PSTAT 5A Practice Worksheet 3

Comprehensive Review: Probability, Counting, an Conditional Probability

Student Name: _____

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1 Instructions and Overview

Time Allocation:

- **Section A (Warm-up):** 8 minutes
- **Section B (Intermediate):** 15 minutes
- **Section C (Advanced):** 12 minutes
- **Section D (Review):** 15 minutes
- **Total:** 50 minutes

Important Instructions:

- Use the formulas provided for guidance
- Round final answers to 4 decimal places unless otherwise specified
- Identify your approach before calculating
- Use calculator as needed

Key Formulas Reference:

Basic Probability:

- **Conditional Probability:** $P(A|B) = \frac{P(A \cap B)}{P(B)}$

- **Law of Total Probability:** $P(A) = \sum P(A|B_i) \cdot P(B_i)$
- **Addition Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Multiplication Rule:** $P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$

Counting:

- **Multiplication Rule:** If a procedure consists of k steps, with n_1 ways for step 1, n_2 for step 2, ..., n_k for step k, then total ways: $n_1 \times n_2 \times \cdots \times n_k$
- **Factorial:** $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$
- **Permutations:** $P(n, r) = \frac{n!}{(n-r)!}$
- **Combinations:** $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

2 Section A: Probability

Estimated time: 8 minutes

Problem A1: Probability Distributions

Each row in the table below is a proposed grade distribution for a class. Identify each as a valid or invalid probability distribution, and explain your reasoning.

Class	A	B	C	D	F
(a)	0.3	0.3	0.3	0.2	0.1
(b)	0	0	1	0	0
(c)	0.3	0.3	0.3	0	0
(d)	0.3	0.5	0.2	0.1	-0.1
(e)	0.2	0.4	0.2	0.1	0.1
(f)	0	-0.1	1.1	0	0

Work Space:

3 Section B: Permutations and Combination

Estimated time: 15 minutes

Problem B1: Permutations and Combinations

A cybersecurity team needs to create a secure access protocol.

Part (a): How many 6-character passwords can be formed using 3 specific letters and 3 specific digits if repetitions are not allowed and letters must come before digits?

> Tip
>
> Since letters must come before digits, think of this as two separate arrangement problems:
> - First, arrange the 3 letters in the first 3 positions

- Then, arrange the 3 digits in the last 3 positions
- Use the multiplication principle to combine these results

Part (b): If the team wants to select 4 people from 12 employees to form a security committee where order doesn't matter, how many ways can this be done?

Tip

Since order doesn't matter, this is a combination problem. Ask yourself:
- Are we arranging people in specific positions, or just selecting a group?
- Which formula should you use: $P(n, r)$ or $C(n, r)$?

Work Space:

4 Section C: Conditional Probability

Estimated time: 12 minutes

Problem C1: Drawing Cards (Without Replacement)

You draw two cards, one after the other, from a standard 52-card deck without putting the first card back. Let

A = {"first card is a heart"},

B = {"second card is an ace"}.

1. P(A)
2. $P(A \text{ and } B)$
3. $P(B \mid A)$
4. P(B)
5. Compare your answers in (3) vs. (4). Why are they different (or the same)? What does this tell you about drawing cards without replacement?

Work Space:

5 Section D: Conditional Probability

Estimated time: 15 minutes

Problem D1: Advanced Counting with Restrictions

A restaurant offers a prix fixe menu where customers must choose:

- 1 appetizer from 6 options
- 1 main course from 8 options
- 1 dessert from 5 options

However, there are restrictions:

- If you choose the seafood appetizer, you cannot choose the vegetarian main course
- If you choose the chocolate dessert, you must choose either the beef or chicken main course (3 of the 8 main courses)

Part (a): How many valid meal combinations are possible?

Part (b): If customers choose randomly among valid combinations, what is the probability someone chooses the chocolate dessert?

Work Space: