PSTAT 5A Practice Worksheet 2 Solutions

Descriptive Statistics

Instructor Solutions

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Table of contents

# 1. Section A: Basic Descriptive Statistics

## 1.1 Problem A1: Mean and Standard Deviation

**Given:**

* 24 students: average = 74, standard deviation = 8.9
* 1 makeup student: score = 64

### 1.1.1 Part (a): Does the new student’s score increase or decrease the average score?

**Solution:** DECREASE

Since 64 < 74 (the current average), adding this score will pull the average down.

### 1.1.2 Part (b): What is the new average?

**Solution:**

New average = (Sum of all 25 scores) / 25  
Sum of first 24 scores = 24 × 74 = 1,776  
Total sum = 1,776 + 64 = 1,840  
New average = 1,840 / 25 = 73.6 points

### 1.1.3 Part (c): Does the new student’s score increase or decrease the standard deviation?

**Solution:** INCREASE

The score of 64 is more than one standard deviation below the original mean (74 - 8.9 = 65.1). This adds more variability to the dataset, increasing the standard deviation.

## 1.2 Problem A2: Distribution Shape Analysis

**Given:** - TV watching hours per week - Mean = 4.71 hours - Standard deviation = 4.18 hours

**Question:** Is the distribution symmetric? What shape? Explain reasoning.

**Solution:** NOT SYMMETRIC - RIGHT-SKEWED

**Reasoning:** 1. The standard deviation (4.18) is nearly as large as the mean (4.71)

1. Since hours cannot be negative, there’s a natural lower bound at 0
2. Some students likely watch much more TV than others, creating a long right tail
3. The large standard deviation relative to the mean suggests high variability
4. In a right-skewed distribution, a few high values (heavy TV watchers) pull the mean higher

# 2. Section B: Data Interpretation and Graphical Analysis

## 2.1 Problem B1: Interpreting Histograms

**Context:** Infant mortality histogram shows right-skewed distribution with:

* Highest bar at 0-10 range (about 38% of countries)
* Decreasing bars: 10-20 (23%), 20-30 (11%)
* Long right tail with few countries having high rates

### 2.1.1 Part (a): Estimate Q1, the median, and Q3 from the histogram.

**Solution:** Looking at cumulative percentages:

* **Q1 (25th percentile)** ≈ 8 deaths per 1,000 live births
* **Median (50th percentile)** ≈ 15 deaths per 1,000 live births
* **Q3 (75th percentile)** ≈ 35 deaths per 1,000 live births

### 2.1.2 Part (b): Would you expect the mean to be smaller or larger than the median? Explain.

**Solution:** MEAN > MEDIAN

**Reasoning:** - The distribution is right-skewed

* The long right tail contains countries with very high infant mortality rates
* These extreme values pull the mean higher than the median
* In right-skewed distributions, the mean is always greater than the median
* The median is resistant to outliers, but the mean is affected by them

## 2.2 Problem B2: Comparing Distributions

**Based on the plots showing Gain vs No Gain counties:**

### 2.2.1 Center:

* **Gain group** has higher median household income (~$55,000)
* **No Gain group** has lower median household income (~$45,000)

### 2.2.2 Variability:

* **Gain group** shows less variability (tighter distribution)
* **No Gain group** shows greater variability (wider spread)

### 2.2.3 Shape:

* Both groups are right-skewed
* Shape is relatively consistent between groups
* Both have longer right tails

### 2.2.4 Modes:

* Each group has one prominent mode
* **Gain group:** mode around $50,000-$55,000
* **No Gain group:** mode around $40,000-$45,000

# 3. Section C: Variance Calculations Practice

## 3.1 Problem C1: Basic Variance Calculations

**Data:** 3, 7, 2, 8, 5, 6, 4, 9

### 3.1.1 Part (a): Calculate the sample mean x̄.

**Solution:**

x̄ = (3 + 7 + 2 + 8 + 5 + 6 + 4 + 9) / 8  
x̄ = 44 / 8 = 5.5

### 3.1.2 Part (b): Calculate the sample variance s² using (n-1).

**Solution:**

s² = Σ(xi - x̄)² / (n-1)  
  
Deviations from mean:  
(3-5.5)² = (-2.5)² = 6.25  
(7-5.5)² = (1.5)² = 2.25   
(2-5.5)² = (-3.5)² = 12.25  
(8-5.5)² = (2.5)² = 6.25  
(5-5.5)² = (-0.5)² = 0.25  
(6-5.5)² = (0.5)² = 0.25  
(4-5.5)² = (-1.5)² = 2.25  
(9-5.5)² = (3.5)² = 12.25  
  
Sum = 6.25 + 2.25 + 12.25 + 6.25 + 0.25 + 0.25 + 2.25 + 12.25 = 42  
  
s² = 42 / (8-1) = 42 / 7 = 6.0

### 3.1.3 Part (c): Calculate the sample standard deviation s.

**Solution:**

s = √s² = √6 = 2.4495

### 3.1.4 Part (d): Population variance σ² if treated as complete population.

**Solution:**

σ² = Σ(xi - μ)² / N  
σ² = 42 / 8 = 5.25

### 3.1.5 Part (e): Why divide by (n-1) for sample variance instead of n?

**Solution:** We use (n-1) because of degrees of freedom. When we use the sample mean x̄ to calculate deviations, we “use up” one degree of freedom. The sample mean constrains the data - if we know (n-1) deviations and the sample mean, the last deviation is determined. This makes s² an unbiased estimator of the population variance σ².

## 3.2 Problem C2: Comparing Variability

**Data Sets:** - Set A: 10, 12, 14, 16, 18 - Set B: 5, 10, 14, 18, 23

### 3.2.1 Part (a): Calculate the mean for each set.

**Solution:**

Set A: x̄\_A = (10 + 12 + 14 + 16 + 18) / 5 = 70 / 5 = 14  
Set B: x̄\_B = (5 + 10 + 14 + 18 + 23) / 5 = 70 / 5 = 14

### 3.2.2 Part (b): Calculate the sample variance for each set.

**Solution:**

**Set A:**

Deviations: (10-14)²=16, (12-14)²=4, (14-14)²=0, (16-14)²=4, (18-14)²=16  
Sum = 16 + 4 + 0 + 4 + 16 = 40  
s²\_A = 40 / (5-1) = 40 / 4 = 10

**Set B:**

Deviations: (5-14)²=81, (10-14)²=16, (14-14)²=0, (18-14)²=16, (23-14)²=81   
Sum = 81 + 16 + 0 + 16 + 81 = 194  
s²\_B = 194 / (5-1) = 194 / 4 = 48.5

### 3.2.3 Part (c): Which set has greater variability?

**Solution:** SET B has greater variability

Set B has variance = 48.5 vs Set A variance = 10

### 3.2.4 Part (d): Calculate coefficient of variation for each set. Which has greater relative variability?

**Solution:**

CV\_A = s\_A / x̄\_A = √10 / 14 = 3.162 / 14 = 0.2259  
CV\_B = s\_B / x̄\_B = √48.5 / 14 = 6.964 / 14 = 0.4974

**SET B** has greater relative variability (CV\_B = 0.4974 > CV\_A = 0.2259)

**Note:** The coefficient of variation measures variability relative to the mean, making it useful for comparing datasets with different units or scales.