CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture 3D: The Binomial Distribution

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UMass Amherst

College of Engineering

October 2, 2025

Outline

Objectives of today's lecture

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Understand and apply the binomial distribution

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Understand and apply the binomial distribution

- PMF

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- PMF
- CDF

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- PMF
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Note about CDF

The median of a distribution is given by the value of X at $F_X(x) = 0.5$.

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The Bernoulli sequence is the basis for the **binomial distribution**

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- The symbol "$\sim$" is shorthand for "distributed as"

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$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, 2, \ldots, n \tag{1}$$

where n and p are the parameters and $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ is the binomial coefficient.

- The symbol "$\sim$" is shorthand for "distributed as"
- $\text{Bin}(n, p)$ is the typical notation for a binomial distribution

PMF of a binomial distribution

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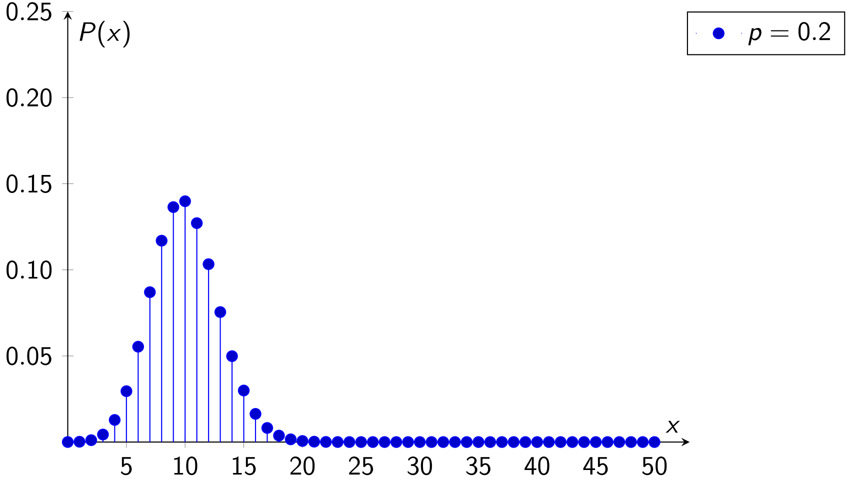

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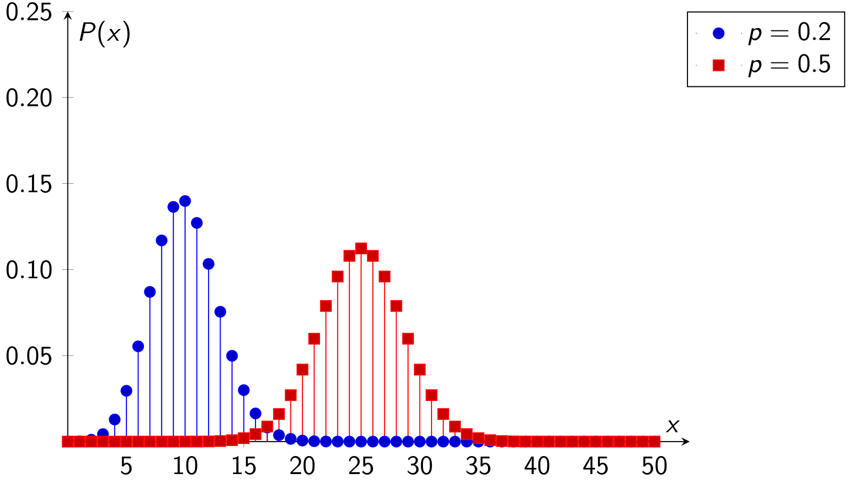

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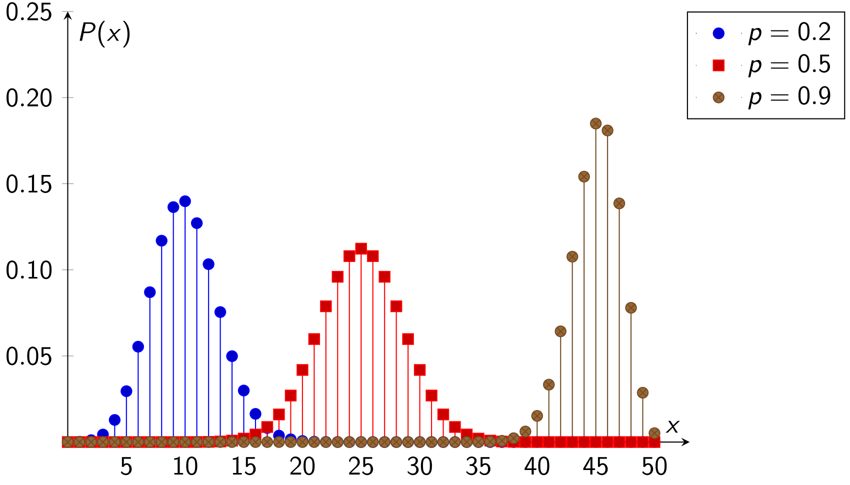

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CDF of a binomial distribution (visualization)

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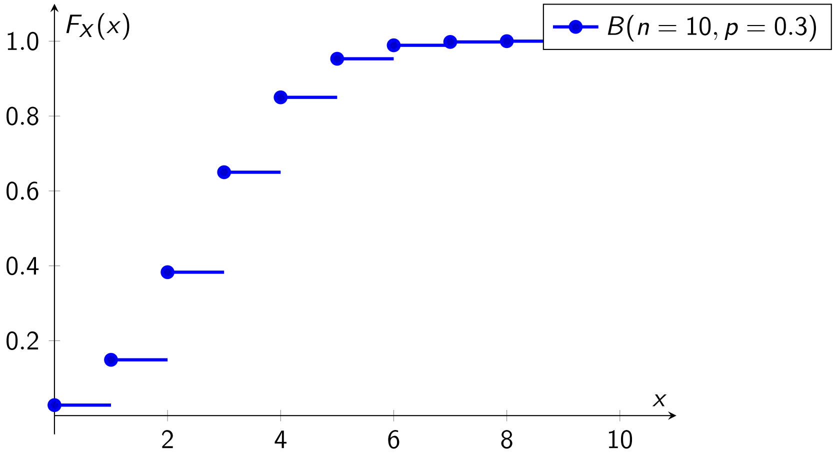

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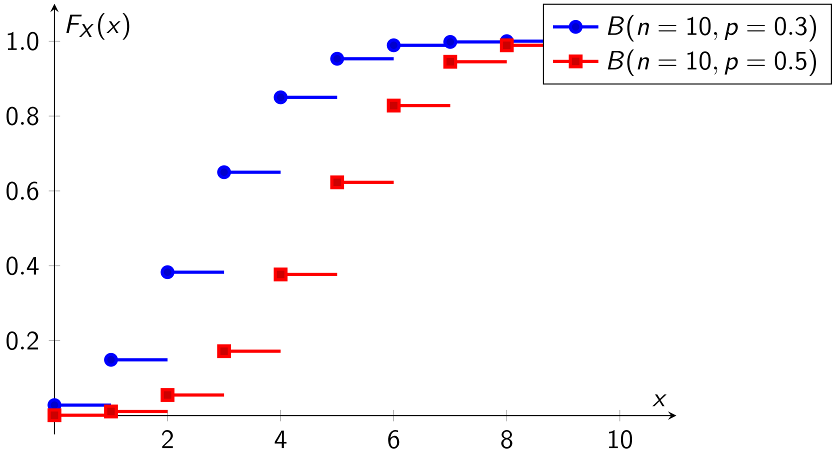

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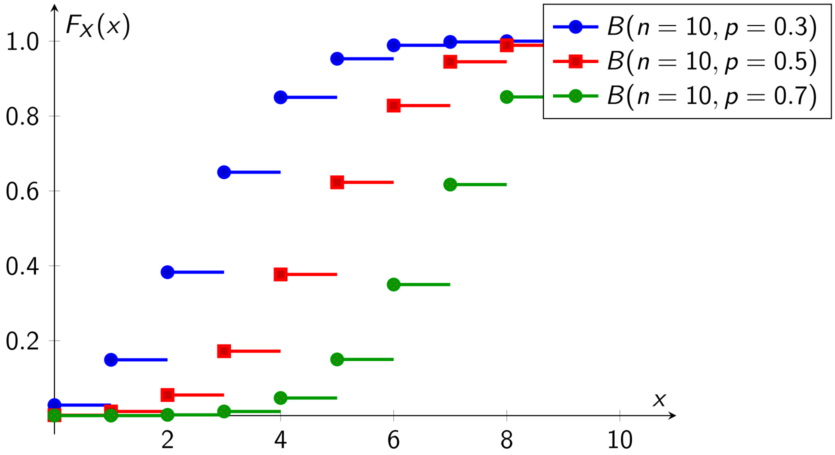

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You can show that variance of a single trial
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(d) Find the probability that at least 3 randomly selected students will be engineering majors.

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$$P(X=2) \;=\; \binom{n}{x} p^x (1-p)^{n-x} = \binom{4}{2}(0.4)^2(0.6)^{4-2}$$

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$$\begin{aligned} P(X=2) &= \binom{n}{x} p^x (1-p)^{n-x} = \binom{4}{2} (0.4)^2 (0.6)^{4-2} \\ &= 6(0.16)(0.36) \end{aligned}$$

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In Python:

```
from scipy.stats import binom
n = 4
p = 0.4
x = 2
prob = binom.pmf(x, n, p)
print(prob)
```

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(d) Find the probability that at least 3 randomly selected students will be engineering majors.

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```
from scipy.stats import binom
n = 4
p = 0.4
x = 3
prob = 1 - binom.cdf(x-1, n, p)  # P(X >= 3)
print(prob)
```

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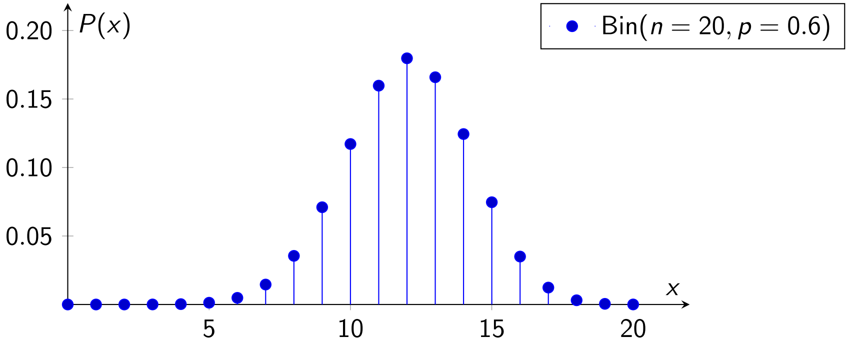

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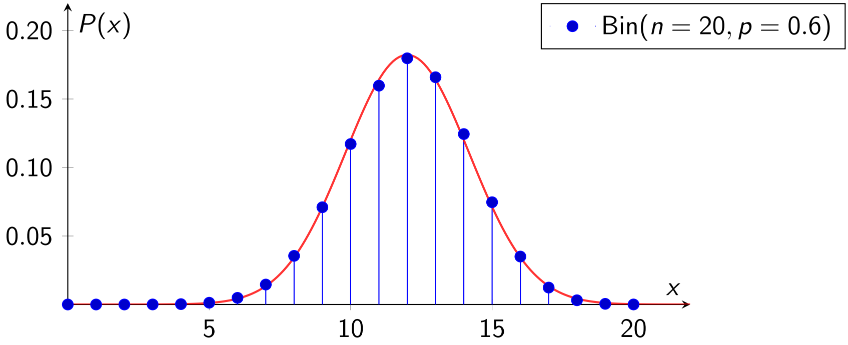
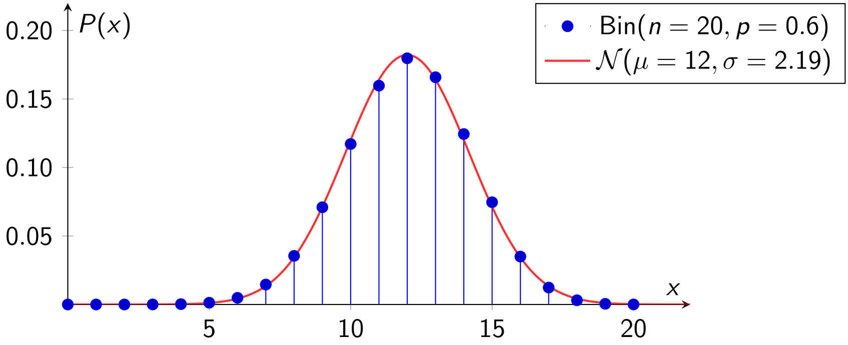


Relationship between binomial and normal distributions

Consider the distribution $B(n = 20, p = 0.6)$.
We see that it can be approximated by $\mathcal{N}(\mu = np, \sigma = \sqrt{npq})$, where $q = 1 - p$.



Relationship between binomial and normal (cont.)

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To check for normality, we can use the following rules of thumb:

$$np \geq 10 \tag{7}$$

$$nq \geq 10 \tag{8}$$

Thus:

$$P(X \leq x) \approx \Phi\left(\frac{x + 0.5 - np}{\sqrt{npq}}\right) \quad np \geq 10; nq \geq 10 \tag{9}$$

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- CDF: $F_X(x) = P(X \leq x) = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$

Reading

- Open Intro Statistics: Section 4.3 (Binomial distribution)