PSTAT 5A Practice Worksheet 5

Continuous Random Variables and Confidence Intervals

Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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# 1. Instructions and Overview

**⏰ Time Allocation:**

* **Quiz Review** : 10 minutes
* **Section A (Continuous Distributions):** 20 minutes
* **Section B (Confidence Intervals):** 25 minutes
* **Optional Questions:** Do on your own
* **Total:** 55 minutes

**📝 Important Instructions:**

* Use the formulas and tables provided for guidance
* Round final answers to 4 decimal places unless otherwise specified
* For confidence intervals, always interpret your results in context
* Use z-table or t-table as appropriate
* Show your work for all calculations

**📚 Key Formulas Reference:**

**Continuous Random Variables:**

**Normal Distribution:**

* **PDF:**
* **Standardization:** where
* **Mean:**
* **Variance:**

**Uniform Distribution:**

* **PDF:** for
* **Mean:**
* **Variance:**

**Exponential Distribution:**

* **PDF:** for
* **Mean:**
* **Variance:**

**Confidence Intervals:**

**For Population Mean (σ known):**

**For Population Mean (σ unknown):**

**Margin of Error:** or

**Sample Size:**

# 2. Section A: Continuous Random Variables

*⏱️ Estimated time: 20 minutes*

**Problem A1: Distribution Identification and Properties**

For each scenario below, identify the appropriate continuous distribution and find the requested values:

**(a)** The time (in minutes) between arrivals at a coffee shop follows an exponential distribution with an average of 2 minutes between arrivals.

* What is the parameter λ?
* What is the probability that the next customer arrives within 1 minute?

**(b)** A random number generator produces values uniformly between 10 and 30.

* What are the parameters a and b?
* What is the expected value and variance?

**Work Space:**

**Problem A2: Normal Distribution Calculations**

The heights of adult women in the US are normally distributed with μ = 64 inches and σ = 2.5 inches.

**(a)** What is the probability that a randomly selected woman is taller than 67 inches?

**(b)** What height represents the 25th percentile?

**(c)** What is the probability that a randomly selected woman has a height between 62 and 68 inches?

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| Tip |
| **Remember to standardize:** Convert to Z-scores using  For part (b), you’re looking for the value x such that P(X ≤ x) = 0.25 |

**Work Space:**

# 3. Section B: Confidence Intervals

*⏱️ Estimated time: 25 minutes*

**Problem B1: Understanding Confidence Intervals**

**(a)** Explain in your own words what a 95% confidence interval means.

**(b)** A 90% confidence interval for the mean weight of apples is (150g, 170g). What is the sample mean and margin of error?

**(c)** True or False: “There is a 95% probability that the population mean lies within our calculated 95% confidence interval.” Explain your reasoning.

**Work Space:**

**Problem B2: Constructing Confidence Intervals**

A sample of 36 students has a mean test score of 78.5 with a standard deviation of 12.

**(a)** Construct a 95% confidence interval for the population mean test score.

**(b)** Interpret this interval in the context of the problem.

**(c)** What would happen to the width of the interval if: - We increased the confidence level to 99%? - We increased the sample size to 144?

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| Tip |
| **Decision Guide:** - Use z-distribution when σ is known OR n ≥ 30 - Use t-distribution when σ is unknown AND n < 30 - For 95% CI: z₀.₀₂₅ = 1.96 |

**Work Space:**

**Problem B3: Sample Size Determination**

A market researcher wants to estimate the average amount customers spend per visit with a margin of error of $5 and 95% confidence. Previous studies suggest the population standard deviation is $25.

**(a)** What sample size is needed?

**(b)** If the researcher wants to reduce the margin of error to $3, what sample size would be needed?

**Work Space:**

# 4. Optional Questions

**Optional Problem 1: Conceptual Understanding**

**(a)** Explain the key difference between discrete and continuous random variables in terms of: - The values they can take - How we calculate probabilities

**(b)** Why do we use P(X = x) = 0 for any specific value x in a continuous distribution?

**(c)** What’s the relationship between PDF and CDF for continuous distributions?

**Optional Problem 2: Advanced Applications**

**(a)** The lifetime of a certain battery follows an exponential distribution with λ = 0.1 per hour. What is the probability that the battery lasts more than 15 hours?

**(b)** A factory claims their light bulbs have an average lifetime of 1000 hours. You test 25 bulbs and find a mean lifetime of 950 hours with a standard deviation of 100 hours. Construct a 90% confidence interval and discuss whether the factory’s claim seems reasonable.

**Work Space:**

**📋 Quick Reference:**

**Common Z-values:** - 90% CI: z₀.₀₅ = 1.645 - 95% CI: z₀.₀₂₅ = 1.96  
- 99% CI: z₀.₀₀₅ = 2.576

**Common t-values (selected):** - df = 24, α = 0.05: t₀.₀₂₅ = 2.064 - df = 35, α = 0.05: t₀.₀₂₅ = 2.030