DMSN Tutorial 1: Networks and Random Graphs

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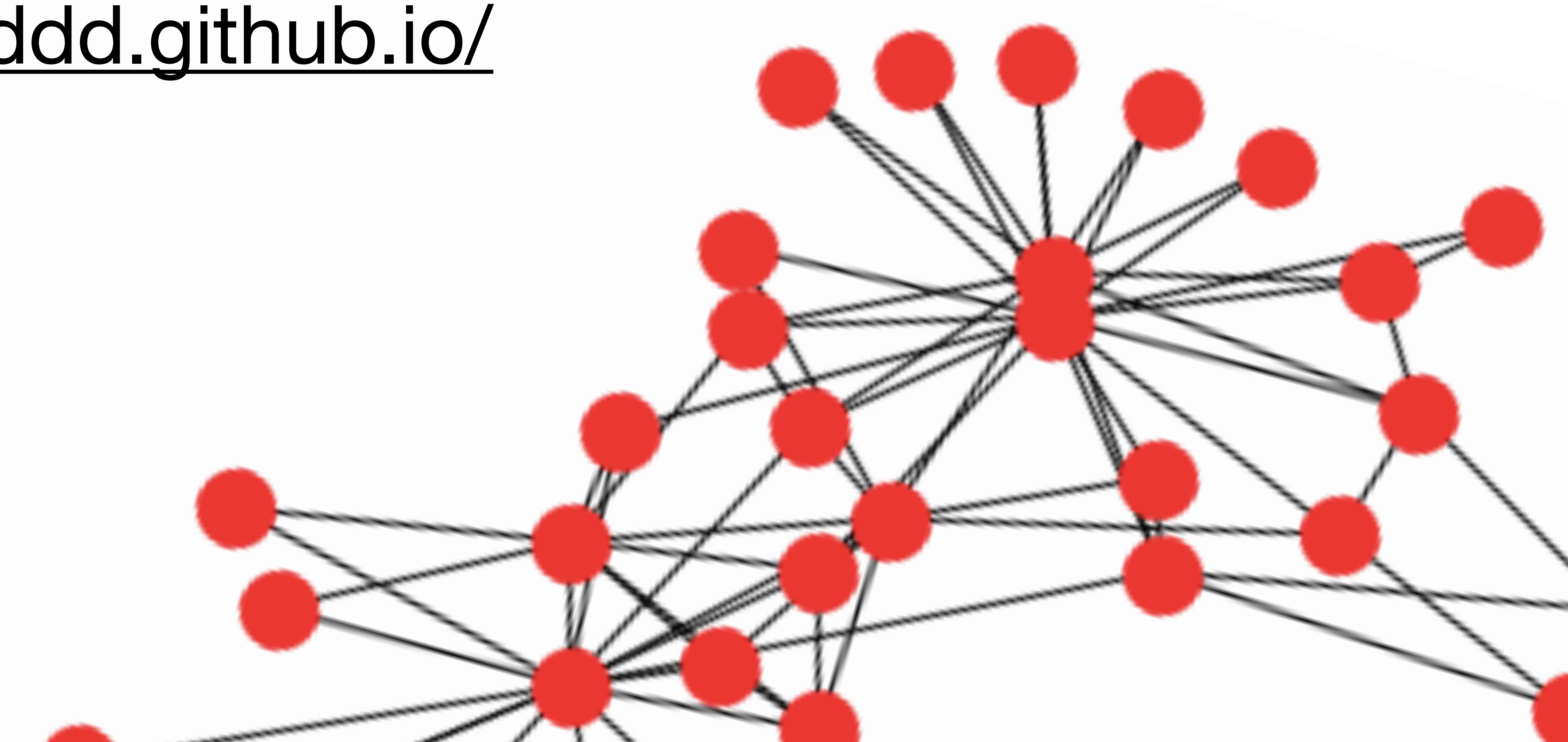
https://narnolddd.github.io/



In this tutorial:

- **Recap** on concepts and metrics covered in the lecture
- **Understand** the Erdos-Renyi random graph model
- **Understand** some of the key similarities and differences between random graphs and real networks

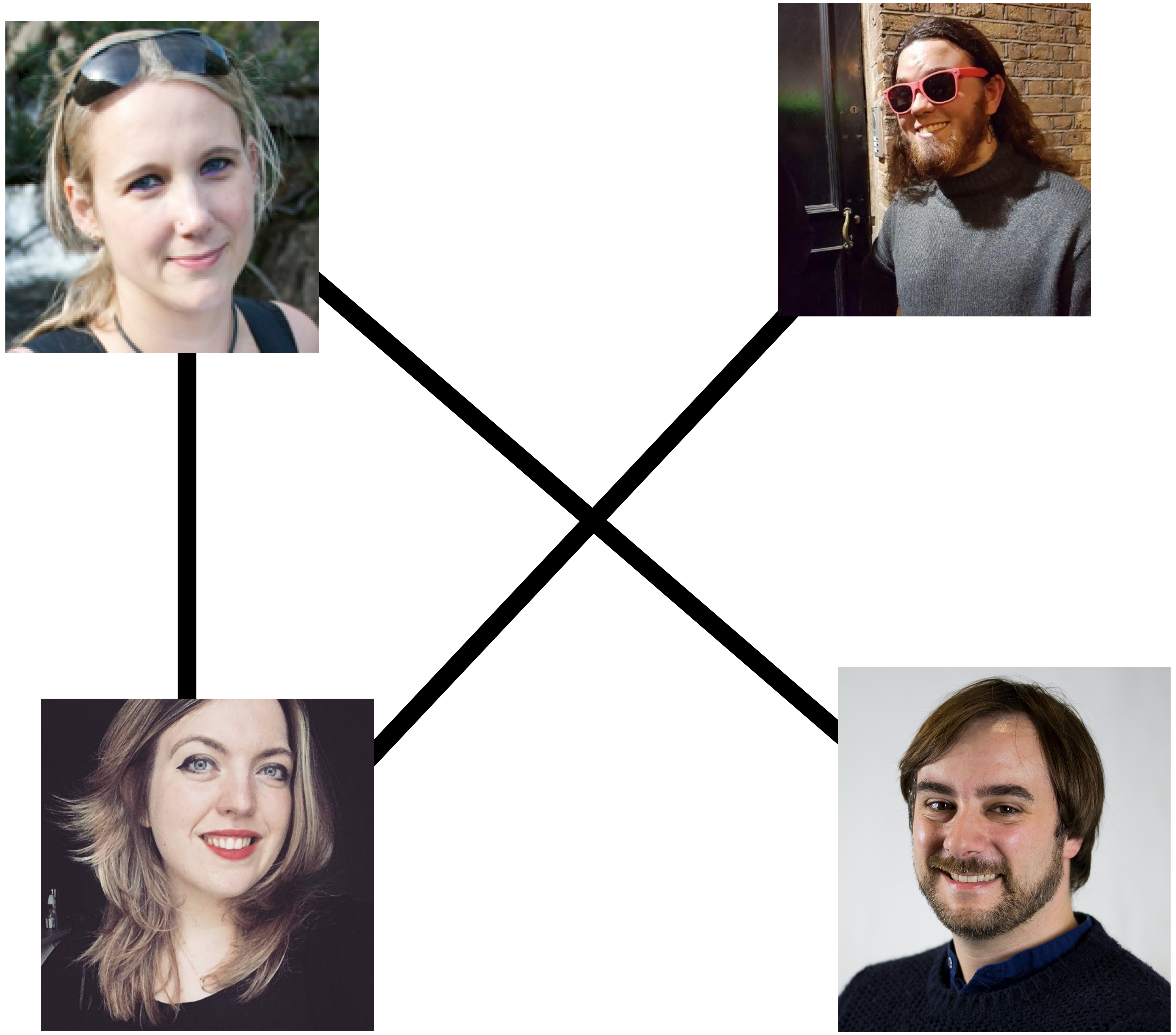
Recap: (Undirected) Graph

A **graph** is a tuple **(V,E)** of a set **V** of vertices and **E** of edges

Vertex (node) set: {Laurissa, Ben, Naomi, Mathieu}

Edge (link) set: { (Laurissa, Naomi), (Laurissa, Mathieu), (Naomi, Ben)}

Here, order doesn't matter as graph is **undirected**

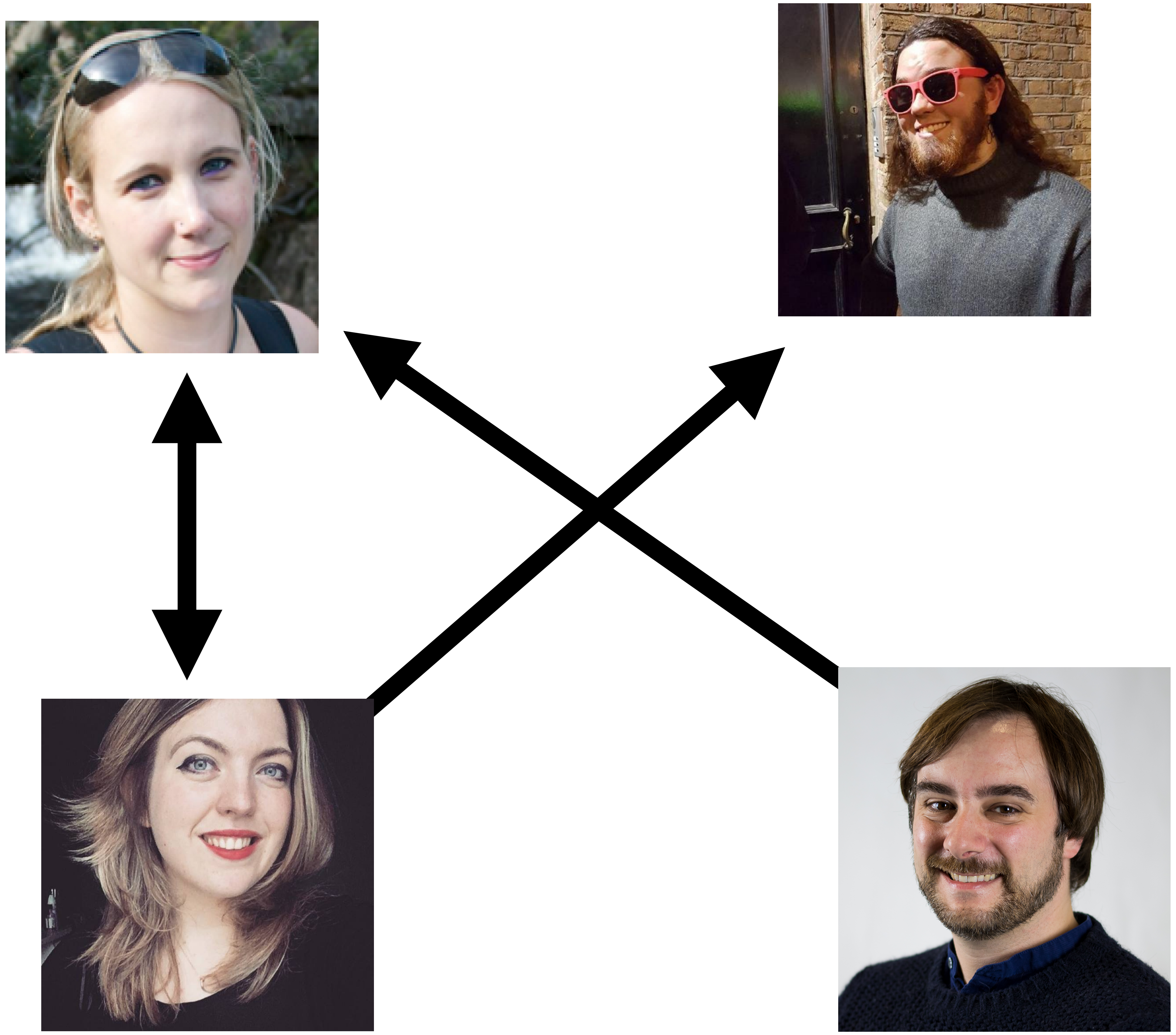


Recap: Directed Graph

Vertex (node) set: {Laurissa, Ben, Naomi, Mathieu}

Edge (link) set: { (Laurissa, Naomi), (Naomi, Laurissa) (Mathieu, Laurissa), (Naomi, Ben)}

Here, order **does** matter as graph is **directed**



How do we measure graphs?
How do we compare them?

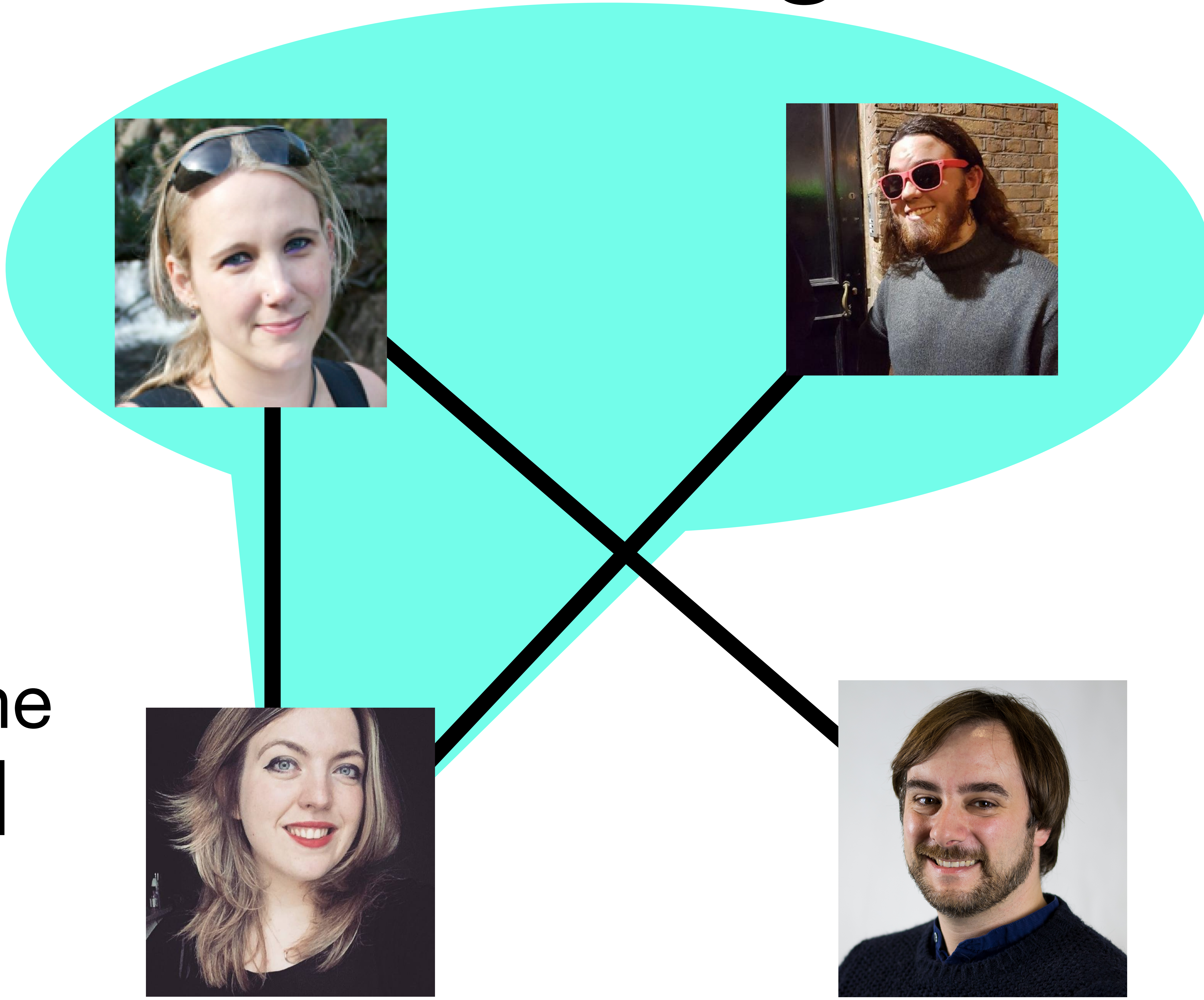
Recap: Neighbourhood and Degree

The **neighbourhood** N(v) of a vertex v is the set of vertices adjacent to V

e.g. N(Naomi) = {Laurissa, Ben}

The **degree** k(v) of a vertex v is the size of the neighbourhood: |N(v)|

e.g. k(Naomi) =2



Recap: Degree Sequence/Average Degree

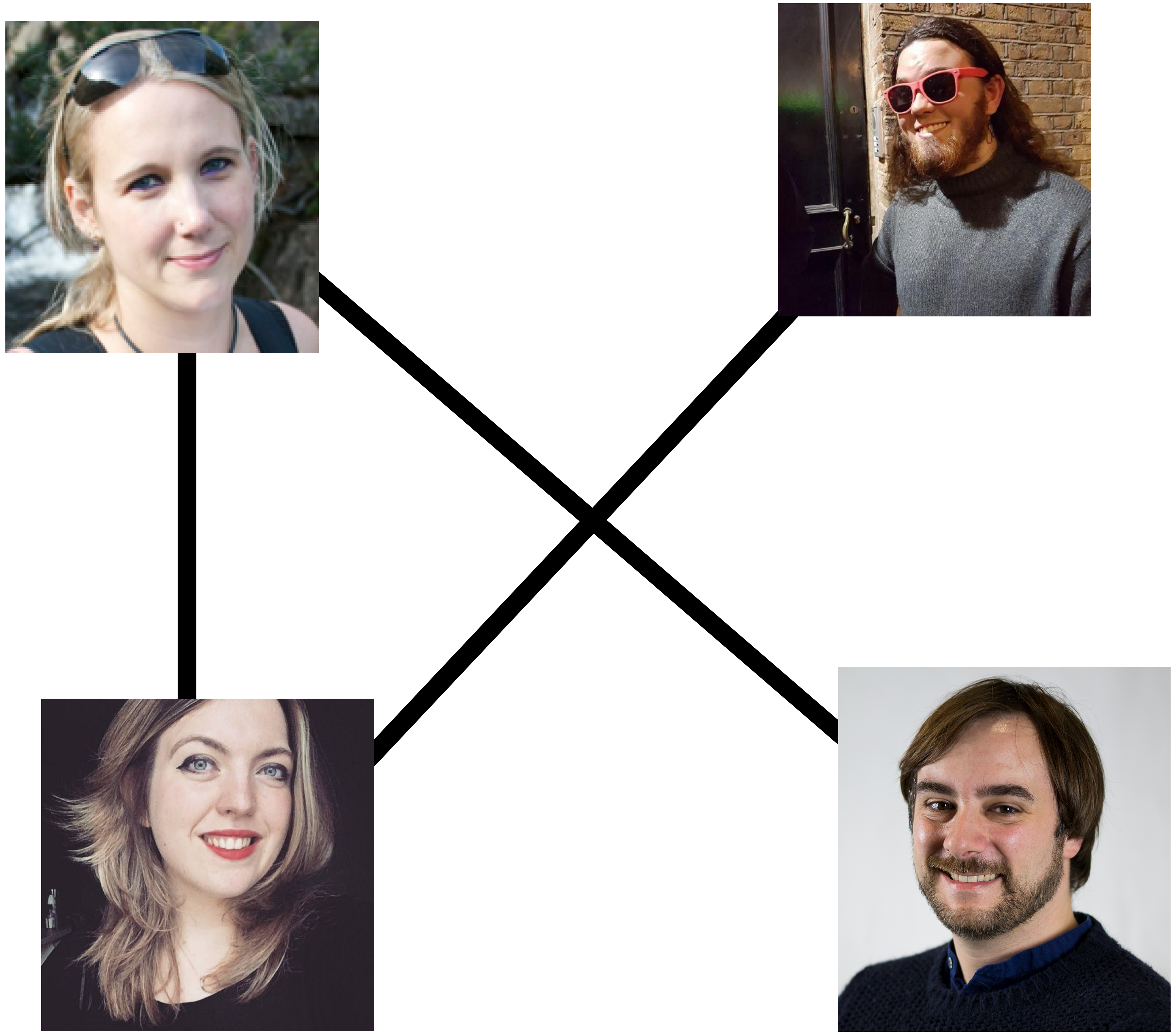
The **degree sequence** of a graph is the list of the vertex degrees for that graph (in decreasing order)

e.g. 2, 2, 1, 1

The **average degree** of a graph **<k>** is the mean of the node degrees

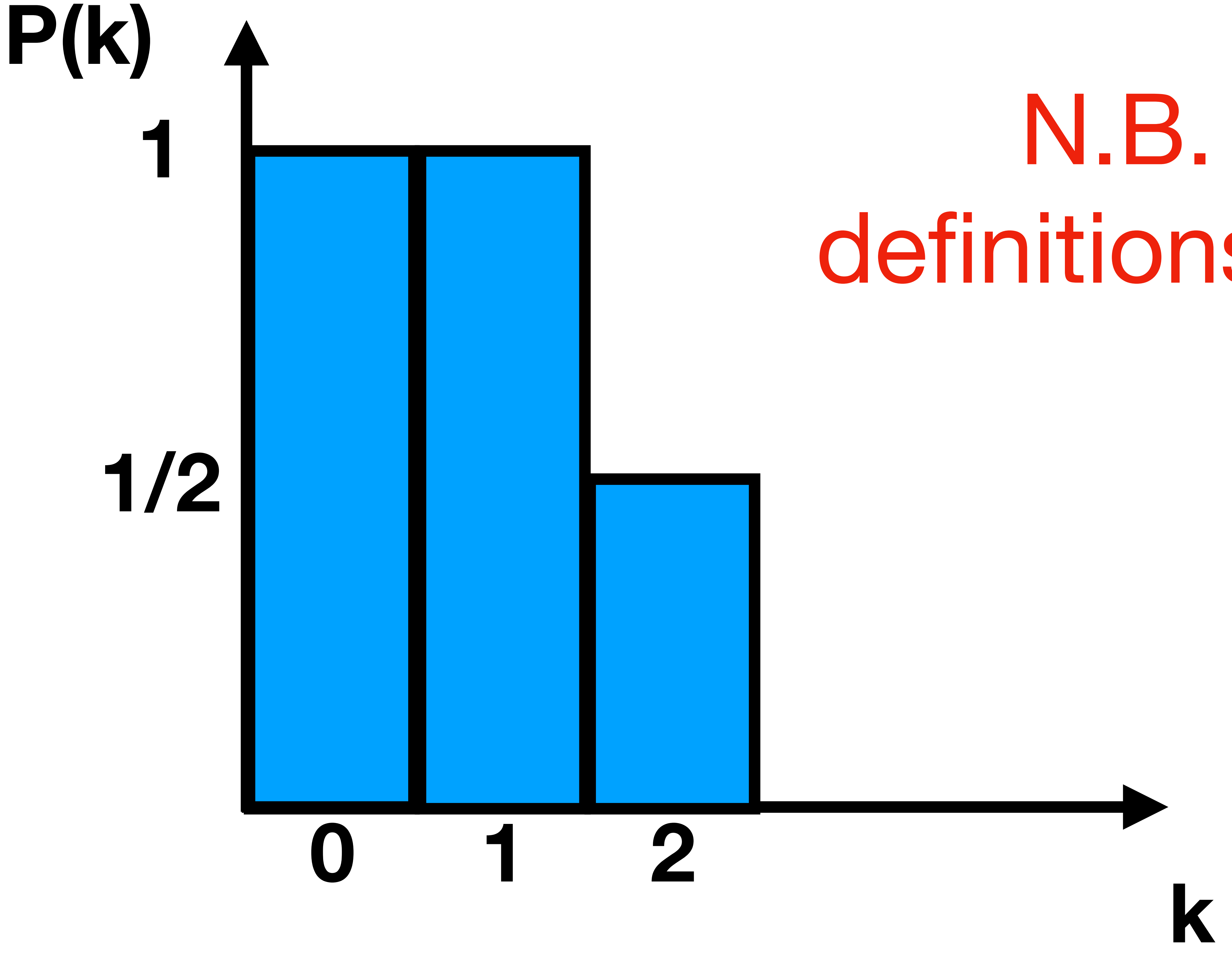
e.g. <k> = (2 + 2 + 1 + 1)/4 = 1.5

(also equal to 2*|edges|/|nodes| … why?)

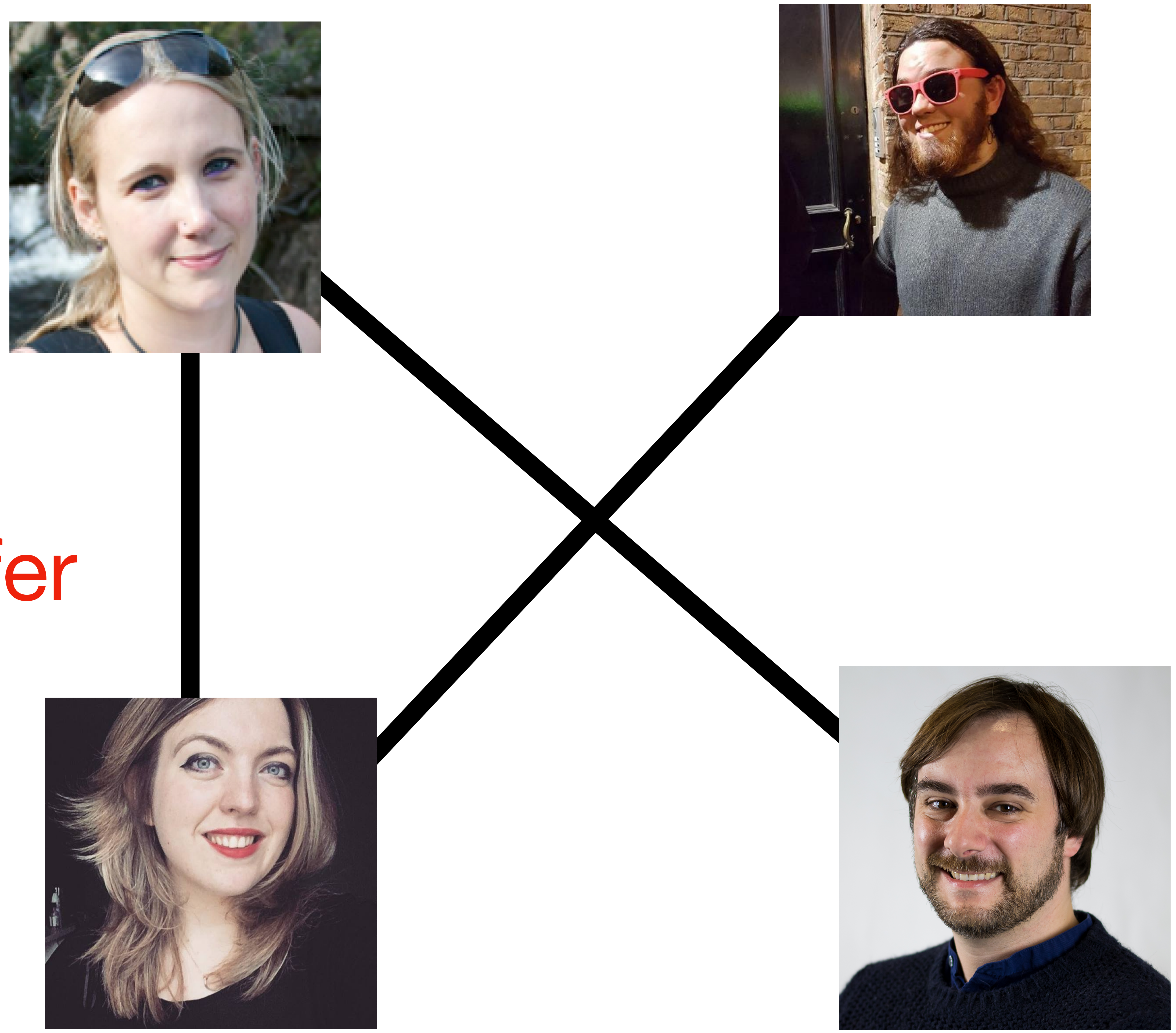


Recap: Degree distribution

The degree distribution **P(k)** is the proportion of nodes with degree **greater than or equal to k**



N.B. some definitions will differ



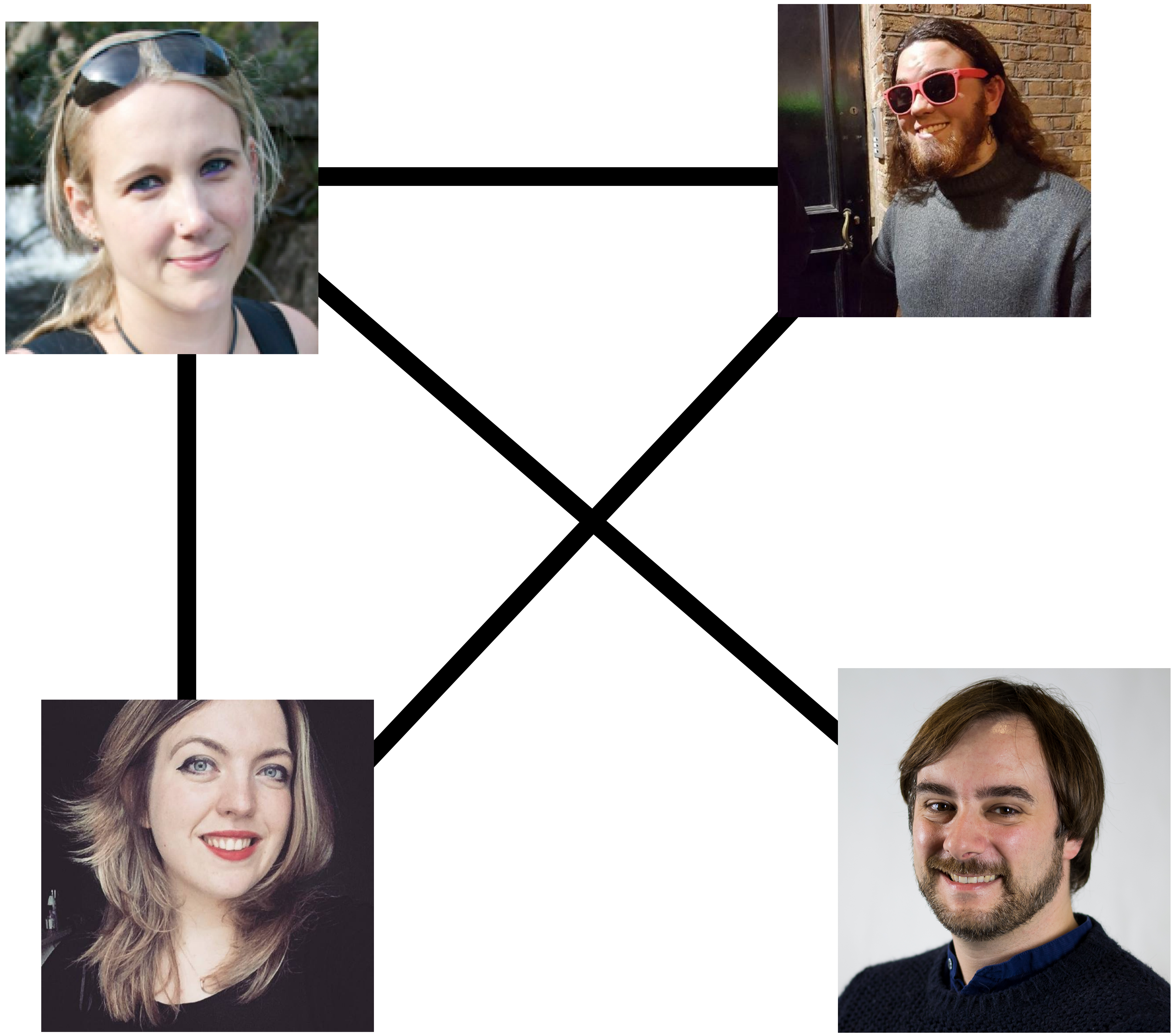
Recap: Clustering Coefficient

The **clustering coefficient C(v)** is the proportion of possible connections between neighbours of **v** that are present

$$C(v) = \frac{|\{(u,w) \mid u,w \in N(v)\}|}{\frac{1}{2}k(v)(k(v)-1)}$$

Pairs of neighbours of v that are connected

Possible pairs of v's neighbours, "k(v) choose 2"



Recap: Clustering Coefficient

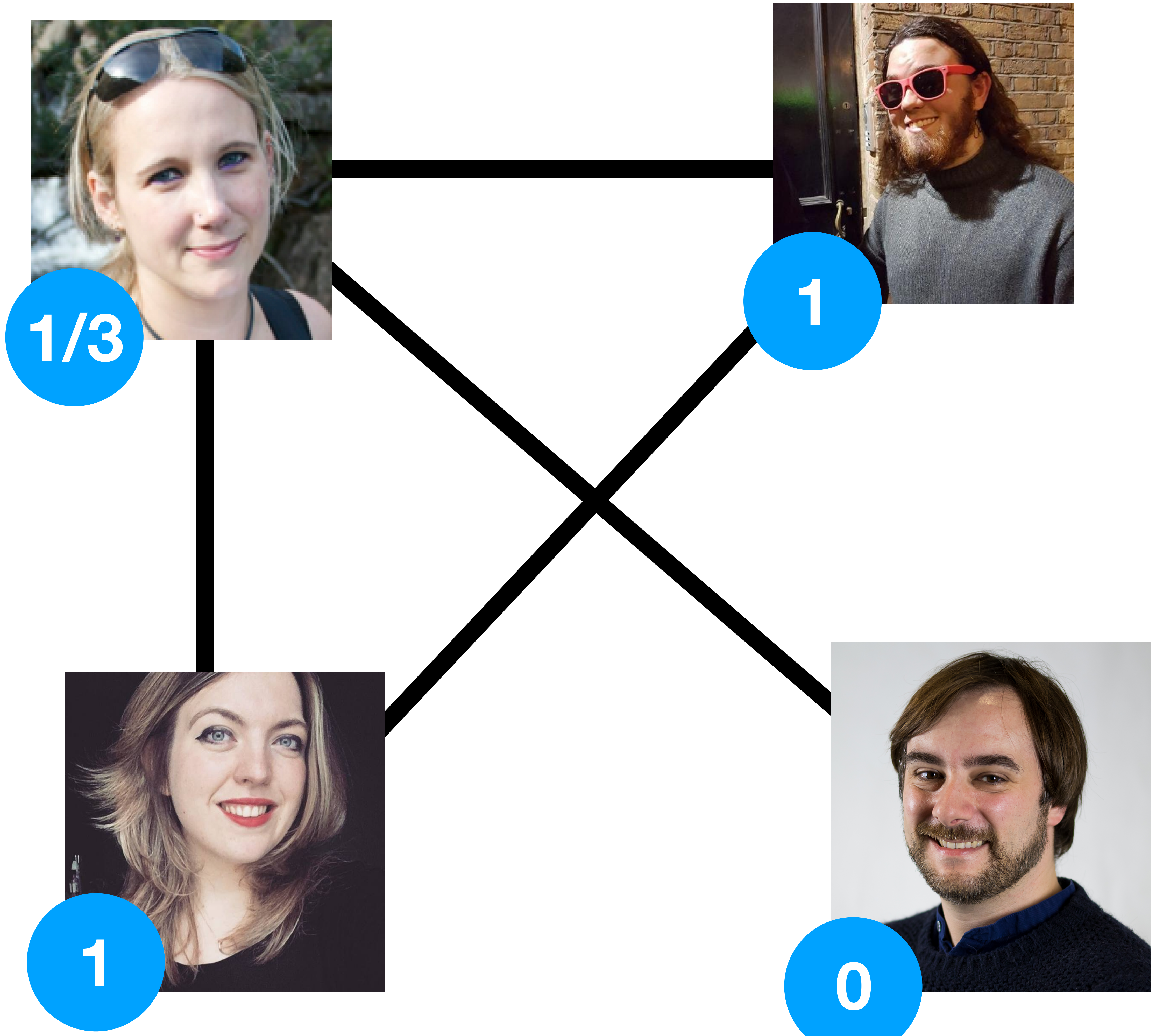
What is Laurissa's clustering coefficient?

Denominator: Laurissa's degree is 3, so 0.5*3*2 = 3

Numerator: Only one pair of Laurissa's neighbours are connected (Naomi, Ben)

So C(Laurissa) = 1/3

Average clustering C(G) = 7/12



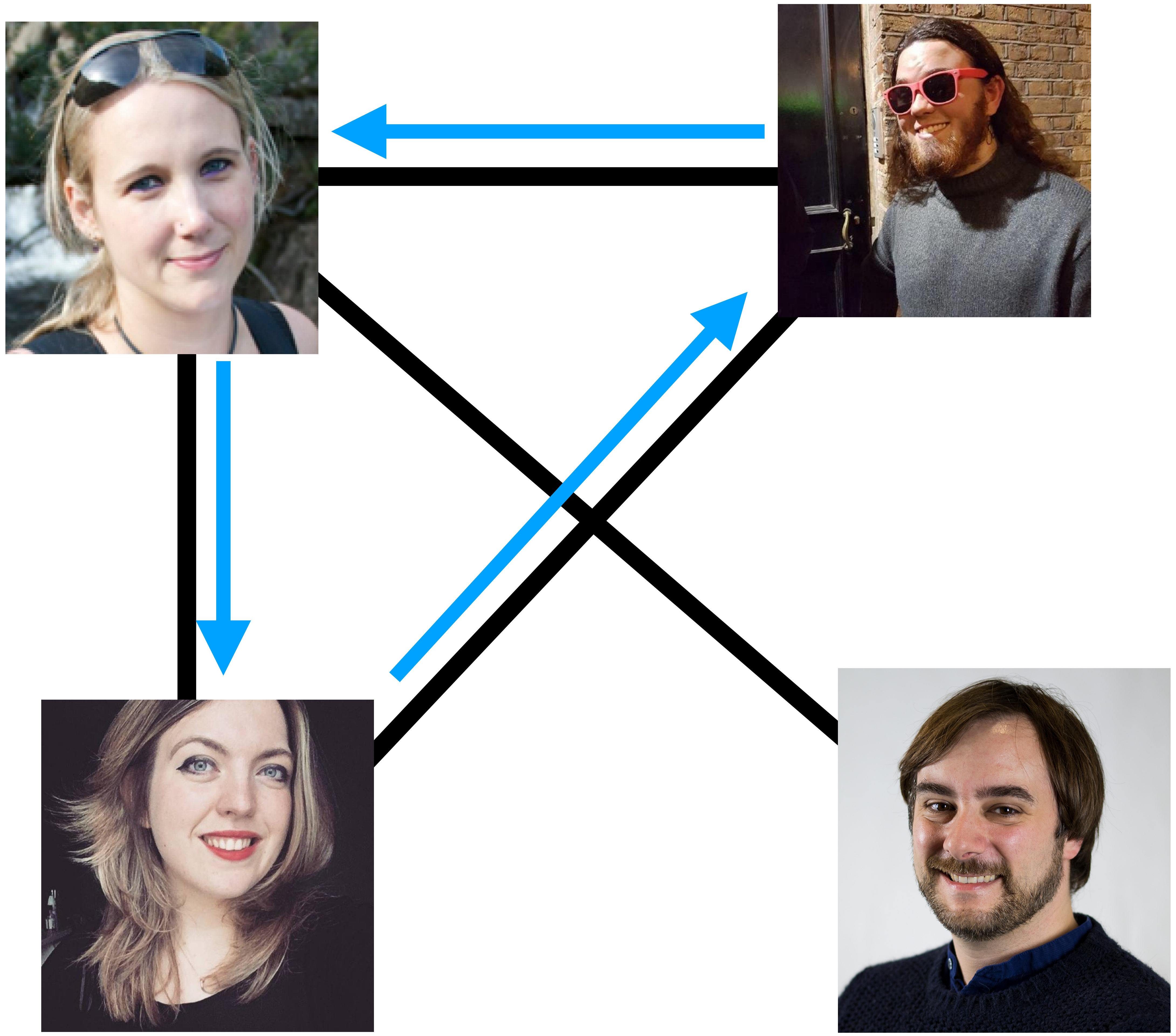
Recap: Paths and Cycles

A **path** is a sequence of nodes where each consecutive pair of nodes is linked by an edge

Ben, Laurissa, Naomi

A **cycle** is a path where the start node is also the end node

Ben, Laurissa, Naomi, Ben



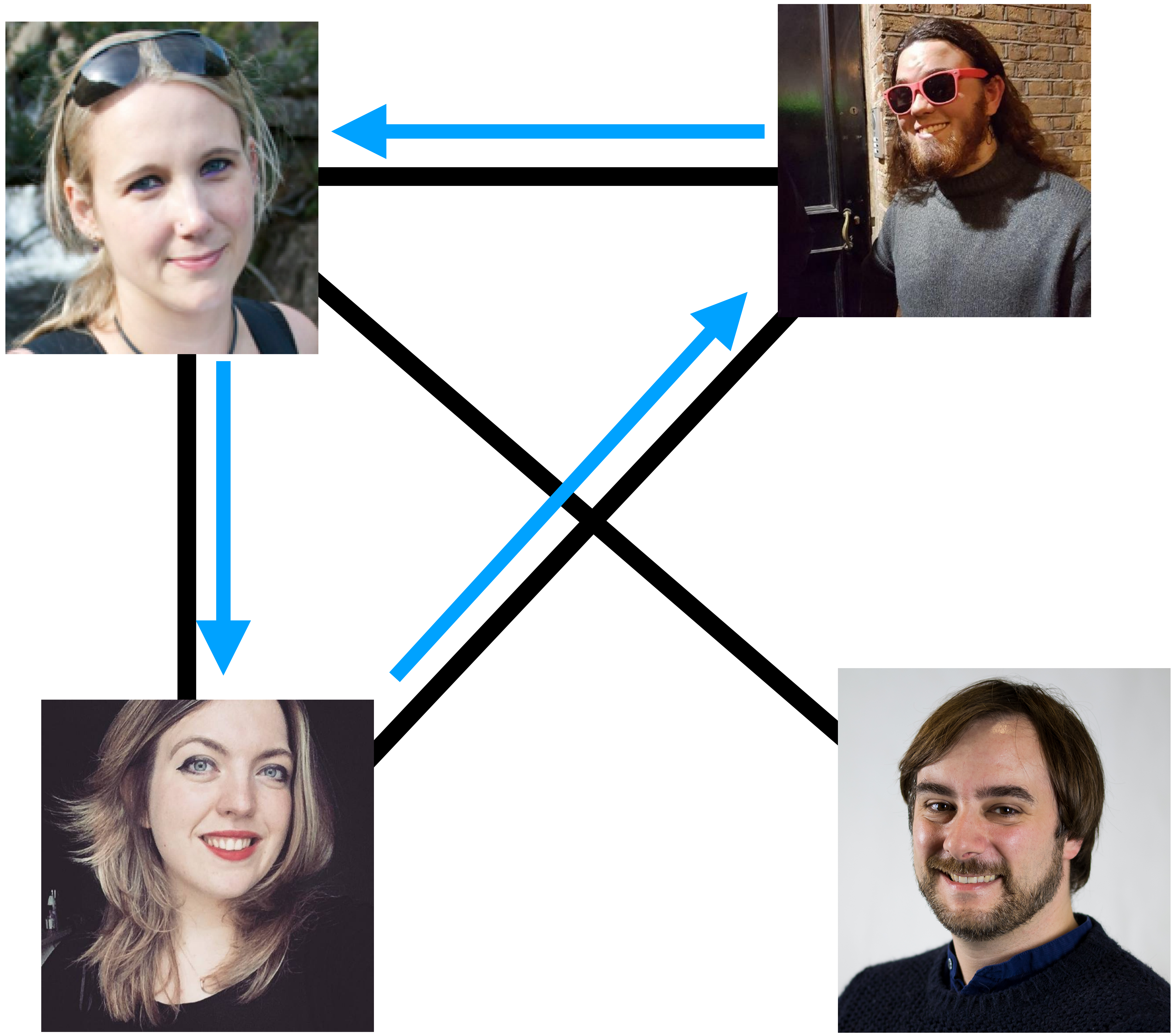
Recap: Paths and Cycles

The **distance** d(u,v) between two nodes is the length of the shortest path connecting them

d(Ben, Mathieu) = 2

The **diameter** of a graph is the largest distance between a pair of nodes in the graph

d(G) = 2

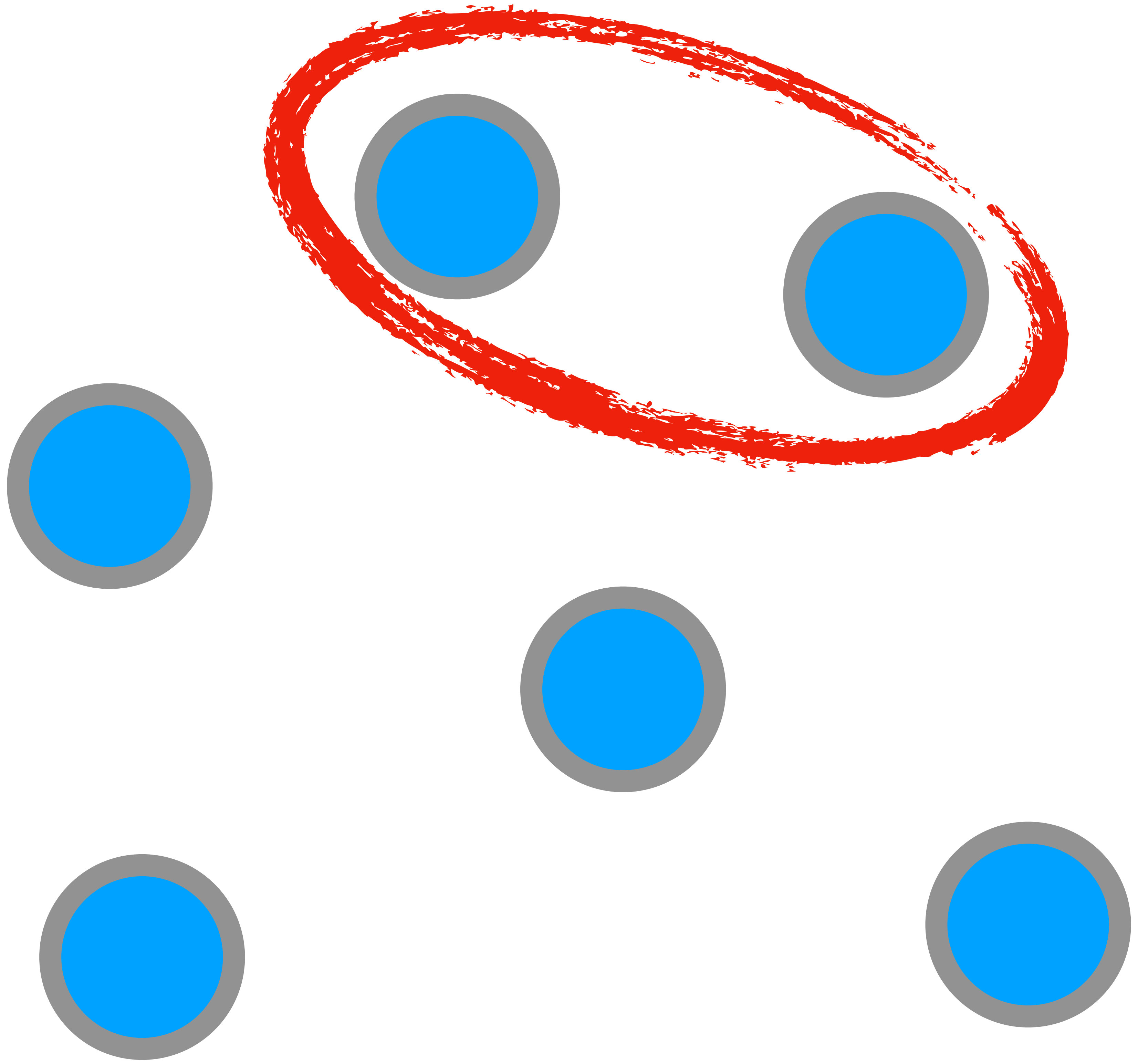


NB: often more meaningful to look at average path length

Erdos-Renyi Random Graph Model

- Want to model real networks, have some **baseline** to compare
- “Is the value of this network metric unusual?” Want a **null model**
- Wha is the **very simplest** model formulation we can look at?

Erdos-Renyi G(n,p) Model



1. Start with an empty graph of **n** nodes

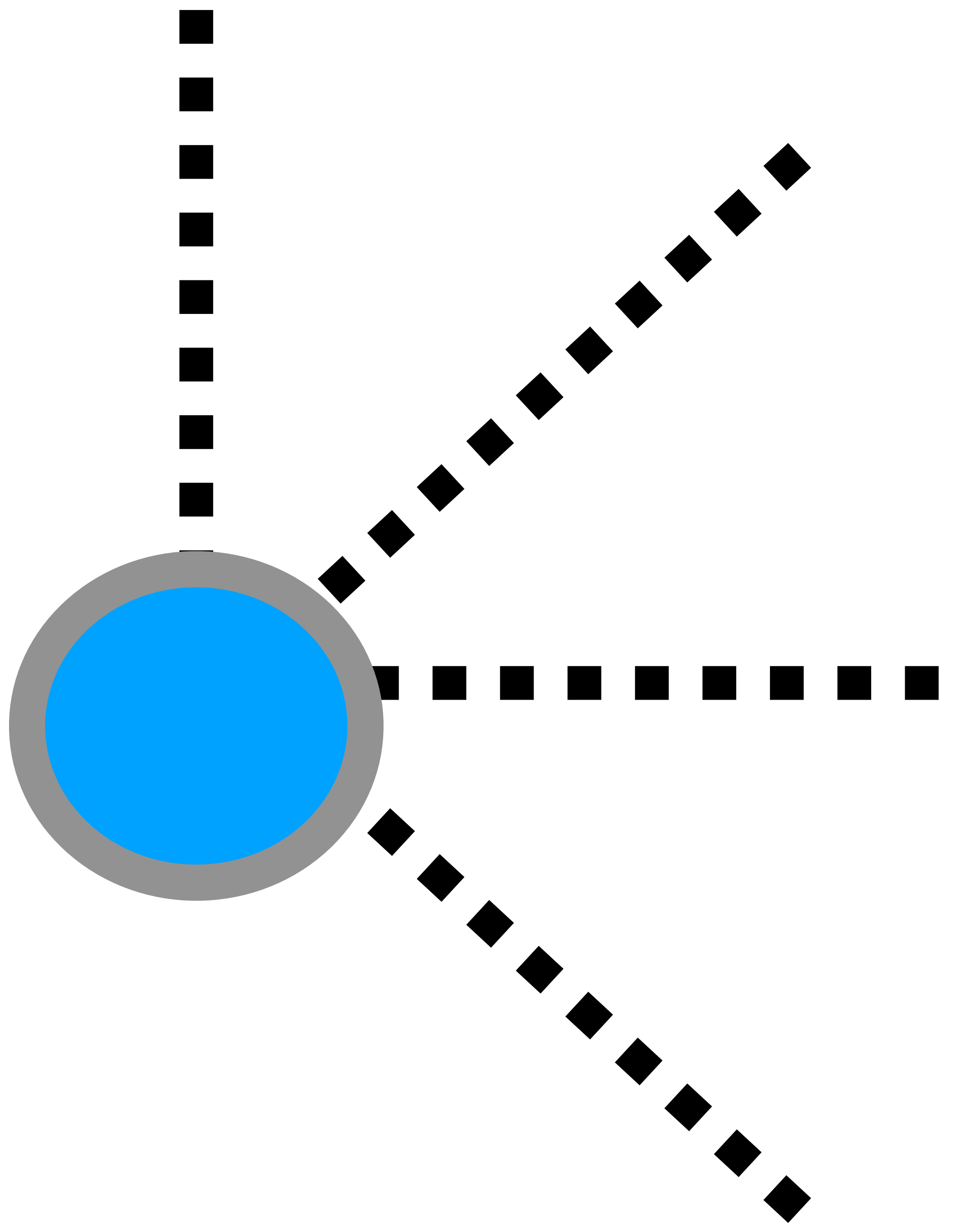
2. Acquire a biased coin with head probability **p**

3. For each pair of nodes, do a coin toss. If heads, draw an edge between them. If not, move on.

Erdos-Renyi G(n,p) model

Increasing **p**

Average degree of ER networks



For each node, there are n-1 others in the graph it could connect to.

Each of those connections can happen with probability p

(If you did Probability and Matrices, this is a binomial with **n-1 trials** and **success probability p**)

So average degree is **(n-1)p**, or approximately **np**

Summary: Random Graphs vs Real Networks

	Real Social Networks	Random Graphs	?
Degree Distribution	Heavy Tailed (most nodes have low degree, small few with high degree)	Light tailed (all nodes have close to the average degree)	?
Clustering Coefficient	High	Low	?
Path Lengths	Low	Low	?
?	?	?	?