Contents

We take the morphism earlier from question 1.6, and create a valid morphism using the method in section 2.

```
sage: K.<x, y, z> = PolynomialRing(QQ)
sage: I = Ideal(x, y - 1)
sage: I
Ideal (x, y - 1) of Multivariate Polynomial Ring in x, y, z over Rational Field
sage: x in I
True
sage: x in I^2
False
sage: y in I^2
False
sage: y in I
False
sage: y - 1 in I^2
False
sage: S = K.quotient(y^2 - x^3 - 1)
sage: I = Ideal(S(x), S(y - 1))
sage: I
Ideal (xbar, ybar - 1) of Quotient of Multivariate Polynomial Ring in x, y, z over Rational Field by th
sage: x in I
True
sage: x in I^2
False
sage: y in I^2
False
sage: y - 1 in I^2
True
sage: x in I
True
sage: x*y in I
True
sage: x*y in I^2
False
sage: S = K.quotient(y^2*z - x^3 - z^3)
sage: I = Ideal(S(x), S(y - 1), S(z))
sage: x in I, x in I^2
(True, False)
sage: y - 1 in I, y - 1 in I^2
(True, False)
sage: y in I, y in I^2
(False, False)
sage: z in I^2
True
sage: z in I^3
True
sage: z in I^6
False
sage: z in I^4
False
sage: z^3 in I^6
True
sage: z^2 in I^6
True
sage: z^2 in I^7
False
sage: x*y in I
True
sage: x*y in I^2
```

```
False
sage: (I^2).gens()
[xbar^2, xbar*ybar - xbar, xbar*zbar, xbar*ybar - xbar, ybar^2 - 2*ybar + 1, ybar*zbar - zbar, xbar*zba
sage: T = S.quotient(I^2)
sage: T
Quotient of Multivariate Polynomial Ring in x, y, z over Rational Field by the ideal (x^2, x*y - x, x*z
sage: T(y^2*z - 2*y*z + z) == T(x^3 + z^3 - 2*y*z + z)
True
sage: T(x^3 + z^3 - 2*y*z + z) == T(-2*y*z + z)
True
sage: T(-2*y*z + z) == T(y*z + y*z - z)
True
sage: T(y*z) == 0
True
sage: T(z) == 0
True
sage: T(y^2 - x^3 - 1) == 0
False
sage: K.<x, y> = PolynomialRing(QQ)
sage: S = K.quotient(y^2 - x^3 - 1)
sage: I = Ideal(S(x), S(y - 1))
sage: T = S.quotient(I^2)
sage: T(y^2 - x^3 - 1) == 0
True
sage: T(y^2 - 2*y + 1) == 0
True
sage: T(x^3 + 1 - 2*y + 1) == 0
True
sage: T( - 2*y + 1) == 0
False
sage: T(1 - 2*y + 1) == 0
True
sage: T(2 - 2*y) == 0
True
sage: T(1 - y) == 0
True
sage: T(y - 1) == 0
True
sage: y - 1 in I, y - 1 in I^2
(True, True)
sage: x in I, x in I^2
(True, False)
sage: y - 1 in I, y - 1 in I^2, y - 1 in I^3
(True, True, True)
sage: y - 1 in I, y - 1 in I^2, y - 1 in I^3
(True, True, True)
sage: y - 1 in I, y - 1 in I^2, y - 1 in I^3, y - 1 in I^4
(True, True, True, False)
sage: I^4
Ideal (xbar*ybar^2 - xbar, ybar^3 - ybar^2 - ybar + 1, ybar^3 - ybar^2 - ybar + 1, xbar^2*ybar^2 - 2*xb
sage: (I^4).groebner_basis()
[xbar*ybar - xbar, ybar^2 - 2*ybar + 1]
sage: (I^3).groebner_basis()
[ybar - 1]
```