CEE 616: Probabilistic Machine Learning

Lecture M1b: Probability

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College of Engineering

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Outline

Random variables

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- *Discrete*
- *Continuous*
- *Mixed* (probability defined over both discrete and range of continuous values)

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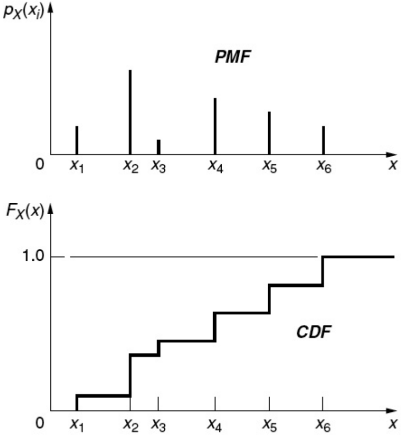

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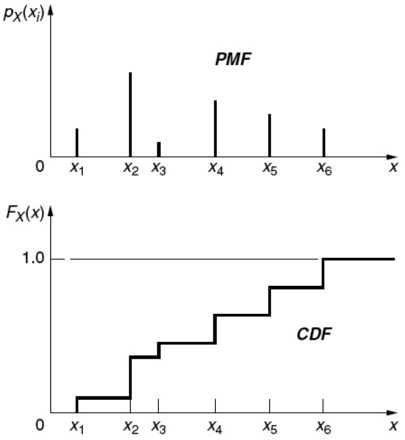

The probability masses in a PMF sum up to 1.

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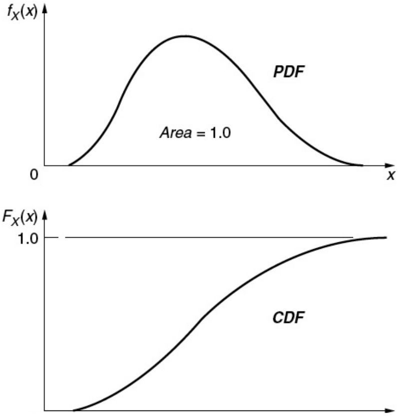

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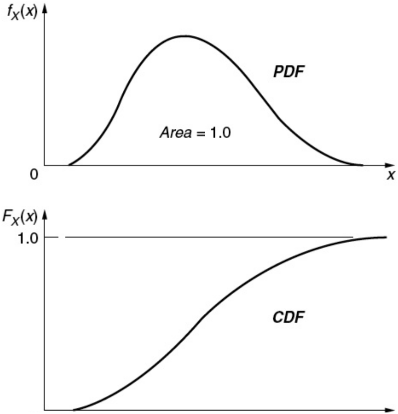

The total area under a PDF is 1.

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$$\delta_X = \frac{\sigma_X}{\mu_X} \tag{12}$$

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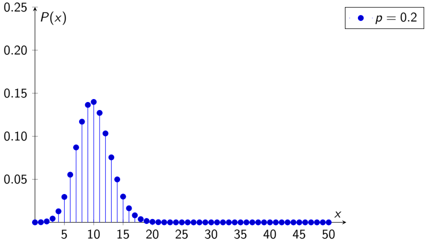

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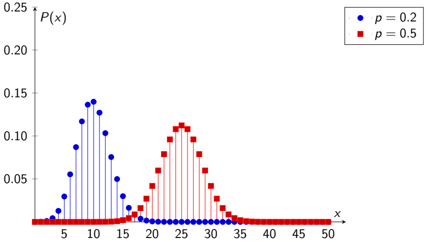

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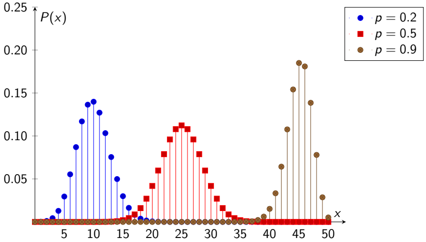

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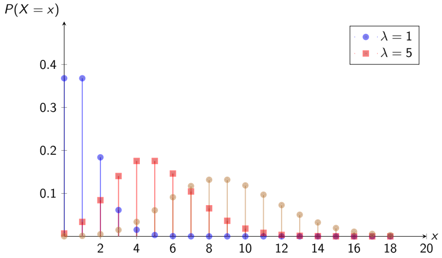

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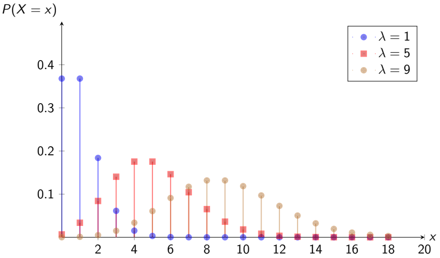

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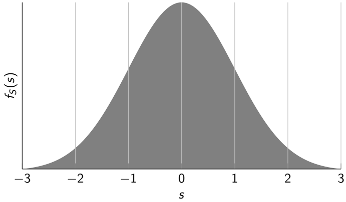

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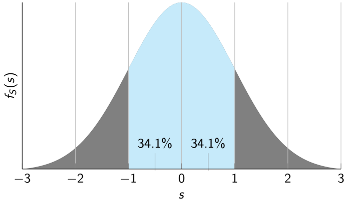

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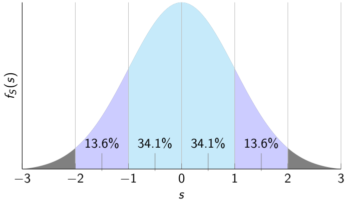

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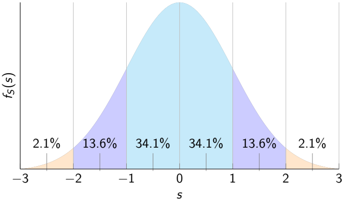

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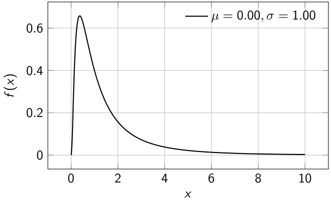

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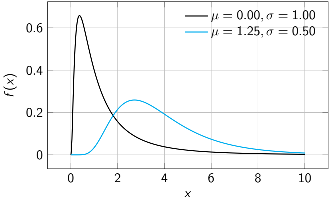

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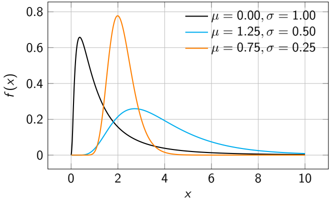

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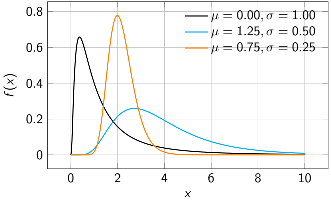

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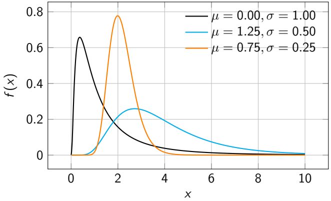

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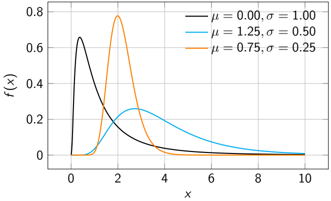

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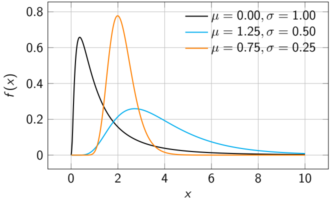

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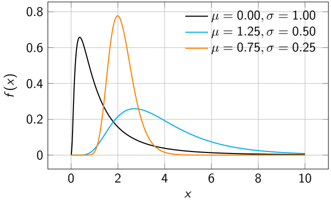

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Exponential distribution

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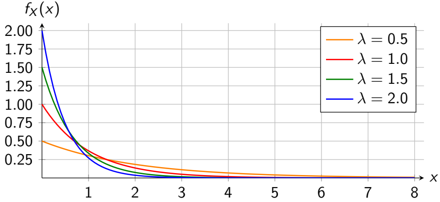

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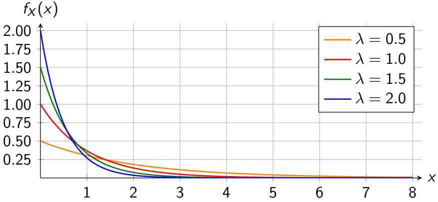

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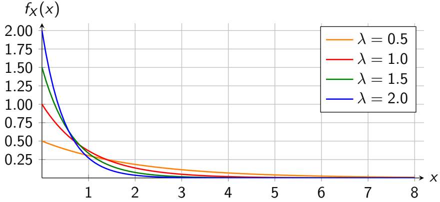

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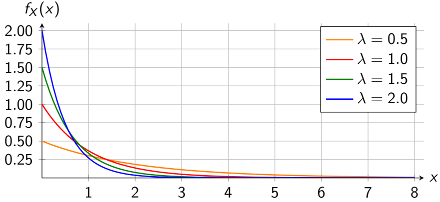

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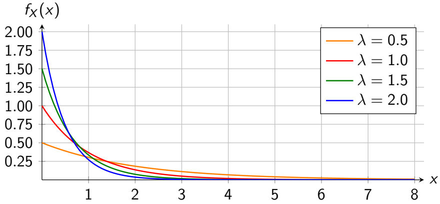

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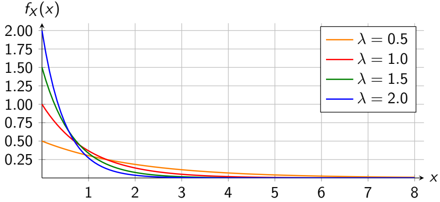

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Covariance and correlation

Recall that the variance of an r.v. X is given by:

$$\mathbb{V}(X) = \mathbb{E}[(X - \mu_X)^2] = \mathbb{E}(X^2) - \mathbb{E}(X)^2 \tag{29}$$

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This is the normalized covariance

$$\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y} := \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathbb{V}(X)\mathbb{V}(Y)}} \tag{31}$$

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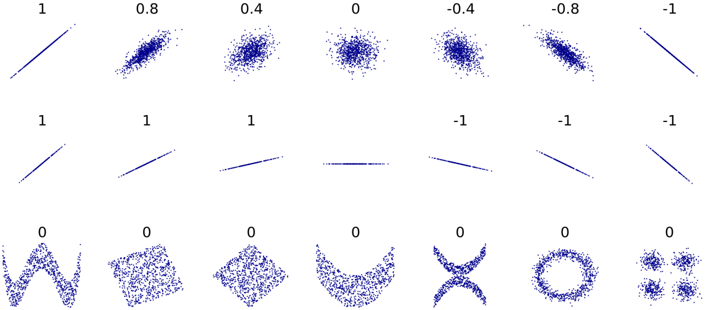

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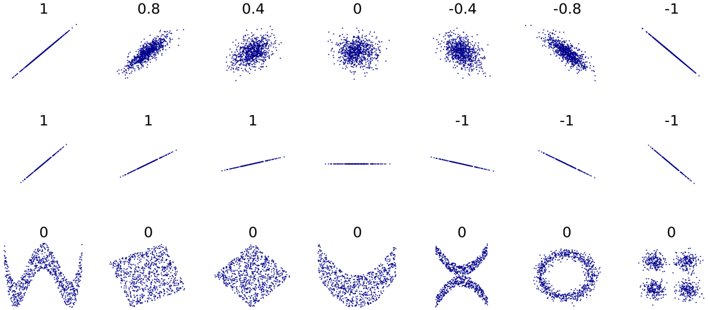

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Note that some with 0 correlation still have functional dependence (but non-linear).

Correlation does not imply causation

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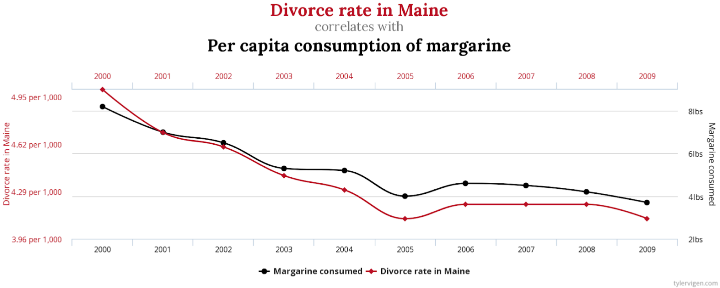

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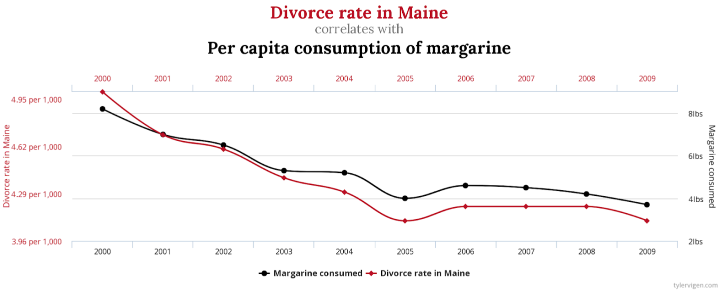

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Visit `https://www.tylervigen.com/spurious-correlations` for more examples.

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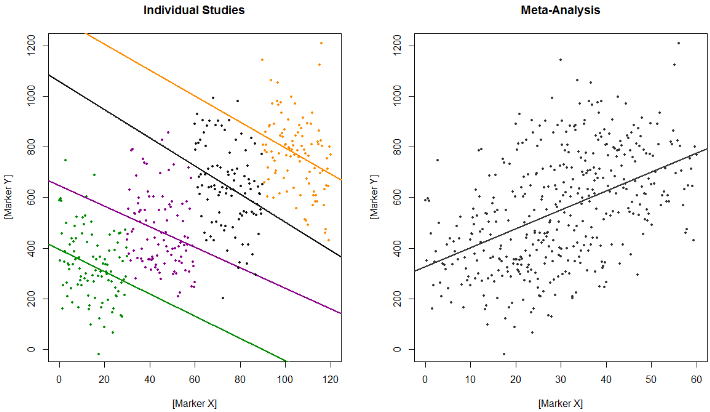

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$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{pmatrix} =$$

Multivariate normal distribution (MVN)

The MVN PDF is given by:

$$\mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) := \frac{1}{(2\pi)^{D/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right] \tag{47}$$

where:

- $\boldsymbol{\mu} = \mathbb{E}[\boldsymbol{x}] \in \mathbb{R}^D$ is the mean vector
- $\boldsymbol{\Sigma} = \mathrm{Cov}[\boldsymbol{x}]$ is the $D \times D$ covariance matrix:

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Bivariate MVN

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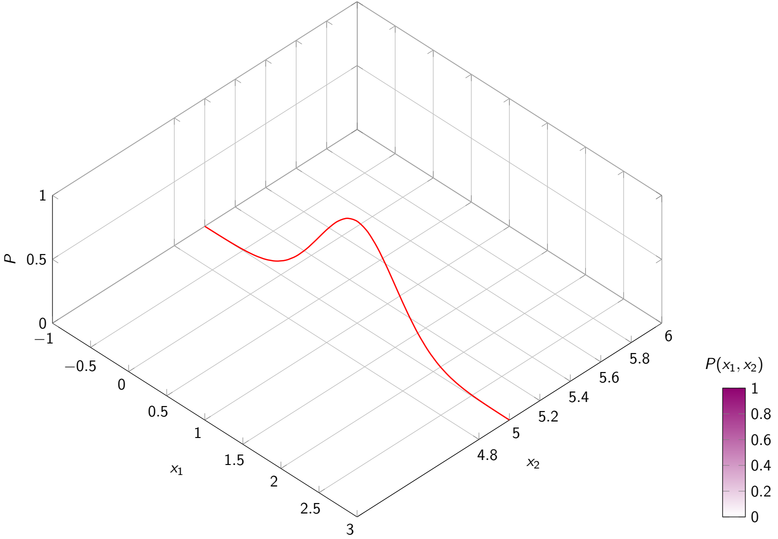

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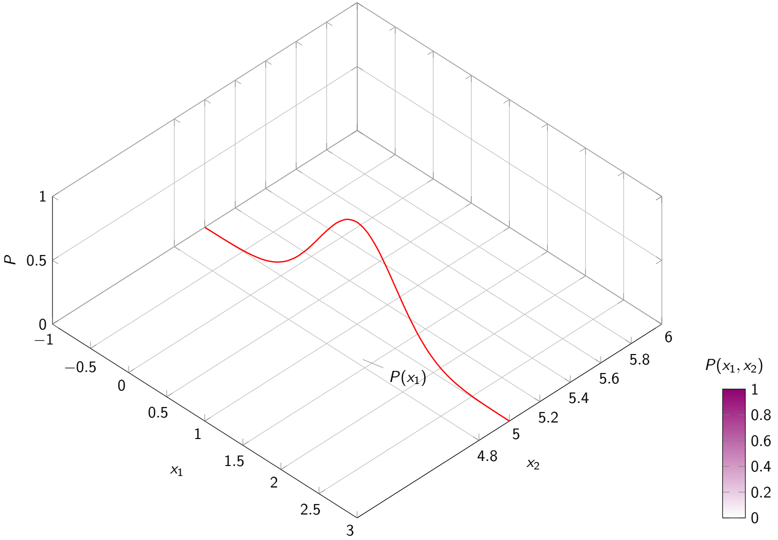

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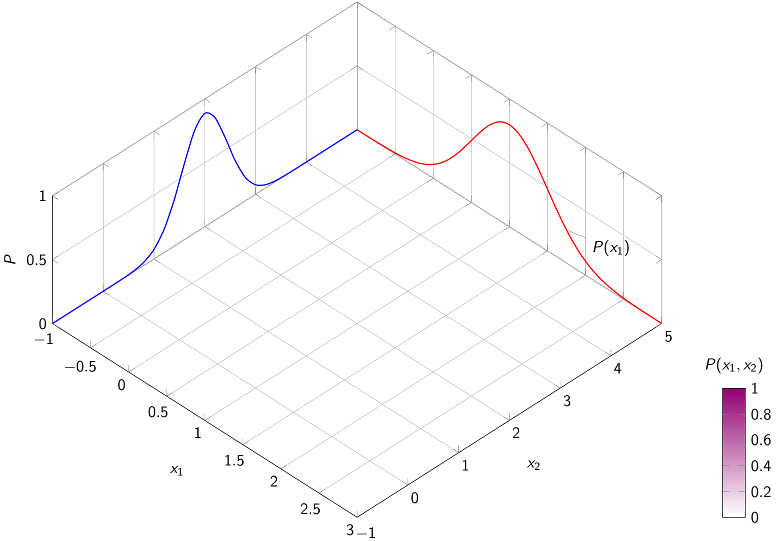

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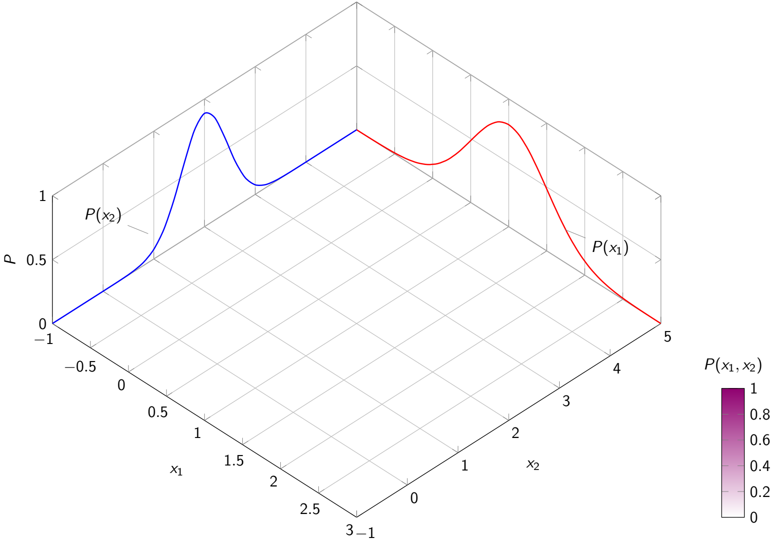

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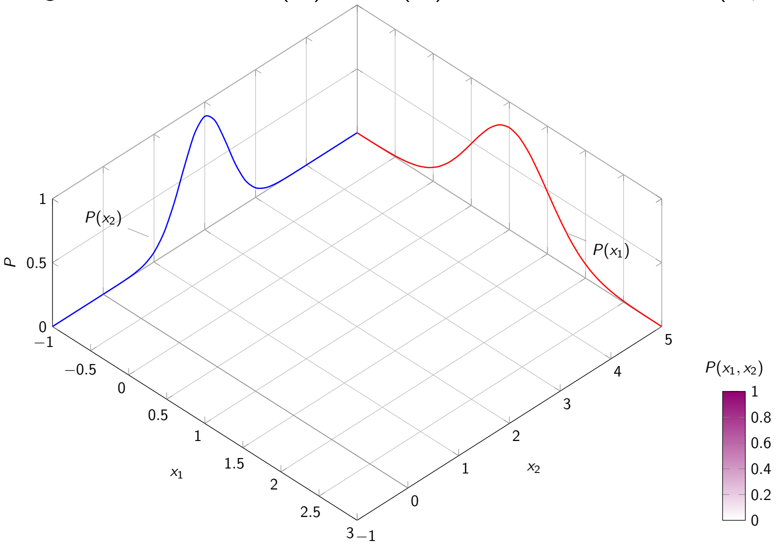

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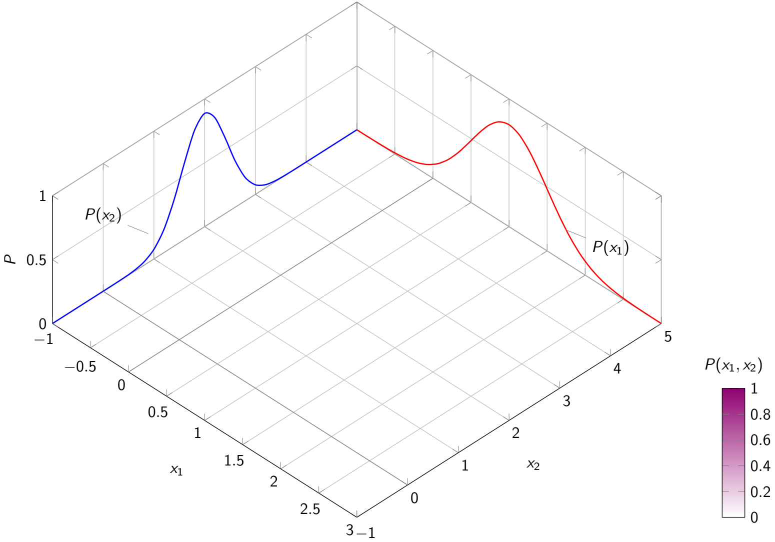

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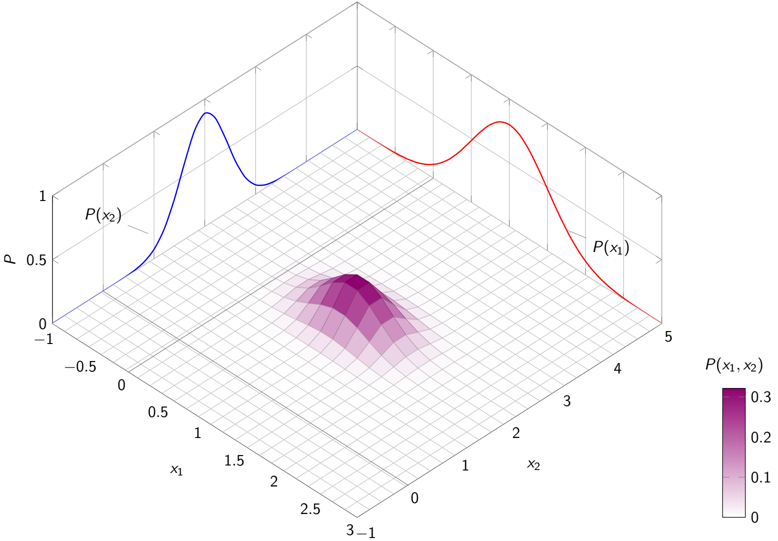

Reading

- PMLI 1, 2, 3
- PMLCE 1, 3, 4