CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture M2c: Conditional Probability and Bayes' Theorem

Prof. Jimi Oke

UMass Amherst

College of Engineering

September 18, 2025

Outline

Recap from Lecture 2b: Theory of Probability

- Three axioms of probability:

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P(S) &= 1 \\
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  - Combinations (distinct; order not important) of group of k items chosen from set of n items: $n!/(k!(n-k)!)$

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- Understand Bayes' Theorem and learn how to apply it to solving inverse probability problems

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Conditional probability (cont.)

We can think of $P(E_1|E_2)$ as the probability of realizing sample points of E_1 within the subsample space of E_2.

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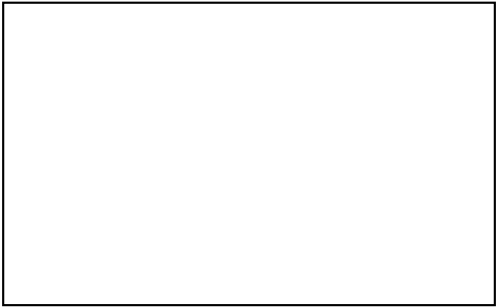

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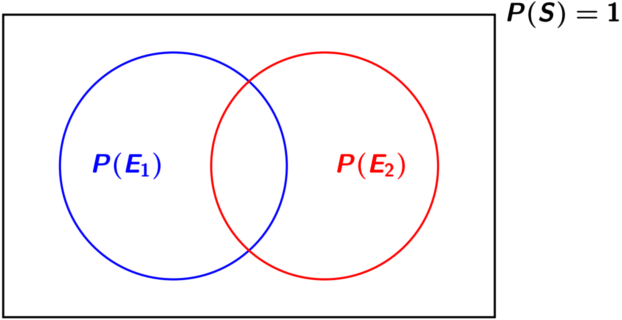

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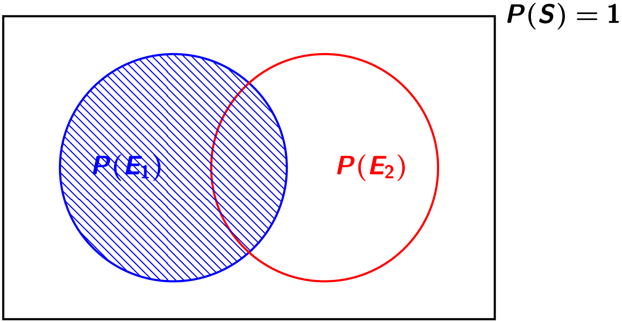

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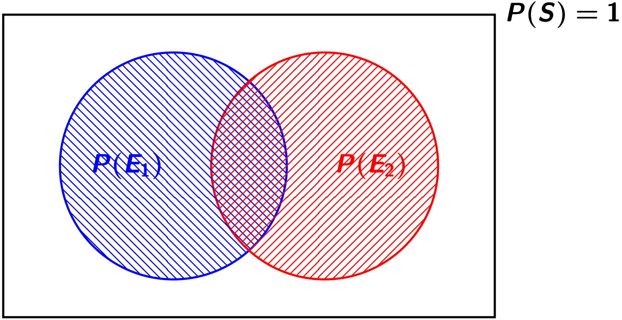

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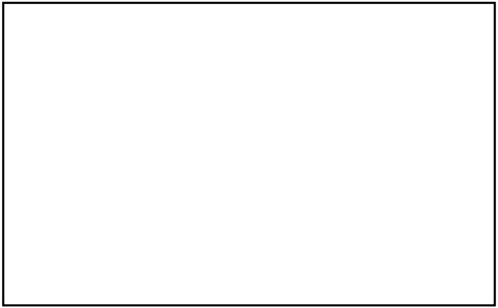

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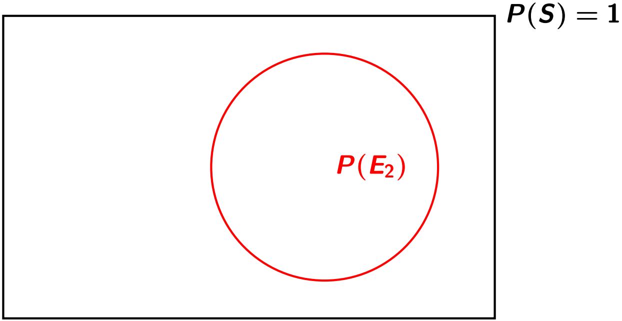

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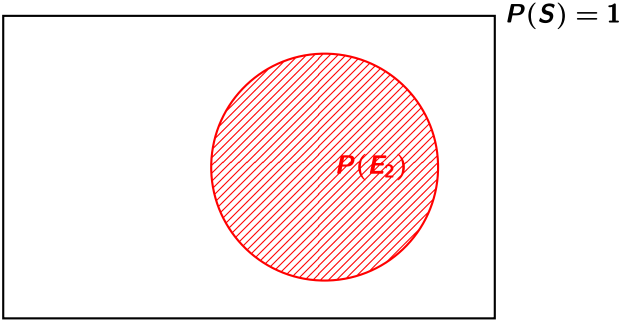

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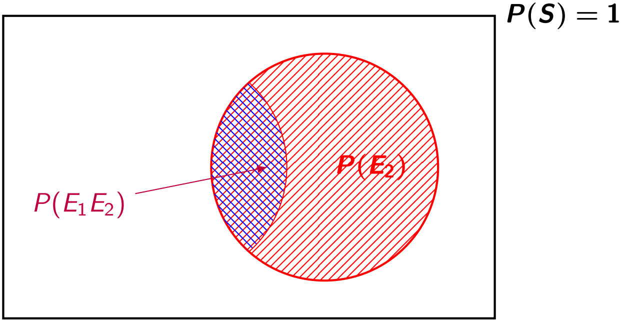

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  - The numerator is simply the intersection of both events

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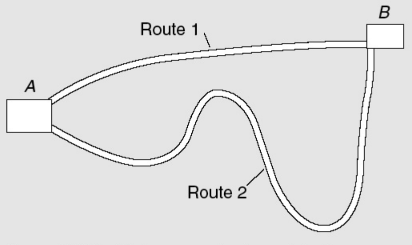

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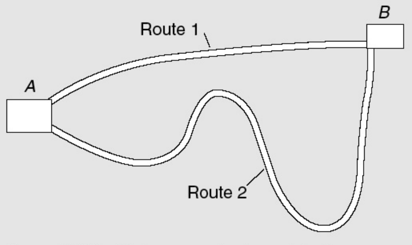

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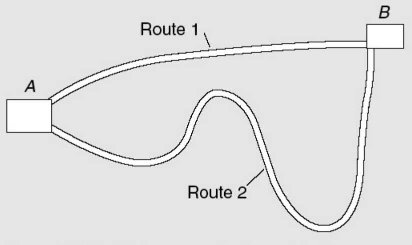

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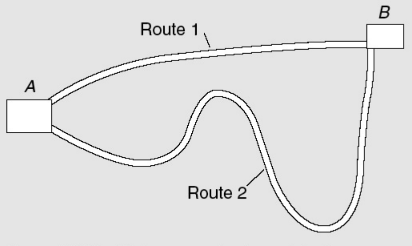

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(b) What is probability that Route 2 is open during a storm conditioned on the event that Route 1 is also open?

(c) What is probability that Route 1 is closed during a storm given that Route 2 is also closed?

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$$\begin{aligned} P(E_2|E_1) &= \frac{P(E_2E_1)}{P(E_1)} \\ &= \frac{0.40}{0.75} = \end{aligned}$$

Example 1: Highway routes (cont.)

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(c) The probability that Route 1 is closed ($\overline{E}_1$) during a storm given that Route 2 is also closed ($\overline{E}_2$) is denoted as:

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&= 1 - P(E_1 \cup E_2)
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Example 2: Rolling two dice

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Example 2: Rolling two dice (cont.)

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(c) Use the conditional probability formula to find $P(Y = 1|X = 1)$

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$$P(Y = 1|X = 1) = \frac{P(Y = 1 \cap Y = 1)}{P(X = 1)}$$

Example 2: Rolling two dice (cont.)

(c) Use the conditional probability formula to find $P(Y = 1|X = 1)$

$$
\begin{aligned}
P(Y = 1|X = 1) &= \frac{P(Y = 1 \cap Y = 1)}{P(X = 1)} \\
&= \frac{P(Y = 1) \times \cancelto{1}{P(X = 1)}}{P(X = 1)}
\end{aligned}
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(d) Why is $P(Y = 1|X = 1) = P(Y = 1)$?

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X and Y are independent events.

The multiplication rule

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Equivalently:

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$$P(AB) = P(B|A)P(A) \tag{11}$$

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IF A and B are statistically independent, the multiplication rule becomes:

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Statistically independent events

IF A and B are statistically independent, the multiplication rule becomes:

$$P(AB) = P(A)P(B) \tag{12}$$

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Equivalently:

$$P(AB) = P(B|A)P(A) \tag{11}$$

Statistically independent events

IF A and B are statistically independent, the multiplication rule becomes:

$$P(AB) = P(A)P(B) \tag{12}$$

That is, the joint probability of two statistically independent events is the product of their individual probabilities.

The multiplication rule: three events

The probability of the joint occurrence of three events is:

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$$
\begin{aligned}
P(ABC) &= P(A|BC)P(BC) \\
&= P(A|BC)P(B|C)P(C) \qquad (13)
\end{aligned}
$$

The multiplication rule: three events

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Statistically independent events

If the three events are statistically independent, then:

$$P(ABC) = P(A)P(B)P(C) \tag{14}$$

The multiplication rule: conditional probabilities

Equally applies to probabilities of events conditioned on the same subsample space.

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$$P(AB|E) = P(A|B|E)P(B|E) \tag{15}$$

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Statistically independent events

If A and B are statistically independent, the multiplication rule for two events conditioned on the same space becomes:

The multiplication rule: conditional probabilities

Equally applies to probabilities of events conditioned on the same subsample space.

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Statistically independent events

If A and B are statistically independent, the multiplication rule for two events conditioned on the same space becomes:

$$P(AB|E) = P(A|E)P(B|E) \tag{16}$$

Example 3: Airline industry strikes

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The airline industry in a certain country is subject to labor strikes by the pilots (event A), mechanics (event B) or flight attendants (event C).

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The probability of strikes by each of the individual groups in the next 3 years is given by:

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$$P(A) = 0.03 \quad P(B) = 0.05 \quad P(C) = 0.06$$

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The airline industry in a certain country is subject to labor strikes by the pilots (event A), mechanics (event B) or flight attendants (event C).

The probability of strikes by each of the individual groups in the next 3 years is given by:

$$P(A) = 0.03 \quad P(B) = 0.05 \quad P(C) = 0.06$$

Assuming that events A, B and C are statistically independent, find

(a) the probability that all groups will strike in the next 3 years

(b) the probability of a labor strike in the airline industry in the next 3 years.

Example 3: Airline industry strikes (cont.)

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$$P(A) = 0.03 \quad P(B) = 0.05 \quad P(C) = 0.06$$

(a) The probability that all 3 groups will strike in the next 3 years is given by:

Example 3: Airline industry strikes (cont.)

$$P(A) = 0.03 \quad P(B) = 0.05 \quad P(C) = 0.06$$

(a) The probability that all 3 groups will strike in the next 3 years is given by:

$$P(ABC) \;=\; P(A)P(B)P(C)$$

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(a) The probability that all 3 groups will strike in the next 3 years is given by:

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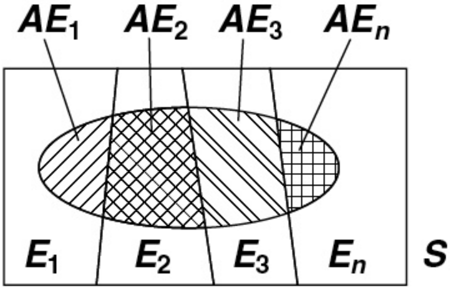

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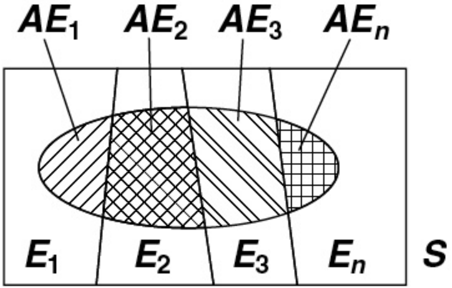

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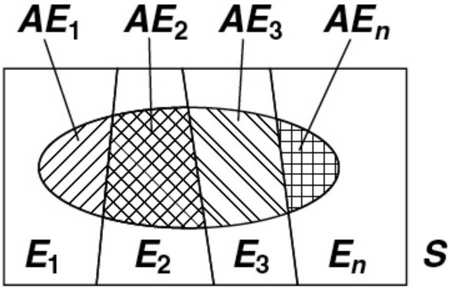

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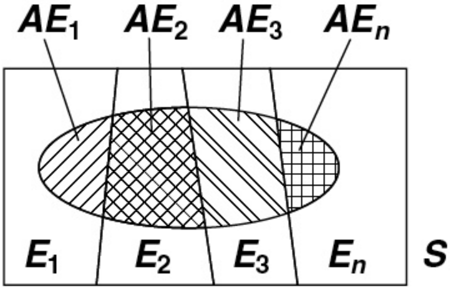

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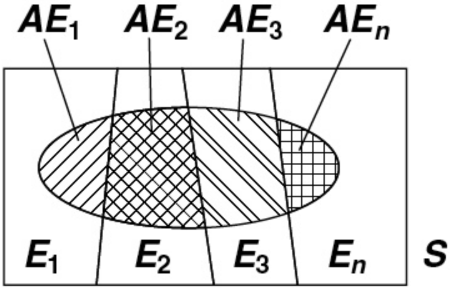

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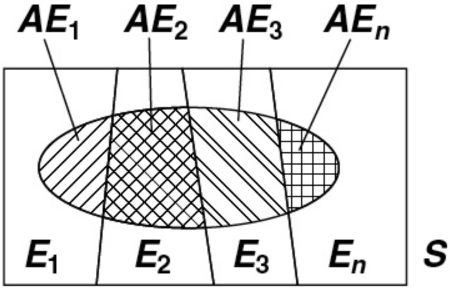

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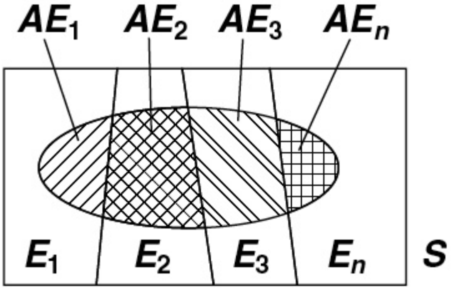

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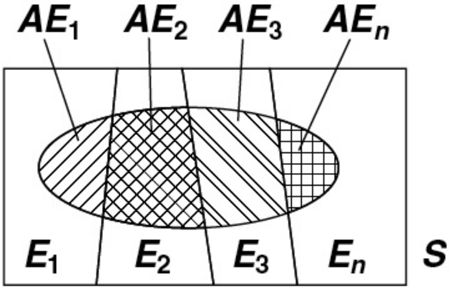

$$P(A) = P(AE_1) + P(AE_2) + P(AE_3) + \cdots + P(AE_n)$$

Note that:
$P(AE_1) = P(A|E_1)P(E_1)$,
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Find the probability of flooding in the river during the spring season $P(F)$.

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$$\begin{aligned} P(F) &= P(F|H)P(H) + P(F|N)P(N) + P(F|L)P(L) \\ &= 0.90(0.20) + 0.40(0.50) + 0.10(0.30) \end{aligned}$$

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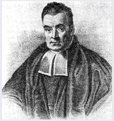
- **posterior probability:** $P(A_i|E)$



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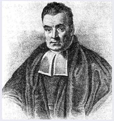
- **posterior probability:** $P(A_i|E)$
- **likelihood:** $P(E|A_i)$



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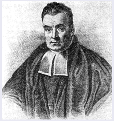
- **likelihood:** $P(E|A_i)$
- **prior:** $P(A_i)$



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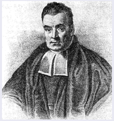

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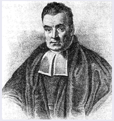

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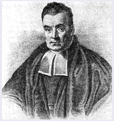

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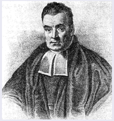

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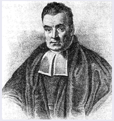

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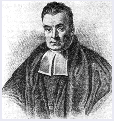

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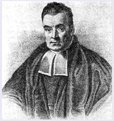

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Aggregates for the construction of a reinforced concrete building are supplied by two companies.

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Example 5: Construction supplies (cont.)

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(a) Draw a Venn diagram and convince yourself that
$P(A) = 0.60, P(B) = 0.40, P(E|A) = 0.03, P(E|B) = 0.01$

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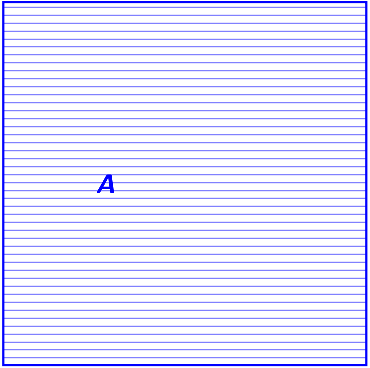

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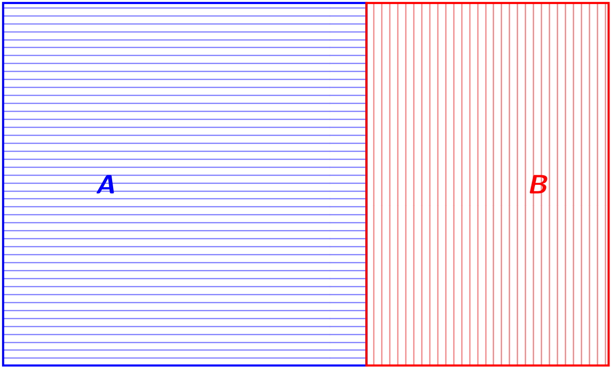

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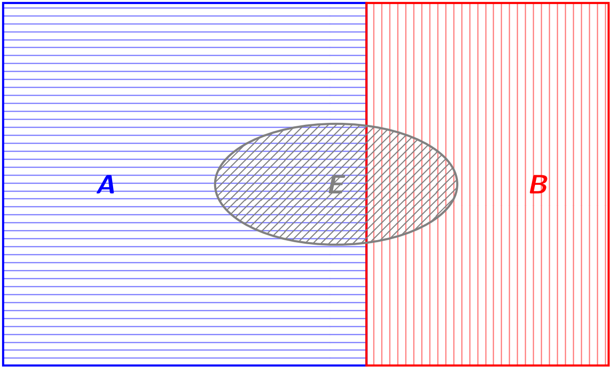

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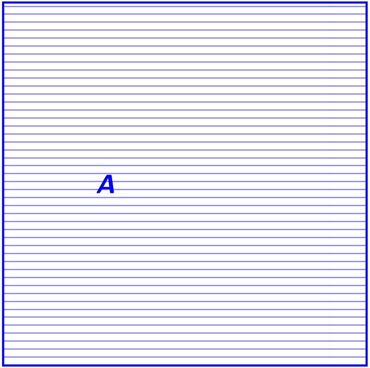

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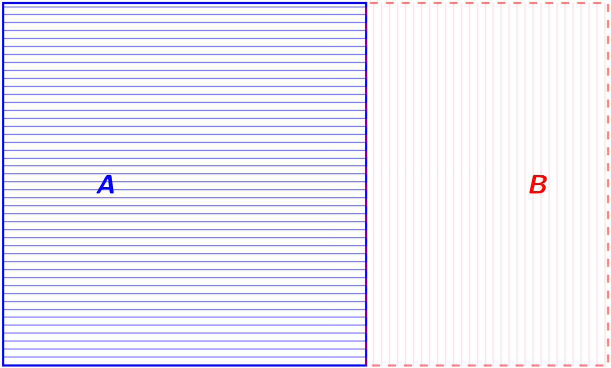

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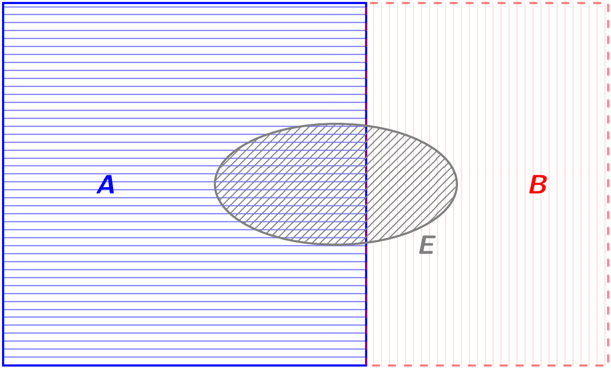

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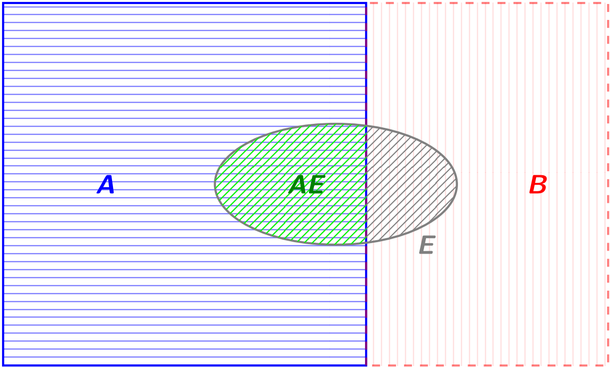

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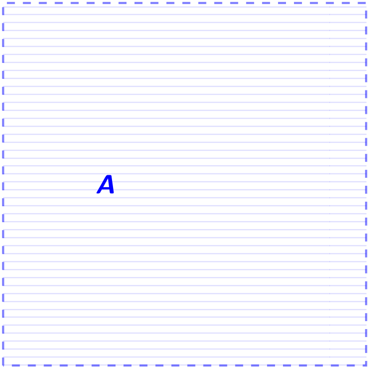

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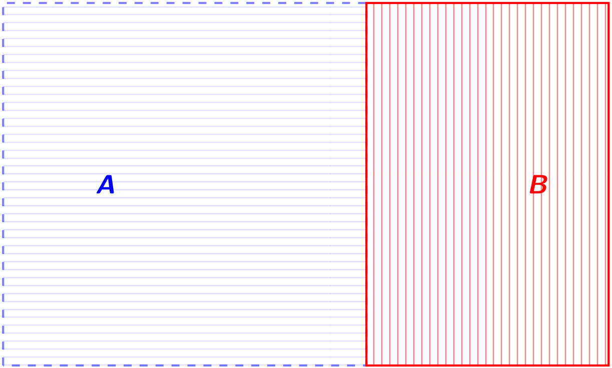

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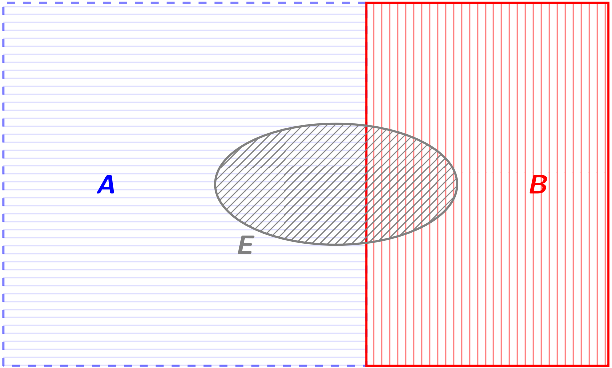

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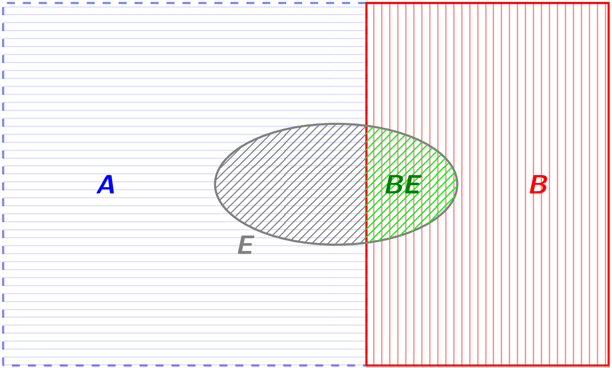

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We use the Theorem of Total Probability:

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(c) Show that the probability $P(A|E) = 0.82$ and discuss its significance

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$P(A|E)$ is the posterior probability of A having observed E. In other words, having prior knowledge of A (i.e. $P(A)$ and the likelihood of of E given A, we can update our knowledge of A (posterior probability) based on these observations.

Recap

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$$P(A|B) = \frac{P(AB)}{P(B)} \tag{25}$$

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Generally, the joint probability (intersection) of any number of independent events is the product of their individual probabilities:

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n) \tag{27}$$

- **Multiplication rule**:

Recap

- **Conditional probability**:

$$P(A|B) = \frac{P(AB)}{P(B)} \tag{25}$$

- **Independent events**:

$$P(AB) = P(A)P(B) \tag{26}$$

Generally, the joint probability (intersection) of any number of independent events is the product of their individual probabilities:

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n) \tag{27}$$

- **Multiplication rule**:

$$P(AB) = P(A|B)P(B) = P(B|A)P(A) \tag{28}$$

Recap (cont.)

- Total probability:

$$P(E) = P(E|A_1)P(A_1) + P(E|A_2)P(A_2) + \cdots + P(E|A_n)P(A_n) \qquad (29)$$

Recap (cont.)

- Total probability:

$$P(E) = P(E|A_1)P(A_1) + P(E|A_2)P(A_2) + \cdots + P(E|A_n)P(A_n) \quad (29)$$

- Bayes' Theorem:

$$P(A_1|E) = \frac{P(E|A_1)P(A_1)}{P(E|A_1)P(A_1) + P(E|A_2)P(A_2) + \cdots + P(E|A_n)P(A_n)} \quad (30)$$