}) = 6e^{2x}$

N 3.

Prove $\frac{d\sigma(x)}{dx} = \sigma(x)(1 - \sigma(x))$;

$\sigma(x) = \frac{1}{1+e^{-x}}$

Solution:

$\frac{d\sigma(x)}{dx} = \frac{d}{dx}\left[\frac{1}{1+e^{-x}}\right] = \frac{d}{dx}\left[(1+e^{-x})^{-1}\right] \Rightarrow$

$= -(-e^{-x})(1+e^{-x})^{-2} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{e^{-x}}{1+e^{-x}} \cdot \frac{1}{1+e^{-x}} =$

$= \left(\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right) \cdot \frac{1}{1+e^{-x}} =$

$= \left(1 - \frac{1}{1+e^{-x}}\right) \cdot \frac{1}{1+e^{-x}} \Rightarrow (1-\sigma(x))\,\sigma(x).$

That has been proved.

N4.

$f(x,y) = 3xy^2 + 2y$

Treating y as a const.

$\frac{d}{dx} f(x,y) = \frac{d}{dx}[3xy^2 + 2y] = \frac{d}{dx}(3xy^2) + \frac{d}{dx}(2y) =$

$= 3y^2 + 0 = 3y^2.$

N5.

$f(x,y) = 3xy^2 + 2y$; Treating x as a const.

$\frac{d}{dy} f(x,y) = \frac{d}{dy}[3xy^2 + 2y] = \frac{d}{dy}(3xy^2) + \frac{d}{dy}(2y) =$

$= 6xy + 2$

N6.

$y = (o-t)^2$; $o = w_1x_1 + w_2x_2 + w_3x_3$

$\frac{dy}{dx_3} = \frac{dy}{do} \cdot \frac{do}{dx_3} = \left[\frac{d}{do}(o-t)^2\right] \cdot \left[\frac{do}{dx_3}\right] =$

$= [2(o-t)] \cdot \left[\frac{do}{dx_3}\right] = 2(o-t) \cdot \left[\frac{d}{dx_3}(w_1x_1 + w_2x_2 + w_3x_3)\right] =$

$= 2(o-t) \cdot w_3 = 2w_3(o-t).$

N7.

$y = -c \ln o + (1-c)\ln(1-o)$; $o = \sigma(x)$

$\frac{dy}{dx} = \frac{dy}{do} \cdot \frac{do}{dx}$. Then,

$$\frac{dy}{dx} = \frac{d}{do}\left[-c\ln o + (1-c)\ln(1-o)\right] \cdot \frac{do}{dx} \Rightarrow$$

$$= \left[-c\,\frac{d(\ln o)}{do} + (1-c)\,\frac{d(\ln(1-o))}{do}\right] \cdot \frac{do}{dx} =$$

$$= \left[-\frac{c}{o} - \frac{1-c}{1-o}\right] \cdot \frac{do}{dx} = \left[-\frac{c}{\sigma(x)} - \frac{1-c}{1-\sigma(x)}\right] \cdot$$

$$\cdot\ \frac{d\,\sigma(x)}{dx} = \left(-\frac{c}{\sigma(x)} - \frac{1-c}{1-\sigma(x)}\right) \cdot \sigma'(x).$$

N8 - Matrix - Vector Product

$$[1 \quad 3] \cdot \begin{bmatrix} 3 & 5 \\ 6 & 2 \end{bmatrix} = [3+18 \quad 5+6] = [21 \quad 11]$$

N9 - Inner Product

$$\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 6+6 & 10+2 \\ 15+12 & 25+4 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 27 & 29 \end{bmatrix}$$

N10 - Transpose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

N11

$\{HH, HT, TH, TT\}$

First toss $P(A) = \frac{1}{2}$

Second toss $P(B) = \frac{1}{2}$

$(A \cap B) = A \cdot B = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

N12

$p(H) = \frac{1}{2}$, you get 1 USD

$p(T) = \frac{1}{2}$, you get 2 USD.

$E(x) = \sum_x p(x) \Rightarrow \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 2 = \frac{1}{2} + 1 = 1,5$

Answer: 1,5 USD.

N13 - Inner product

$u \cdot v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 1 \cdot 3 + 2 \cdot 4 = 11$

N14.

L2 norm:

$\|A\|_2 = \sqrt{(1)^2 + (2)^2} = \sqrt{5} \approx 2,236$

N15.

About 30 min.

Problem set 2.

Confusion matrix:

TP 2	FP 2
FN 2	TN 1

Accuracy: $\frac{TP + TN}{TP + TN + FP + FN} = \frac{2+1}{2+1+2+2} = \frac{3}{7} = 0,43$

Recall: $\frac{TP}{TP+FN} = \frac{2}{2+2} = \frac{1}{2} = 0,5$

Precision: $\frac{TP}{TP+FP} = \frac{2}{2+2} = 0,5$

F1-score: $\frac{2\,TP}{(2TP + FP + FN)} = \frac{4}{4+2+2} = \frac{1}{2} = 0,5$

Specificity: $\frac{TN}{FP+TN} = \frac{1}{2+1} = \frac{1}{3} = 0,33$