PSTAT 5A Practice Worksheet 3 - SOLUTIONS

Comprehensive Review: Probability, Counting, and Conditional Probability

Solution Key

2025-07-08

Table of contents

# 1. Section A: Probability - SOLUTIONS

*⏱️ Estimated time: 8 minutes*

**Problem A1: Probability Distributions - SOLUTION**

For a valid probability distribution, two conditions must be met:

1. All probabilities must be non-negative (≥ 0)
2. The sum of all probabilities must equal 1

**Analysis:**

**(a) Invalid**

* Sum = 0.3 + 0.3 + 0.3 + 0.2 + 0.1 = 1.2 > 1 The probabilities sum to more than 1, violating the second condition.

**(b) Valid**

* Sum = 0 + 0 + 1 + 0 + 0 = 1 All probabilities are non-negative and sum to 1. This represents a class where everyone receives a C.

**(c) Invalid**

* Sum = 0.3 + 0.3 + 0.3 + 0 + 0 = 0.9 < 1 The probabilities sum to less than 1, violating the second condition.

**(d) Invalid**

* Contains F = -0.1 < 0 Although the sum would equal 1.0, the probability for grade F is negative, violating the first condition.

**(e) Valid**

* Sum = 0.2 + 0.4 + 0.2 + 0.1 + 0.1 = 1.0 All probabilities are non-negative and sum to 1.

**(f) Invalid**

* Contains B = -0.1 < 0 Although the sum equals 1.0, the probability for grade B is negative, violating the first condition.

# 2. Section B: Permutations and Combinations - SOLUTIONS

*⏱️ Estimated time: 15 minutes*

**Problem B1: Permutations and Combinations - SOLUTION**

**Part (a):** How many 6-character passwords can be formed using 3 specific letters and 3 specific digits if repetitions are not allowed and letters must come before digits?

**Solution:** Since letters must come before digits, we have a fixed structure: LLL DDD

* Step 1: Arrange 3 letters in the first 3 positions
  + This is a permutation: P(3,3) = 3! = 6 ways
* Step 2: Arrange 3 digits in the last 3 positions
  + This is a permutation: P(3,3) = 3! = 6 ways
* Step 3: Apply multiplication principle
  + Total passwords = 6 × 6 = **36 passwords**

**Part (b):** If the team wants to select 4 people from 12 employees to form a security committee where order doesn’t matter, how many ways can this be done?

**Solution:** Since order doesn’t matter, this is a combination problem.

# 3. Section C: Conditional Probability - SOLUTIONS

*⏱️ Estimated time: 15 minutes*

**Problem B1: Conditional Probability and Medical Testing - SOLUTION**

**Given Information:**

* P(has variant) = 0.03
* P(test positive | has variant) = 0.95 (sensitivity)
* P(test negative | no variant) = 0.92 (specificity)
* Therefore: P(test positive | no variant) = 1 - 0.92 = 0.08

**Part (a):** What is the probability that a randomly selected person tests positive?

**Solution:**

Using the Law of Total Probability:

**Part (b):** If someone tests positive, what is the probability they actually have the variant?

**Solution:** Using Bayes’ Theorem:

**Part (c):** If someone tests negative, what is the probability they actually don’t have the variant?

**Solution:** First, find P(test negative):

Using Bayes’ Theorem:

**Part (d) [Challenge]:** Two consecutive positive tests - what is the probability they actually have the variant?

**Solution:** Assuming independence between tests:

**Problem C1: Advanced Counting with Restrictions - SOLUTION**

**Part (a):** How many valid meal combinations are possible?

**Solution:** We need to consider cases based on the restrictions.

**Case 1: Seafood appetizer is chosen**

* 1 appetizer option (seafood)
* 7 main course options (cannot choose vegetarian)
* 5 dessert options
* Combinations: 1 × 7 × 5 = 35

**Case 2: Non-seafood appetizer + chocolate dessert**

* 5 appetizer options (non-seafood)
* 3 main course options (only beef or chicken allowed with chocolate)
* 1 dessert option (chocolate)
* Combinations: 5 × 3 × 1 = 15

**Case 3: Non-seafood appetizer + non-chocolate dessert** - 5 appetizer options (non-seafood)

* 8 main course options (no restrictions)
* 4 dessert options (non-chocolate)
* Combinations: 5 × 8 × 4 = 160

**Total valid combinations:** 35 + 15 + 160 = **210 combinations**

**Part (b):** If customers choose randomly among valid combinations, what is the probability someone chooses the chocolate dessert?

**Solution:** Combinations with chocolate dessert: 15 (from Case 2 above) Total valid combinations: 210

# 4. Section D: Review - SOLUTIONS

*⏱️ Estimated time: 12 minutes*

**Problem B3: Daily Expenses - SOLUTION**

**Given:**

* Coffee: Mean = $1.40, SD = $0.30
* Muffin: Mean = $2.50, SD = $0.15
* Prices are independent

**Part (a):** What is the mean and standard deviation of the amount she spends on breakfast daily?

**Solution:** For the sum of independent random variables:

**Mean of daily expenses:**

**Variance of daily expenses:**

**Standard deviation of daily expenses:**

**Part (b):** What is the mean and standard deviation of the amount she spends on breakfast weekly (7 days)?

**Solution:** For the sum of 7 independent daily expenses:

**Mean of weekly expenses:**

**Variance of weekly expenses:**

**Standard deviation of weekly expenses:**