PSTAT 5A Practice Worksheet 3 - SOLUTIONS

Comprehensive Review: Probability, Counting, and Conditional Probability

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**📚 Key Formulas Reference:**

**Basic Probability:**

* **Conditional Probability:**
* **Law of Total Probability:**
* **Addition Rule:**
* **Multiplication Rule:**

**Counting:**

* **Multiplication Rule:** If a procedure consists of steps, with ways for step 1, for step 2, …, for step , then total ways:
* **Factorial:**
* **Permutations:**
* **Combinations:**

# 1. Section A: Probability - SOLUTIONS

*⏱️ Estimated time: 8 minutes*

**Problem A1: Probability Distributions - SOLUTION**

For a valid probability distribution, two conditions must be met:

1. All probabilities must be non-negative (≥ 0)
2. The sum of all probabilities must equal 1

**Analysis:**

**(a) Invalid**

The probabilities sum to more than 1, violating condition 2.

**(b) Valid**

All probabilities are non-negative and sum to 1. This represents a class where everyone receives a C.

**(c) Invalid**

The probabilities sum to less than 1, violating condition 2.

**(d) Invalid** Contains Although the sum would equal 1.0, the probability for grade F is negative, violating condition 1.

**(e) Valid**

All probabilities are non-negative and sum to 1.

**(f) Invalid** Contains Although the sum equals 1.0, the probability for grade B is negative, violating condition 1.

# 2. Section B: Permutations and Combinations - SOLUTIONS

*⏱️ Estimated time: 15 minutes*

**Problem B1: Permutations and Combinations - SOLUTION**

**Part (a):** How many 6-character passwords can be formed using 3 specific letters and 3 specific digits if repetitions are not allowed and letters must come before digits?

**Solution:** Since letters must come before digits, we have a fixed structure: **LLL DDD**

**Step 1:** Arrange 3 letters in the first 3 positions

* This is a permutation: ways

**Step 2:** Arrange 3 digits in the last 3 positions

* This is a permutation: ways

**Step 3:** Apply multiplication principle

**Part (b):** If the team wants to select 4 people from 12 employees to form a security committee where order doesn’t matter, how many ways can this be done?

**Solution:** Since order doesn’t matter, this is a **combination** problem.

# 3. Section C: Conditional Probability - SOLUTIONS

*⏱️ Estimated time: 12 minutes*

**Problem C1: Drawing Cards (Without Replacement) - SOLUTION**

**Given Information:**

* Standard 52-card deck
* Drawing two cards without replacement

**Solution:**

**1. P(A)**

There are 13 hearts in a 52-card deck.

**2. P(A and B)**

We need both events to occur: first card is a heart AND second card is an ace.

**Case 1:** First card is the Ace of Hearts -

* (3 aces left)

**Case 2:** First card is a non-ace heart - (12 non-ace hearts)

* (4 aces left)

**3. P(B|A)**

Using the definition of conditional probability:

**Alternative approach:** Given that the first card is a heart:

* If it’s the Ace of Hearts: 3 aces remain out of 51 cards
* If it’s a non-ace heart: 4 aces remain out of 51 cards

**4. P(B)**

Using the Law of Total Probability. Let = “first card is not a heart”

We know:

* and
* (from part 3)
* (if first card isn’t a heart, all 4 aces remain)

**5. Comparison of P(B|A) vs P(B)**

**Analysis:**

**Explanation:** The probability of getting an ace on the second draw is slightly **lower** when we know the first card is a heart compared to when we don’t know anything about the first card. This happens because:

* When the first card is a heart, there’s a chance it’s the Ace of Hearts, removing one ace from the deck
* This makes it slightly less likely to draw an ace on the second draw
* This demonstrates **dependence** - the events are not independent because drawing without replacement creates dependence between successive draws

# 4. Section D: Advanced Counting with Restrictions - SOLUTIONS

*⏱️ Estimated time: 15 minutes*

**Problem D1: Advanced Counting with Restrictions - SOLUTION**

**Given:**

* 6 appetizer options (including 1 seafood)
* 8 main course options (including 1 vegetarian, and 3 that are beef or chicken)
* 5 dessert options (including 1 chocolate)

**Restrictions:**

1. Seafood appetizer → cannot choose vegetarian main course
2. Chocolate dessert → must choose beef or chicken main course (3 specific options)

**Part (a):** How many valid meal combinations are possible?

**Solution using cases:**

**Case 1: Seafood appetizer is chosen**

* 1 appetizer choice (seafood)
* 7 main course choices (8 total minus 1 vegetarian)
* 5 dessert choices (no restrictions)
* Total: combinations

**Case 2: Non-seafood appetizer + chocolate dessert**

* 5 appetizer choices (6 total minus 1 seafood)
* 3 main course choices (only beef or chicken allowed with chocolate)
* 1 dessert choice (chocolate)
* Total: combinations

**Case 3: Non-seafood appetizer + non-chocolate dessert**

* 5 appetizer choices (6 total minus 1 seafood)
* 8 main course choices (no restrictions since no seafood appetizer)
* 4 dessert choices (5 total minus 1 chocolate)
* Total: combinations

**Total valid combinations:**

**Verification using complementary counting:**

* Total unrestricted combinations:
* Invalid combinations to subtract:
  + Seafood + vegetarian + any dessert:
  + Non-seafood + chocolate + non-beef/chicken:
* Valid combinations: ✓

**Part (b):** If customers choose randomly among valid combinations, what is the probability someone chooses the chocolate dessert?

**Solution:** From our case analysis, combinations with chocolate dessert come only from Case 2:

* Combinations with chocolate dessert: 15
* Total valid combinations: 210

**Alternative verification:**

We can also calculate this directly:

* Non-seafood appetizers: 5 choices
* With chocolate dessert, must choose from 3 main courses
* Valid chocolate combinations:
* Probability: ✓

# 5. Summary of Key Concepts

**Probability Distributions**

* Valid distributions require: all probabilities and sum
* Check both conditions systematically

**Permutations vs Combinations**

* Permutations: Order matters, use
* Combinations: Order doesn’t matter, use
* Multiplication principle: Combine independent choices

**Conditional Probability**

* Without replacement: Creates dependence between events
* Use definition:
* Law of Total Probability: For calculating marginal probabilities

**Advanced Counting**

* Case analysis: Break complex problems into manageable parts
* Handle restrictions: Consider what’s allowed vs. not allowed
* Verification: Use complementary counting or direct calculation