Discrete Mathematics
R204GA05401

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Objectives

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Objectives

- This course will introduce and illustrate in the elementary discrete mathematics for computer science and engineering students.
- To equip the students with standard concepts like formal logic notation, methods of proof, induction, sets, relations, graph theory, permutations and combinations, counting principles.

Course Outcomes

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Course Outcomes

1. Illustrate discrete mathematic components like statements, logic, sets, structures, numbers and combinatorics.
2. Evaluate and simplify propositional and predicate calculus using inference theory.
3. Perform the operations on Sets, Relations and functions and their properties.
4. Identify algebraic systems and use general properties on number theory.
5. Use combinatorics solving the counting problems.
6. Use graph algorithms for representing, identifying, generating and evaluating the Graphs.

Unit IV
Combinatorics

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Unit IV

Combinatorics: Basic of Counting, Permutations, Permutations with Repetitions, Circular Permutations, Restricted Permutations, Combinations, Restricted Combinations, **Generating Functions of Permutations and Combinations,** Binomial and Multinomial Coefficients, **Binomial and Multinomial Theorems,** The Principles of Inclusion–Exclusion, Pigeonhole Principle and its Application

Combinatorics

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Combinatorics

Basic of Counting:

There are two rules:

1. **Product Rule**
2. **Sum Rule**

Combinatorics

Basic of Counting:

There are two rules:

1. **Sum Rule**

- Let us consider two tasks:
  - m is the number of ways to do **task 1**
  - n is the number of ways to do **task 2**
  - Tasks are <u>independent</u> of each other, i.e.,
    - Performing **task 1** does not accomplish **task 2** and vice versa.
- *<u>Sum rule</u>*: the number of ways that "**either** task 1 **or** task 2 can be done, but **not both**", is $m+n$.
- Generalizes to multiple tasks ...

Combinatorics

Basic of Counting:

Examples:

- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects respectively. How many possible projects are there to choose from?

- If A is the set of ways to do task 1, and B the set of ways to do task 2, and if A and B are disjoint, then:

  "the ways to do either task 1 or 2 are $A \cup B$, and $|A \cup B| = |A| + |B|$"

Combinatorics

Basic of Counting:

There are two rules:

1. **Product Rule**

- Let us consider two tasks:
  - m is the number of ways to do **task 1**
  - n is the number of ways to do **task 2**
  - Tasks are independent of each other, i.e.,
    - Performing task 1does not accomplish task 2 and vice versa.
- P*roduct rule*: the number of ways that "**both** tasks 1 and 2 can be done" in mn.
- Generalizes to multiple tasks ...

Combinatorics

Basic of Counting:

There are two rules:

1. Product Rule (Example)

- **The chairs of an auditorium are to be labeled with a letter and a positive integer not to exceed 100. What is the largest number of chairs that can be labeled differently?**

> - If A is the set of ways to do task 1, and B the set of ways to do task 2, and if A and B are disjoint, then:
> - The ways to do both task 1 and 2 can be represented as $A \times B$, and $|A \times B| = |A| \cdot |B|$

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Basic of Counting:

- Main computer addresses are in one of 3 types:
  - *Class A:* address contains a 7-bit "netid" $\neq 1^7$, and a 24-bit "hostid"
  - *Class B:* address has a 14-bit netid and a 16-bit hostid.
  - *Class C:* address has 21-bit netid and an 8-bit hostid.

Bit Number	0	1	2	3 4	8	16	24	31
Class A	0	netid			hostid			
Class B	1	0	netid			hostid		
Class C	1	1	0	netid			hostid	

  - Hostids that are all 0s or all 1s are **not allowed**.
- How many valid computer addresses are there?

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Basic of Counting:

- (# addrs)
  = (# class A) + (# class B) + (# class C)
  (by sum rule)
- # class A = (# valid netids)·(# valid hostids)
  (by product rule)
- (# valid class A netids) = $2^7 - 1 = 127$.
- (# valid class A hostids) = $2^{24} - 2 = 16,777,214$.
- Continuing in this fashion we find the answer is:
  3,737,091,842 (3.7 billion IP addresses)

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Permutations:

- A *permutation* of a set *S* of objects is an ordered arrangement of the elements of S where each element appears only once:
  e.g., 1 2 3, 2 1 3, 3 1 2
- An ordered arrangement of *r* distinct elements of *S* is called an *r-permutation*.
- The number of *r*-permutations of a set *S* with $n=|S|$ elements is

$$\boldsymbol{P(n,r)} = n(n-1)\ldots(n-r+1) = n!/(n-r)!$$

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Permutations with repetitions:

- Frequently we want to know the number of permutations of a multiset, that is, a set of objects some of which are alike. We will

$$P(n;\ n_1,\ n_2,\ \ldots,\ n_r)$$

- Let denote the number of permutations of n objects of which n_1 are alike, n_2 are alike, . . ., n_r are alike. The general formula follows:

$$P(n;\ n_1,\ n_2,\ldots,\ n_r) = \frac{n!}{n_1!\,n_2!\ldots n_r!}$$

Combinatorics

Permutations with repetitions:

EXAMPLE 5.5 Find the number m of seven-letter words that can be formed using the letters of the word "*BENZENE*."

We seek the number of permutations of 7 objects of which 3 are alike (the three E's), and 2 are alike (the two N's). By Theorem 5.6,

$$m = P(7;\ 3,\ 2) = \frac{7!}{3!2!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot2\cdot1} = 420$$

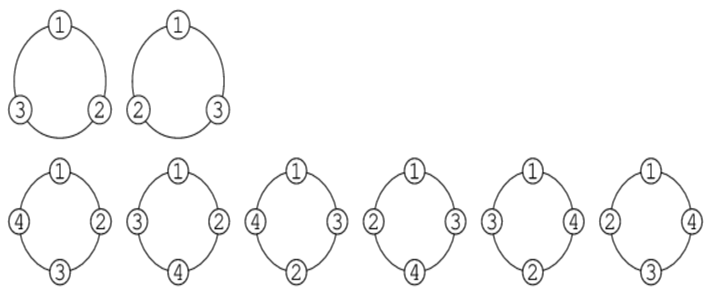
Combinatorics

Circular Permutations:

The number of ways to arrange n distinct objects along a fixed (i.e., cannot be picked up out of the plane and turned over) circle is

$$P_n = (n-1)!.$$

The number is $(n-1)!$ instead of the usual factorial $n!$ since all cyclic permutations of objects are equivalent because the circle can be rotated.



Combinatorics

Circular Permutations:

- **In how many ways 8 students be seated in a circle and in a line.**
- In a circle is (8-1)!=7!=5070
- In a line is 8! = 40320

Combinatorics

Restricted Permutations:

- The permutation is a way of filtering and selecting a set of objects, where the arrangement of objects does matter. However, the arrangement of objects may be done by imposing certain restrictions in the order of selection. For instance, the order of arrangement of articles, such that an article is always included or excluded from the set of given objects. Imposing the restrictions implies that not all the objects from the given set need to be ordered. There are different types of common restrictions that may be imposed on the permutation:
- Inclusion of a set of objects
- Exclusion of a set of objects
- Certain objects that always occur together
- Certain objects that stay apart

Combinatorics

Restricted Permutations:

- The common types of restricted permutations are:
  1. Formation of numbers with digits with some digits at fixed positions.
  2. Word building with some letters with a fixed position.
  3. Vowels or consonants in the set of alphabets occur together.
  4. A set of objects always occurring together
  5. A set of objects that never occur together
  6. Restrictions for circular permutations
  7. The choice of dress to wear from a set of dresses
  8. The order of eating
  9. The combinations of the colors to make

Combinatorics

Restricted Permutations:

- The permutation is a way of filtering and selecting a set of objects, where the arrangement of objects does matter. However, the arrangement of objects may be done by imposing certain restrictions in the order of selection. For instance, the order of arrangement of articles, such that an article is always included or excluded from the set of given objects. Imposing the restrictions implies that not all the objects from the given set need to be ordered. There are different types of common restrictions that may be imposed on the permutation:
- Inclusion of a set of objects
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Combinatorics

Formula of Restricted Permutations:

- Number of permutations of 'n' things taking 'r' at a time, corresponding to the case where a particular thing always occurs = $\boldsymbol{r}.(\boldsymbol{n-1}.\boldsymbol{P_{r-1}})$
- Number of permutations of 'n' things taking 'r' at a time, corresponding to the case where a particular thing never occurred = $\boldsymbol{n-1}.\boldsymbol{P_r}$
- **Sample Questions:**

1. Find out how many 4 digits numbers without any repetition can be made using 1, 2, 3, 4, 5, 6, 7 if 4 will always be there in the number?
2. How many 5 digit numbers can be formed by 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. So that 2 is always there in the number?

Combinatorics

Formula of Restricted Permutations:

- **Sample Questions:**

1. Find out how many 4 digits numbers without any repetition can be made using 1, 2, 3, 4, 5, 6, 7 if 4 will always be there in the number?
2. How many 5 digit numbers can be formed by 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. So that 2 is always there in the number?
3. How many different three-letter words can be made by 5 vowels if 'a' is never included?
4. . How many four-digit numbers without any repetition can be made by using 1, 2, 3, 4, 5, 6, 7 if 4 will never be included?

Combinatorics

Combinations:

- The number of ways of choosing *r* elements from S (order does not matter).

S={1,2,3}

e.g., 1 2 , 1 3, 2 3

- The number of *r*-combinations *C(n,r)* of a set with *n*=|*S*| elements is

$$C(n,r)=\binom{n}{r}=\frac{n!}{r!(n-r)!}$$

Combinatorics

Combinations:Example

- How many distinct 7-card hands can be drawn from a standard 52-card deck?
  - The order of cards in a hand doesn't matter.
- Answer $C(52,7) = P(52,7)/P(7,7)$
  $= 52\cdot 51\cdot 50\cdot 49\cdot 48\cdot 47\cdot 46 \,/\, 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

$$52\cdot 17\cdot 10\cdot 7\cdot 47\cdot 46 = 133{,}784{,}560$$

Combinatorics

Combinations: Example

- Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

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Restricted Combinations:

(a) Number of combinations of 'n' different things taken 'r' at a time, when 'p' particular things are always included = $^{n-p}C_{r-p.}$

(b) Number of combination of 'n' different things, taken 'r' at a time, when 'p' particular things are always to be excluded = $^{n-p}C_r$

Example:

In how many ways can a cricket-eleven be chosen out of 15 players? if

(i) A particular player is always chosen,

(ii) A particular is never chosen.

Combinatorics

Generating Permutations:

- To generate a complete list of permutations for the set $\{1, 2, \cdots, n\}$. We assign a direction to each integer $K \in \{1, 2, \cdots, n\}$by writing an arrow above it pointing to the left or to the right:

$$\overleftarrow{k} \quad \text{or} \quad \overrightarrow{k}.$$

- If permutations of each integer is represented with a direction, we call them as directed permutations.
- An integer k in a directed permutations is called **mobile** if its arrow points to smaller integer adjacent to it.

$$\overrightarrow{3}\,\overrightarrow{2}\,\overleftarrow{5}\,\overrightarrow{4}\,\overrightarrow{6}\,\overrightarrow{1}$$

Combinatorics

Generating Permutations:

- The integer n is mobile, except two cases:

(i) n is the first integer and its arrow points to the left, i.e., $\overleftarrow{n}\cdots$;

(ii) n is the last integer and its arrow points to the right, i.e., $\cdots\overrightarrow{n}$.

Algorithm 1.1. Algorithm for Generating Permutations of $\{1, 2, \ldots, n\}$:

Step 0. Begin with $\overleftarrow{1}\,\overleftarrow{2}\cdots\overleftarrow{n}$.

Step 1. Find the largest mobile integer m.

Step 2. Switch m and the adjacent integer its arrow points to.

Step 3. Switch the directions for all integers $p > m$.

Step 4. Write down the resulting permutation with directions and return to Step 1.

Step 5. Stop if there is no mobile integer.

Combinatorics

Generating Permutations:

For $n = 4$, the algorithm produces the list

$\overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{3}\ \overleftarrow{\mathbf{4}}$	$\overleftarrow{3}\ \overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{\mathbf{4}}$	$\overleftarrow{2}\ \overrightarrow{3}\ \overleftarrow{1}\ \overleftarrow{\mathbf{4}}$
$\overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{\mathbf{4}}\ \overleftarrow{3}$	$\overleftarrow{3}\ \overleftarrow{1}\ \overleftarrow{\mathbf{4}}\ \overleftarrow{2}$	$\overleftarrow{2}\ \overrightarrow{3}\ \overleftarrow{\mathbf{4}}\ \overleftarrow{1}$
$\overleftarrow{1}\ \overleftarrow{\mathbf{4}}\ \overleftarrow{2}\ \overleftarrow{3}$	$\overleftarrow{3}\ \overleftarrow{\mathbf{4}}\ \overleftarrow{1}\ \overleftarrow{2}$	$\overleftarrow{2}\ \overleftarrow{\mathbf{4}}\ \overrightarrow{3}\ \overleftarrow{1}$
$\overleftarrow{4}\ \overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{\mathbf{3}}$	$\overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{1}\ \overleftarrow{\mathbf{2}}$	$\overleftarrow{4}\ \overleftarrow{2}\ \overrightarrow{\mathbf{3}}\ \overleftarrow{1}$
$\overrightarrow{\mathbf{4}}\ \overleftarrow{1}\ \overleftarrow{3}\ \overleftarrow{2}$	$\overrightarrow{\mathbf{4}}\ \overrightarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}$	$\overrightarrow{\mathbf{4}}\ \overleftarrow{2}\ \overleftarrow{1}\ \overrightarrow{3}$
$\overleftarrow{1}\ \overrightarrow{\mathbf{4}}\ \overleftarrow{3}\ \overleftarrow{2}$	$\overrightarrow{3}\ \overrightarrow{\mathbf{4}}\ \overleftarrow{2}\ \overleftarrow{1}$	$\overleftarrow{2}\ \overrightarrow{\mathbf{4}}\ \overleftarrow{1}\ \overrightarrow{3}$
$\overleftarrow{1}\ \overleftarrow{3}\ \overrightarrow{\mathbf{4}}\ \overleftarrow{2}$	$\overrightarrow{3}\ \overleftarrow{2}\ \overrightarrow{\mathbf{4}}\ \overleftarrow{1}$	$\overleftarrow{2}\ \overleftarrow{1}\ \overrightarrow{\mathbf{4}}\ \overrightarrow{3}$
$\overleftarrow{1}\ \overleftarrow{\mathbf{3}}\ \overleftarrow{2}\ \overrightarrow{4}$	$\overrightarrow{\mathbf{3}}\ \overleftarrow{2}\ \overleftarrow{1}\ \overrightarrow{4}$	$\overleftarrow{2}\ \overleftarrow{1}\ \overrightarrow{3}\ \overrightarrow{4}$

Combinatorics

Generating Permutations:

For $n = 4$, we have the list

	1		2		3	**4**		1	2	3	4
	1		2	**4**	3			1	2	4	3
	1	**4**	2		3			1	4	2	3
4	1		2		3			4	1	2	3
4	1		3		2			4	1	3	2
	1	**4**	3		2			1	4	3	2
	1		3	**4**	2			1	3	4	2
	1		3		2	**4**		1	3	2	4
	3		1		2	**4**		3	1	2	4
	3		1	**4**	2			3	1	4	2
	3	**4**	1		2			3	4	1	2
4	3		1		2		$\Rightarrow$	4	3	1	2
4	3		2		1			4	3	2	1
	3	**4**	2		1			3	4	2	1
	3		2	**4**	1			3	2	4	1
	3		2		1	**4**		3	2	1	4
	2		3		1	**4**		2	3	1	4
	2		3	**4**	1			2	3	4	1
	2	**4**	3		1			2	4	3	1
4	2		3		1			4	2	3	1
4	2		1		3			4	2	1	3
	2	**4**	1		3			2	4	1	3
	2		1	**4**	3			2	1	4	3
	2		1		3	**4**		2	1	3	4

Combinatorics

Binomial and Multinomial Coefficients:

In mathematics, the **binomial coefficients** are the positive integers that occur as coefficients in the binomial theorem. Commonly, a binomial coefficient is indexed by a pair of integers $n \geq k \geq 0$ and is written $\binom{n}{k}$. It is the coefficient of the x^k term in the polynomial expansion of the binomial power $(1+x)^n$, and is given by the formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

For example, the fourth power of $1 + x$ is

$$\begin{aligned}(1+x)^4 &= \binom{4}{0}x^0 + \binom{4}{1}x^1 + \binom{4}{2}x^2 + \binom{4}{3}x^3 + \binom{4}{4}x^4 \\ &= 1 + 4x + 6x^2 + 4x^3 + x^4,\end{aligned}$$

Combinatorics

Binomial and Multinomial Coefficients:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

Combinatorics

Binomial and Multinomial Coefficients:

Definition

Suppose that $n_1, \ldots, n_r$ are positive integers, and $n_1 + \cdots + n_r = n$. Then

$$\binom{n}{n_1, n_2, \ldots, n_r} := \frac{n!}{n_1!\, n_2! \,\cdots\, n_r!} = \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots\binom{n-\sum_{i=1}^{r-i} n_i}{n_r}$$

is called a multinomial coefficient. Binomial coefficients are the special case of $r = 2$.

Exercise

A police department of 10 officers wants to have 5 patrol the streets, 2 doing paperwork, and 3 at the dohnut shop. How many ways can this be done?

Answer. $\binom{10}{5}\binom{5}{2}\binom{3}{3} = \frac{10!}{5!\,5!} \cdot \frac{5!}{2!\,3!} \cdot \frac{3!}{3!\,0!} = \frac{10!}{5!\,2!\,3!} = 2520.$

Combinatorics

Binomial and Multinomial Coefficients:

Multinomials and words

Consider an alphabet with r letters: $\{s_1, \dots, s_r\}$.

The number of length-n "words" (i.e., strings) that you can write using exactly n_i instances of s_i (where $n_1 + \cdots + n_r = n$) is

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1!\ n_2!\ \cdots\ n_r!}.$$

Examples

(i) The number of distinct permutations of the letters in the word MISSISSIPPI is

$$\binom{11}{1,4,4,2} = \frac{11!}{1!\ 4!\ 4!\ 2!} = 34650.$$

(ii) How many length-13 strings can be made using 6 instances of $*$ ("star") and 7 instances of | ("bar")? Examples include:

$*||***||||**|, \qquad ******|||||||, \qquad |*|*|*|*|*|*|.$

Answer: $\binom{13}{6,7} = \frac{13!}{6!\ 7!} = \binom{13}{6} = 1716.$

Combinatorics

Binomial and Multinomial Theorem:

The binomial theorem

We will motivate the following theorem with an example:

$$\begin{aligned}(x+y)^6 &= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6 \\ &= \tbinom{6}{0}x^6 + \tbinom{6}{1}x^5y + \tbinom{6}{2}x^4y^2 + \tbinom{6}{3}x^3y^3 + \tbinom{6}{4}x^2y^4 + \tbinom{6}{5}xy^5 + \tbinom{6}{6}y^6.\end{aligned}$$

Theorem

For any x, y and $n \geq 1$,

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Combinatorics

Binomial and Multinomial Theorem:

The multinomial theorem

Multinomial coefficients generalize binomial coefficients (the case when $r = 2$).

Not surprisingly, the Binomial Theorem generalizes to a Multinomial Theorem.

Theorem

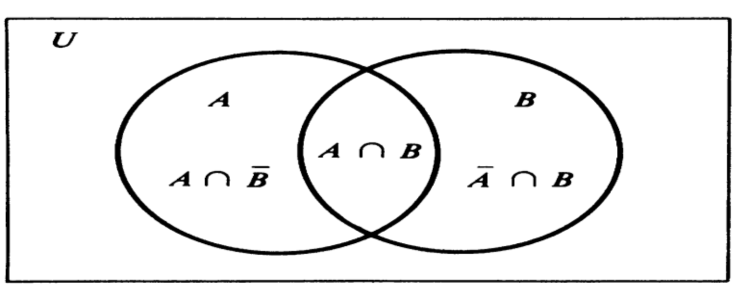
For any $x_1, \ldots, x_r$ and $n > 1$,

$$(x_1 + \cdots + x_r)^n = \sum_{\substack{(n_1,\ldots,n_r) \\ n_1+\cdots+n_r=n}} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}.$$

Combinatorics

Principle of Inclusion and Exclusion

- If the sets are not disjoint we must refine the sum rule called Principles of inclusion-Exclusion. Sometimes called Sieve method.
- **Theorem 1: if A and B are subsets of some universe set U, then** $|AUB| = |A| + |B| - |A \cap B|$



Combinatorics

Principle of Inclusion and Exclusion

- **Theorem 1: if A and B are subsets of some universe set U, then** $|\boldsymbol{AUB}| = |\boldsymbol{A}| + |\boldsymbol{B}| - |\boldsymbol{A} \cap \boldsymbol{B}|$
- **Proof**: From Diagram, We can Simply the following one is |AUB| and |A| + |B|
- AUB is the sum of 3 Disjoint Sets as Follows
- $|AUB| = |A \cap \bar{B}| + |A \cap B| + |\bar{A} \cap B|$ (1)
- |A| + |B| = $|A \cap \bar{B}| + |A \cap B| + |\bar{A} \cap B| + |\,A \cap B|$(2)
- From 1 and 2 We prove that $|\boldsymbol{AUB}|=|\boldsymbol{A}|+|\boldsymbol{B}|-|\boldsymbol{A}\cap\boldsymbol{B}|$

Eg:1 Suppose that 200 faculty members can speak French and 50canspeakRussian, while only 20 can speak both French and Russian. How many faculty members can speak either French or Russian?

Combinatorics

Principle of Inclusion -Exclusion

- **Eg: 2** From a group of 10 professors how many ways can a committee of 5 members be formed so that at least one of Professor A and Professor B will be included?
- Sum rule: C(8,3) + 2C(8,4)
- Indirect Counting: C(10,5) – C(8,5)
- Principle of Inclusion and Exclusion: C(9,4) + C(9,4) – C(8,3)
- **Theorem 2: If A, B, C are finite sets,**

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \quad (2.8.7)$$

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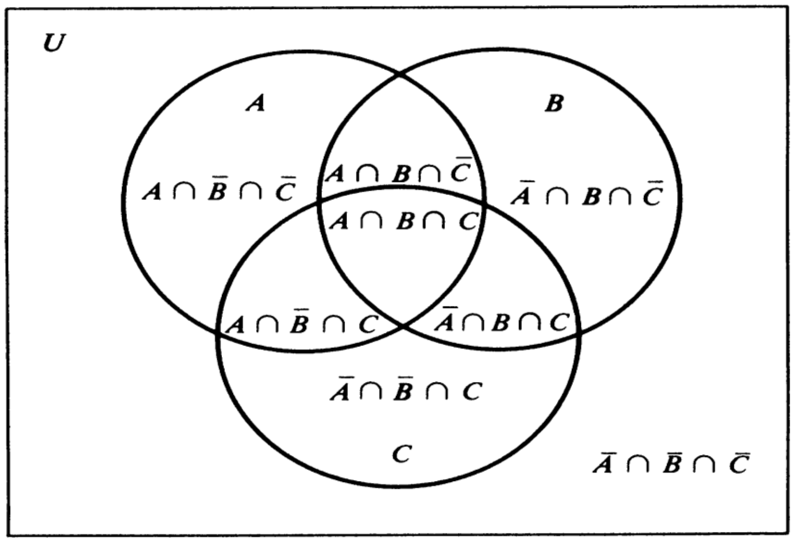
- **Proof: From Diagram we Calculate the Following things**



Combinatorics

- **Proof: From Diagram we Calculate the Following things**

$$|A| = |A \cap \bar{B} \cap \bar{C}| + A \cap B \cap \bar{C}| + |A \cap \bar{B} \cap C| + |A \cap B \cap C|, \tag{2.8.4}$$

$$|B| = |\bar{A} \cap B \cap \bar{C}| + |A \cap B \cap \bar{C}| + |\bar{A} \cap B \cap C| + |A \cap B \cap C|, \tag{2.8.5}$$

$$|C| = |\bar{A} \cap \bar{B} \cap C| + |A \cap \bar{B} \cap C| + |\bar{A} \cap B \cap C| + |A \cap B \cap C|, \tag{2.8.6}$$

The first 7 of these sets make up $A \cup B \cup C$, the next 2 make up $A \cap B$, and the next 2 give $A \cap C$. Thus, we have $|A| + |B| + |C| = |A \cup B \cup C| + |A \cap B| + |A \cap C| + |\bar{A} \cap B \cap C|$.

By rearranging terms, we have $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |\bar{A} \cap B \cap C|$.

But we would like an expression free of complements. We note that $|\bar{A} \cap B \cap C| + |A \cap B \cap C| = |B \cap C|$ so that we have the following theorem.

Combinatorics

- If there are 200 faculty members that speak
- **Eg:** French, 50 that speak Russian,100 that speak Spanish, 20 that speak French and Russian, 60 that speak French and Spanish, 35 that speak Russian and Spanish, while only 10 speak French, Russian, and Spanish, how many speak either French or Russian or Spanish?

Combinatorics

Theorem 2.8.1. (General statement of the principle of inclusion-exclusion). If A_i are finite subsets of a universal set U, then

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{i,j} |A_i \cap A_j| + \sum_{i,j,k} |A_i \cap A_j \cap A_k| + \cdots + (-1)^{n-1} |A_1 \cap A_2 \cdots \cap A_n|, \tag{2.8.10}$$

where the second summation is taken over all 2-combinations $\{i,j\}$ of the integers $\{1,2,\ldots,n\}$, the third summation is taken over all 3-combinations {i,j,k} of $\{1,2,\ldots,n\}$, and so on.

For $n = 4$ there are $4 + C(4,2) + C(4,3) + 1 = 2^4 - 1 = 15$ terms and the theorem states that

Combinatorics

Pigeonhole Principle and its Application:

- In Discrete Mathematics, the pigeonhole principle states that if we must put N + 1 or more pigeons into N Pigeon Holes, then some pigeonholes must contain two or more pigeons.

Generalized Pigeonhole Principle

- If Kn + 1 (where k is a positive integer) pigeons are distributed among n holes than some hole contains at least k + 1 pigeons.
- This principle is applicable in many fields like Number Theory, Probability, Algorithms, Geometry, etc.

Combinatorics

Pigeonhole Principle and its Application:

1. If 6 color are used to paint 37 home. Show that at least 7 home of them will be of same colour.
2. A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?
3. Find the minimum number of teachers in a college to be sure that four of them are born in the same month,

Combinatorics

Pigeonhole Principle and its Application:

1. A box contain 10 blue ball, 20 red balls, 8 green balls, 15 yellow balls and 25 white balls. How many ball we have chosen to ensure that we have 12 balls of the same color.
2. Prove that among 1, 00, 000 there are two who are born on the same time.

Combinatorics

Pigeonhole Principle and its Application:

- **Theorem:** Let q1, q2, . . . , qn be positive integers. If q1+ q2+ . . . + qn − n + 1 objects are put into n boxes, then either the 1st box contains at least q1 objects, or the 2nd box contains at least q2 objects, . . ., the nth box contains at least qn objects.

Combinatorics

Pigeonhole Principle and its Application:

1. In a computer science department, a student club can be formed with either 10 members from first year or 8 members from second year or 6 from third year or 4 from final year. What is the minimum no. of students we have to choose randomly from department to ensure that a student club is formed?