Review of origin-destination estimation methods for public transit systems

February 9, 2021

1 A brief background to travel demand modeling

Estimating travel demand is a key input to service planning. An origin-destination model that predicts the decisions made by riders based on good assumptions is essential for designing an efficient transit system. However, a huge amount of Data is needed to achieve this. In the past, the only methods of collecting this data were direct methods such as home surveys and road surveys (direct sampling estimation) (Viti, 2008; Neto et al., 2017). These methods are useful in describing some characteristics of the respondents that are otherwise not collected such as age, gender, income, occupation, and detailed travel information like trip purpose and mode choice (Zannat and Choudhury, 2019). However, such methods are not without flaws. The data needed is collected at big intervals, maybe once or twice in a decade, because an enormous sample data is needed which is expensive (Neto et al., 2017). the process is also subject to biases and may take a very long time for the data to be collected, analyzed, and finally be used to develop a model.

Early classical modeling included two major approaches, the most commonly used are the Four Stage (step) Model (FSM), and the Activity Based Model (Rasouli et al., 2017; Shan et al., 2012). FSM was first employed in the 1950s and was developed for large scale infrastructure projects. Each of its four stages (Trip generation, trip distribution, modal split, and trip assignment) can be carried out using many different models which range from simple to very sophisticated and comprehensive.

the first step of FSM is known as Trip Generation. It is a zone-wise analysis to give an idea of how many trips are generated and attracted from and to different zones in the study area. It uses methods like the Growth factor model and Regression model. In the second step, the trip distribution, Casey introduced the Gravity Model, which employs the physics-based law of gravitational attraction to spatially assign trips in a network (Willumsen, 1978; Ekowicaksono et al., 2016). Many researchers such as Högberg (1976), Robillard (1975), and Low (1972) used different types of the gravity model for their research (Willumsen, 1978). In the third step of FSM, the modal split, models including the binary logit model and the multinomial logit model are used to determine the choice of passengers and split the trips mode-wise . Then in the Traffic Assignment, which is the final step, assignment models such as the all-or-nothing, user-equilibrium, and system-optimum are commonly utilized to distribute trips within the network (*Trip Assignment* 2020).

In the 1970s, researchers took advantage of big data generated from traffic monitoring systems to develop indirect mathematical models such as the ones based on maximum entropy and minimum information principles developed by van Zuylen and Willumsen (1980) and Willumsen (1978) to obtain OD matrices.

FSM dominated the history of demand forecasting and measuring the performance of transit systems (Mcnally, 2000). However, while FSM and other trip-based models are effective for estimating the impact of the transportation infrastructure development and expansion, they do not provide a valid representation of the underlying travel behavior (they can tell what people do but not why they do it) (McNally and Rindt, 2007). They are, therefore, unable to be responsive to the changing policies oriented towards management rather than expansion. This means that they cannot tell us how people might react to new approaches or travel alternatives. As a result of these weaknesses, the development of alternative approaches was stimulated. The study of Mitchell and Rapkin (1954) has established the link of travel and activities and called for a comprehensive framework for travel behavior. And in the 1970s, the study of the activity based approach was first analyzed in depth. The motivation behind this approach was that the reflection of the link between travel and activities and understanding of activity behavior is fundamental, and understanding travel behavior comes second (McNally and Rindt, 2007).

2 Origin Destination Transfer Inference

Origin, destination, and interchange inference (ODX) method is a recent research outcome that utilizes automatic data collection systems which provide a very rich new data source that is both realistic and up to date.

Analyzing a transit network using ODX provides a more accurate analysis of the network usage and a level of detailed geographical disaggregate data on passengers trips that was unavailable with previous data collection methods. This offers a complete and reliable picture of riders' activity and route choices that does not rely on assumptions as is the case with older models.

It can also provide planners with specific information about where to target their available resources and make adjustments to achieve maximum efficiency of the transit network and what the possible effects of those adjustments are (Vanderwaart, 2016).

In recent years, various OD estimation methods using the trip-chaining approach have attracted the attention of researchers. And a new method of inferring the origin destination matrix using automated data collection systems (ADCS) called origin destination transfer (interchange) inference coined by Gordon as (ODX) was developed (Vanderwaart, 2016; Sánchez-Martínez, 2017). The process is divided into three steps: origin inference, destination inference, and transfer (i.e., interchange) which mainly aims to find out whether the inferred alighting location is the real destination for that particular trip.

A number of studies used different methodologies to infer OD matrices for transit trips using smart card data such as the study by Wang (2010), Nassir et al. (2011), and Gordon (2012)

The validity of these estimation methods has not been extensively investigated, because passengers are often required to tap their smart cards only when boarding a public transport service (in open systems), resulting in datasets that lack data about passengers' alighting (Alsger, Assemi, et al., 2016). However, with ADCS data which are a major improvement over the traditional data collection methods, inferring origin destination matrices is possible with minimal costs.

ODX uses key ADCS like automatic vehicle location, automatic fare collection, and automatic passenger counting instead of manual surveys to infer destinations and transfers in open systems, which is usually the case, or only to infer transfers in closed systems where each OD pair is given. Some trips cannot be inferred however due to errors, incomplete data, assumptions of the model not being fulfilled, and passengers who do not use farecards. Therefore, the OD matrix needs to be scaled up to full demand using method such as iterative proportional fitting (IPF) (Vanderwaart, 2016; Cui, 2006).

2.1 Automated Data Collection Systems (ADCS)

The origin destination matrix is the primary input for the public transit systems modeling approaches (Wong, Wong, et al., 2005). In the past, OD matrices were only calculated when a network level passenger survey was being conducted which were taken in very long intervals. Also, OD estimation was not the primarily purpose for conducting them (Ben-Akiva and Morikawa, 1989) which meant that a relatively recent single route or network matrix is typically unavailable (Cui, 2006).

In recent years, there have been rapid advancement of information and communication technology which range from sources specific to public transport, like smart cards used for automatic fare collection and global positioning system (GPS) for automatic vehicle location (AVL), to more generic data like digital footprints of mobile phone users (Zannat and Choudhury, 2019). This resulted in systems that can continuously collect and store raw data that can explain human mobility patterns at much lower costs without needing any human intervention (Cui, 2006) which led the resolution of the collected data to reach the level of the individual passenger (Nassir et al., 2011). These sources have the advantage of near or real time data collection with larger sample sizes that contains much more detailed and accurate individual level data which opens the possibility for more dynamic research in this field (Zannat and Choudhury, 2019). However, the public transit industry has yet to implement the full potential of these new sources (Cui, 2006).

The main automated data collection systems are Automatic Vehicle Location, Automatic Passenger Count system, and Automatic Passenger Count system. Since these sources cover different parts of transit. They can give a much more reliable and accurate information if they are used together. However, it isn't a straight forward process. these systems are not usually designed to be used together and provide integrated data or for generating an OD matrix at all. Hence, additional processing of data is required to make it useful (Zhao, 2004) and up to date OD matrices can be expected to be available much more frequently as a result due to the reduced costs, larger sample sizes, and suitability for automation (Cui, 2006).

- Automatic Fare Collection system AFC

  The automated fare collection system has been increasingly popular in transit systems around the world (Huang et al., 2020). Many Cities have their own Smart fare card system such as London (Oyster card), New York (Smart Link), Boston (Charlie card), Beijing (Yika tong), and Hong Kong (Octopus card) (Zannat and Choudhury, 2019). Passengers are required to tap their cards during entry and/or exit depending on the specific transit agency's arrangement. This information is collected by the AFC system and can be used for OD inference (Cui, 2006).

- Automatic Vehicle Location system AVL

  The location of vehicles in a network is very important for estimating OD matrices. In bus systems, AFC systems do not usually know their location and only collect payment records. GPS is typically used for AVL, sometimes supplemented by odometer readings(Vanderwaart, 2016). If It is installed in the vehicle, time and location of each payment can be collected (Cui, 2006)(Zannat and Choudhury, 2019). But if AVL data in not available, other method to infer the location of the bus are used such as using the bus schedule (Cui, 2006)

- Automatic Passenger Count system APC

  With sensors installed on the doors of buses, the number of passengers entering and exiting at each stop is recorded. The main purpose of APC is to provide the time, location, and total number of passengers boarding and alighting and the total number of passengers on the vehicle (Vanderwaart, 2016) (Cui, 2006).

2.2 Summary of ODX relevant work

Wang (2010) successfully implemented ADCS archived data to analyze the bus passengers' travel behavior. He based his work on the trip chaining principles applied in Chicago by Cui (2006) to infer boardings and alightings and to analyze the interchange patterns of London's TFL bus network passengers and distinguish between linked and unlinked trips. Wang distinguished his results by being the first to validate his work using manual surveys (the Bus Passenger Origin and Destination(BODS) survey data). Similarly, Nassir et al. (2011) proposed two new models to estimate the alighting stop considering passenger trip chaining relying on AFC, APC, and transit schedule data (GTFS), in the Minneapolis–Saint Paul, Minnesota, area, to estimate passenger origins and destinations at the level of individual stops. They validated their output by comparing it to APC data with vehicle location data (APC-VL). Building upon the work done by Wang (2010), Gordon (2012) expanded the methods used by Wang to include the rail system and presented methods for inferring the full journeys of all passengers on a large public transit system which was scaled up to include the passengers who do not use farecards using a modified IPF method. His results showed that reliance on small and infrequent sample can be avoided and that the process can be implemented on a daily basis for the whole network.

Chen and Liu (2013) inferred boarding and alighting locations in Chongqing, China, without GPS data. They were the first to attempt to create an algorithm to use stop Geographic Information System (GIS) and dual-direction bus stop group to estimate the traveling direction and boarding location. Zeng et al. (2015) take a unique and novel approach of inferring each passenger's full journey in the New York City called "Dynamic Transfer Link". It creates an "automatic, error-tolerant, and analyst-free daily re-calibration" of the the shortest path model. The research showed the feasibility of automation in large scale transit planning.

Alsger, Mesbah, et al. (2015) and Alsger, Assemi, et al. (2016) used smart card fare data from Queensland, Australia, with the advantage of having both boarding and alighting information, to assess the validity of previously used methods of estimating OD matrices and the effects of their assumptions (allowable transfer time, walking distance, last destination). They also proposed an algorithm in 2015 and a revised algorithm in 2016 for using individual user transactions for OD matrix generation. Their findings indicated that the assumed allowable transfer time does not have a significant impact on the OD matrices and that most passengers (90 percent) are willing to walk less than 10 minutes for transfers and most of them return to their first origin.

On the other hand, Vanderwaart et al. (2017) focused on the applications of ODX rather than improving the algorithm. They used an ODX data set provided by the MBTA from 2014 to propose a new service planning procedure to determine needed service changes in specific locations of interest by calculating the number of passengers who would benefit from these changes. The results of this procedure would provide planners with better information than what is available otherwise and would support decision making when allocating the scarce resources available.

More recently and unlike earlier models that disregarded the number of transfers on path choice or the effect of the time spent waiting or in vehicles, Sánchez-Martínez (2017) developed a dynamic programming model based on a generalized disutility minimization objective to try to find the path that the passenger will most likely take. This model is being used to infer OD matrices in Boston, Massachusetts, and is producing better results than previously applied models.

2.3 General OD matrix estimation approach using ADCS

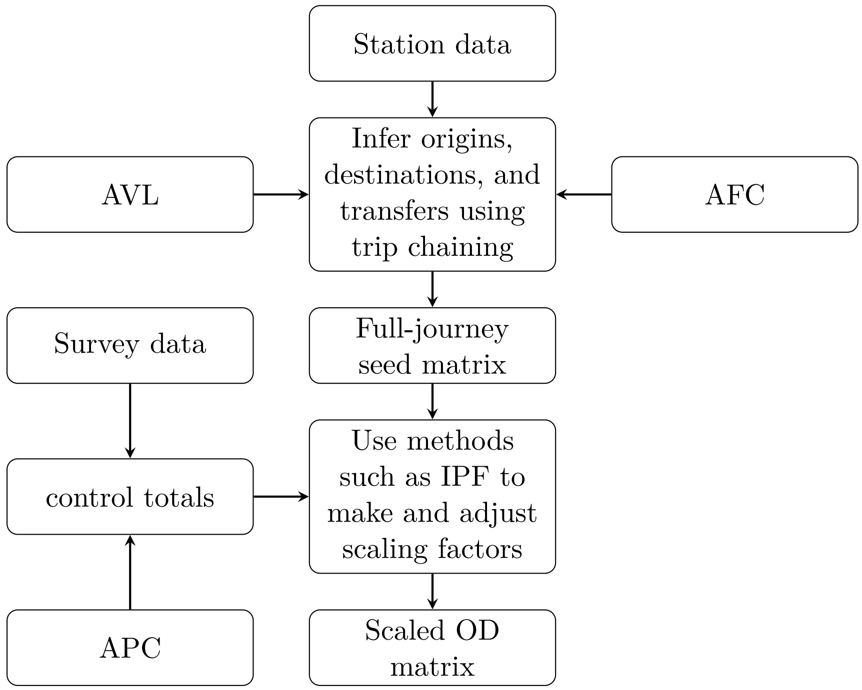

FIGURE 1 Flow chart of general OD matrix estimation approach.

3 Summary of key methods for origin-destination inference in transit systems

ADCS have become increasingly available in recent years, and with the different characteristics and types data available to different transit agencies, different methodologies have been developed.

3.1 Trip Chaining

Trip chaining is a process where passengers take at least one connection from one travel vehicle to another (Huang et al., 2020). This method is used to infer alighting stops based on entry-only fare card transaction and AVL data which is an essential step in obtaining a seed matrix. (Cui, 2006).

Trip Chaining infers passenger trips to by connecting trip legs when the criteria for a transfer between them are met. It creates buffer zones around each boarding location to infer the alighting location that preceded it. These buffer zones are based on the following general assumptions(Alsger, Assemi, et al., 2016):

- Allowable transfer time

  This value ranges from 30 min to 90 min in previous research (Alsger, Assemi, et al., 2016).

- Allowable walking distance

  Similarly, different walking distances were used ranging from 400 m to 1100 m (Alsger, Assemi, et al., 2016).

- Last destination

  The most common of the two assumptions used for the last destination is that the last alighting location of the day is the same as the fist boarding location of that day. But the more realistic assumption is to choose the closest stop on the last trip's route to the first boarding location as the last destination (Alsger, Assemi, et al., 2016).

3.2 Minimum Information/ Maximum Entropy

The maximum entropy approach is based on the statistical theory of probability and is equivalent to the gravity models (Wilson, 1967). This approach is based on the idea that there are many possible trip distributions and that the most probable estimation of the OD matrix is the one that maximizes the total entropy (randomness) (Ali and Eliasson, n.d.).

3.3 Iterative Proportional Fitting (IPF)

IPF, also referred to as biproportional fitting and RAS algorithm, is a process in which all the rows are factored proportionately to match their row totals, followed by factoring all the columns. The process is repeated until it converges (Navick and Furth, 1994). it is related to the entropy maximization technique and considered to be a subset of it (Lovelace et al., 2015). Its original formulation is attributed to(Deming and Stephan, 1940).

IPF is usually used to estimate a single route OD matrix from a seed matrix and the total boarding and alighting counts as control totals. A modified IPF or a Proportional distribution methods are used to estimate OD matrices on the network level (Cui, 2006).

IPF is more desirable than other matrix expansion methods due to ”its computational ease without loss of accuracy” (Ben-Akiva, Macke, et al., 1985).

3.4 Maximum Likelihood Estimation (MLE)

Similar to IPF, this method is used to estimate a single route OD matrix. But to use MLE, only a sample of Passenger flows and a sample of boarding and alighting counts, instead of totals, are needed (Cui, 2006).

3.5 Bayesian Approach

(“Bayesian methods allow us to estimate model parameters, to construct model forecasts and to conduct model comparisons.” ”In the Bayesian context, a model is defined by a likelihood function and a prior.” “Under the maximum likelihood approach, the model parameters are interpreted as fixed and the observed data represents a particular draw from the likelihood function. Parameter estimation then requires the maximization of the likelihood function.” “By contrast, the Bayesian approach interprets the parameters as random variables.” (**BayesianMethodOverview**).)<>

Table 1 Summary of Articles that used the above OD estimation methods

Method	**Article**	**Case Study**
Bayesian Approach	(Zhu and Levinson, 2018) (Hazelton, 2010) (Li, 2005) (Perrakis et al., 2012)	The Flanders Belgian region Bus service in San Francisco Bay RA region, Leicester The Flanders region, Belgium
IPF	(Cui, 2006) (Zhao et al., 2007) (Navick and Furth, 1994) (Gordon, 2012)	Chicago Transit Authority (CTA) bus network Chicago Transit Authority (CTA) rail system The Boston area London public transit system
MLE	(Cui, 2006) (Navick and Furth, 1994)	Chicago Transit Authority (CTA) bus network The Boston area
MI/ME/GM	(Wong, Wong, et al., 2005) (Ge and Fukuda, 2016)	Hong Kong multimodal transit network Tokyo City

Method	Article	Case Study
	(Tong et al., 2001)	The MTR system in Hong Kong
	(Wong, Tong, et al., 2005)	Hong Kong highway network
	(Ekowicaksono et al., 2016)	Bogor City
Trip Chaining	(Widyawan et al., 2017)	Public transport of Jakarta, Indonesia
	(Zhao et al., 2007)	Chicago Transit Authority (CTA) rail system
	(Huang et al., 2020)	Suzhou,China
	(Nassir et al., 2011)	Minneapolis–Saint Paul, Minnesota
	(Wang, 2010)	London TFL network

4 Impact on Sustainability

Public transit is considered an important tool for sustainability (Miller et al., 2016). Therefore, when planning transit networks, there is a need to know how to use the available infrastructure more efficiently and what is the most effective investment to make given the available resources (Peterson et al., 2007).

Big data based OD estimation has the potential to be a very efficient and effective approach because it uses real data instead of relying on samples provided by surveys. That helps planners understand the exact details of what modes people are using, how long they take while traveling, and where they are traveling to and from. This makes the decisions made by agencies more accurate and therefore can help in making transportation more sustainable by cutting unnecessary transit schedules and reducing travel time.

This sustainability means the improvement to the quality of life, the reduction of emissions and energy consumption, and has many economic benefits as a result of the increased efficiency (Henke and Carten, 2020).

5 The PVTA System

"The Pioneer Valley Transit Authority (PVTA) was created in 1974 under M.G.L. Chapter 161B and is the second largest transit agency in Massachusetts. PVTA provides public transportation services to 24 communities in Hampden and Hampshire Counties, and serves 21 Opportunity Zones. PVTA's service area covers over 600 sq. mi. and a population of almost 600,000. In FY19, PVTA provided over 10 Million fixed route rides and over 250,000 paratransit trips. PVTA's fixed route system is split into three main areas: Springfield Area Transit Company (SATCo), which operates in the southern portion of Hampden County and typically serves 60 percent of riders, UMass Transit Services (UMTS/UMass), which operates PVTA's routes in the Five College area in eastern Hampshire County and serves 31 percent of riders on average, and Valley Area Transit Company (VATCo), which operates in the northwest portion of PVTA's service area and serves approximately 9percent of riders."

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- A Brief History
- Motivation

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