PSTAT 5A - Quiz 1

Detailed Solutions

Q1: *Mean of* $\{2, 4, 6, 8, 10\}$.

$$\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6.$$

Answer: $\boxed{6}$

Q2: *Measure of central tendency most affected by outliers.*
The arithmetic mean shifts the most when extreme values are added. **Answer: Mean**

Q3: *Median of* $\{1, 3, 3, 5, 7, 8, 12\}$.
For 7 ordered values, the median is the 4^{th}: 5. **Answer:** $\boxed{5}$

Q4: *Sign of the standard deviation.*
Standard deviation is the square root of a variance and is therefore ≥ 0. It equals 0 only for constant data and is positive otherwise. **Answer: Always non-negative (positive)**

Q5: *Standard deviation given variance* 16.

$$\sigma = \sqrt{16} = 4.$$

Answer: $\boxed{4}$

Q6: *Fill-in-the-blanks.*

(a) The **mode** is the value that appears most frequently.
Answer: mode

(b) The difference between the maximum and minimum is the **range**.
Answer: range

(c) The **50**$^{\text{th}}$ percentile equals the **median**.
Answer: median

Q7: $P(A) = 0.3,\ P(B) = 0.4$ are mutually exclusive.

$$P(A \cup B) = P(A) + P(B) = 0.7.$$

Answer: $\boxed{0.7}$

Q8: Valid probabilities satisfy $0 \leq P(E) \leq 1$. **Answer: Between 0 and 1**

Q9: Even outcomes on a fair die: $\{2, 4, 6\}$.

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2}.$$

Answer: $\boxed{\frac{1}{2}}$

Q10: Complement of 0.25: $1 - 0.25 = 0.75$. **Answer:** $\boxed{0.75}$

Q11: $P(A) = 0.6,\ P(B) = 0.5,\ P(A \cup B) = 0.8$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.3.$$

Answer: $\boxed{0.3}$

Q12: Bag totals: $5 + 3 + 2 = 10$.

$$P(\text{blue}) = \frac{3}{10} = 0.300.$$

Answer: $\boxed{0.300}$

Q13: The notation $P(A \mid B)$ means "probability of A given B." **Answer: "probability of A given B"**

Q14: $P(A) = 0.4,\ P(B) = 0.3,\ P(A \cap B) = 0.12$.

$$P(A \mid B) = \frac{0.12}{0.30} = 0.4.$$

Answer: $\boxed{0.4}$

Q15: $P(\text{King} \mid \text{face})$. There are 4 kings in the 12 face cards:

$$P = \frac{4}{12} = \frac{1}{3}.$$

Answer: $\boxed{\frac{4}{12}}$

Q16: If $P(A \mid B) = P(A)$, events A and B are **independent**. **Answer: independent**

Q17: *True/False.* Two independent events cannot be mutually exclusive unless one has probability 0. **Answer: True**

Q18: *Select all that are true for independence.*

$$P(A \cap B) = P(A)P(B), \qquad P(A \mid B) = P(A).$$

Answer: $\boxed{P(A \cap B) = P(A)P(B)}$

Q19: For independent A, B: $P(A \cap B) = 0.4 \times 0.6 = 0.24$. **Answer:** $\boxed{0.24}$

Q20: *Mutually exclusive but not independent.*
"Drawing a heart" vs. "drawing a spade" in one card draw: $P(\text{heart} \cap \text{spade}) = 0$.
We draw one card at random from a standard deck of 52.

$$\Omega = \{\text{all 52 distinct cards}\}, \qquad \Pr(\{\text{each card}\}) = \frac{1}{52}.$$

The two events are :

$$H = \{\text{the card is a Heart}\}, \qquad S = \{\text{the card is a Spade}\}.$$

Each suit has 13 cards:

$$\Pr(H) = \frac{13}{52} = \frac{1}{4}, \qquad \Pr(S) = \frac{13}{52} = \frac{1}{4}.$$

Hearts and spades share no common card, so

$$H \cap S = \varnothing \quad \Longrightarrow \quad \Pr(H \cap S) = 0.$$

Events H and S would be independent iff

$$\Pr(H \cap S) = \Pr(H)\ \Pr(S).$$

Checking the independence criterion gives

$$\Pr(H)\ \Pr(S) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = 0.0625 \quad \neq \quad \Pr(H \cap S) = 0.$$

Since the equality fails, H and S are *not* independent.

$$\boxed{\text{“Drawing a heart” and “drawing a spade” are mutually exclusive but not independent.}}$$

Answer: Drawing a heart and drawing a spade.

Q21: A factory produces widgets with a 5% defect rate. You randomly select three widgets, each selection independent.
Let

$$p = \Pr(\text{defective}) = 0.05, \qquad q = 1 - p = \Pr(\text{non-defective}) = 0.95.$$

a) *Probability all three widgets are non-defective.*

$$\Pr(\text{all 3 good}) \;=\; q^3 \;=\; 0.95^3 \;=\; 0.857375 \;\approx\; 0.857.$$

Answer: $\boxed{0.857}$ (to three decimals)

Why? For each widget the chance of being good is q. Independence $\Longrightarrow$ multiply the three identical factors.

b) *Probability that at least one widget is defective.*
"At least one defective" means the complement of "all three good," so

$$\Pr(\text{at least 1 defective}) \;=\; 1 - \Pr(\text{all 3 good}) \;=\; 1 - q^3 \;=\; 1 - 0.857375 \;=\; 0.142625 \;\approx\; 0.143.$$

Answer: $\boxed{0.143}$