H.T.No: | | | | | | | | | | |

Course Code: **201BS3T13**

ADITYA ENGINEERING COLLEGE (A)

B.Tech – III Semester End Examinations Regular & Supplementary (AR20) – JAN 2023

DISCRETE MATHEMATICS
(Common to CSE, IT & AIML)

Time: 3 hours | **Max. Marks: 70**

Answer ONE question from each unit
All Questions Carry Equal Marks
All parts of the questions must be answered at one place only

UNIT – I

1	a	Verify that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent or not Justify?	K3	CO1	[7M]
	b	Obtain a conjunctive normal form of $\neg(p \vee q) \leftrightarrow (p \wedge q)$	K3	CO1	[7M]
		OR			
2	a	Show that $r \vee s$ follows logically from the premises $c \vee d, (c \vee d) \rightarrow \neg h$, $\neg h \rightarrow (a \wedge \neg b)$ and $(a \wedge \neg b) \rightarrow (r \vee s)$	K3	CO1	[7M]
	b	Express each of these statements using quantifiers. i) Every koala can climb. ii) No monkey can speak French	K2	CO1	[7M]

UNIT – II

3	a	Determine whether the relation $R=\{(1,2),(1,4),(2,1),(2,3),(2,4),(3,2),(4,1),(4,2)\}$ is an equivalence relation.	K3	CO2	[7M]
	b	Find the transitive closure of R if $R=\{(a,b),(b,c),(c,d),(d,e)\}$	K3	CO2	[7M]
		OR			
4	a	Let $X=\{1,2,3,..,7\}$ and $R=\{<x,y>/x-y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation.	K3	CO2	[7M]
	b	Draw the Hasse diagram for the partial ordering $\{(A,B) \mid A \subseteq B\}$ on the power set P(S), where $S=\{a,b,c\}$..	K3	CO2	[7M]

UNIT – III

5	a	A person deposits Rs. 10,000 in a savings account at a bank yielding 11% per year with interest compounded annually. How much will be in the account after 30 years.	K2	CO3	[7M]
	b	Solve the recurrence relation $a_n=3a_{n-1}+2^n$, $a_0=1$	K3	CO3	[7M]
		OR			
6	a	By using generating function solve the recurrence relation $a_n-7a_{n-1}+10a_{n-2}=0$, for $n \geq 2$.	K3	CO3	[7M]
	b	Solve the recurrence relation $a_n+7a_{n-1}+12a_{n-2}=n^2$, for $n \geq 2$.	K3	CO3	[7M]

UNIT – IV

7	a	Represent the graph $K_{2,3}$ with an adjacency matrix.	K2	CO4	[7M]
	b	Draw the graphs represented by the following adjacency matrix:	K3	CO4	[7M]

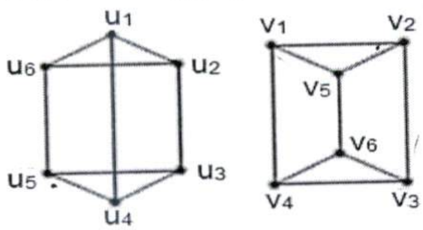
$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

OR

8	a	Give an example of a graph which is Eulerian but not Hamiltonian.	K2	CO4	[7M]
	b	Define planar graph and non-planar graph and give one example to each.	K2	CO4	[7M]

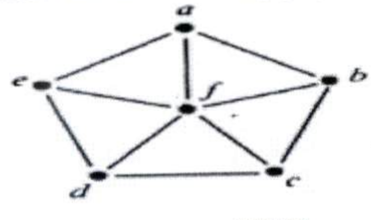
UNIT – V

9	a	Show that a tree with n vertices has exactly n-1 edges.	K2	CO5	[7M]
	b	Determine whether the following graphs are isomorphic.	K3	CO5	[7M]



OR

10	a	Determine the chromatic number of a cycle graph C_n for (n>2) with even number of vertices.	K2	CO5	[7M]
	b	Find the chromatic number of the following graph	K3	CO5	[7M]



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B. Krishna veni
(Dr. B. Krishnaveni
Emp ID: 2249)

366

ADITYA ENGINEERING COLLEGE (A)

Name of the Examination : B.Tech - III sem End examinations Regular (AR 20) - JAN 2023
Name of the Course : Discrete Mathematics
Course Code : 201BS3T13
Name of the Expert : Dr. B. Krishnaveni
Designation : Assoc. professor
Department : H & BS
Contact Number : 9493716422

S.No	Sub Q.No		Marks
1	(a)	Let $X = (P \to Q) \wedge (P \to R)$, $Y = P \to (Q \wedge R)$	
		<u>using truth table</u>	
		see table below	4M
		The two formulas have same truth values (or) $X \leftrightarrow Y$ is a tautology. $\therefore$ Given two formulas are equivalent	3M
		<u>Note</u>: The same can be verified by using equivalent laws	

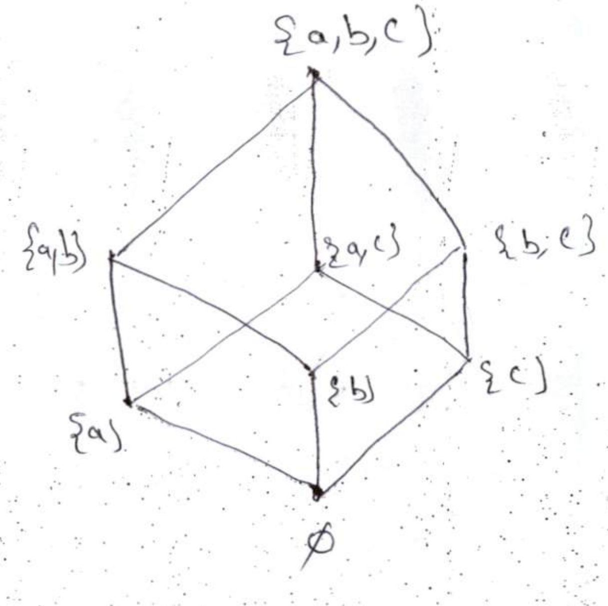
P	Q	R	$Q \wedge R$	$P \to Q$	$P \to R$	X	Y
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

S.No	Sub Q.No		Marks
1	(b)	$\neg(p\vee q) \leftrightarrow (p\wedge q)$ $\Leftrightarrow [\neg(p\vee q) \to (p\wedge q)] \wedge [(p\wedge q) \to \neg(p\vee q)]$ $\Leftrightarrow [(p\vee q) \vee (p\wedge q)] \wedge [(\neg p\vee \neg q) \vee (\neg p\wedge \neg q)]$	3M
		$\Leftrightarrow (p\vee q\vee p) \wedge (p\vee q\vee q) \wedge (\neg p\vee \neg q\vee \neg p)$ $\wedge (\neg p\vee \neg q\vee \neg q)$ $\Leftrightarrow (p\vee q) \wedge (p\vee q) \wedge (\neg p\vee \neg q) \wedge (\neg p\vee \neg q)$ $\Leftrightarrow (p\vee q) \wedge (\neg p\vee \neg q)$	4M
		which is required conjunctive normal form	
2	(a)	Premises: $c\vee d$, $(c\vee d)\to \neg h$, $\neg h \to (a\wedge \neg b)$, $(a\wedge\neg b) \to (r\vee s)$ Conclusion: $r\vee s$ $\{1\}$ (1) $c\vee d$ (P rule) $\{2\}$ (2) $(c\vee d)\to \neg h$ (P rule) $\{1,2\}$ (3) $\neg h$ (T rule) $\{4\}$ (4) $\neg h\to(a\wedge\neg b)$ (P rule) $\{1,2,4\}$ (5) $(a\wedge \neg b)$ (T rule) $\{6\}$ (6) $(a\wedge\neg b)\to(r\vee s)$ (P rule) $\{1,2,4,6\}$ (7) $r\vee s$ (T rule) Hence Proved.	7M

S.No	Sub Q.No		Marks
2	(b)	i, let $K(x)$: x is Koala $C(x)$: x can climb The symbolic form of "Every Koala can climb" is $\forall x \left[K(x) \rightarrow C(x) \right]$	3M
		ii, let $M(x)$: x is a monkey $F(x)$: x can speak French The symbolic form of "No monkey can speak French" is $\forall (x) \left[M(x) \rightarrow \neg F(x) \right]$	4M
3	(a)	Given $R = \{ (1,2), (1,4), (2,1), (2,3), (2,4), (3,2), (4,1) (4,2) \}$ To show that R is an equivalence relation R must be reflexive, symmetric and transitive Since $(1,1), (2,2), (3,3), (4,4) \notin R$ $\Rightarrow R$ is not reflexive	3M

S.No	Sub Q.No		Marks
		Hence R is not reflexive i.e, it is not satisfying one of the property of equivalence relation ∴ R is not an equivalence relation	4M
3	(b)	Given $R = \{(a,b), (b,c), (c,d), (d,e)\}$ The transitive closure of R is $R^* = R \cup R^2 \cup R^3 \cup R^4 \cup R^5$ or by using warshall's algorithm given by	4M
		$R^* = \{(a,b), (a,c), (a,d), (a,e), (b,c),$ $(b,d), (b,e), (c,d), (c,e), (d,e)$	3m
4	(a)	Given $X = \{1, 2, 3, \cdots, 7\}$ $= \{1, 2, 3, 4, 5, 6, 7\}$ $(x,y) \in R$ suchthat $x-y$ divisible by 3 ~~Then R = {(1,1), (1,4), (1,7), (2,2)~~	

S.No	Sub Q.No		Marks
		$\therefore R = \{(1,1), (1,4), (1,7), (2,2), (2,5), (3,3), (3,6), (4,1), (4,4), (4,7), (5,2), (5,5), (6,3), (6,6), (7,1), (7,4), (7,7)\}$	2 M
		Since $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7) \in R$	
		Hence R is reflexive	
		Symmetric : In given relation R whenever $(a,b) \in R \Rightarrow (b,a) \in R$ for example $(1,4) \in R \Rightarrow (4,1)$ and is satisfied for every element R	
		$\therefore$ R is symmetric	
		Transitive : If (a,b) & $(b,c) \in R \Rightarrow (a,c) \in R$	
		For example $(1,7), (7,4) \in R \Rightarrow (1,4) \in R$ and is satisfied in R	
		$\therefore$ R is transitive	5M
		Hence R is an equivalence relation	

S.No	Sub Q.No		Marks
4	(b)	Give $S = \{a,b,c\}$	
		$P(S) = \{ (\emptyset, \{a\}), (\emptyset, \{b\}), (\emptyset, \{c\}),$	
		$(\{\emptyset\}, \{a,b\}), (\emptyset, \{b,c\}), (\emptyset, \{a,c\}), (\emptyset, \{a,b,c\}),$	
		$(\{a\},\{a,b\}), (\{a\}, \{a,c\}), (\{a\}, \{a,b,c\}),$	
		$(\{b\}, \{b,c\}), (\{b\}, \{a,b,c\}), (\{c\}, \{b,c\}),$	
		$(\{c\}, \{a,c\}), (\{c\}, \{a,b,c\}), (\{a,b\}, \{a,b,c\}),$	3m
		$(\{b,c\}, \{a,b,c\}), (\{a,c\}, \{a,b,c\}) \}$	
		Hasse diagram	
			4m

S.No	Sub Q.No		Marks
5	(a)	Let a_n be the amount in savings after 'n' years $a_n = a_{n-1} + \frac{11}{100} a_{n-1}$, $a_0 = 10,000$ $a_n = \left(1 + \frac{11}{100}\right) a_{n-1} = 1.11 a_{n-1}$ $\Rightarrow a_n - 1.11 a_{n-1} = 0$ Characteristic equation is $r - 1.11 = 0$ $\Rightarrow r = 1.11$ Solution is $a_n = \alpha_1 (1.11)^n$ $a_0 = 10,000$, $a_0 = \alpha_1 (1.11)^0 \Rightarrow \alpha_1 = 10,000$ $\therefore a_n = \alpha_1 (1.11)^n$ $\Rightarrow a_n = 10,000 (1.11)^n$ ~~After~~ 30 years amount in account is $a_{30} = 10,000 (1.11)^{30}$ $= 2,28,922$	2M 5M
5	(b)	Given $a_n - 3a_{n-1} = 0 \rightarrow (1)$ Characteristic equation is $r - 3 = 0 \Rightarrow r = 3$ Solution is $a_n^{(H)} = \alpha \cdot 3^n$ Assume $a_n^{(P)} = d2^n$	2M

S.No	Sub Q.No		Marks
		From ①, $d2^n - 3(d2^{n-1}) = 1$	
		$\Rightarrow d = -2$	
		$\therefore a_n^{(P)} = -2 \cdot 2^n = -2^{n+1}$	
		Hence the solution is	
		$a_n = a_n^{(H)} + a_n^{(P)}$	
		$a_n = \alpha_1 \cdot 3^n - 2 \cdot 2^n$	
		Given $a_0 = 1 \Rightarrow \alpha_1 = 3$	
		The solution is	
		$a_n = 3 \cdot 3^n - 2 \cdot 2^n$	5M
		$= 3^{n+1} - 2^{n+1}$	
6	(a)	we have $G(z) = \sum_{n=0}^{\infty} a_n z^n$	
		Given $a_n - 7a_{n-1} + 10a_{n-2} = 0,\ n \geq 2$	
		$\sum_{n=2}^{\infty} a_n z^n - 7\sum_{n=2}^{\infty} a_{n-1} z^n + 10\sum_{n=2}^{\infty} a_{n-2} z^n = 0$	
		$\Rightarrow \sum_{n=2}^{\infty} a_n z^n - 7z\sum_{n=2}^{\infty} a_{n-1} z^{n-1} + 10z^2 \sum_{n=2}^{\infty} a_{n-2} z^{n-2} = 0$	
		$\Rightarrow (G(z) - a_0 - a_1 z) - 7z[G(z) - a_0]$	
		$+ 10z^2 G(z) = 0$	

S.No	Sub Q.No		Marks

$$\Rightarrow G(z) = \frac{a_0 + a_1 z - 7z a_0}{1 - 7z + 10z^2}$$

let $a_0 = k_0$, $a_1 = k_1$

$$G(z) = \frac{k_0 + (k_1 - 7k_0)z}{(1 - 7z + 10z)^2} = \frac{k_0 + (k_1 - 7k_0)z}{(1-2z)(1-5z)}$$

By Partial fractions.

$$G(z) = \frac{5k_0 - k_1}{3}(1-2z)^{-1} + \left(\frac{k_1 - 2k_0}{3}\right)(1-5z)$$ 4M

$$\Rightarrow \sum_{n=0}^{\infty} a_n z^n = \frac{5k_0 - k_1}{3}\sum_{n=0}^{\infty}(2z)^n + \frac{k_1 - 2k_0}{3}\sum_{n=0}^{\infty}(5z)^n$$

$$\therefore a_n = \left(\frac{5k_0 - k_1}{3}\right)2^n + \left(\frac{k_1 - 2k_0}{3}\right)5^n$$ 3M

is the solution.

6 (b) Given $a_n + 7a_{n-1} + 12a_{n-2} = n^2$, $n \geq 2$

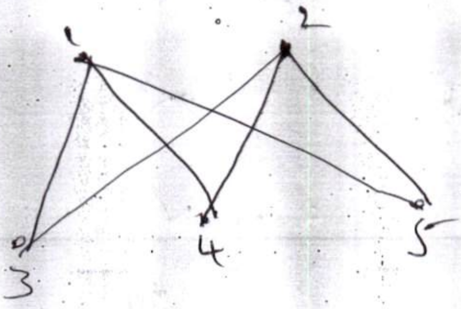
Characteristic Equation is $r^2 + 7r + 12 = 0$

$\Rightarrow r = -3, -4$

$$a_n^{(H)} = \alpha_1(-3)^n + \alpha_2(-4)^n$$ 2M

S.No	Sub Q.No		Marks
		Let $a_n^{(P)} = d_0 + d_1 n + d_2 n^2$ which satisfies given recurrence relation On simplification, $d_0 = \frac{-189}{4000}$, $d_1 = \frac{31}{200}$, $d_0 = \frac{1}{20}$ $\therefore a_n^{(P)} = \frac{-189}{4000} + \frac{31}{200} n + \frac{1}{20} n^2$ Hence the solution is $a_n = a_n^{(H)} + a_n^{(P)}$ $= \alpha_1 (-3)^n + \alpha_2 (-4)^n - \frac{189}{4000} + \frac{31}{200} n + \frac{1}{20} n^2$ Note: Finding d_0, d_1, d_2 in $a_n^{(P)}$ becomes very lengthy. So once the student did the process correctly give full marks	5M

S.No	Sub Q.No		Marks
7	(a)	The graph of $K_{2,3}$ is	2M

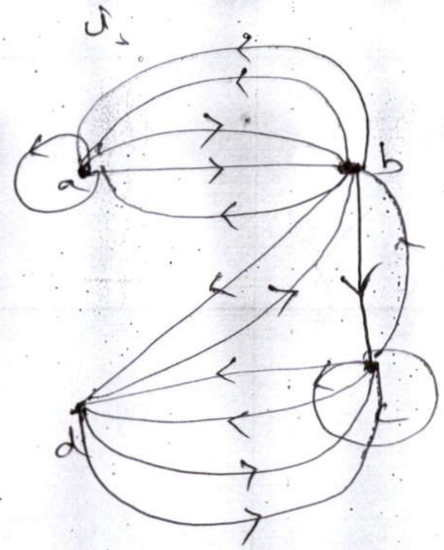


Adjacency matrix is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

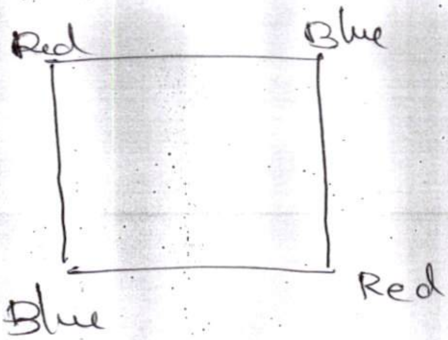
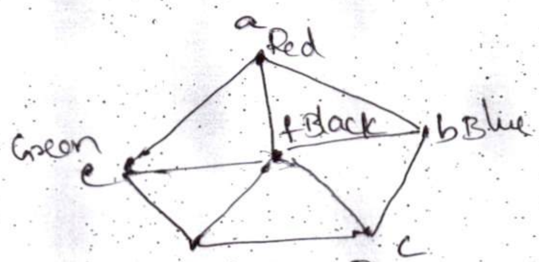
5M

S.No	Sub Q.No		Marks
7	(b)	Since Given matrix is not Symmetric Hence the graph is directed graph	
		Let a, b, c, d be the vertices of given graph	2M

S.No	Sub Q.No		Marks
		the graph of given adjacency matrix 	5M
8	(a)	If the Student give example of the graph which is Eulerian but not Hamilton give full marks i.e, 7 Marks	7M
8	(b)	A graph is called planar if it can be drawn in the plane without any edges crossing	

S.No	Sub Q.No		Marks
		Any example of planar graph	4M
		A graph which is not planar is called nonplanar graph	
		Example of Non planar graph	3M
9	(a)	We have show that a tree with 'n' vertics have $(n-1)$ edges	
		this can be proved by mathematical induction on no of vertics 'n'	2M
		If $n=1$, then there are no edgs $\Rightarrow$ The result is trivial	
		~~If~~ the trees with 'n' vertics have exactly $(n-1)$ edgs	
		Consider a tree with $(n+1)$ vstics	
		Now from tree remove the vstex & associated edge e	
		$T^1 = T - V$ (say)	
		Then T^1 has $(n-1)$edges	

S.No	Sub Q.No		Marks
		Hence T has n edgs ∴ By mathematical induction, the result is true for positive integers 'n'.	5M
9	(b)	Given graphs G&H(say) have six vertics and 9 edges All the vertics in both the graph have degree is $\underline{3}$ ∴ The degree sequence is same in both the graphs	3M
		Define a function $f: V(G) \to V(H)$ such that $f(u_1) = v_1$, $f(u_2) = v_2$, $f(u_3) = v_3$, $f(u_4) = v_4$, $f(u_5) = v_6$, $f(u_6) = v_5$ then whenever $(a,b) \in G \Rightarrow (f(a), f(b)) \in H$	4M
		Hence the two graphs are isomorphic	

S.No	Sub Q.No		Marks
10	(a)	Consider any cyclic grap C_n with even number of vertics C_4 —  Hence the Chromatic number of C_4 is 2 In this way, if C_n is a cyclic graph with even number of vertices, then the chromatic number of C_n is <u>2</u>	7M
10	(b)	 ~~Hen~~	3M

S.No	Sub Q.No		Marks
		The minimum number of colors required to color a given graph are 4 Hence the chromatic number of the given graph is <u>4</u>	4M
		B. Krishnaveni (Dr. B. Krishnaveni Emp ID: 2249)	