Final Project (Option A): Solar Panel Angle

Prof. Oke

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *12.6.2025*

Contents

1 Project Overview

Objective: Determine the optimal tilt angle for solar panel installation by measuring power output at different angles, then use linear regression to model the relationship between angle and energy production.

Engineering Context: Solar panel angle optimization is crucial for maximizing energy yield in photovoltaic installations. Civil engineers design mounting structures, environmental engineers assess renewable energy systems, and mechanical engineers develop tracking mechanisms. A poorly angled panel can lose 25–40% of potential energy production.

What You'll Learn:

- How solar panel angle affects power generation
- The relationship between tilt angle and energy output
- Linear regression modeling of trigonometric relationships
- Cost-benefit analysis for solar installations
- Design considerations for fixed vs. adjustable mounting systems

2 Background: Solar Energy and Angles

Key Concepts

- **Tilt Angle (θ):** The angle between the solar panel surface and the horizontal plane (0° = flat, 90° = vertical)
- **Insolation:** The amount of solar radiation received per unit area. Maximum when the panel is perpendicular to the sun's rays.
- **Optimal Angle Rule of Thumb:** For fixed installations, optimal tilt angle ≈ latitude of location
  - Amherst, MA latitude: 42.4°N
  - Expected optimal angle: ∼40–45° for year-round performance
  - Varies seasonally: steeper in winter, shallower in summer

Why Does Angle Matter?

Solar panels generate maximum power when sunlight hits them at 90° (perpendicular). As the angle deviates from perpendicular, the effective area receiving direct sunlight decreases, following a cosine relationship.

3 Materials & Equipment

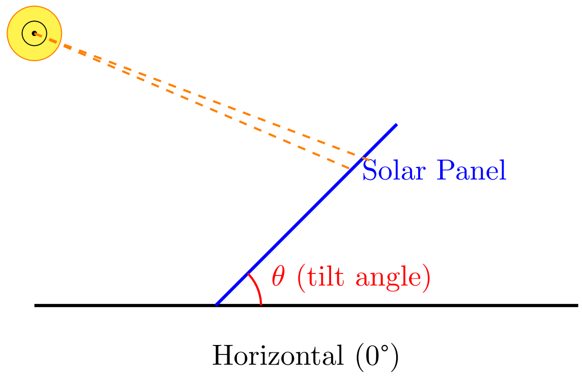
Provided by Instructor:
- Small solar panel or solar calculator
- Multimeter (voltage measurement)
- Protractor or angle-measuring app

Bring Your Own:
- Ruler/measuring tape
- Mounting setup (e.g., cardboard, books)
- Laptop for data analysis

Location Information: You are conducting this experiment at approximately 42.4°N latitude (Amherst, MA). This will be important for comparing your experimental results to theoretical predictions!

4 Experimental Setup

Panel Angle Measurement Convention



Setup Instructions:

1. Work between 11am–1pm when sun angle is most consistent; face panel toward the sun
2. Create adjustable mount using cardboard, books, or provided stand
3. Use protractor to measure angle accurately (±2° precision)
4. Connect Drok USB tester to solar panel USB jack
5. Keep all setup parameters constant except tilt angle

▷ **Critical Controls:**

- **Indoor:** Do NOT move lamp or change bulb brightness
- **Outdoor:** Complete all measurements within 30 minutes
- **Both:** Keep panel orientation (compass direction) constant
- **Both:** Ensure no shadows fall on the panel during any measurement
- **Both:** Allow 30 seconds for voltage to stabilize before recording

5 Experimental Procedure

Calibration Phase

1. Set up your light source and measurement area
2. Position panel at 45° as a test
3. Verify multimeter is reading voltage correctly (should see 0.5–6V depending on panel)
4. Practice adjusting angle and verifying with protractor

Data Collection Phase

1. **Start by testing the following angles:** 0°, 15°, 30°, 45°, 60°, 75°, 90°
2. For each angle:
   - Carefully adjust panel to target angle
   - Verify angle with protractor
   - Wait 30 seconds for voltage stabilization
   - Record voltage reading (to nearest 0.01V)

- Take 3 measurements per angle

3. Test additional angles near your predicted optimal angle (e.g., 35°, 40°, 50°, 55°) for better resolution
4. Aim for at least **10 data points**

6 Data Collection

Use the template available on the Google Drive to aid this effort. Also record your test conditions.

Test Conditions:
Testing Location: ☐ Indoor (lamp) ☐ Outdoor (sun)
Date: __________ Start Time: ________ End Time: ________
Weather conditions: ________________ Temperature: ________°F
Light Source Distance (indoor): __________ cm
Panel Orientation (compass): ________________ (e.g., facing South)

Angle (deg)	Trial 1 Voltage (V)	Trial 2 Voltage (V)	Trial 3 Voltage (V)	Average Voltage (V)	Light	cos(angle)
0						1.000
15						0.966
30						0.866
45						0.707
60						0.500
75						0.259
90						0.000

Observed Maximum Output: • Angle with highest voltage: _____ degrees • Maximum voltage measured: _____ V • Theoretical optimal angle (≈ latitude): 42.4° • Difference from theoretical: _____ degrees

7 Data Analysis Requirements

7.1 Create Scatter Plot

Plot **Angle** on x-axis vs. **Average Voltage (V)** on y-axis

- Include all data points from your experiment[1]
- Label axes clearly with units
- Add descriptive title

[1] To improve your results, you may also focus on the **"linear region"** of your data (angles from 20° to 60°) where the relationship is approximately linear. This is not absolutely necessary.

7.2 Linear Regression Analysis

Calculate the least-squares regression line: $y = \beta_0 + \beta_1 x$ where:

- y = voltage (V)
- x = tilt angle (degrees)
- β_0 = y-intercept (baseline voltage)
- β_1 = slope (voltage change per degree)

```
Report the following: • Regression equation: y = _____ + _____ x • Slope: _____
V/degree • Intercept: _____ V • Correlation coefficient (r): _____ • Coefficient
of determination (R²): _____
```

7.3 Plot Regression Line

Add your regression line to the scatter plot created in step 1

7.4 Residual Analysis

Create a residual plot (residuals vs. predicted values) to check if linear model is appropriate

8 Discussion Questions

8.1 Part A: Statistical Analysis *(Answer all questions: Q1–Q5)*

1. What angle produced the maximum power output in your experiment? How close is this to the theoretical optimal angle ($\approx 42.4°$ for Amherst)?

2. Interpret the slope of your linear regression. What does it tell you about how voltage changes with angle in your tested range?

3. The p-values of the model coefficients (slope and intercept) indicate their respective statistical significance.

4. What are the p-values for your slope and intercept? Are they statistically significant at the 0.05 level? Explain what this means in the context of your experiment.

5. What is your R^2 value for the linear model? Is the linear approximation appropriate for your selected angle range? Support with evidence from your residual plot.

8.2 Part 2: Experimental Design *(Answer Q4 AND either Q5 or Q6)*

6. What were the three most significant sources of error in your experiment? How did you minimize them?

7. Calculate the absolute percent difference between your optimal angle and the theoretical optimal (latitude):

$$\% \text{ difference} = \frac{|\text{experimental} - \text{theoretical}|}{\text{theoretical}} \times 100\%$$

8. Your panel produced maximum output at angle θ_{opt}. Calculate the power loss at $\theta_{\text{opt}} \pm 10°$:

$$\% \text{ loss} = \frac{V_{\theta_{\text{opt}}} - V_{\theta_{\text{opt}} \pm 10°}}{V_{\max}} \times 100\%$$

This tells you how forgiving the optimal angle is!

8.3 Part 3: Engineering Applications *(Answer Q7 AND any one of Q8–Q10)*

9. You're designing a solar installation on a residential roof in Amherst, MA. The roof is sloped at 25°. Based on your regression model, what percentage of potential energy production is lost compared to optimal angle? Would you recommend a mounting system to adjust the angle, or is 25° acceptable?

10. Research the typical energy gain from single-axis tracking systems (which adjust angle throughout the day) vs. fixed installations. If tracking adds \$800 to a \$4,000 system cost and increases annual energy by 25%, calculate the payback period assuming \$0.15/kWh electricity rate and 5 kWh/day base production.

11. Flat roofs (0° panels) are common in commercial buildings despite being far from optimal. Research and explain THREE reasons why building owners might choose flat installation despite the efficiency loss.

12. Snow accumulation is a major concern in Massachusetts. How does panel angle affect: (a) snow shedding (self-cleaning) and (b) wind load on mounting structures. Make a recommendation for angle considering both energy production AND structural/maintenance factors.

8.4 Part 4: Extra Credit

The true relationship between voltage and tilt angle is given by: $V = V_{\max} \cdot \cos(\theta - \theta_{\text{optimal}})$. Thus, repeat the regression analysis using $\cos(\theta)$ as the independent variable. Does this model have a better fit (i.e. higher R^2)? Include a scatter plot and regression line for this model.

9 Submission Requirements

9.1 Deliverables

1. Submit a PDF report including:
   (a) **Data Collection (Section 6)** with all measurements
   (b) **Regression Analysis Requirements (Section 7)**
   (c) **Answers to Discussion Questions (Section 8)**
2. Submit a notebook file (e.g., .ipynb) with your regression analysis code
3. Submit your presentation slides as a PDF

9.2 Grading Criteria

- Data collection quality and completeness (20%)
- Regression analysis completeness and correctness (25%)
- Discussion response depth and accuracy (20%)
- Professional presentation (aim for approx 6 slides; 4 minutes): (20%)
  - Organization: Introduction, Data, Methods, Results+Discussion, Conclusion (10%)
  - Quality of graphs and visualizations (5%)
  - Clarity of delivery (5%)
- Code quality and documentation (15%)
- Extra Credit (10% bonus)

10 Additional Resources

Recommended Research Topics:

- NREL (National Renewable Energy Laboratory) PVWatts Calculator
- Solar path diagrams and sun angle calculations
- International Building Code (IBC) wind and snow loads
- Massachusetts solar incentive programs (SMART, SREC)
- Bifacial solar panels (capture reflected light from ground)
- Agrivoltaics (combining agriculture with solar installations)
- PVsyst or SAM (System Advisor Model) for real solar design

Software for Analysis:

- Excel/Google Sheets
- Python

Useful Equations:

Optimal fixed angle ≈ Latitude (annual average) Summer optimal ≈ Latitude - 15°
Winter optimal ≈ Latitude + 15°
Energy production $\propto \cos(\theta_{\text{panel}} - \theta_{\text{sun}})$
Payback period = Initial cost / Annual savings