PSTAT 5A Practice Worksheet 4

Comprehensive Review: Discrete Random Variables and Distributions

Student Name: ____________________________________

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Table of contents

1 Instructions and Overview

Time Allocation:

- **Quiz Review** : 15 minutes
- **Section A (Warm-up):** 15 minutes
- **Section B (Intermediate):** 20 minutes
- **Optional Question:** Do on your own
- **Total:** 50 minutes

Important Instructions:

- Use the formulas provided for guidance
- Round final answers to 4 decimal places unless otherwise specified
- Identify the distribution type before calculating
- Show your work for expected value and variance calculations
- Use calculator as needed for factorials and combinations

Key Formulas Reference:

General Random Variable Properties:

- **Expected Value:** $E[X] = \sum_k k \cdot P(X = k)$
- **Variance:** $\text{Var}(X) = E[X^2] - (E[X])^2 = \sum_k k^2 \cdot P(X = k) - \mu^2$

- **Standard Deviation:** $\sigma = \sqrt{\text{Var}(X)}$

Discrete Distributions:

Bernoulli Distribution: $X \sim \text{Bernoulli}(p)$

- **PMF:** $P(X = k) = p^k(1-p)^{1-k}$ for $k \in \{0, 1\}$
- **Mean:** $E[X] = p$
- **Variance:** $\text{Var}(X) = p(1-p)$

Binomial Distribution: $X \sim \text{Binomial}(n, p)$

- **PMF:** $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$ for $k = 0, 1, 2, ..., n$
- **Mean:** $E[X] = np$
- **Variance:** $\text{Var}(X) = np(1-p)$

Geometric Distribution: $X \sim \text{Geometric}(p)$

- **PMF:** $P(X = k) = (1-p)^{k-1}p$ for $k = 1, 2, 3, ...$
- **Mean:** $E[X] = \frac{1}{p}$
- **Variance:** $\text{Var}(X) = \frac{1-p}{p^2}$

Poisson Distribution: $X \sim \text{Poisson}(\lambda)$

- **PMF:** $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ for $k = 0, 1, 2, ...$
- **Mean:** $E[X] = \lambda$
- **Variance:** $\text{Var}(X) = \lambda$

2 Section A: Basic Concepts and Identification

Estimated time: 15 minutes

Problem A1: Distribution Identification

For each scenario below, identify the appropriate discrete distribution and state the parameter(s). **Do not calculate probabilities yet.**

(a) A fair coin is flipped until the first head appears. Let X = number of flips needed.

(b) A quality control inspector tests 20 randomly selected items from a production line where 5% are defective. Let X = number of defective items found.

(c) A website receives visitors at an average rate of 3 per minute. Let X = number of visitors in a 2-minute period.

(d) A basketball player shoots one free throw with an 80% success rate. Let X = 1 if successful, 0 if unsuccessful.

(e) A student keeps taking a driving test until they pass. The probability of passing on any attempt is 0.7. Let X = number of attempts needed to pass.

Work Space:

Problem A2: Probability Mass Function

The random variable X has the following probability distribution:

X	1	2	3	4	5
P(X=k)	0.1	0.3	0.4	a	0.1

(a) Find the value of a.

(b) Calculate $P(X \leq 3)$.

(c) Calculate $P(X > 2)$.

Work Space:

3 Section B: Expected Value and Variance

Estimated time: 20 minutes

Problem B1: Manual Calculations

Using the probability distribution from Problem A2, calculate:

(a) The expected value $E[X]$

(b) The variance $\text{Var}(X)$

(c) The standard deviation σ

> Tip
>
> **Calculation Strategy:**
> For expected value: $E[X] = \sum k \cdot P(X = k)$
> For variance: First find $E[X^2] = \sum k^2 \cdot P(X = k)$, then use $\text{Var}(X) = E[X^2] - (E[X])^2$
> Show your work step by step!

Work Space:

Problem B2: Bernoulli and Binomial Applications

A manufacturing process produces items that are defective with probability 0.15.

(a) If you select one item randomly, what is the expected value and variance of X = number of defective items?

(b) If you select 25 items randomly, what is the expected number of defective items and the standard deviation?

> Tip
>
> Part (a) is a Bernoulli distribution. Part (b) is a Binomial distribution. Use the formulas from the reference box!

Work Space:

4 Optional Questions

Optional Problem : Conceptual Understanding

(a) Explain the key difference between a Binomial distribution and a Geometric distribution in terms of what they count.

(b) When would you use a Poisson distribution instead of a Binomial distribution?

(c) If $X \sim \text{Binomial}(n, p)$, under what conditions would the variance be maximized?

Work Space: