Contents

Liouville (theorem 2.3) **2**

```
sage: v = matrix([[1, a, b, a*b, b^2, a*b^2]]).transpose()
sage: v
[    1]
[    a]
[    b]
[  a*b]
[  b^2]
[a*b^2]
sage: A = matrix([
....:     [0, 1, 0, 0, 0, 0],
....:     [2, 0, 0, 0, 0, 0],
....:     [0, 0, 0, 1, 0, 0],
....:     [0, 0, 2, 0, 0, 0],
....:     [0, 0, 0, 0, 0, 1],
....:     [0, 0, 0, 0, 2, 0]
....: ])
sage: A*v
[    a]
[    2]
[  a*b]
[  2*b]
[a*b^2]
[2*b^2]
sage: B = matrix([
....:     [0, 0, 1, 0, 0, 0],
....:     [0, 0, 0, 1, 0, 0],
....:     [0, 0, 0, 0, 1, 0],
....:     [0, 0, 0, 0, 0, 1],
....:     [2, 0, 0, 0, 0, 0],
....:     [0, 2, 0, 0, 0, 0]
....: ])
sage: B*v
[    b]
[  a*b]
[  b^2]
[a*b^2]
[    2]
[  2*a]
sage: A_B = A + B
sage: matrix.identity(6)
[1 0 0 0 0 0]
[0 1 0 0 0 0]
[0 0 1 0 0 0]
[0 0 0 1 0 0]
[0 0 0 0 1 0]
[0 0 0 0 0 1]
sage: x*matrix.identity(6)
[x 0 0 0 0 0]
[0 x 0 0 0 0]
[0 0 x 0 0 0]
[0 0 0 x 0 0]
[0 0 0 0 x 0]
[0 0 0 0 0 x]
sage: ( x*matrix.identity(6) - A_B ).determinant()
x^6 - 6*x^4 - 4*x^3 + 12*x^2 - 24*x - 4
```

Liouville (theorem 2.3)

$$|\alpha - p/q| > 1 > 1/q^n$$

So lets take $|a - p/q| \leq 1$.

Mean value theorem gives us $f'(\gamma)$.

$q^n f(p/q)$ is an integer means $|f(p/q)| \geq 1/q^n$.

$$\alpha < \gamma < p/q, |\alpha - p/q| \leq 1 \Rightarrow |\gamma - \alpha| < 1$$

Then observe

$$|f(\alpha) - f(p/q)| < C|\alpha - p/q| < C$$

So then $f'(\gamma) < C = 1/c_0$.

Combine these

$$\left|\alpha - \frac{p}{q}\right| = \left|\frac{p/q}{f'(\gamma)}\right| > \frac{c_0}{q^n}$$