Assignment

January 4, 2024

CBSE 2019 Mathematics Questions: 56.5.3

1. Find the order of the differential equation of the family of circle of radius 3 units.
2. Find the angle between the lines $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane. $\overrightarrow{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.
3. Find the co-ordinate of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cut the yz-plane.
4. If A=$\begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$,then find$\left|adj.A\right|$.
5. If $y = 2\sqrt{\sec\left(e^{2x}\right)}$, than find $\frac{dy}{dx}$.
6. Four card are drawn one by one with replacement from a well-shuffled deck of playing card. Find the probability that at least three cards are of diamonds.
7. The probability of two students A and B coming to school on time are $\frac{2}{7}$and $\frac{4}{7}$ respectively. assuming that the event A coming on the time and B coming on time are independent, Find the probability of only one of them coming to school on time.

8. Using vectors, prove that the points $(2,-1,3)$, $(3,-5,1)$and $(-1,11,9)$ are co-linear.

9. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$,prove that $\left(\overrightarrow{a} x \overrightarrow{b}\right)^2 = \overrightarrow{a}^2 \overrightarrow{b}^2 - \left(\overrightarrow{a} . \overrightarrow{b}\right)^2$.

10. Find

$$\int \frac{x-1}{(x-2)(x-3)} dx.$$

11. Integrate:

$$\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$$

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12. If $P(A) = 0.6, P(B) = 0.5$ and $P(B|A) = 0.4$, Find $P(A \bigcup B)$ and $P(A|B)$.

13. Find the value of $(x-y)$ from the matrix equation $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$.

14. Find

$$\int e^x (\cos x - \sin x) \csc^2 x dx. \tag{1}$$

15. Find the differential equation of the family of curves represented by $y^2 = a\left(b^2 - x^2\right)$.

16. Let an operation *on the set of natural number N be defined by $a * b = a^b$.Find (i) where the *is a binary or not, and (ii) if it is a binary, then is it commutative or not.

17. Find the particular solution of the differential equation :

$$\left(1+e^{2x}\right) dy + \left(1+y^2\right) e^x dx = 0.$$

given that $y(0) = 1$.

18. Find the particular solution of differential equation :

$$x\frac{dy}{dx}\sin\frac{y}{x} + x - y\sin\frac{y}{x} = 0$$

,given that$y(1) = \frac{\pi}{2}$.

19. Prove that the relation r in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = (a, b)$: $(|a - b|)$is even,an equivalence relation.

20. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$defined as $f(x) = \frac{4x+3}{6x-4}$is one-one and onto.hence,find f^{-1}.

21. Find whether the function f$(x) = \cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

22. Using vector find the value of x such that that four points $A(x, 5, -1)$, $B(3, 2, 1)$, $c(4, 5, 5)$ and D$(4, 2, -2)$ are co-planar.

23. Prove that :

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}. \tag{2}$$

24. If $x^P y^Q = (x + y)^{P+Q}$,prove that $\frac{dy}{dx} + \frac{y}{x}$and $\frac{d^2y}{dx^2} = 0$.

25. Find

$$\int (\sin x. \sin 2x. \sin 3x)\, dx. \tag{3}$$

26. Differentiate $\tan^{-1}\frac{3x - x^3}{1 - 3x^2}, |x| < \frac{1}{\sqrt{3}}$ w.r.t.tan $-1\frac{x}{\sqrt{1 - x^2}}$.

27. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, $|x| < 1, |y| < 1$,show that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

28. Using properties of determinants, prove that following:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

29. Evaluate :

$$\int_{-1}^{2} |x^2 - x| dx. \tag{4}$$

30. Find the equation of planes passing through the intersection of planes $\vec{r}.\left(3\hat{i}+6\hat{j}\right)+12=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}+4\hat{k}\right)=0$ and are at a unit distance from origin.

31. Using integration, find the area of following region : $\left\{(x, y) : x^2 + y^2 \leq 16a^2 and y^2 \leq 6ax\right\}$.

32. Using integration, find the area of $\triangle ABC$ bounded by the lines

$$4x - y + 5 = 0, \tag{5}$$
$$x + y - 5 = 0 \tag{6}$$
$$x - 4y + 5 = 0. \tag{7}$$

33. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line.

$$\vec{r} = \left(\hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)\right). \tag{8}$$

Also, find the distance between these two lines.

34. Find the coordination of the foot Q of the perpendicular drawn from the point $P(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

35. Using elementary row transformation, find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

36. Using matrix,solve the following system of linear equations:

$$2x + 3y + 10z = 4 \tag{9}$$
$$4x - 6y + 5z = 1 \tag{10}$$
$$6x + 9y - 20z = 2. \tag{11}$$

37. The sum of the perimeters of circle and a square is K is some constant. Prove that the sum of their area is least when the side of the square is twice the radius of the radius of the circle.

38. A bag contains 5 red and 3 black balls and another bag contain 2 red and 6 black balls.Two balls are drawn at random ($without replacement$) from one of the bags an both are found to be red. Find the probability that balls are drawn from the first bag.

39. If x, y, z are different and $\delta = \begin{bmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{bmatrix} = 0$, then using properties of determination,show that $xyz = 1$.

40. Using vectors, find the value f x such that the four option $A\left(x, 5, -1\right), B\left(3, 2, 1\right), C\left(4, 5, 5\right)$ and D $(4, 2, -2)$are co-planar.

41. differentiate $\tan^{-1} \dfrac{3x - x^3}{1 - 3x^2}, |x| < \dfrac{1}{\sqrt{3}}$ w.r.t. $\tan^{-1} \dfrac{x}{\sqrt{1 - x^2}}$.

42. If $y = (\sin x)^x + \sin -1 \left(\sqrt{1 - x^2} \right)$, then find $\dfrac{dy}{dx}$.

43. Find

$$\int (\cos 2x \cos 4x \cos 6x)\, dx. \tag{12}$$

44. Find the interval in which the function f given by

$$f(x) = \sin 2x + \cos 2x, 0 \leq x \leq \pi \tag{13}$$

is strictly decreasing.

45. A company manufactures two types of novelty souvenirs made of plywood. souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling.souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. there are 3 hours 20 minutes available for cutting and 4hours for assembling. the profit for type *A* souvenirs is ₹100 each and for type *B* souvenirs, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximum the profit ? Formulate the problem as a LPP and then solve it graphically.

46. A card from a pack of 52 playing cards is lost. from the remaining cards of the pack,two cards are drawn at random (*without*) and both are found to be spades. Find the probability of the lost card being a spade.

47. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.