PSTAT 5A Practice Worksheet 4

Comprehensive Review: Discrete Random Variables and Distributions

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# 1. Instructions and Overview

**⏰ Time Allocation:**

* **Section A (Warm-up):** 8 minutes
* **Section B (Intermediate):** 15 minutes
* **Section C (Advanced):** 12 minutes
* **Section D (Applications):** 15 minutes
* **Total:** 50 minutes

**📝 Important Instructions:**

* Use the formulas provided for guidance
* Round final answers to 4 decimal places unless otherwise specified
* Identify the distribution type before calculating
* Show your work for expected value and variance calculations
* Use calculator as needed for factorials and combinations

**📚 Key Formulas Reference:**

**General Random Variable Properties:**

* **Expected Value:**
* **Variance:**
* **Standard Deviation:**

**Discrete Distributions:**

**Bernoulli Distribution:**

* **PMF:** for
* **Mean:**
* **Variance:**

**Binomial Distribution:**

* **PMF:** for
* **Mean:**
* **Variance:**

**Geometric Distribution:**

* **PMF:** for
* **Mean:**
* **Variance:**

**Poisson Distribution:**

* **PMF:** for
* **Mean:**
* **Variance:**

# 2. Section A: Basic Concepts and Identification

*⏱️ Estimated time: 8 minutes*

**Problem A1: Distribution Identification**

For each scenario below, identify the appropriate discrete distribution and state the parameter(s). **Do not calculate probabilities yet.**

**(a)** A fair coin is flipped until the first head appears. Let X = number of flips needed.

**(b)** A quality control inspector tests 20 randomly selected items from a production line where 5% are defective. Let X = number of defective items found.

**(c)** A website receives visitors at an average rate of 3 per minute. Let X = number of visitors in a 2-minute period.

**(d)** A basketball player shoots one free throw with an 80% success rate. Let X = 1 if successful, 0 if unsuccessful.

**(e)** A student keeps taking a driving test until they pass. The probability of passing on any attempt is 0.7. Let X = number of attempts needed to pass.

**Work Space:**

**Problem A2: Probability Mass Function**

The random variable X has the following probability distribution:

| X | 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- | --- |
| P(X=k) | 0.1 | 0.3 | 0.4 | a | 0.1 |

**(a)** Find the value of .

**(b)** Calculate .

**(c)** Calculate .

**Work Space:**

# 3. Section B: Expected Value and Variance

*⏱️ Estimated time: 15 minutes*

**Problem B1: Manual Calculations**

Using the probability distribution from Problem A2, calculate:

**(a)** The expected value

**(b)** The variance

**(c)** The standard deviation

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| Tip |
| **Calculation Strategy:**  For expected value:  For variance: First find , then use  Show your work step by step! |

**Work Space:**

**Problem B2: Bernoulli and Binomial Applications**

A manufacturing process produces items that are defective with probability 0.15.

**(a)** If you select one item randomly, what is the expected value and variance of X = number of defective items?

**(b)** If you select 25 items randomly, what is the expected number of defective items and the standard deviation?

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| Tip |
| Part (a) is a Bernoulli distribution. Part (b) is a Binomial distribution. Use the formulas from the reference box! |

**Work Space:**

# 4. Section C: Distribution Calculations

*⏱️ Estimated time: 12 minutes*

**Problem C1: Binomial Distribution**

A multiple-choice quiz has 10 questions, each with 4 choices. A student guesses randomly on all questions.

**(a)** What is the probability the student gets exactly 3 questions correct?

**(b)** What is the probability the student gets at least 2 questions correct?

**(c)** What is the expected number of correct answers?

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| Tip |
| This is binomial with and (since 1 out of 4 choices is correct).  For part (b): |

**Work Space:**

**Problem C2: Poisson Distribution**

A call center receives an average of 4 calls per minute.

**(a)** What is the probability of receiving exactly 6 calls in a given minute?

**(b)** What is the probability of receiving no calls in a given minute?

**(c)** What is the probability of receiving more than 2 calls in a given minute?

**Work Space:**

# 5. Section D: Advanced Applications

*⏱️ Estimated time: 15 minutes*

**Problem D1: Geometric Distribution**

A software company releases updates that fix critical bugs with probability 0.6 per update.

**(a)** What is the probability that the first successful bug fix occurs on the 3rd update?

**(b)** What is the expected number of updates needed to get the first successful bug fix?

**(c)** What is the probability that it takes more than 4 updates to get the first successful bug fix?

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| Tip |
| For part (c): where  This represents the probability of 4 consecutive failures. |

**Work Space:**

**Problem D2: Mixed Applications**

A quality assurance team at a pharmaceutical company is testing a new batch of medications.

* Each pill has a 2% chance of being defective
* They test pills one by one until they find the first defective pill
* They also want to know about defects in batches of 50 pills

**(a)** What is the probability that the first defective pill is found on the 5th test?

**(b)** What is the expected number of pills they need to test to find the first defective one?

**(c)** In a batch of 50 pills, what is the probability that exactly 2 pills are defective?

**(d)** In a batch of 50 pills, what is the expected number of defective pills?

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| Tip |
| Parts (a) and (b) use geometric distribution. Parts (c) and (d) use binomial distribution.  Be careful with the parameter for both distributions. |

**Work Space:**

# 6. Reflection Questions

**Problem E: Conceptual Understanding**

**(a)** Explain the key difference between a Binomial distribution and a Geometric distribution in terms of what they count.

**(b)** When would you use a Poisson distribution instead of a Binomial distribution?

**(c)** If , under what conditions would the variance be maximized?

**Work Space:**