CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture 2a: Events and Set Operations

Prof. Oke

UMass Amherst

College of Engineering

September 11, 2025

Outline

Module 2: Probability

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Key goals for this module:

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- Understand basic set theory and operations

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- Understand basic set theory and operations
- Understand introductory probability theory
- Learn the fundamentals of conditional probability and Bayes' theorem

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- Understand events and sample spaces
- Use set theory to express combinations of events
- Understand the concepts of mutual exclusivity and collective exhaustivity of events

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The symbol "$\subset$" means "is a subset of"

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$\varnothing$	empty/null set

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A \cup A &= A && (4)\\
A \cap A &= A && (5)\\
A \cup S &= S && (6)
\end{aligned}
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$$(A \cup B) \cup C = A \cup (B \cup C) \quad (10)$$

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Given the sets:

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3. If the sample space is given by the integers in the interval $[0, 10]$, find $(A \cup B)^c$:

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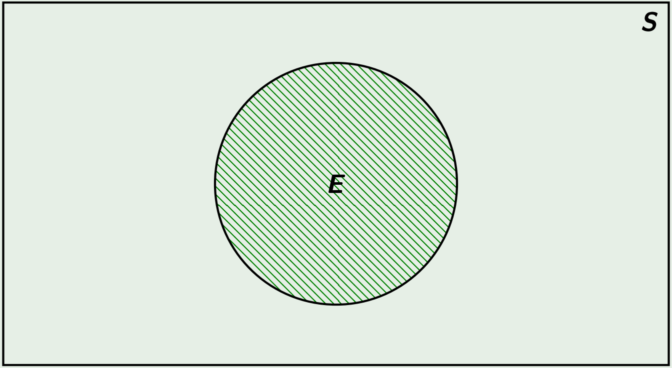

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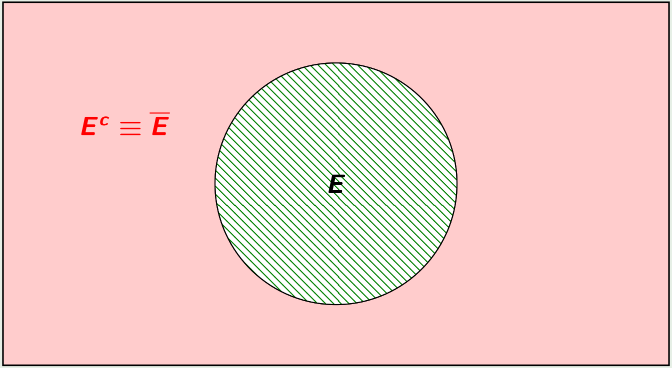

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- $A =$ students who like pepperoni
- $B =$ students who like mushrooms
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Activity

- Stand and sort yourselves physically into regions of the room
- Start with just sets A and B, creating a human Venn diagram
- Add set C and watch the complexity emerge
- Count each region and calculate: $|A \cup B|$, $|A \cap B|$, $|A^c|$, $|B^c|$, $|C^c|$.

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$$\overline{E} = S \setminus E \tag{17}$$

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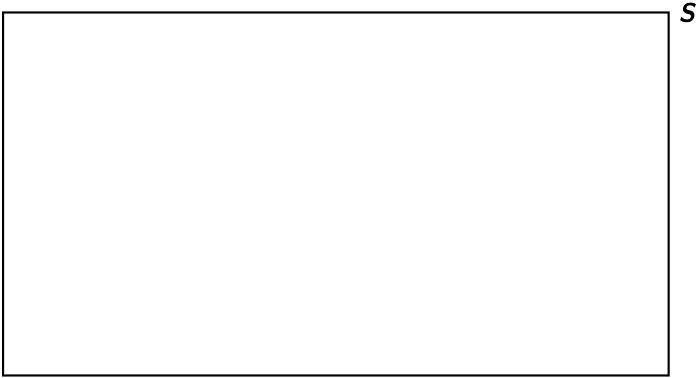

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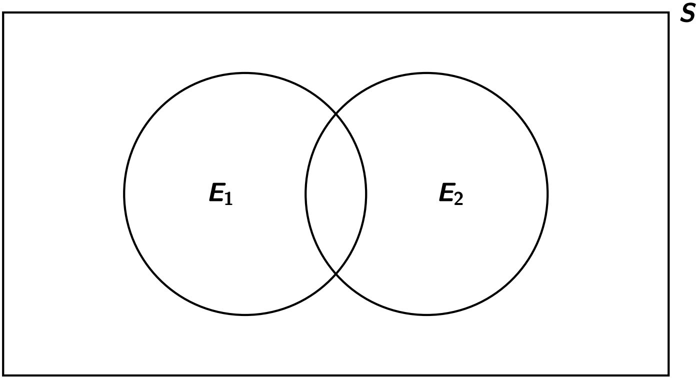

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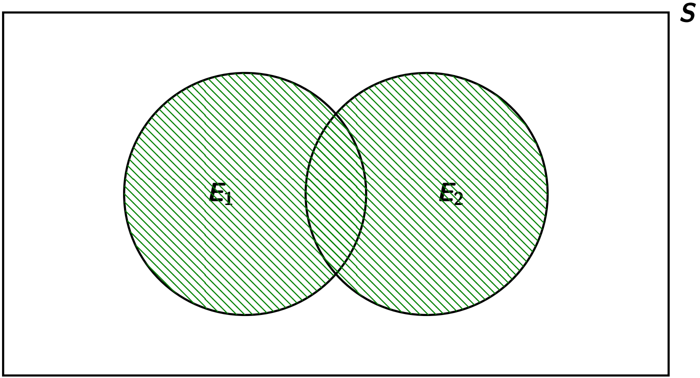

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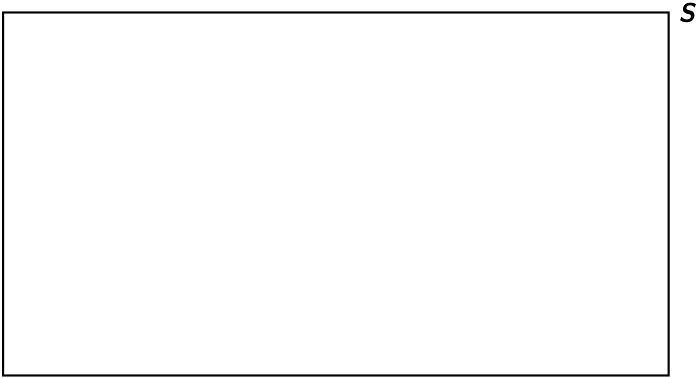

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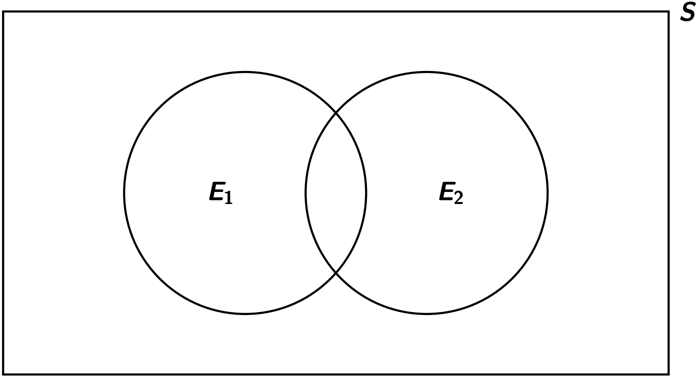

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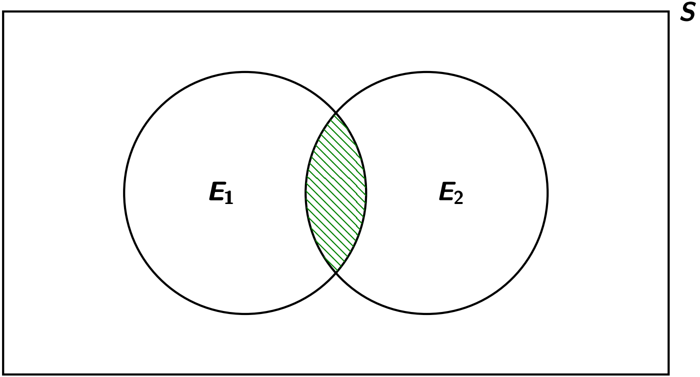

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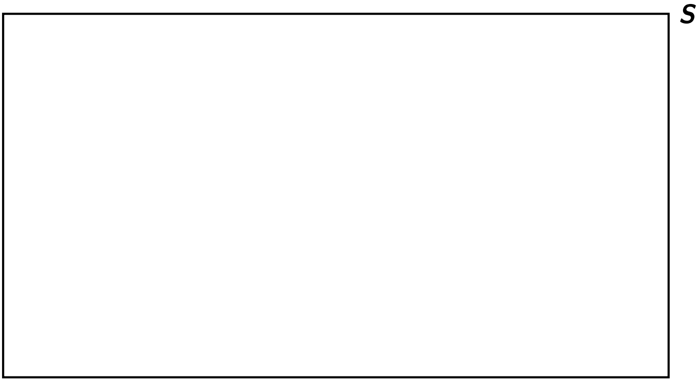

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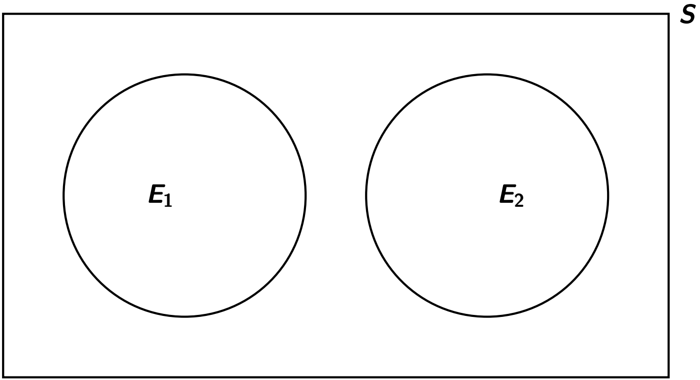

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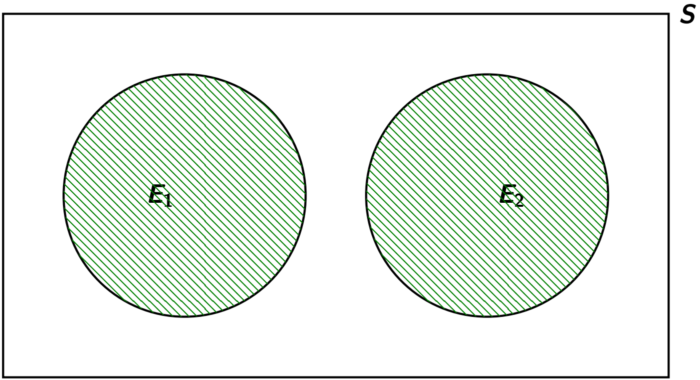

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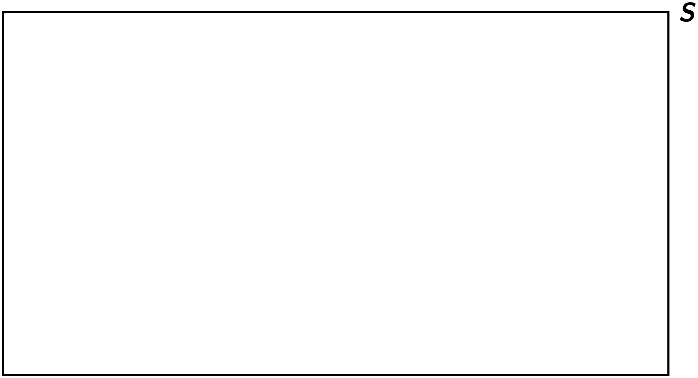

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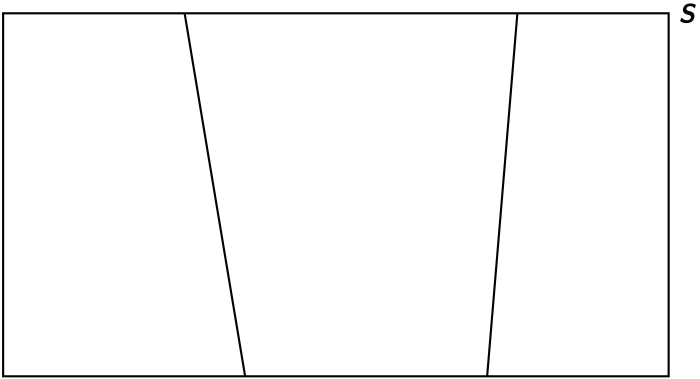

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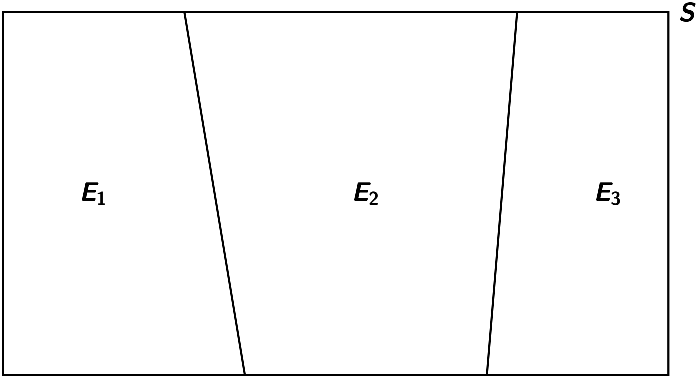

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(iii) Company *a* and company *b* are the only bidders for the single project

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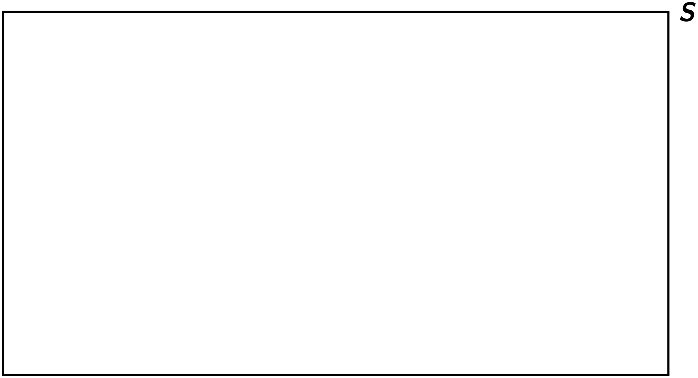

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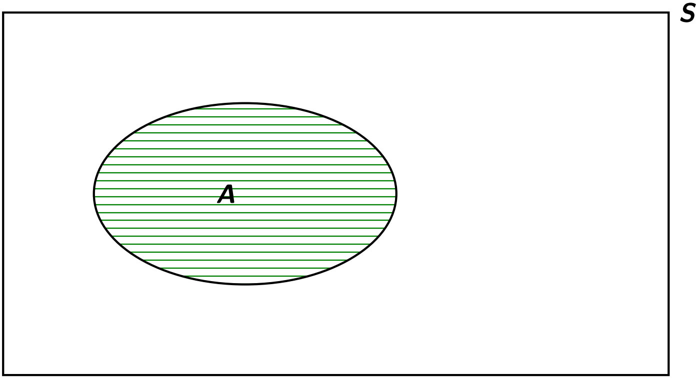

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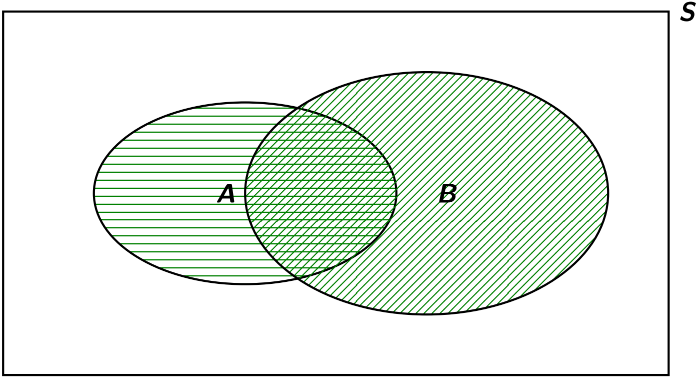

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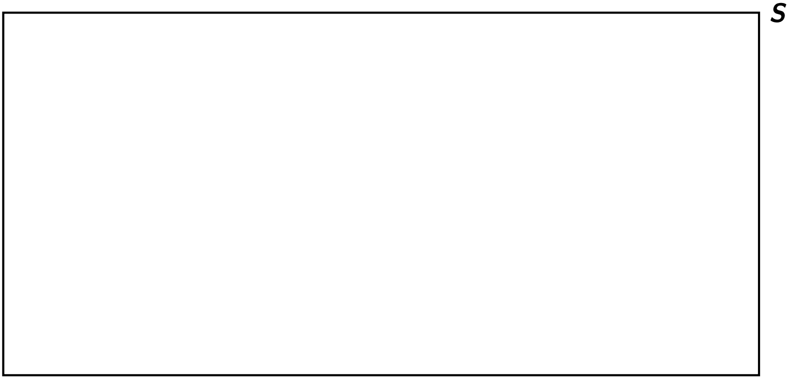

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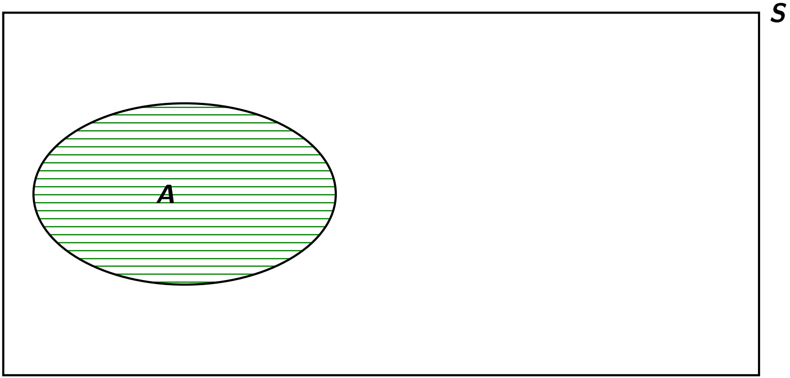

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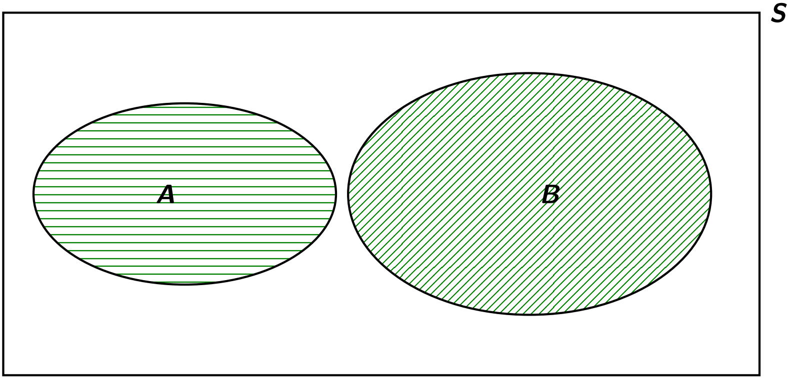

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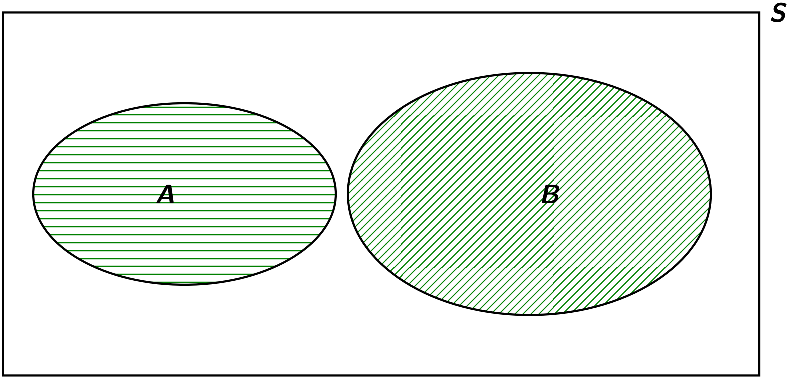

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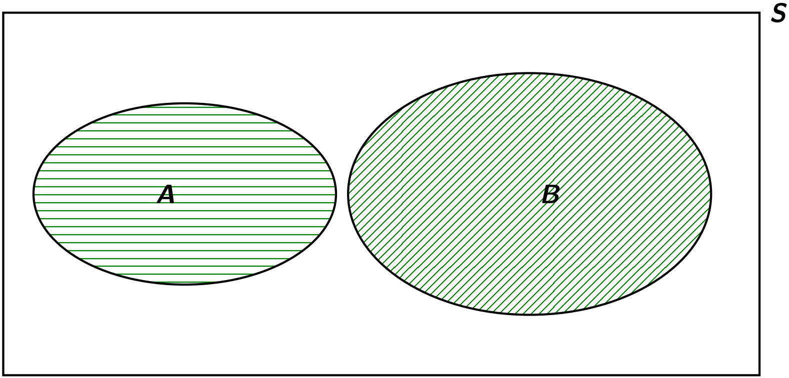

These events are mutually exclusive, as both companies cannot win the same project.

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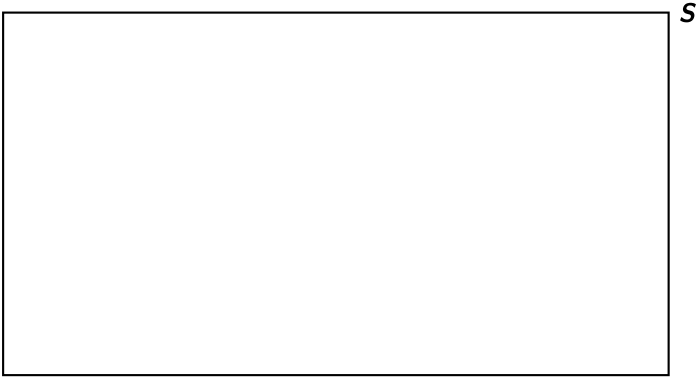

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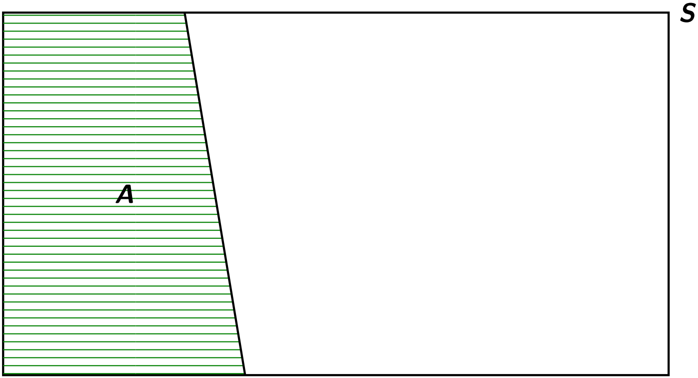

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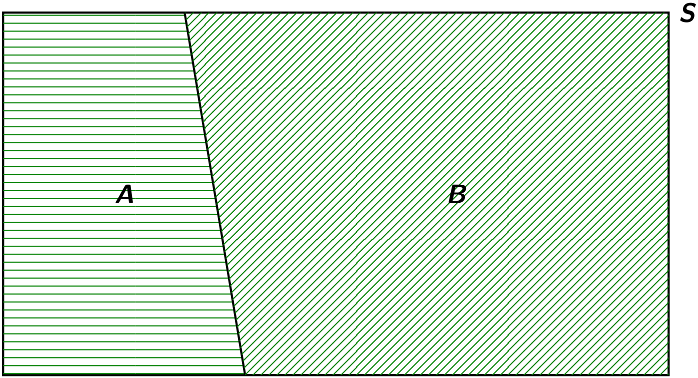

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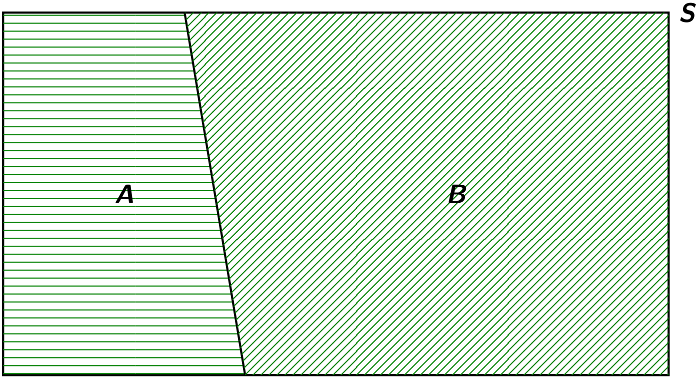

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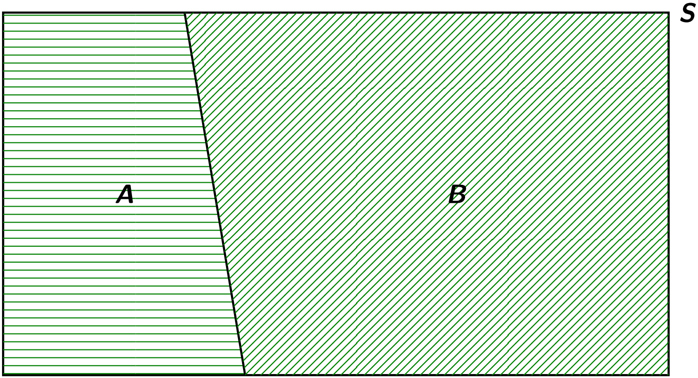

These events are both mutually exclusive and collectively exhaustive.

Recap

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Play around with set operations: https://seeing-theory.brown.edu/compound-probability/index.html#section1

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Equivalently:

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The complement of the intersection of a given number of sets/events is the union of their complements:

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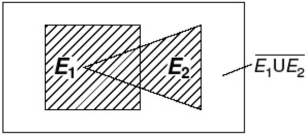
Venn diagram demonstrating de Morgan's rule



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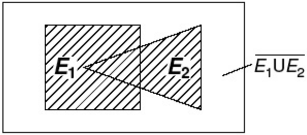
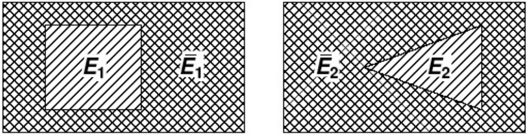
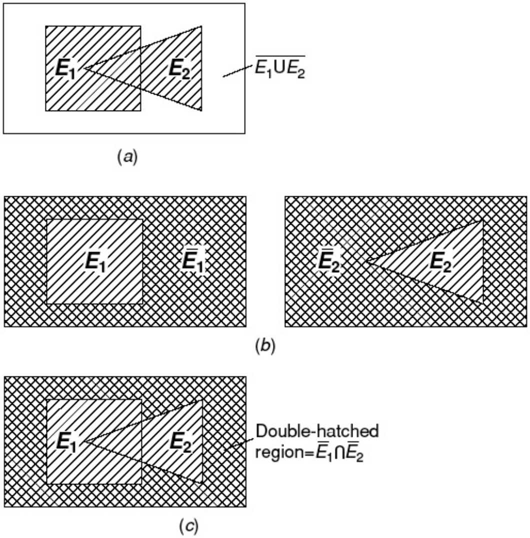


(a)



(b)

Venn diagram demonstrating de Morgan's rule



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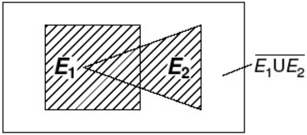
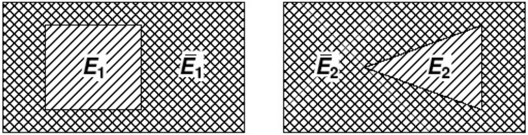
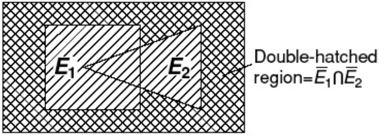


(*a*)

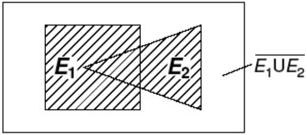


(*b*)



(*c*)

Venn diagram demonstrating de Morgan's rule



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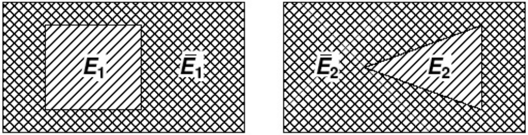

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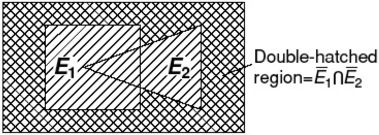

(c)

$\overline{E_1 \cup E_2} =$

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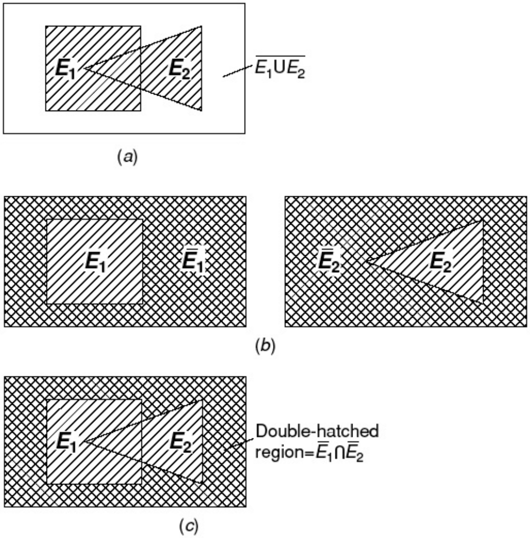

$$\overline{E_1 \cup E_2} = \overline{E_1} \cap \overline{E_2}$$

Example 4: Water supply

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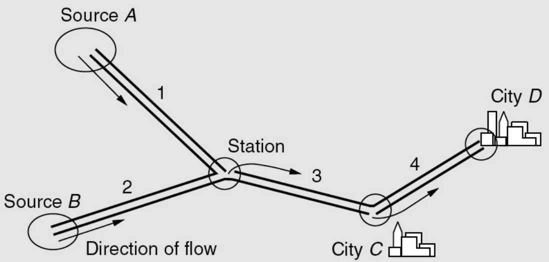
The water supply for two cities C and D comes from the two sources A and B. Water is transported by pipelines 1, 2, 3 and 4. Assume that either one of the two sources by itself is sufficient to supply the water for both cities. Also, denote E_1, E_2, E_3, E_4 as the failure of branches 1, 2, 3 and 4, respectively.



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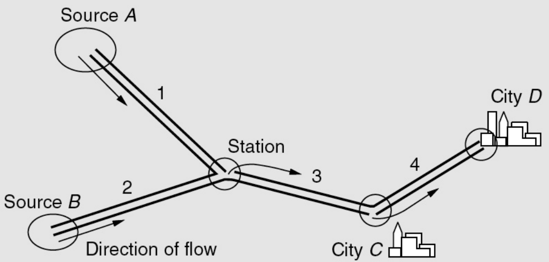
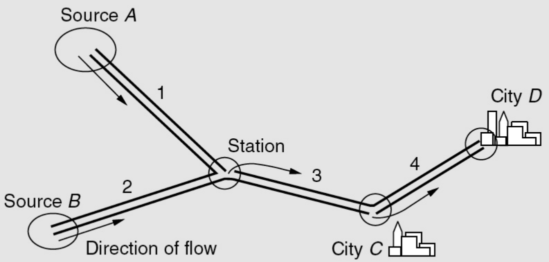


(a) Denote the event that there is no shortage of water in C.

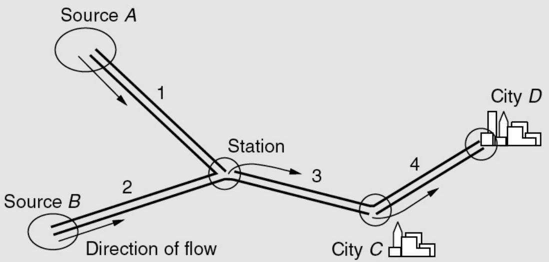
(b) Denote the event that there is no shortage of water in D.

Simplify your answers using De Morgan's rule.

Example 4: Water supply (cont.)

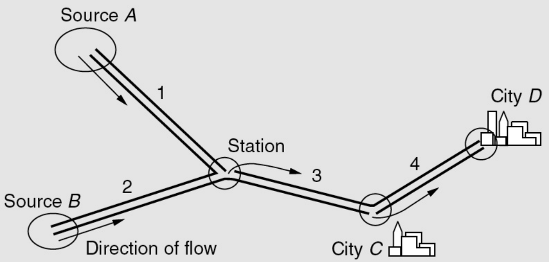


Example 4: Water supply (cont.)



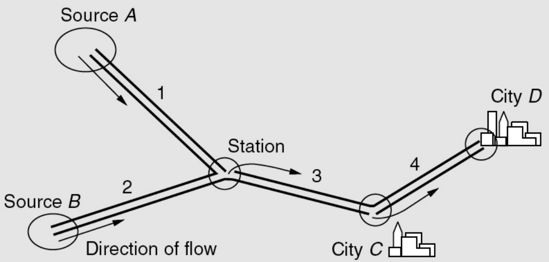
Shortage of water in C is represented by

Example 4: Water supply (cont.)



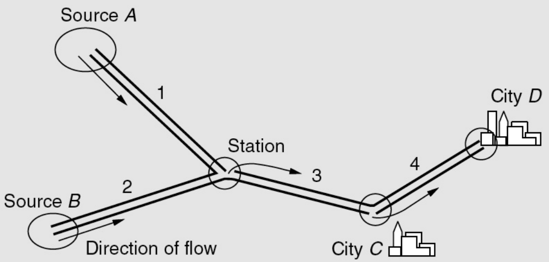
Shortage of water in C is represented by $E_1 \cap E_2 \cup E_3$.

Example 4: Water supply (cont.)



Shortage of water in C is represented by $E_1 \cap E_2 \cup E_3$. Its complement $\overline{E_1 E_2 \cup E_3}$ means there is no shortage of water in C.

Example 4: Water supply (cont.)



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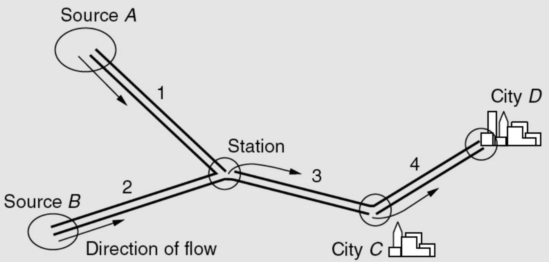

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Example 4: Water supply (cont.)

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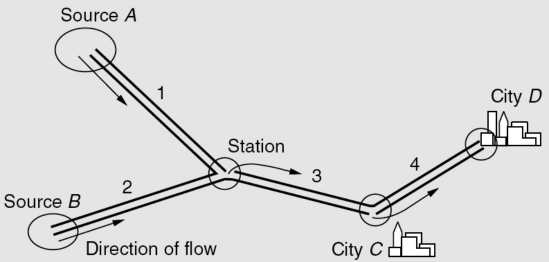
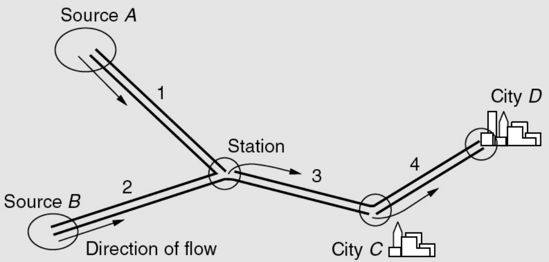


Example 4: Water supply (cont.)



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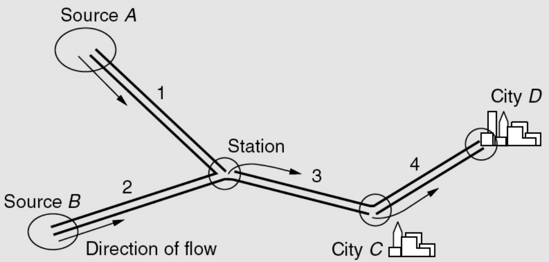
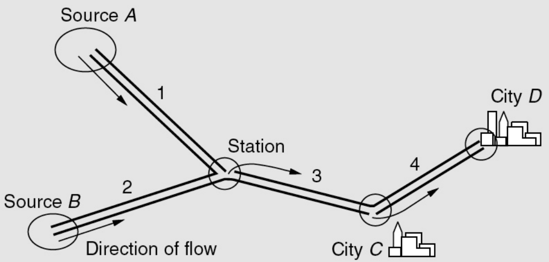
Example 4: Water supply (cont.)



No shortage of water in D is represented by $\overline{E_1 E_2 \cup E_3 \cup E_4}$.

Example 4: Water supply (cont.)



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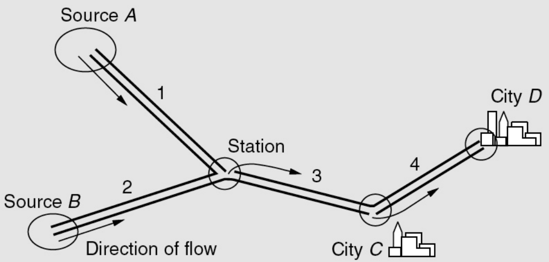
Simplified using de Morgan's rule, this becomes

Example 4: Water supply (cont.)



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Simplified using de Morgan's rule, this becomes $(\overline{E_1} \cup \overline{E_2})\overline{E_3}\,\overline{E_4}$