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Exercise 2.2.8

$$f(x, y) = (bx - ay)^k g(x, y)$$

Differentiating f we get

$$f'(x, y) = k(bx - ay)^{k-1} g(x, y) + (bx - ay)^k g'(x, y)$$

Note that constants don't matter to us so we can abstract this system

$$f = u^k g, \quad u' = 1, \quad A + A = A$$

$$f' = (u^k g)' = u^{k-1} g + u^k g'$$

Applying the rule recursively, we see that eventually g is a term. But we know that $g(P) \neq 0$ and $u(P) = 0$.

$$\begin{aligned} f &= u^k g \\ f' &= u^{k-1} g + u^k g' \\ f'' &= u^{k-2} g + u^{k-1} g' + u^k g'' \\ f''' &= u^{k-3} g + u^{k-2} g' + u^{k-1} g'' + u^k g''' \\ f'''' &= u^{k-4} g + u^{k-3} g' + u^{k-2} g'' + u^{k-1} g''' + u^k g'''' \end{aligned}$$

Exercise 2.2.12

$$\begin{aligned} f(x, y, z) &= (t_0 s - s_0 t)^k g(a_1 s + b_1 t, a_2 s + b_2 t, a_3 s + b_3 t) \\ &= (t_0 s - s_0 t)^k g(c_1 u + d_1 v, c_2 u + d_2 v, c_3 u + d_3 v) \\ &= (u_0 v - v_0 u)^j g(c_1 u + d_1 v, c_2 u + d_2 v, c_3 u + d_3 v) \end{aligned}$$

Because (s, t) and (u, v) define the same line, there is a transform between them defined by

$$\begin{aligned} u &= \alpha_1 s + \beta_1 t \\ v &= \alpha_2 s + \beta_2 t \end{aligned}$$

Now we saw that $(t_0 s - s_0 t)^k = (u_0 v - v_0 u)^j$ so

$$\begin{aligned} (t_0 s - s_0 t)^k &= (u_0 v - v_0 u)^j \\ &= (u_0(\alpha_2 s + \beta_2 t) - v_0(\alpha_1 s + \beta_1 t))^j \\ &= (s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0))^j \end{aligned}$$

Taking $s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0)$ which is a linear equation and substituting (s_0, t_0) , see that that it's zero and so conclude

$$\begin{aligned} t_0 s - s_0 t &= s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0) \\ &\Rightarrow k = j \end{aligned}$$

Alternatively we can argue by expanding both sides out and observing the degrees of s and t that $k = j$.