

# Lab 6: Basic Hypothesis Testing & Simple Regression

PSTAT 5A - Summer Session A 2025

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**Total Lab Time:** 50 minutes

Welcome to Lab 6! This lab focuses on two fundamental areas of statistical analysis that you'll use throughout your data science journey: **hypothesis testing** and **simple linear regression**. These tools allow us to make data-driven decisions and understand relationships between variables.



## What You'll Learn Today

By the end of this lab, you'll be able to:

- **Conduct hypothesis tests** to determine if sample data provides evidence against a claim
- **Model relationships** between variables using simple linear regression
- **Make predictions** based on data patterns
- **Interpret statistical results** in plain English for real-world applications

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## Getting Started

**Estimated time:** 5 minutes

### **i** Setup

Navigate to our class [Jupyterhub Instance](#). Create a new notebook and rename it “**lab6**” (for detailed instructions view [lab1](#)).

First, let's load our tools! Copy the below code to get started! We'll be using the following core libraries:

- **NumPy**: Fundamental package for fast array-based numerical computing.

- **Matplotlib** (pyplot): Primary library for creating static 2D plots and figures.
- **SciPy** (stats): Collection of scientific algorithms, including probability distributions and statistical tests.
- **Pandas**: High-performance data structures (DataFrame) and tools for data wrangling and analysis.
- **Statsmodels**: Econometric and statistical modeling for regression analysis, time series, and more.
- **Seaborn**: Seaborn is a Python data visualization library based on `matplotlib`. It provides a high-level interface for drawing attractive and informative statistical graphics.

```
# Install any missing packages (will skip those already installed)
#!%pip install --quiet numpy matplotlib scipy pandas statsmodels seaborn

# Load our tools (libraries)
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
import pandas as pd
import statsmodels.api as sm
import seaborn as sns

# Make our graphs look nice
plt.style.use('seaborn-v0_8-whitegrid')
sns.set_palette("husl")

# Set random seed for reproducible results
np.random.seed(42)

print(" All tools loaded successfully!")
```

---

## Task 1: One-Sample T-Test

**Estimated time:** 20 minutes

### What is a One-Sample T-Test?

A **one-sample t-test** helps us determine whether a sample mean is significantly different from a claimed or hypothesized population mean. It's one of the most common statistical tests you'll encounter.

**Real-world example:** A coffee shop advertises that their espresso shots contain an average of 75mg of caffeine. As a health-conscious consumer (or maybe a caffeine researcher!), you want to test this claim. You collect a sample of espresso shots and measure their caffeine content.

**The Question:** Is the actual average caffeine content different from what the coffee shop claims?

### Scenario

A coffee shop claims their average espresso shot contains **75 mg** of caffeine. You suspect it's actually **higher**. You test **20 shots** and want to test at  $\alpha = 0.05$  significance level.

**Your Goal:** Determine if there's sufficient evidence that the actual caffeine content exceeds the coffee shop's claim.

### Step 1: Explore the Data

```
# Generate caffeine data for our analysis
np.random.seed(123)
caffeine_data = np.random.normal(78, 8, 20) # Sample data: n=20 espresso shots

print(" Coffee Shop Caffeine Analysis")
```

```

print("=" * 40)
print(f" Sample size: {len(caffeine_data)}")
print(f" Sample mean: {np.mean(caffeine_data):.2f} mg")
print(f" Sample std dev: {np.std(caffeine_data, ddof=1):.2f} mg")
print(f" Coffee shop's claim: 75 mg")

# Let's look at our raw data
print(f"\n First 10 caffeine measurements:")
print([f"{x:.1f}" for x in caffeine_data[:10]])

```

## Step 2: Set Up Your Hypotheses

**Think about this carefully:** - What does the coffee shop claim? (This becomes your null hypothesis) - What do you suspect? (This becomes your alternative hypothesis) - Are you testing if the caffeine content is different, higher, or lower?

```

print(" STEP 1: Setting Up Hypotheses")
print("=" * 35)

# TODO: Complete these hypotheses
print("$H_0$ (Null Hypothesis): $\mu$ = ____ mg")          # What is the coffee shop's claim?
print("$H_1$ (Alternative Hypothesis): $\mu$ ____ mg")    # What do you suspect? (>, <, =)

# TODO: What type of test is this?
print("Test type: ____-tailed test")    # Right, left, or two-tailed?

print("\n Explanation:")
print("• $H_0$ represents the coffee shop's claim (status quo)")
print("• $H_1$ represents what we suspect is actually true")
print("• We use $\alpha$ = 0.05 as our significance level")

```

## Step 3: Calculate the Test Statistic

The t-statistic formula is:  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

```

print(" STEP 2: Calculating Test Statistic")
print("=" * 38)

# Calculate the components
sample_mean = np.mean(caffeine_data)
sample_std = np.std(caffeine_data, ddof=1) # ddof=1 for sample std dev
n = len(caffeine_data)
claimed_mean = 75

print(f"Sample mean ( $\bar{x}$ ): {sample_mean:.3f} mg")
print(f"Sample std dev (s): {sample_std:.3f} mg")
print(f"Sample size (n): {n}")
print(f"Claimed mean ( $\mu_0$ ): {claimed_mean} mg")

# TODO: Calculate the t-statistic using the formula above
# Hint: t = (sample_mean - claimed_mean) / (sample_std / sqrt(n))
t_statistic = (sample_mean - claimed_mean) / (sample_std / np.sqrt(n))

degrees_freedom = n - 1

print(f"\n Formula:  $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ ")
print(f" Calculation: t = ({sample_mean:.3f} - {claimed_mean}) / ({sample_std:.3f} /  $\sqrt{{n}}$ )")
print(f" t-statistic: {t_statistic:.3f}")
print(f" Degrees of freedom: {degrees_freedom}")

```

#### Step 4: Find the P-Value

For a **right-tailed test**, the p-value is the probability of getting a t-statistic as extreme or more extreme than what we observed.

##### What exactly is a p-value?

Loosely speaking, the p-value answers the question:

*“If the null hypothesis were true, how surprising would my sample be?”*

Formally, it is the probability, calculated under the assumption that the null hypothesis is correct; of obtaining

a test statistic **as extreme or more extreme** than the one observed.

- **Small p-value** (e.g.,  $< 0.05$ )  $\rightarrow$  data are rare under  $H$   $\rightarrow$  strong evidence *against*  $H$ .
- **Large p-value**  $\rightarrow$  data are plausible under  $H$   $\rightarrow$  little or no evidence against  $H$ .

**Important:** A p-value does **not** give the probability that the null hypothesis is true; it quantifies how incompatible your data are with  $H$ .

```
print(" STEP 3: Finding the P-Value")
print("=" * 32)

# TODO: Calculate p-value for right-tailed test
# Hint: For right-tailed test, p-value = 1 - stats.t.cdf(t_statistic, df)
p_value = 1 - stats.t.cdf(t_statistic, degrees_freedom)

print(f" P-value calculation:")
print(f"    P(t > {t_statistic:.3f}) = {p_value:.4f}")
print(f"\n Interpretation:")
print(f"    If the coffee shop's claim is true ( = 75),")
print(f"    there's a {p_value:.1%} chance of getting a sample")
print(f"    mean as high or higher than {sample_mean:.2f} mg")
```

## Step 5: Make Your Decision

Compare your p-value to  $\alpha = 0.05$  and make a statistical decision.

```
print(" STEP 4: Making the Decision")
print("=" * 31)

alpha = 0.05
print(f" Significance level ($\\alpha$): {alpha}")
print(f" P-value: {p_value:.4f}")
print(f" Decision rule: Reject $H_0$ if p-value < $\\alpha$")
```

```

print(f"\n Comparison:")
if p_value < alpha:
    print(f"    {p_value:.4f} < {alpha} ")
    print(f"    Decision: **REJECT $H_0$**")
    print(f"    Conclusion: There IS sufficient evidence that")
    print(f"                the average caffeine content > 75 mg")
    print(f"    The coffee shop's claim appears to be FALSE")
else:
    print(f"    {p_value:.4f} {alpha} ")
    print(f"    Decision: **FAIL TO REJECT $H_0$**")
    print(f"    Conclusion: There is NOT sufficient evidence that")
    print(f"                the average caffeine content > 75 mg")
    print(f"    We cannot conclude the coffee shop's claim is false")

# TODO: Write your conclusion in your own words
print(f"\n Your conclusion in plain English:")
print(f"    -----")
print(f"    -----")

```

## Step 6: Verify with Python

Let's double-check our work using Python's built-in statistical functions.

```

print(" VERIFICATION using scipy.stats")
print("=" * 35)

# Use scipy's one-sample t-test function
t_stat_scipy, p_val_scipy = stats.ttest_1samp(caffeine_data, 75, alternative='greater')

print(f" Your calculations:")
print(f"    t-statistic: {t_statistic:.3f}")
print(f"    p-value: {p_value:.4f}")

print(f"\n Python's calculations:")
print(f"    t-statistic: {t_stat_scipy:.3f}")
print(f"    p-value: {p_val_scipy:.4f}")

print(f"\n Match? {abs(t_statistic - t_stat_scipy) < 0.001 and abs(p_value - p_val_scipy) < 0.001}")

```



## Step 7: Visualize Your Results

```
# Create visualizations to understand our test
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(16, 6))

# Plot 1: Sample data histogram with means
ax1.hist(caffeine_data, bins=8, density=True, alpha=0.7, color='lightblue',
         edgecolor='black', label='Sample Data')
ax1.axvline(sample_mean, color='red', linestyle='-', linewidth=3,
            label=f'Sample Mean = {sample_mean:.1f}mg')
ax1.axvline(claimed_mean, color='orange', linestyle='--', linewidth=3,
            label=f'Claimed Mean = {claimed_mean}mg')
ax1.set_xlabel('Caffeine Content (mg)', fontsize=12)
ax1.set_ylabel('Density', fontsize=12)
ax1.set_title('Sample vs Claimed Caffeine Content', fontsize=14, fontweight='bold')
ax1.legend(fontsize=11)
ax1.grid(True, alpha=0.3)

# Plot 2: t-distribution with test statistic and p-value
x = np.linspace(-4, 4, 1000)
y = stats.t.pdf(x, degrees_freedom)
ax2.plot(x, y, 'b-', linewidth=2, label=f't-distribution (df={degrees_freedom})')
ax2.fill_between(x, y, alpha=0.3, color='lightblue')

# Shade the rejection region (right tail)
x_reject = x[x >= t_statistic]
y_reject = stats.t.pdf(x_reject, degrees_freedom)
ax2.fill_between(x_reject, y_reject, alpha=0.7, color='red',
                label=f'p-value = {p_value:.4f}')

ax2.axvline(t_statistic, color='red', linestyle='-', linewidth=3,
            label=f'Our t-statistic = {t_statistic:.3f}')
ax2.set_xlabel('t-value', fontsize=12)
ax2.set_ylabel('Density', fontsize=12)
ax2.set_title('T-Distribution with Test Statistic', fontsize=14, fontweight='bold')
ax2.legend(fontsize=11)
ax2.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```

## Reflection Questions

Answer these questions to check your understanding:

1. **Hypotheses:** What were your null and alternative hypotheses? Why did you choose a right-tailed test?
  2. **Test Choice:** Why did you use a t-test instead of a z-test for this problem?
  3. **Results:** What was your t-statistic and p-value? What do these numbers mean?
  4. **Decision:** What was your final conclusion at  $\alpha = 0.05$ ? Do you reject or fail to reject the null hypothesis?
  5. **Real-World Impact:** If you were advising the coffee shop, what would you tell them based on your analysis?
- 

## Task 2: Simple Linear Regression

Estimated time: 25 minutes

### What is Simple Linear Regression?

Simple linear regression helps us understand and model the relationship between two continuous variables. Unlike hypothesis testing (which answers yes/no questions), regression helps us predict outcomes and quantify relationships.

**Real-world example:** As a student, you've probably wondered: "If I study more hours, how much will my exam score improve?" Linear regression can help answer this question by finding the relationship between study time and exam performance.

**The Question:** Can we predict exam scores based on hours studied? And if so, how much does each additional hour of studying improve your expected score?

💡 At a glance — what you'll do

1. Explore & visualize the data
2. Measure correlation ( $r$ ) and  $R^2$
3. Fit the regression line  $\hat{y} = \beta_0 + \beta_1 x$
4. Test if the slope is significant
5. Predict new values & quantify error
6. Check model assumptions
7. Visualize diagnostics
8. Write a plain-English conclusion

i Key Concepts:

- **Correlation:** How strongly two variables move together (-1 to +1)
- **Slope:** How much  $Y$  changes when  $X$  increases by 1 unit
- **Intercept:** The predicted value of  $Y$  when  $X = 0$
- $R^2$ : What percentage of the variation in  $Y$  is explained by  $X$

! Important

**Remember:** Correlation does not imply causation! Just because two variables are related doesn't mean one causes the other.

## Scenario

You want to investigate the relationship between **study hours** and **exam performance**. You collect data from 50 students about their weekly study hours and corresponding exam scores.

**Your Goal:** Create a statistical model to predict exam scores based on study hours and determine how much each additional hour of studying helps.

## Step 1: Explore the Data

```
# Generate realistic study data
np.random.seed(101)
n_students = 50

# Study hours (predictor variable X)
study_hours = np.random.uniform(1, 20, n_students)

# Exam scores with linear relationship plus noise
# True relationship: score = 65 + 2*hours + noise
true_intercept = 65
true_slope = 2
noise = np.random.normal(0, 8, n_students)
exam_scores = true_intercept + true_slope * study_hours + noise

# Create DataFrame for easier handling
study_data = pd.DataFrame({
    'hours_studied': study_hours,
    'exam_score': exam_scores
})

print(" Study Hours vs Exam Scores Analysis")
print("=" * 45)
print(f" Sample size: {len(study_data)} students")
print(f" Study hours range: {study_hours.min():.1f} to {study_hours.max():.1f} hours")
print(f" Exam scores range: {exam_scores.min():.1f} to {exam_scores.max():.1f} points")

print(f"\n First 10 students:")
print(study_data.head(10).round(2))
```

### Quick Questions:

- Do you see any obvious pattern in the data?
- Which variable is the predictor (X) and which is the response (Y)?

## Step 2: Calculate and Interpret Correlation

Correlation measures how strongly two variables move together.

```
print(" STEP 1: Measuring the Relationship")
print("=" * 40)

# TODO: Calculate the correlation coefficient
# Hint: Use np.corrcoef(x, y)[0, 1] to get correlation between x and y
correlation = _____

print(f" Correlation coefficient: r = {correlation:.3f}")

# TODO: Interpret the correlation strength
print(f"\n Interpretation:")
if abs(correlation) < 0.3:
    strength = "weak"
elif abs(correlation) < 0.7:
    strength = "moderate"
else:
    strength = "strong"

direction = "positive" if correlation > 0 else "negative"
print(f" This indicates a {strength} {direction} relationship")
print(f" between study hours and exam scores.")

print(f"\n What this means:")
print(f" r = {correlation:.3f} means the variables are strongly related")
print(f" As study hours increase, exam scores tend to increase")
print(f" About {correlation**2:.1%} of the variation in scores")
print(f" can be explained by study hours alone")
```

### Check Your Understanding:

- What does  $r = 0.8$  vs  $r = 0.3$  tell you?
- If  $r = -0.9$ , what would that mean?

### Step 3: Fit the Linear Regression Model

Now we'll find the "line of best fit" through our data points.

```
print(" STEP 2: Fitting the Regression Line")
print("=" * 42)

# Set up the regression (add constant for intercept)
X = sm.add_constant(study_hours) # Add intercept term

# TODO: Fit the OLS (Ordinary Least Squares) model
# Hint: Use sm.OLS(y_variable, X_variable).fit()
model = -----

print(f" Regression Equation:")
print(f" Exam Score = $\beta_0$ + $\beta_1$ × Hours Studied")
print(f" Exam Score = {model.params[0]:.2f} + {model.params[1]:.2f} × Hours")

print(f"\n Model Coefficients:")
print(f" Intercept ($\beta_0$): {model.params[0]:.3f}")
print(f" Slope ($\beta_1$): {model.params[1]:.3f}")
print(f" R-squared ($R^2$): {model.rsquared:.3f}")

# TODO: Complete these interpretations
print(f"\n What These Numbers Mean:")
print(f" Intercept ({model.params[0]:.1f}): Expected score with 0 hours of study")
print(f" Slope ({model.params[1]:.2f}): Each additional hour increases score by {model.params[1]:.2f}")
print(f" $R^2$ ({model.rsquared:.3f}): Study hours explain {model.rsquared:.1%} of score variance")
```

### Step 4: Test Statistical Significance

Is the relationship we found statistically significant, or could it be due to chance?

```
print(" STEP 3: Testing Statistical Significance")
print("=" * 46)

# Check if the slope is significantly different from zero
slope_pvalue = model.pvalues[1] # p-value for the slope
alpha = 0.05
```

```

print(f" Hypothesis Test for Slope:")
print(f"   $H_0$:  $\beta_1 = 0$  (no relationship)")
print(f"   $H_1$:  $\beta_1 \neq 0$  (there is a relationship)")
print(f"    $\alpha = \{\alpha\}$ ")

print(f"\n Test Results:")
print(f"   Slope p-value: {slope_pvalue:.6f}")

# TODO: Make the decision
if slope_pvalue < alpha:
    print(f"   Decision: REJECT $H_0$")
    print(f"   Conclusion: The relationship IS statistically significant")
    significance = "IS"
else:
    print(f"   Decision: FAIL TO REJECT $H_0$")
    print(f"   Conclusion: The relationship is NOT statistically significant")
    significance = "IS NOT"

print(f"\n Bottom Line:")
print(f"   Study hours {significance} a significant predictor of exam scores")

# Show confidence intervals
conf_int = model.conf_int(alpha=0.05)
print(f"\n 95% Confidence Intervals:")
print(f"   Intercept: [{conf_int[0,0]:.2f}, {conf_int[0,1]:.2f}]")
print(f"   Slope: [{conf_int[1,0]:.2f}, {conf_int[1,1]:.2f}]")

```

## Step 5: Make Predictions

Now let's use our model to predict exam scores for different study scenarios.

```

print(" STEP 4: Making Predictions")
print("=" * 32)

# TODO: Calculate predictions for different study hours
# Hint: prediction = intercept + slope * hours
example_hours = [5, 10, 15, 20]

```

```

print(f" Prediction Examples:")
for hours in example_hours:
    # TODO: Calculate predicted score
    pred_score = model.params[0] + model.params[1] * hours
    print(f"      {hours:2d} hours → Predicted score: {pred_score:.1f} points")

print(f"\n Your Turn:")
# TODO: Pick your own study hours and make a prediction
your_hours = _____ # Enter a number between 1-20
your_prediction = model.params[0] + model.params[1] * your_hours
print(f"      {your_hours} hours → Predicted score: {your_prediction:.1f} points")

# Calculate residuals for analysis
y_predicted = model.predict(X)
residuals = exam_scores - y_predicted
residual_std = np.std(residuals, ddof=2)

print(f"\n Prediction Accuracy:")
print(f"      Average prediction error: ±{residual_std:.1f} points")
print(f"      This means most predictions are within ±{residual_std:.1f} points of actual scores")

```

## Step 6: Check Model Assumptions

Before trusting our model, we need to verify it meets the assumptions of linear regression.

```

print(" STEP 5: Checking Model Assumptions")
print("=" * 42)

print(" Linear Regression Assumptions:")
print("   1 Linear relationship between X and Y")
print("   2 Residuals are normally distributed")
print("   3 Residuals have constant variance (homoscedasticity)")
print("   4 Residuals are independent")

# Calculate residuals
y_predicted = model.predict(X)
residuals = exam_scores - y_predicted

```



```

print(f"\n Residual Analysis:")
print(f"    Mean residual: {np.mean(residuals):.6f} (should be 0)")
print(f"    Std of residuals: {np.std(residuals, ddof=2):.3f}")

# TODO: Check normality of residuals using Shapiro-Wilk test
from scipy.stats import shapiro
shapiro_stat, shapiro_p = shapiro(residuals)
print(f"\n Normality Test (Shapiro-Wilk):")
print(f"    p-value: {shapiro_p:.4f}")
if shapiro_p > 0.05:
    print("    Residuals appear normally distributed")
else:
    print("    Residuals may not be normally distributed")

```

## Step 7: Visualize Your Results

```

# Create comprehensive visualization
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(16, 12))

# Plot 1: Scatter plot with regression line
ax1.scatter(study_hours, exam_scores, alpha=0.6, color='blue', s=60,
            label='Student Data')
sorted_hours = np.sort(study_hours)
sorted_predictions = model.params[0] + model.params[1] * sorted_hours
ax1.plot(sorted_hours, sorted_predictions, color='red', linewidth=3,
        label=f'y = {model.params[0]:.1f} + {model.params[1]:.2f}x')

ax1.set_xlabel('Study Hours', fontsize=12)
ax1.set_ylabel('Exam Score', fontsize=12)
ax1.set_title(f' Study Hours vs Exam Scores\n $R^2 = {model.rsquared:.3f}$ ',
            fontsize=14, fontweight='bold')
ax1.legend(fontsize=11)
ax1.grid(True, alpha=0.3)

# Plot 2: Residuals vs Fitted values
ax2.scatter(y_predicted, residuals, alpha=0.6, color='purple', s=50)
ax2.axhline(y=0, color='red', linestyle='--', linewidth=2)
ax2.set_xlabel('Fitted Values', fontsize=12)

```

```

ax2.set_ylabel('Residuals', fontsize=12)
ax2.set_title(' Residuals vs Fitted\n(Should show no pattern)',
              fontsize=14, fontweight='bold')
ax2.grid(True, alpha=0.3)

# Plot 3: Q-Q plot for normality of residuals
stats.probplot(residuals, dist="norm", plot=ax3)
ax3.set_title(' Q-Q Plot of Residuals\n(Should be roughly linear)',
              fontsize=14, fontweight='bold')
ax3.grid(True, alpha=0.3)

# Plot 4: Histogram of residuals
ax4.hist(residuals, bins=12, density=True, alpha=0.7, color='lightgreen',
         edgecolor='black')
ax4.set_xlabel('Residuals', fontsize=12)
ax4.set_ylabel('Density', fontsize=12)
ax4.set_title(' Distribution of Residuals\n(Should look normal)',
              fontsize=14, fontweight='bold')
ax4.grid(True, alpha=0.3)

# Overlay normal curve
x_norm = np.linspace(residuals.min(), residuals.max(), 100)
y_norm = stats.norm.pdf(x_norm, np.mean(residuals), np.std(residuals))
ax4.plot(x_norm, y_norm, 'r-', linewidth=2, label='Normal curve')
ax4.legend()

plt.tight_layout()
plt.show()

```

## Step 8: Interpret Your Model

```

print(" FINAL INTERPRETATION")
print("=" * 25)

print(f" Our Model: Exam Score = {model.params[0]:.1f} + {model.params[1]:.2f} × Study Hours")
print(f"\n Key Findings:")
print(f"     Strong positive relationship (r = {correlation:.3f})")
print(f"     Study hours explain {model.rsquared:.1%} of score variation")

```

```

print(f"    Each extra hour → {model.params[1]:.1f} point increase")
print(f"    Relationship is statistically significant (p < 0.001)")

print(f"\n Practical Insights:")
print(f"    Going from 5 to 10 hours of study:")
pred_5 = model.params[0] + model.params[1] * 5
pred_10 = model.params[0] + model.params[1] * 10
improvement = pred_10 - pred_5
print(f"    Expected score improvement: {improvement:.1f} points")

print(f"\n Important Limitations:")
print(f"    • Correlation Causation")
print(f"    • Model only explains {model.rsquared:.1%} of variation")
print(f"    • Other factors matter too (sleep, prior knowledge, etc.)")
print(f"    • Predictions have uncertainty: ±{residual_std:.1f} points")

# Show full model summary
print(f"\n Full Statistical Summary:")
print("=" * 30)
print(model.summary())

```

## Reflection Questions

Test your understanding by answering these questions:

### 1. Correlation vs Causation:

- What was your correlation coefficient?
- Does this prove that studying more **causes** higher exam scores? Why or why not?

### 2. Model Interpretation:

- What does the slope coefficient mean in practical terms?
- What does the intercept represent, and does it make sense?

### 3. Prediction Quality:

- What percentage of exam score variation is explained by study hours?
- How accurate are your predictions (what's the typical error)?

4. **Statistical Significance:**

- Is the relationship statistically significant?
- What would it mean if the p-value for the slope was 0.20?

5. **Assumptions:**

- Based on your diagnostic plots, are the regression assumptions satisfied?
- What would you do if the assumptions were violated?

1. **Practical Application:**

- If you were advising a student, what would you tell them based on this analysis?
- What other variables might improve your prediction model?

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**Congratulations!** You've successfully completed Lab 6 !  
and learned fundamental statistical analysis techniques: