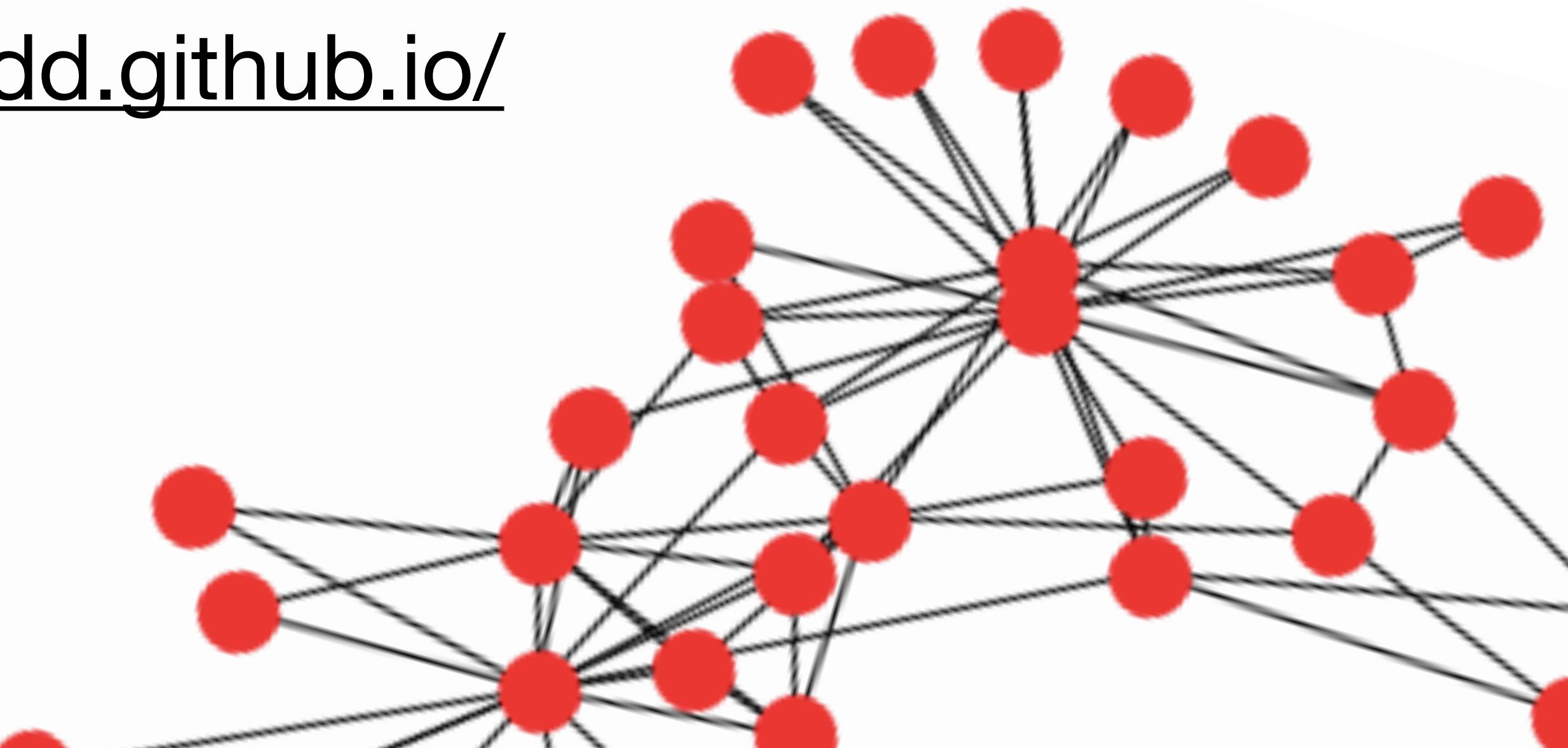


# Week 9: Epidemics and mobility

Naomi Arnold

<https://narnolddd.github.io/>



# Tutorial aims

- Recap different **epidemic models** for networks
- Discuss different epidemic **intervention measures** in the context of networks and epidemic models
- Look at some **numerical simulations** implemented in Python

# What might be modelled as an epidemic?



**Infectious diseases**

<https://covid19obs.fbk.eu/>

[Wang, L., Wood, B. C., An epidemiological approach to model the viral propagation of memes, *Applied Mathematical Modelling*, 2011]

[Weng, L., Flammini, A., Vespignani, A. & Menczer, F. Competition among memes in a world with limited attention, *Scientific Reports*, 2012]



**Information spread**

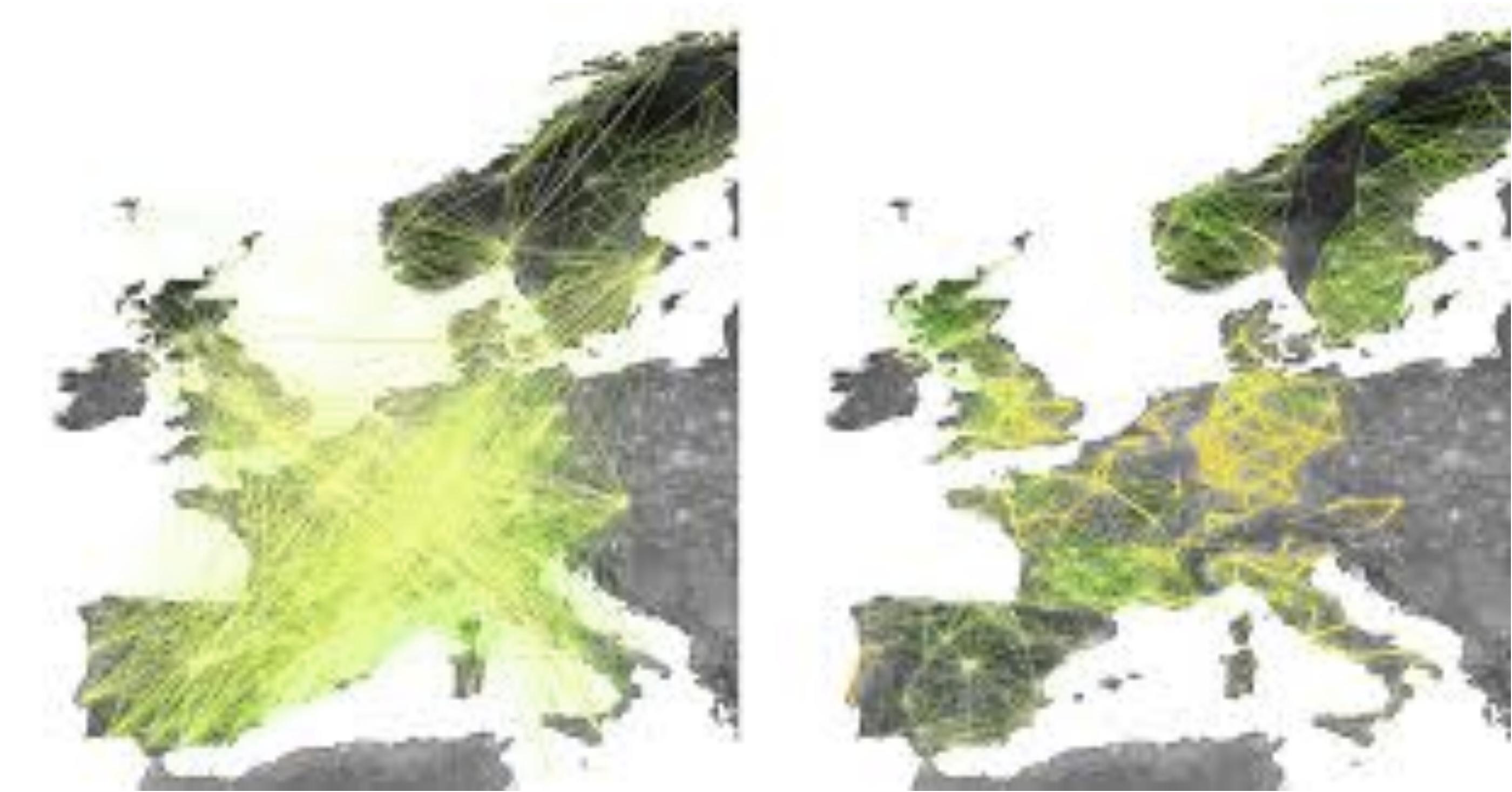


**Spread of memes**

# Underlying network



**Face-to-face contact network**

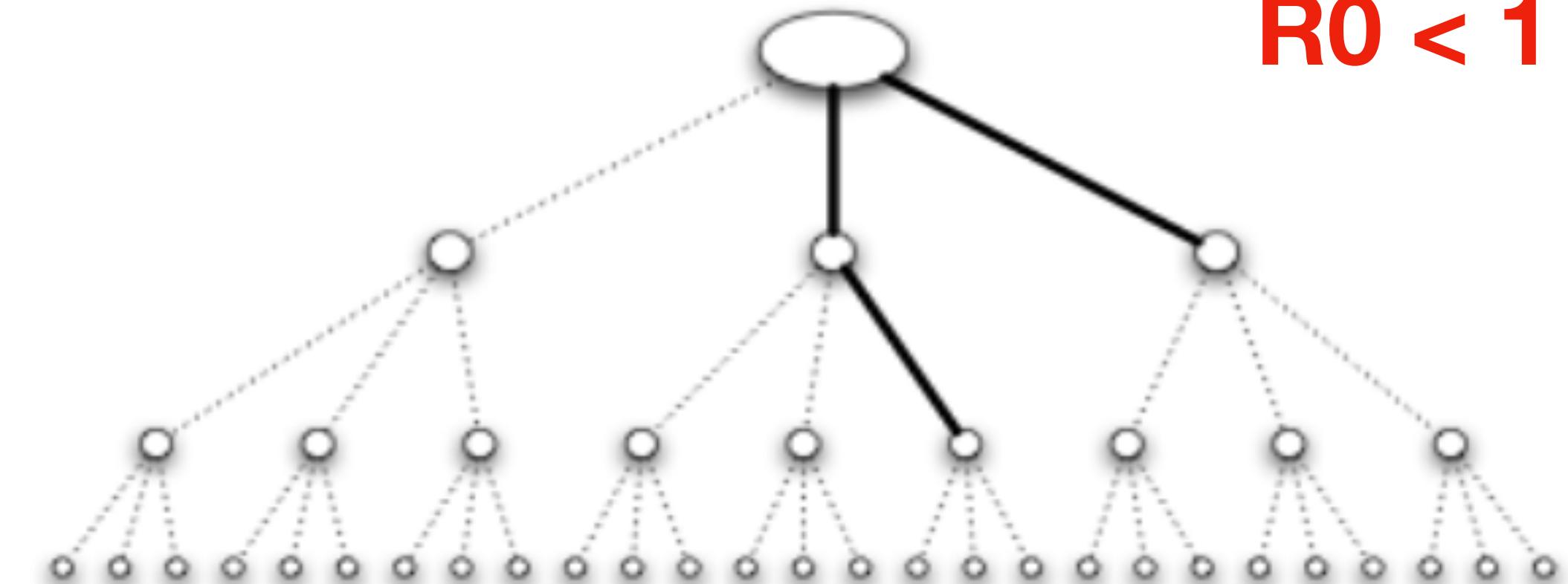
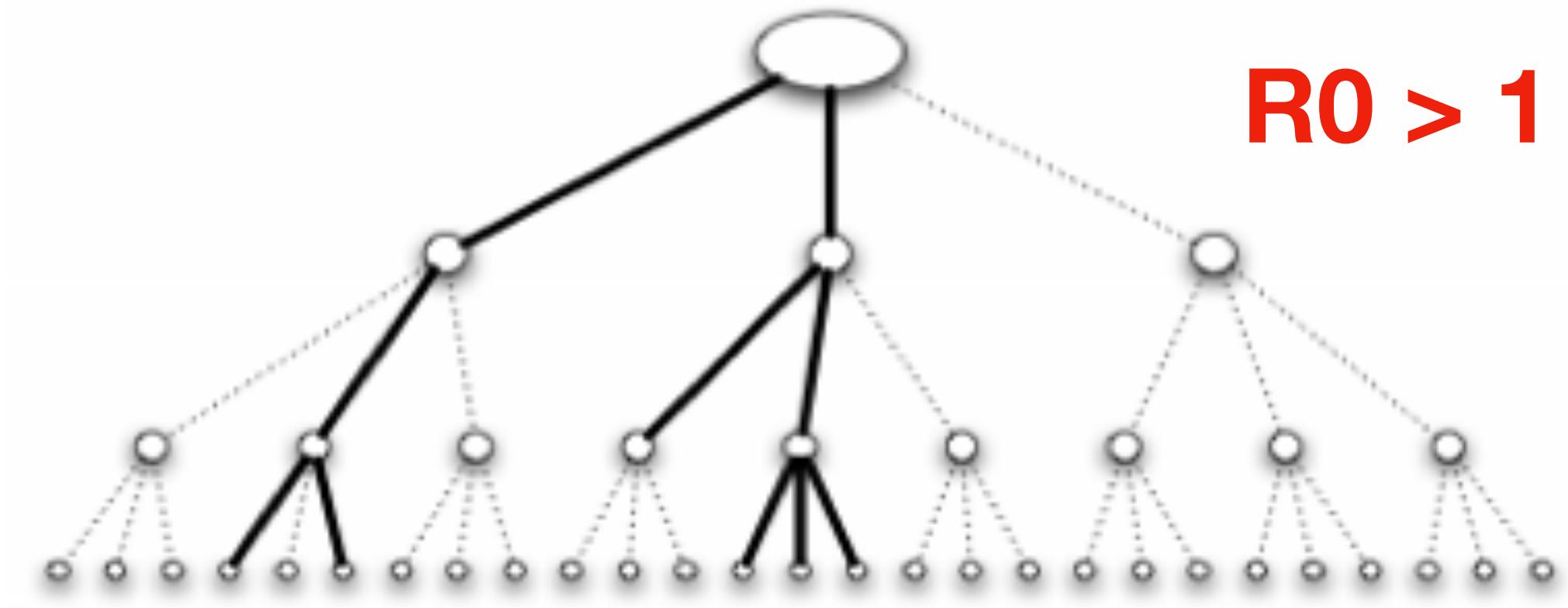


**Airport network (providing more global picture)**

See GLEAM – Global Epidemic and Mobility Model

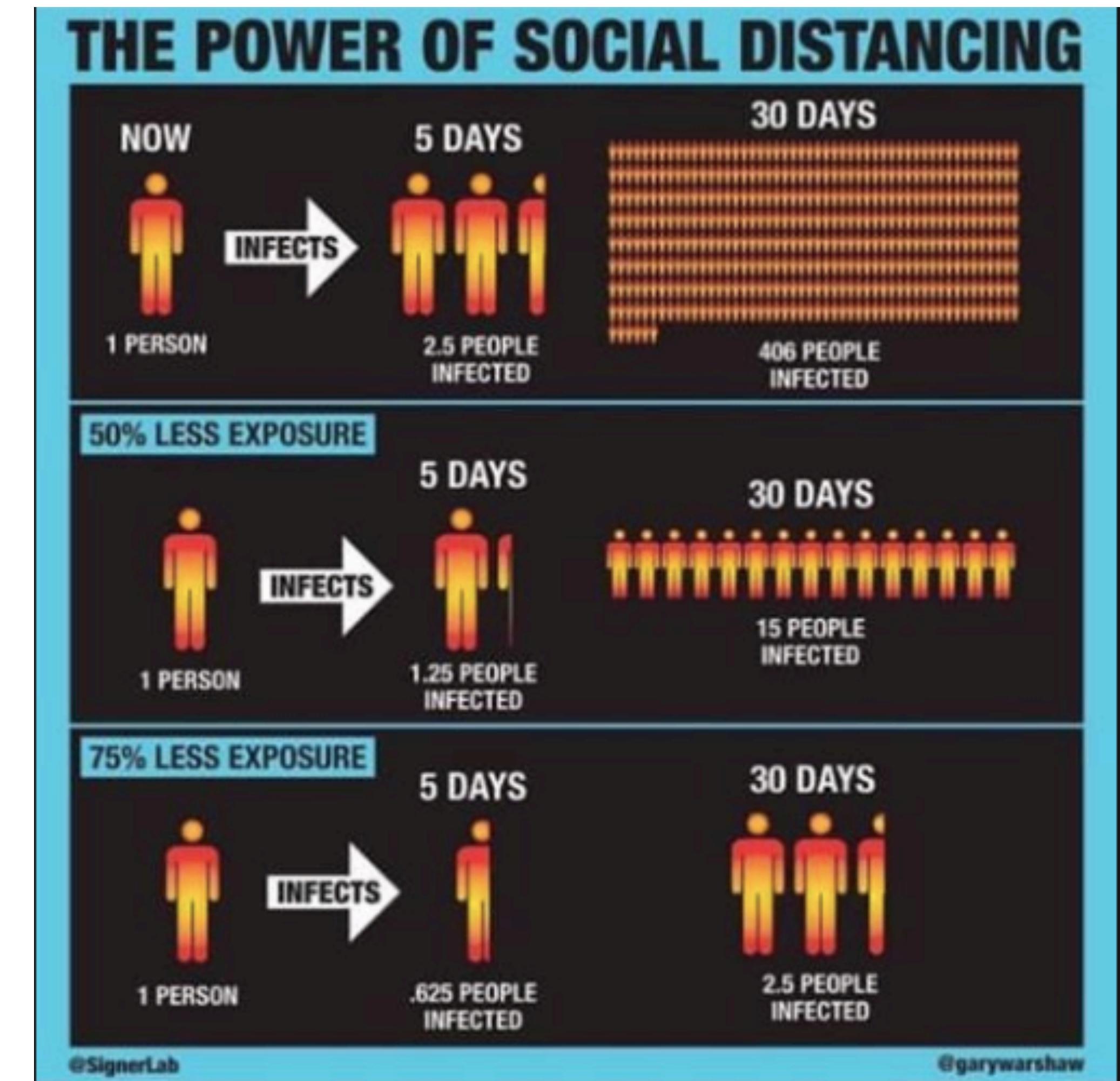
# Simple Epidemic Model

- Start with single infected person in the population.
- **First wave:** Infected person meets  $k$  people, infecting each with probability  $p$ , so  $kp$  new infected individuals after this wave.
- $kp = R_0$  (basic reproductive number) – **number of cases infected by one person**



# Simple Epidemic Model

- **Second wave:** each of these  $k_p$  infected individuals goes on to meet  $k$  people, again infecting each with probability  $p$ .
- $k_p \times k_p = (k_p)^2$  new infected individuals.
- Or in terms of  $R_0$ ,  $(R_0)^2$  in second wave.



# Example: Total # infected

Disease with **R<sub>0</sub> = 2** using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays infected**)

**Initial  
infected  
person**

# Example: Total # infected

Disease with **R<sub>0</sub> = 2** using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays infected**)

1

**Initial  
infected  
person**

# Example: Total # infected

Disease with **R<sub>0</sub> = 2** using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays infected**)

$$1 + 2$$

Initial  
infected  
person



**Two people  
infected in  
first wave**

# Example: Total # infected

Disease with **R<sub>0</sub> = 2** using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays infected**)

$$1 + 2 + 2^2$$

Initial infected person

Two people infected in first wave

Each of those infects two more

# Example: Total # infected

Disease with  $R_0 = 2$  using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual stays infected)

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$$

Initial infected person

Two people infected in first wave

Each of those infects two more

Subsequent waves

# Example: Total # infected

Disease with **R<sub>0</sub> = 2** using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays infected**)

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$$

Initial infected person

Two people infected in first wave

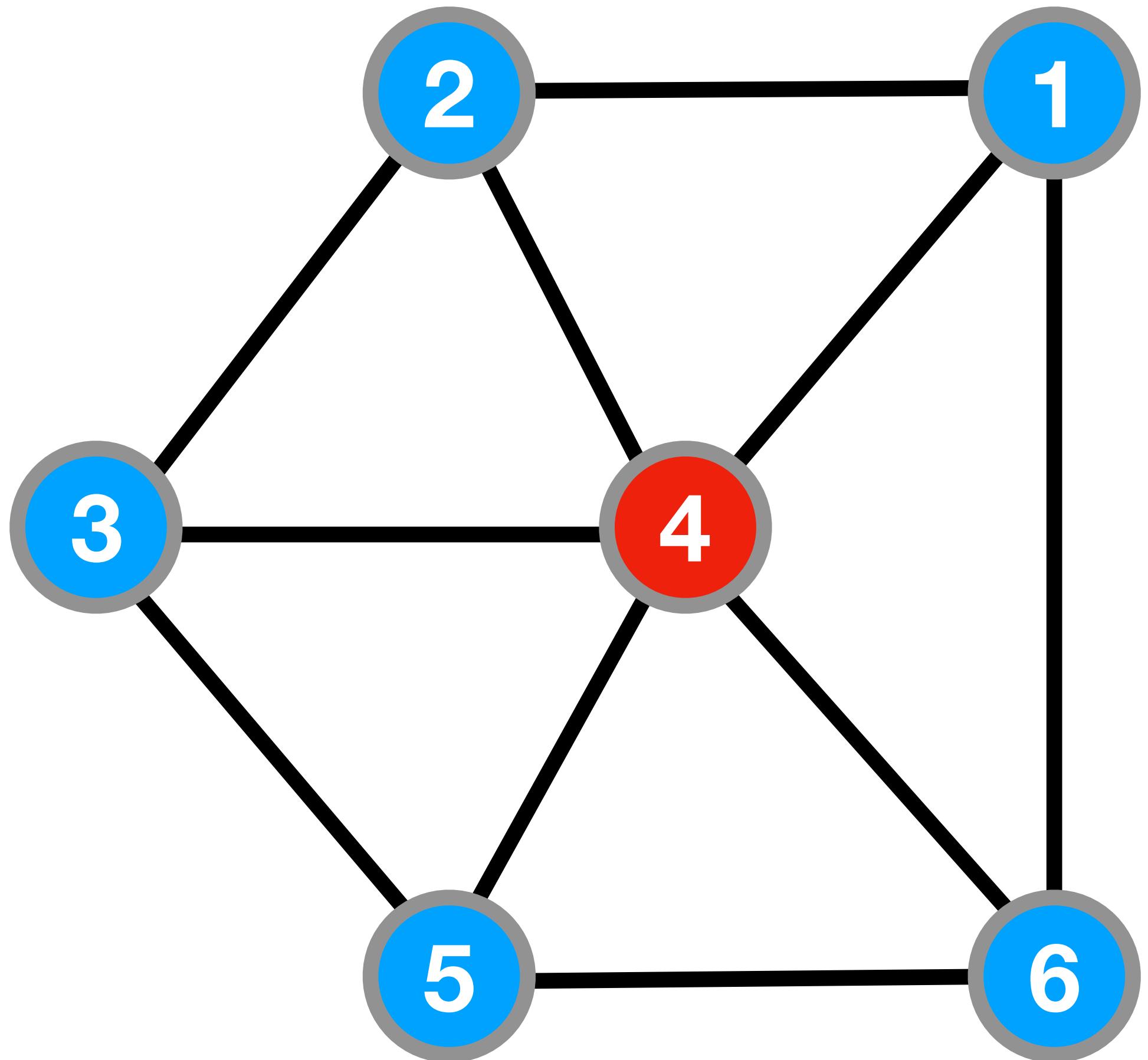
Each of those infects two more

Subsequent waves

Total infected: 63

# Epidemics on networks (SI)

- Nodes are either **susceptible** (S) or **infected** (I). Once infected will never recover.
- An infected node infects its neighbours with a **rate  $\beta$** .
- Ultimately the **whole network** will become infected (provided it's connected).



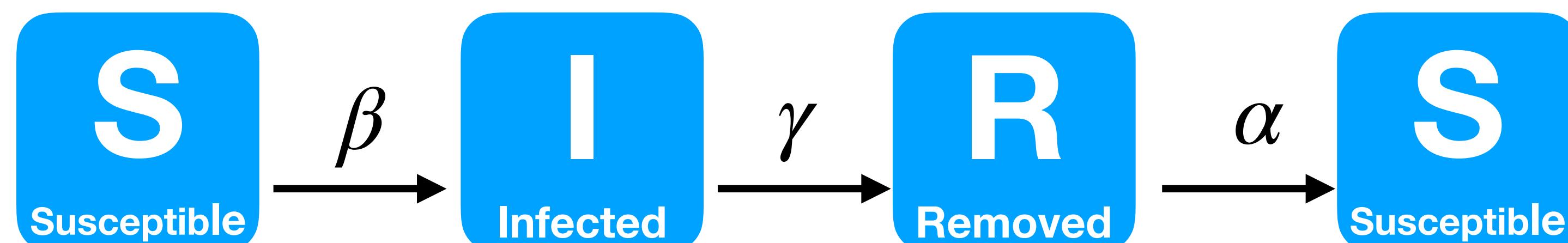
# More realistic models



Node recovers and becomes **susceptible** again after being infected, so can be infected **multiple times**.



Nodes are **recovered/removed** after being infected. This means they are **immune** to the disease.

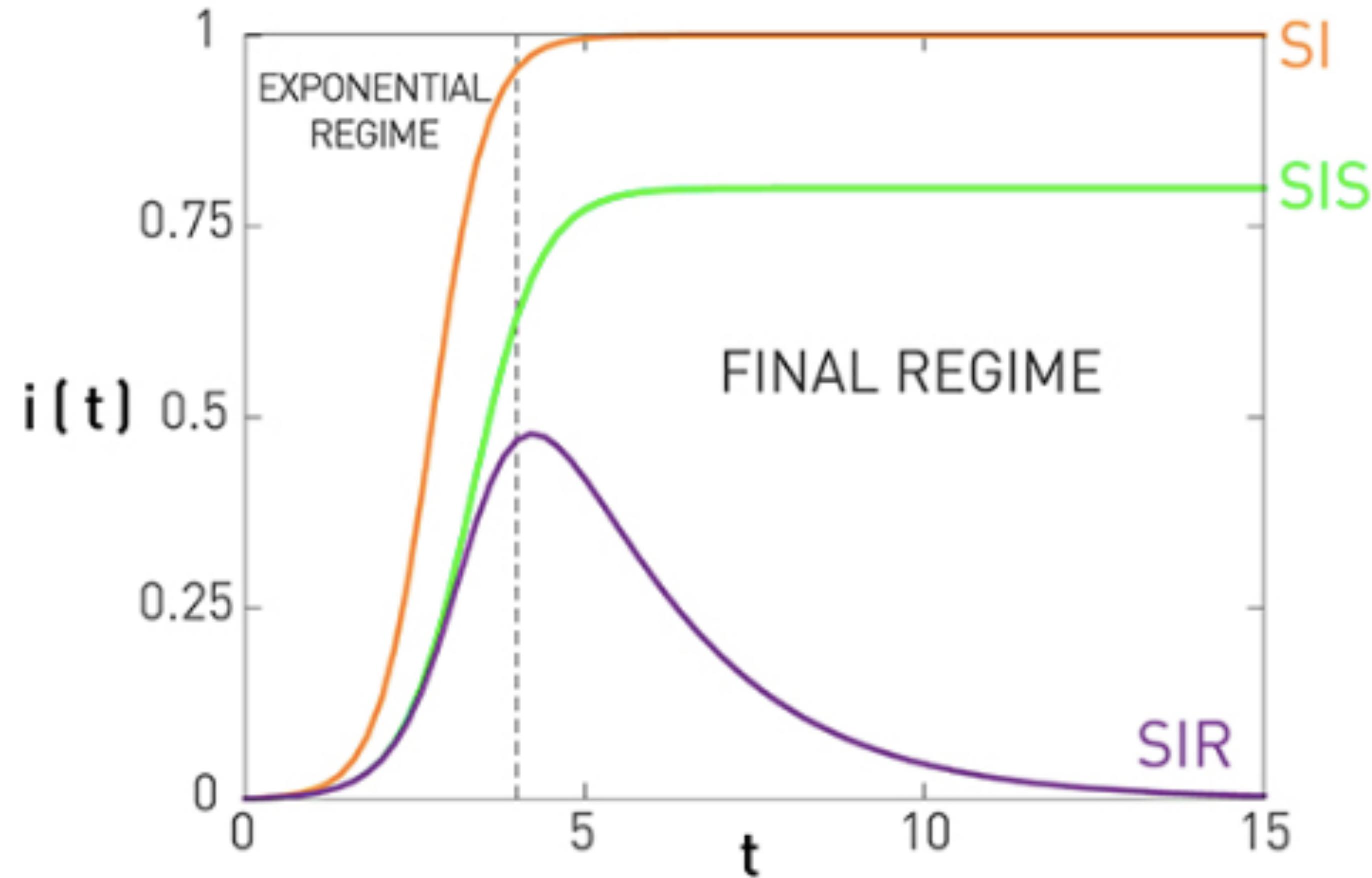


Nodes have **temporary immunity** after being infected

Which to use? Depends on the **disease** and the **application**...

# Infection curves

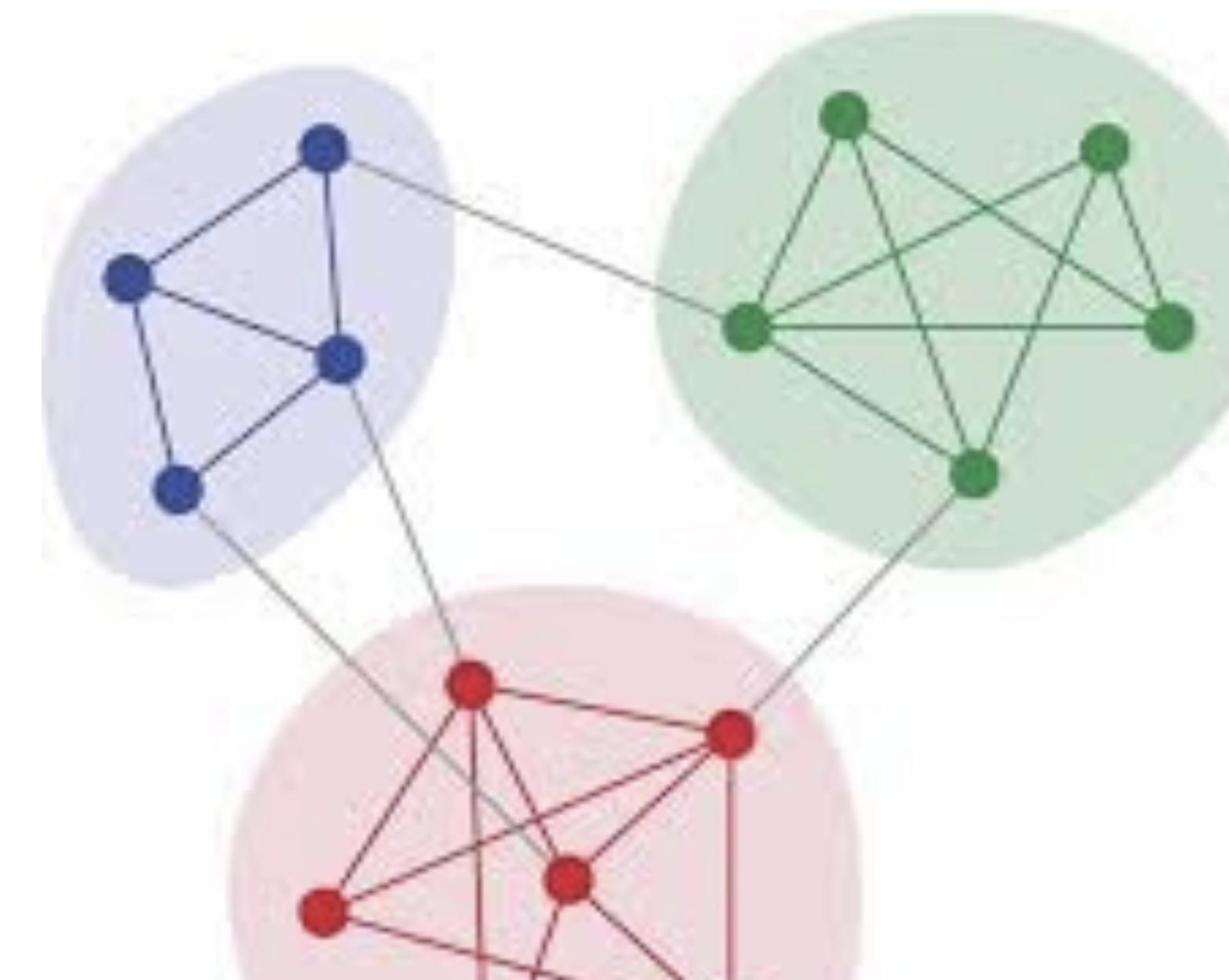
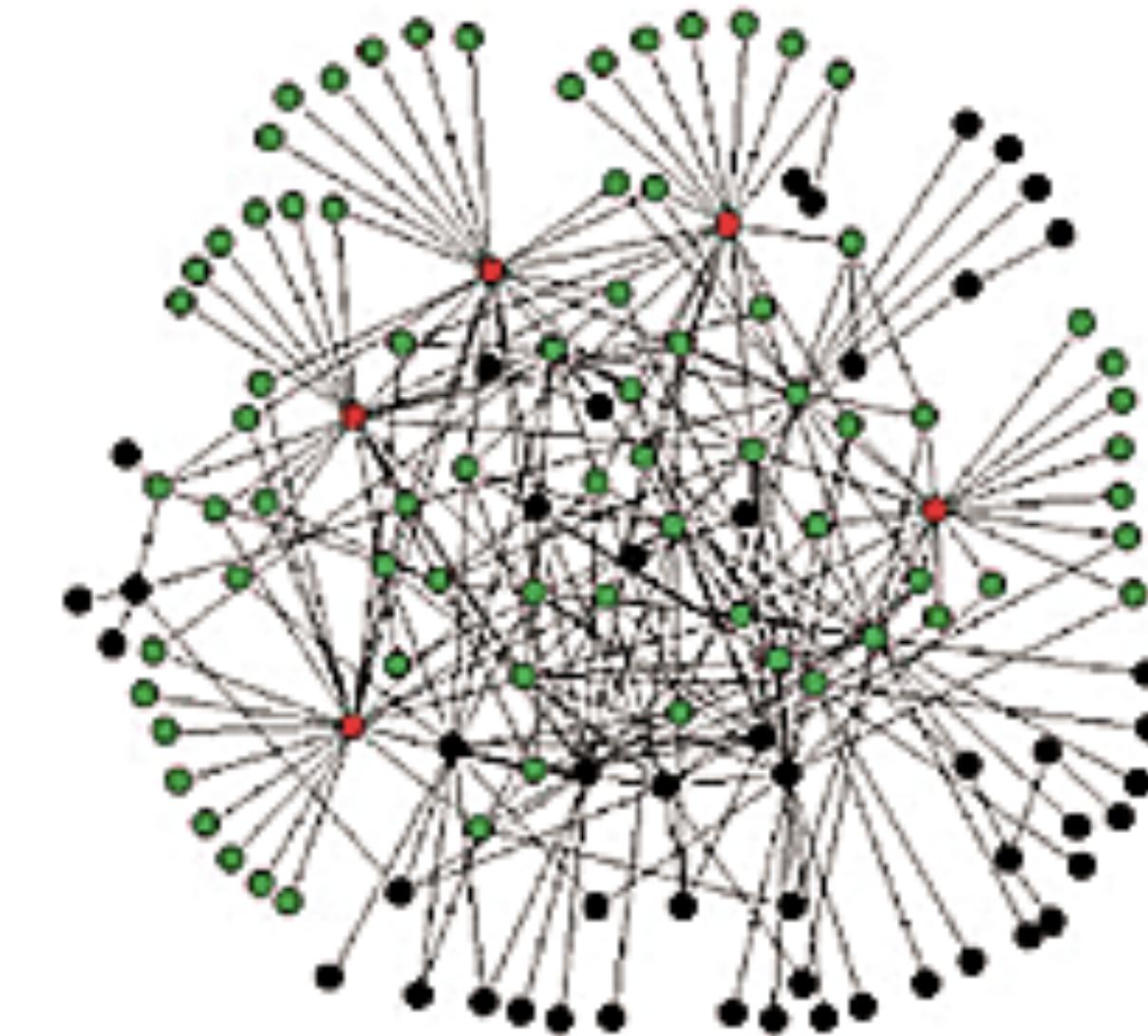
Plot of number of infected individuals over time for different models  
(taken from Barasi's Network Science Book)



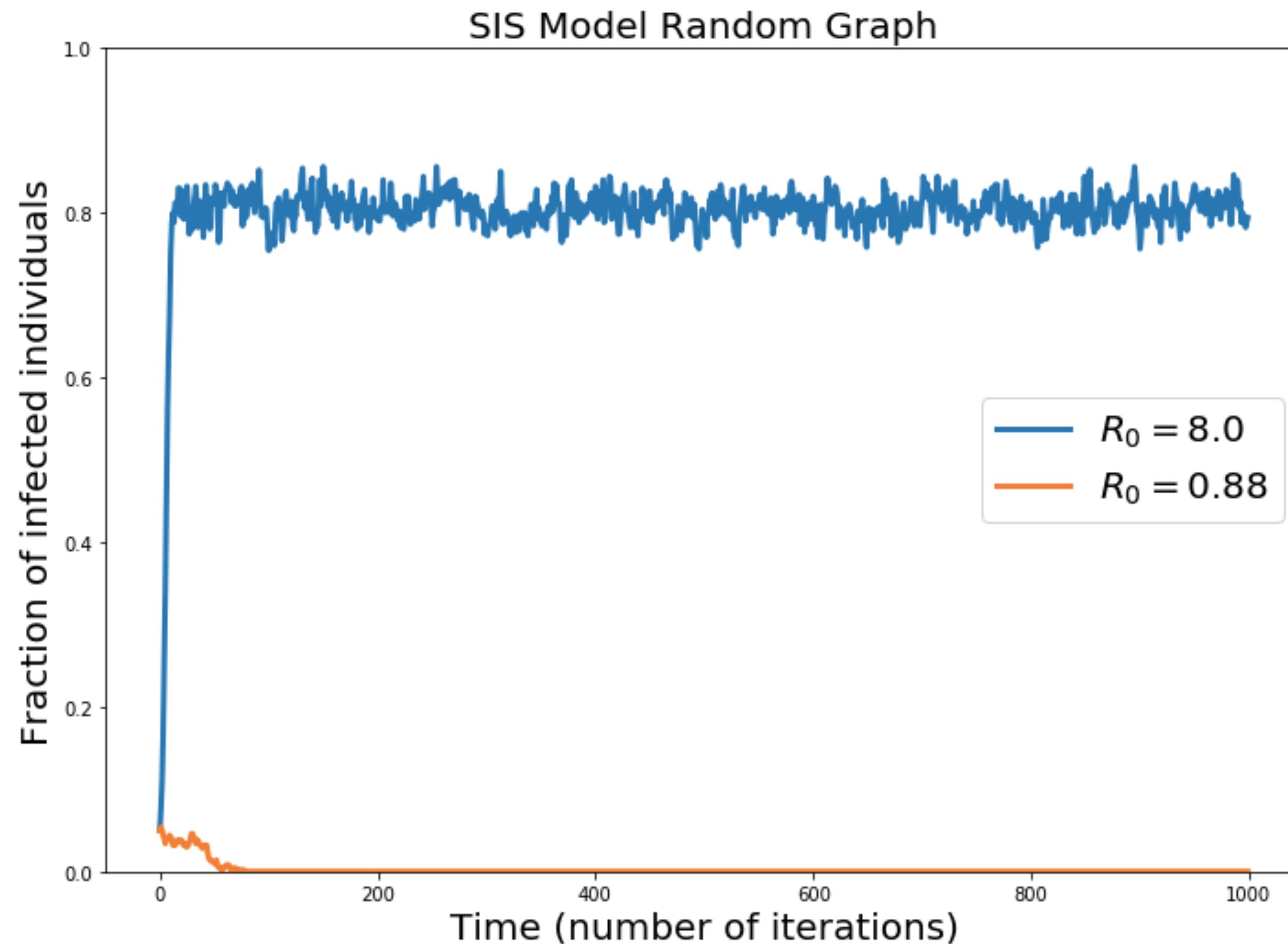
- **SI:** whole population becomes infected
- **SIS:** disease reaches endemic state, where a constant proportion of people infected
- **SIR:** disease hits a peak, after which enough people are immune that the disease dies out

# Role of network structure

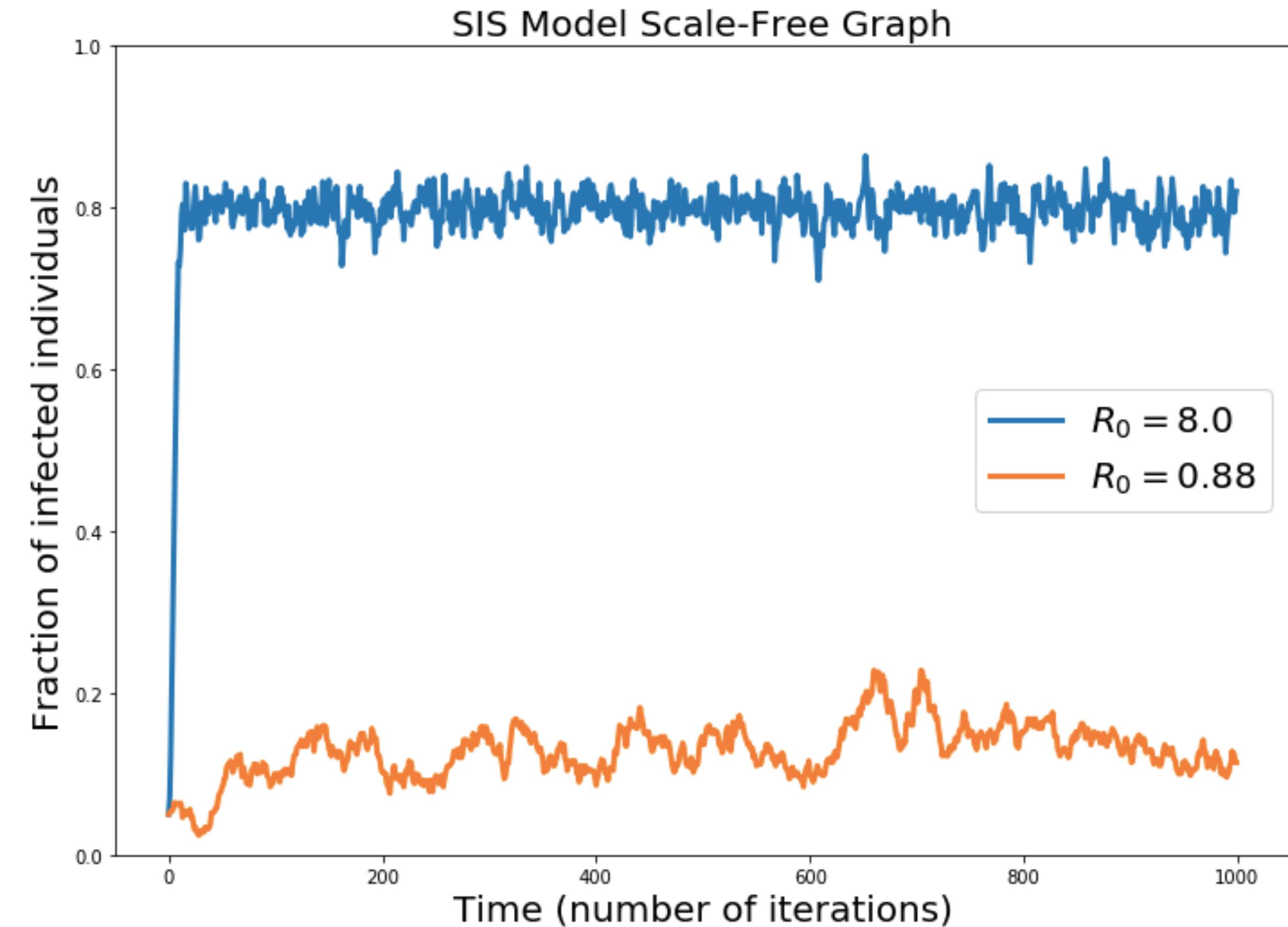
- **Heterogeneous degree distribution** (e.g. scale-free networks, and often real social networks) can **speed up** the spread of diseases, and make them **persist** even if they have **low infection rate**
- Largely due to presence of **highly connected hubs** – perhaps (??) why we seem to have seen so many celebrities who have tested positive for COVID-19
- **Modular structure** (tightly knit communities with few links between) can help slow down spread



# SIS: Random vs Scale-free



Erdos-Renyi graph, disease with  
 $R_0 < 1$  dies out quickly



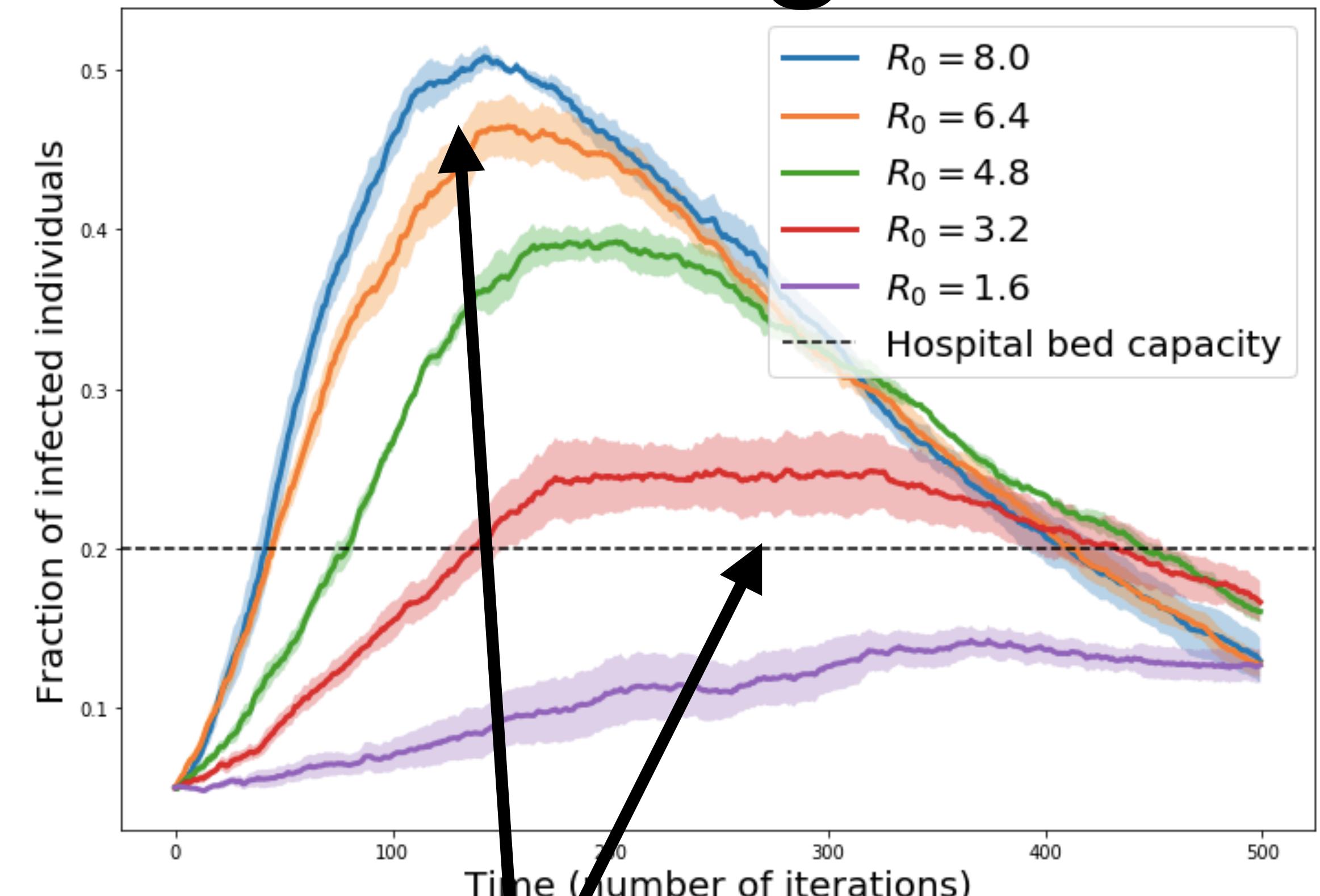
Scale free graph, same disease  
persists

# Modelling epidemic prevention measures

- **Reducing infection probability:** encouraging handwashing, cleaning surfaces, wearing masks
- **Removal of nodes from network:** quarantine, vaccination
- **Reduction of average node degree:** encouraging social distancing
- **Removal of edges between communities:** travel restriction

# Epidemic prevention: Reducing “R<sub>0</sub>”

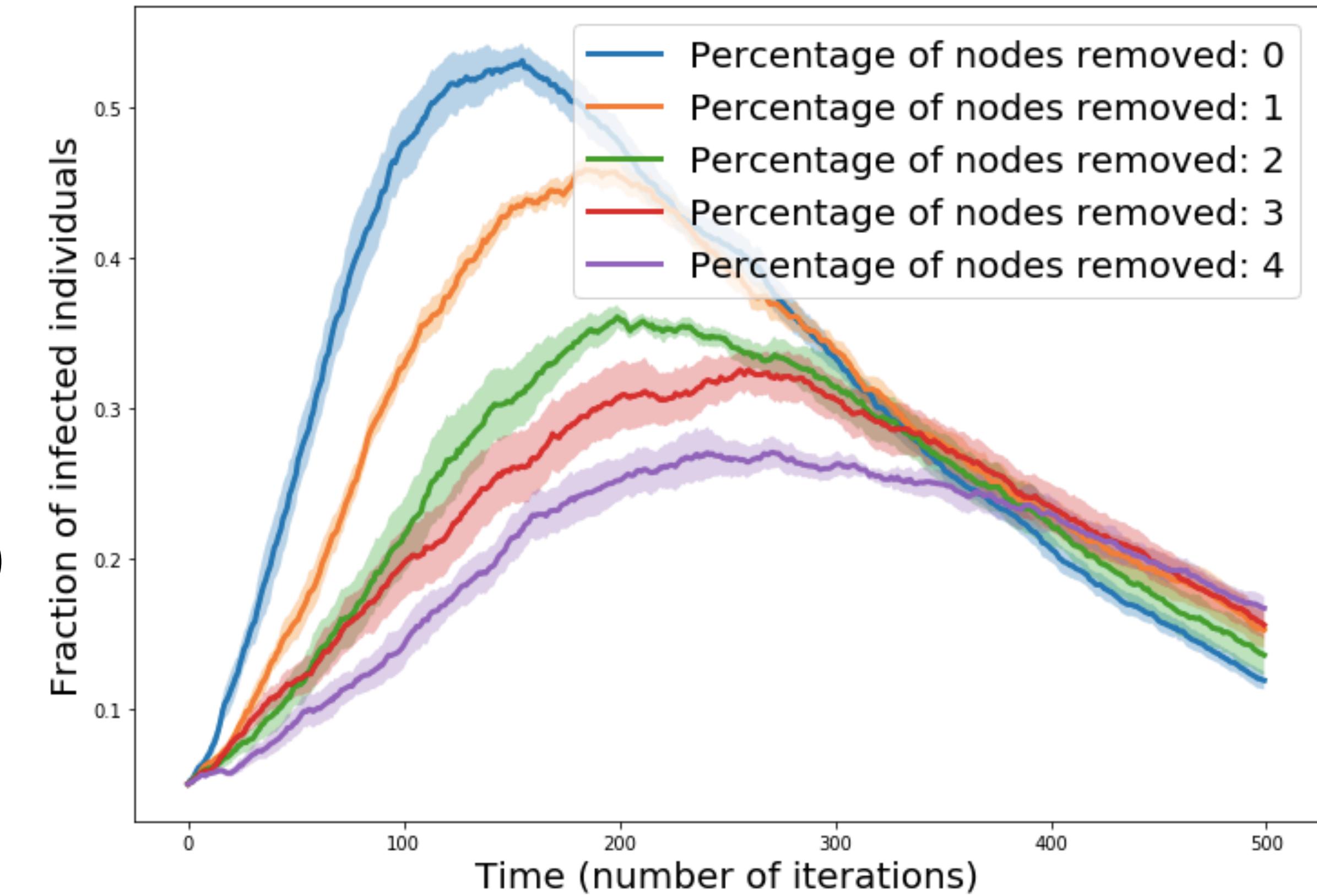
- Reducing the chance of transmission from person to person.
- Handwashing techniques, wearing a mask, keeping 1m+ apart



Reducing R<sub>0</sub> not only reduces the **size of peak**, but pushes it **later in time**

# Epidemic prevention: removing nodes/links

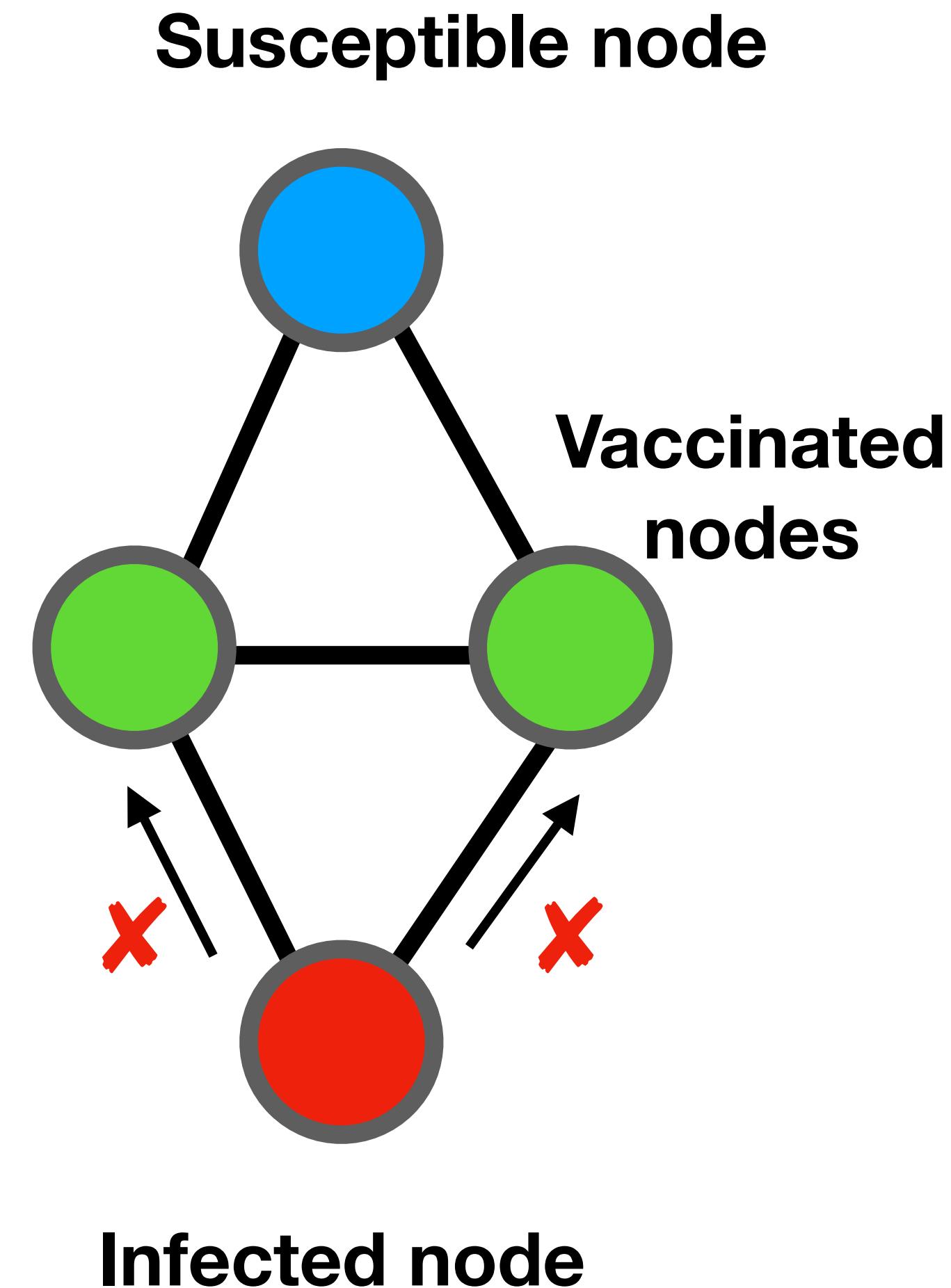
- Remove **nodes** – vaccination or quarantine of certain individuals
- Remove **edges** – pair of individuals cut contact altogether
- (in transport network, mobility restrictions)



**Targeted immunisation in scale-free network (Pastor-Satorras et al) – removing nodes reduces and pushes back the infection peak, but gains decrease af**

# Herd immunity

- Cannot vaccinate **whole** of population (some too **vulnerable** to be vaccinated)
- Depending on network structure, if **enough people** are immune, the disease will die out (immune people act as blockers)



# Conclusions

- **Models of epidemics** on networks can be key for providing insights into how a disease spreads through a population.
- **More complex** models available which can give more **precise guidance** on measures to suppress or mitigate epidemics.
- **Network structure** plays a huge role (scale-free vs random, modular structure)
- **Challenges:** the true “network” is often **unknowable** and spreading processes are complex, **economic and social consequences** to whichever course of action taken that are hard to predict.