assignment

January 6, 2024

CBSE 2019 Mathematics Questions: 56.5.3

Vector Algebra

1. Find the equation of planes passing through the intersection of planes $\overrightarrow{r}.\left(3\hat{i}+6\hat{j}\right)+12=0$ and $\overrightarrow{r}.\left(3\hat{i}-\hat{j}+4\hat{k}\right)=0$ and are at a unit distance from origin.
2. Find the angle between the line $\overrightarrow{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ and the plane $\overrightarrow{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=3$.
3. Find the co-ordinates of the point where the line $\frac{x+2}{1}=\frac{y-5}{3}=\frac{z+1}{5}$ cuts the yz-plane.
4. Using vectors, prove that the points $(2,-1,3)$, $(3,-5,1)$ and $(-1,11,9)$ are co-linear.
5. For any two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ prove that
$$\left(\overrightarrow{\mathbf{a}}\times\overrightarrow{\mathbf{b}}\right)^2=\overrightarrow{\mathbf{a}}^2\overrightarrow{\mathbf{b}}^2-\left(\overrightarrow{\mathbf{a}}.\overrightarrow{\mathbf{b}}\right)^2$$
6. Using integration, find the area of $\triangle ABC$ bounded by the lines
$$\begin{aligned}4x-y+5&=0,\\x+y-5&=0,\\x-4y+5&=0.\end{aligned}$$

Matrices

7. If A=$\begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$, then find$\left|\text{adj}.A\right|$.

8. Using properties of determinants, prove that following:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

9. Using matrices solve the following system of linear equations:

$$\begin{aligned} 2x+3y+10z &= 4 \\ 4x+6y+5z &= 1 \\ 6x+9y-20z &= 2 \end{aligned}$$

10. Find the value of $(x-y)$ from the matrix equation

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Functions

11. Let an operation *on the set of natural number N be defined by $a * b = a^b$. Find (i) where the *is a binary or not and (ii) if it is a binary, then is it commutative or not.

12. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

13. Find the interval in which the function f given by

$$f(x) = \sin 2x + \cos 2x, 0 \leq x \leq \pi$$

is strictly decreasing.

Integration

14. Using integration, find the area of following region :$\left\{(x, y) : x^2 + y^2 \leq 16a^2\right\}$ and $\left\{y^2 \leq 6ax\right\}$

15. Evaluate :

$$\int_{-1}^{2} |x^2 - x| \, dx.$$

16. Find

$$\int (\cos 2x \cos 4x \cos 6x) \, dx.$$

17. Find :

$$\int e^x \left(\frac{2 + \sin 2x}{2 \cos^2 x} \right) dx$$

18. Solve the equation differential equation

$$\left(y + 3x^2\right) \frac{dx}{dy} = x \, dx$$

Differentiation

19. If $y = 2\sqrt{\sec\left(e^{2x}\right)}$, than find $\frac{dy}{dx}$.

20. If $y = (\sin x)^x + \sin^{-1}\left(\sqrt{1 - x^2}\right)$, then find $\frac{dy}{dx}$.

21. If

$$y = 5e^{7x} + 6e^{-7x}$$

show that $\frac{d^2y}{dx^2} = 49y$

22. Find the differential equation of the family of curves represented by $y^2 = a\left(b^2 - x^2\right)$.

23. Find the order of differential equation of the family of circles of radius 3 units.

24. Find the differential equation representing the family of curves

$$y = -A\cos 3x + B\sin 3x$$

Probability

25. A bag contains 5 red and 3 black balls and another bag contain 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags an both are found to be red. Find the probability that balls are drawn from the first bag.

26. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

27. In answering a question on a multiple choice questions with four choices in each question out of which only one is correct a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question given that he correctly answered it.

Optimization

28. The sum of the perimeters of circle and a square is K is some constant. Prove that the sum of their area is least when the side of the square is twice the radius of the radius of the circle.

29. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

30. A company manufactures two types of novelty souvenirs made of plywood. souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4hours for assembling. The profit for type *A* souvenirs is ₹100 each and for type *B* souvenirs, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximum the profit ? Formulate the problem as a LPP and then solve it graphically.