PSTAT 5A Practice Worksheet 5 - SOLUTIONS

Continuous Random Variables and Confidence Intervals

Instructor Solutions

2025-07-23

Table of contents

# 1. Section A Solutions: Continuous Random Variables

*Solution*. **Solution A1: Distribution Identification and Properties**

**(a) Exponential Distribution**

Since the average time between arrivals is minutes, we have:

* **Parameter :** The rate parameter arrivals per minute
* **Probability calculation:** where
* For exponential distribution:

**(b) Uniform Distribution**

* **Parameters:**
* **Expected Value:**
* **Variance:**

*Solution*. **Solution A2: Normal Distribution Calculations**

Given:

**(a)**

**Step 1: Standardize**

**Step 2: Find probability**

**(b) 25th percentile**

Step 1: Find -value for -th percentile

, so

Step 2: Convert back to

**(c) P(62 < X < 68)**

Step 1: Standardize both values

Step 2: Find probability

# 2. Section B Solutions: Confidence Intervals

*Solution*. **Solution B1: Understanding Confidence Intervals**

**(a) Explanation of Confidence Interval:**

A confidence interval means that if we were to repeat our sampling process many times (say times) and construct a confidence interval each time using the same method, approximately of those intervals would contain the true population mean. **It does NOT mean there’s a probability that the population mean lies in any one specific interval.**

**(b) Sample mean and margin of error:**

Given : ()

* **Sample mean:**
* **Margin of error:**

**(c) True or False statement:**

**FALSE.** Once we calculate a specific confidence interval, the population mean either is or isn’t in that interval, there’s no probability involved for that specific interval. The refers to the long-run success rate of the method, not the probability for any individual interval.

*Solution*. **Solution B2: Constructing Confidence Intervals**

Given:

**(a) 95% Confidence Interval:**

Step 1: Check conditions

* , so we can use -distribution
* For

Step 2: Calculate margin of error

Step 3: Construct interval

**(b) Interpretation:**

We are confident that the true population mean test score is between and points.

**(c) Effects on interval width:**

* **Increasing confidence level to 99%:** The interval would become **wider** because we need
* **Increasing sample size to 144:** The interval would become **narrower** because the margin of error would be (smaller than )

*Solution*. **Solution B3: Sample Size Determination**

Given: , confidence =

**(a) Required sample size:**

Step 1: Use sample size formula

Step 2: Substitute values

Step 3: Round up

customers (**always round up for sample size**)

**(b) For margin of error = $3:**

customers

# 3. Optional Problem Solutions

*Solution*. **Optional Solution 1: Conceptual Understanding**

**(a) Differences between discrete and continuous:**

**Values they can take:**

* Discrete: Countable values (integers, specific points)
* Continuous: Uncountably infinite values (any real number in an interval)

**How we calculate probabilities:**

* Discrete: can be non-zero; we sum probabilities
* Continuous: for any specific ; we integrate over intervals

**(b) Why for continuous distributions:**

In continuous distributions, there are infinitely many possible values in any interval. The probability of hitting any one exact value is infinitesimally small, hence zero. We instead calculate by integrating the PDF over the interval .

**(c) Relationship between PDF and CDF:**

* **PDF (f(x)):** The probability density function gives the “density” of probability at each point
* **CDF (F(x)):** The cumulative distribution function gives
* **Relationship:**

**Key Takeaways:**

1. **Always standardize** normal distribution problems using
2. **Interpret confidence intervals** in context, they’re about the method’s reliability, not individual interval probabilities
3. **Choose the right distribution** use when is unknown and
4. **Round up sample sizes** to ensure you meet the margin of error requirement
5. **For continuous distributions**, focus on intervals, not individual points