

E1: MIDTERM EXAM

CEE 260/MIE 273: Probability & Statistics in Civil Engineering

October 16, 2024

TIME LIMIT: TWENTY-FOUR HOURS

Name

Please print your name clearly in the box below.

Turn to the next page to read the instructions.

Instructions

This exam contains **22 pages** (including the front and back pages) and **10 problems, 106 points** (with 5 points extra credit). You have **24 hours** to complete it. You may print out the PDF, complete it and upload as a PDF on Moodle, or *neatly* answer the questions on blank pages of paper, scan and upload.

This is an **open resource examination**. You are expected to complete the exam individually. Asking anyone (colleague, friend, tutor, etc) questions about the exam is *not allowed*. If any questions arise during the exam, direct them to me (via email).

The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- **Show ALL your work where appropriate**. The work you show will be evaluated as well as your final answer. Thus, provide ample justification for each step you take. Indicate when you have used a probability table or MATLAB/Python to obtain a result. In the case of MATLAB/Python, briefly include the function or statement you used to arrive at your result. In the long response questions, simply putting down an answer without showing your steps will not merit full credit. **EXCEPTION:** For short response or “True/False” questions, *no explanations are required*. However, the more work you show, the greater your chance of receiving partial credit if your final answer is incorrect.
- If you need more space, use the blank pages at the end, and clearly indicate when and where you have done this.
- Questions are roughly in order of the lectures, so later questions may not necessarily be harder. If you are stuck on a problem, it may be better to skip it and get to it later.
- Manage your time wisely.

Problem 1 *True/False questions (15 points)*

Respond “T” (*True*) or “F” (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point. Note that a statement can only be regarded as true in this framework if it always holds in all circumstances. If a statement does not hold under a given condition not already explicitly excluded, then it should be regarded as false.

- (i) ☐ The probability of the union of mutually exclusive events is 1.
- (ii) ☐ If two events A and B are collectively exhaustive, then $P(A) - P(B) = 1$.
- (iii) ☐ For a given online account, your password must have seven small or capital letters. Assuming you are the first user to create an account, the number of possibilities for your password is 52^7 .
- (iv) ☐ The area under the curve of a PDF can be less than 1.
- (v) ☐ You begin your turn in a board game by rolling a six-sided die. If all outcomes are equally likely, then the probability of rolling a “2” is $\frac{2}{6}$.
- (vi) ☐ In a right-skewed distribution, the mean is not equal to the median.
- (vii) ☐ $P(A \cup B) = P(A) + P(B)$ for two events A and B that are mutually exclusive.
- (viii) ☐ If A and B are statistically independent events, then $P(AB)$ is always equal to $P(A)P(B)$.
- (ix) ☐ The complement of the union of collectively exhaustive events yields an empty set.

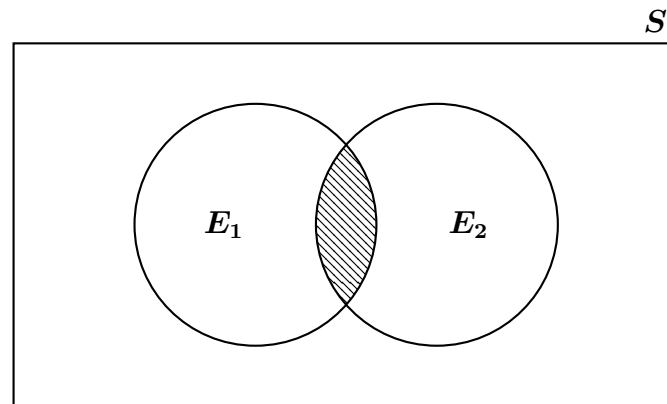
- (x) ☐ The maximum value of any cumulative distribution function is 1.
- (xi) ☐ Under certain conditions, a binomial distribution with parameters (n, p) can be approximated by a normal distribution with $\sigma = np(1 - p)$.
- (xii) ☐ The probability of the number of tails that occur in 100 tosses of a coin (which can yield either a head or a tail) can be appropriately modeled by a binomial distribution.
- (xiii) ☐ Given a normal distribution with parameters μ and σ , the median of the distribution is μ^2 .
- (xiv) ☐ The standard normal variate Z has a variance of 1.
- (xv) ☐ The probability of the elapsed time between events that constitute a Poisson process can be modeled using an exponential distribution.

Problem 2 *Venn diagrams (11 points)*

Write the combination of events (using set notation) depicted in each of the figures below.

(i)

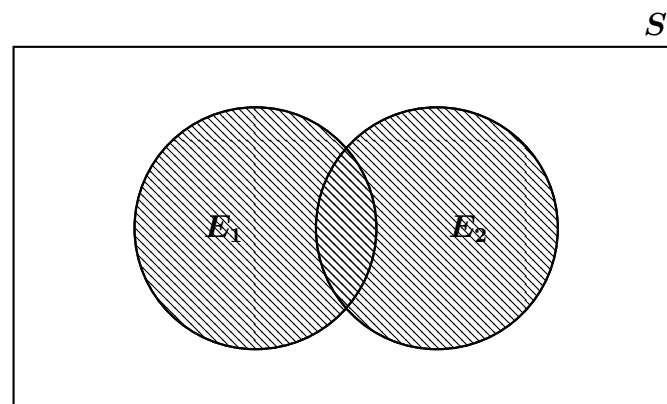
[1]



Answer:

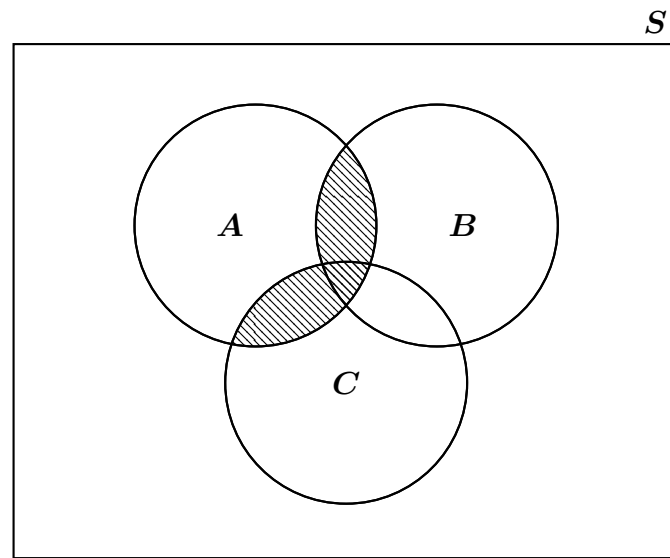
(ii)

[2]



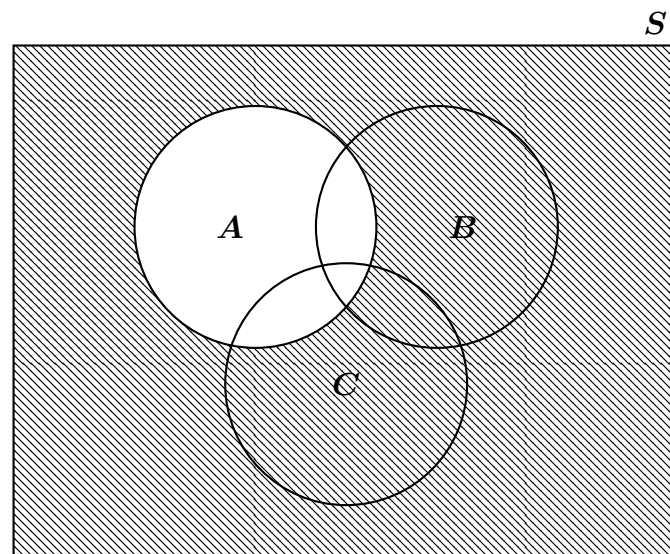
Answer:

[2] (iii)



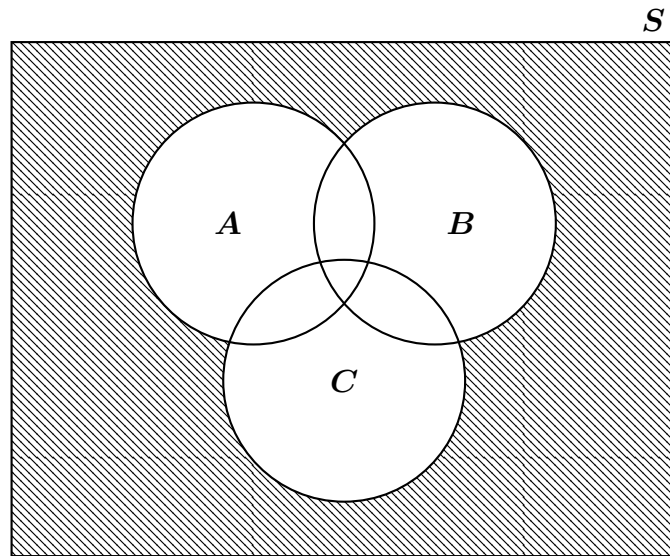
Answer:

[2] (iv)



(v)

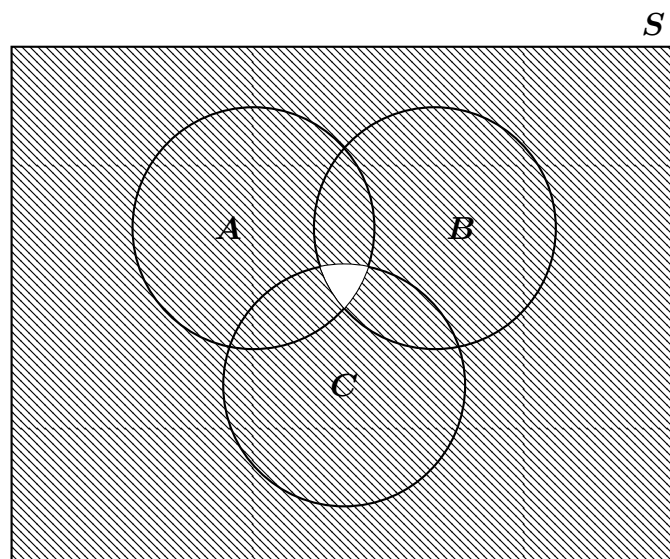
[2]



Answer:

(vi)

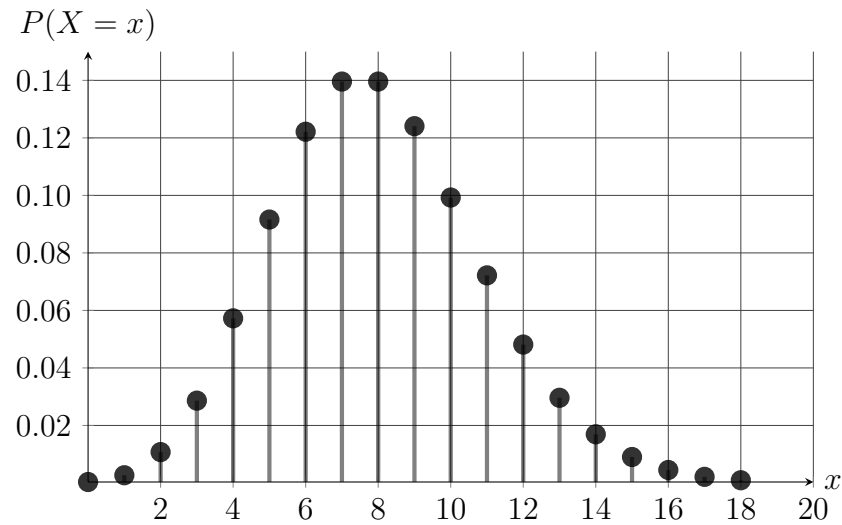
[2]



Answer:

Problem 3 *Short answer questions (22 points)*

- (a) The PMF of a random variable X is given in the figure below. Use the figure to answer the following questions.



- [1] (i) Estimate the probability $P(X = 4)$.

Answer:

- [2] (ii) Estimate the probability $P(6 < X \leq 8)$.

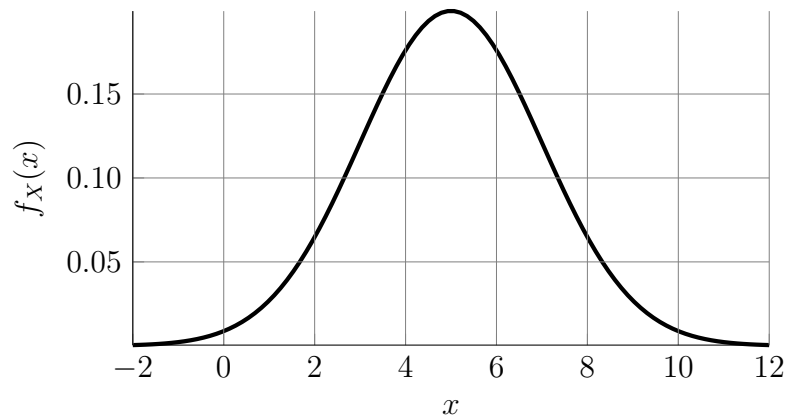
Answer:

- [1] (iii) If the distribution shown has a single parameter λ , which of the following would be your best guess for λ ? (Circle or underline the correct answer.)

- A. 2
- B. 8
- C. 10
- D. 12

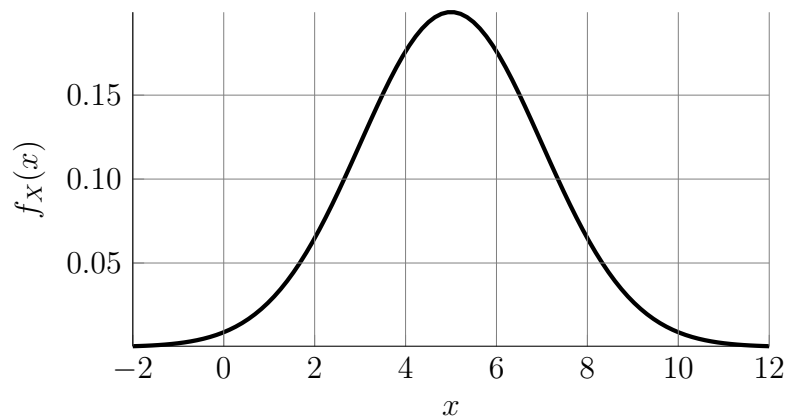
(b) Shade the area under the curve that represents the probability $P(X > 8)$.

[1]



(c) Shade the area under the curve that represents the probability $P(4 \leq X < 6)$.

[1]

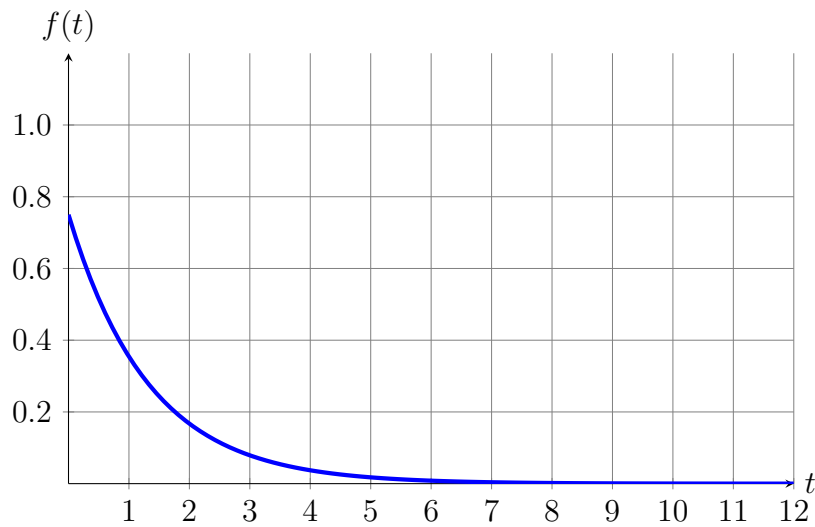


(d) If the curve in part (c) is the PDF of a normal distribution, what is its mean value?

[1]

Answer:

- [2] (e) Shade the area under the curve that gives you the probability $P(X \leq 1 \cap X > 2)$.



- [1] (f) The figure shows the graph of the CDF of an exponential random variate. What is the 30th percentile of this distribution?

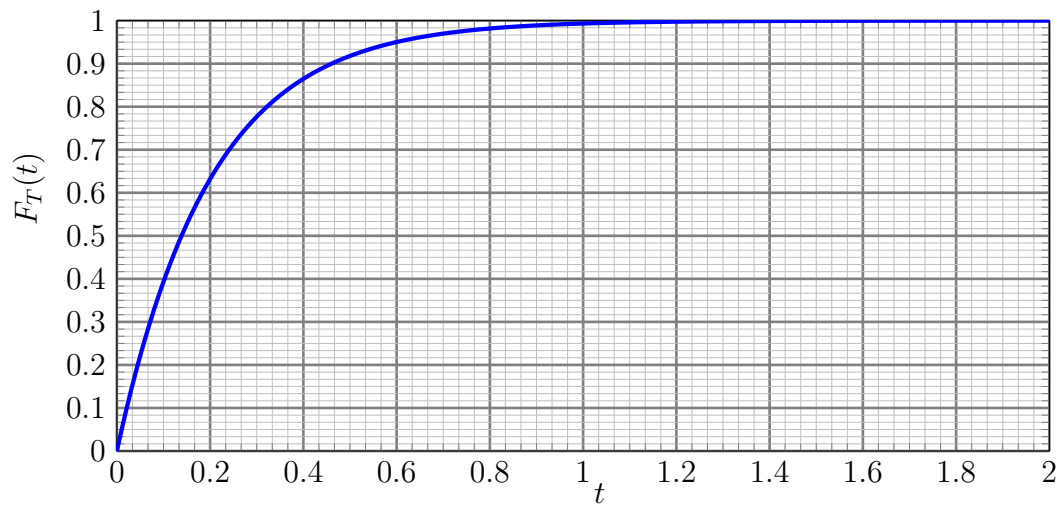
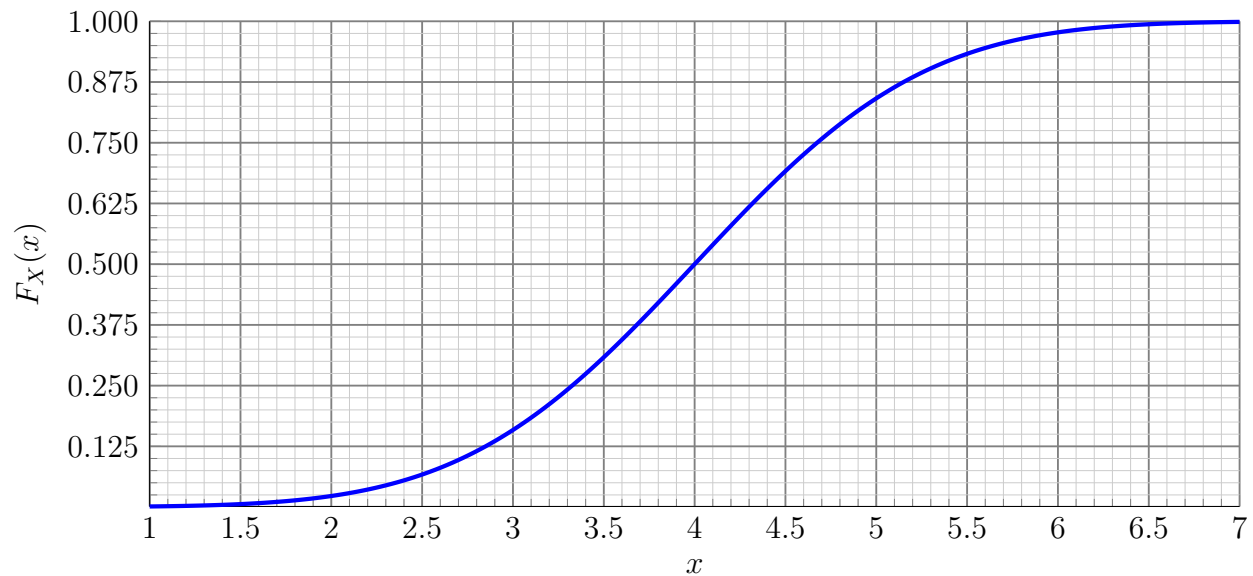


Figure 1: CDF of an exponential random variate

Answer:

- (g) Below is the CDF of a given normal distribution. Use the figure to answer the following 5 questions (i) – (v).



- (i) What is the mean of this distribution?

[1]

Answer:

- (ii) What is the mode of this distribution?

[1]

Answer:

- (iii) Estimate the first quartile.

[1]

Answer:

- (iv) Estimate the third quartile.

[1]

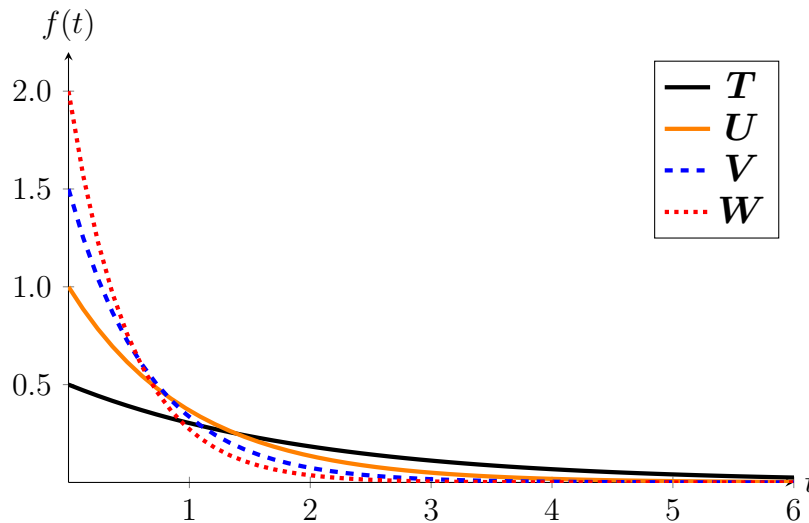
Answer:

- (v) Compute the interquartile range (IQR).

[1]

Answer:

- [1] (h) Consider the PDFs of the exponential random variates T , U , V and W (measured in hours) shown in the figure below. Which of them has the greatest mean?

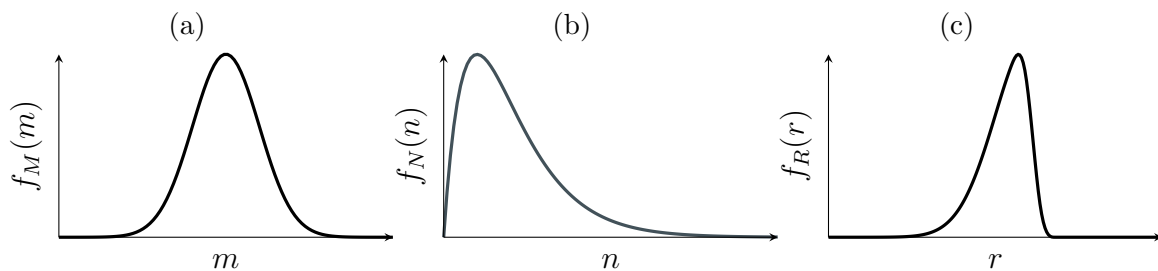


Answer:

- [1] (i) In the figure above, which random variable has standard deviation of 2 hours?

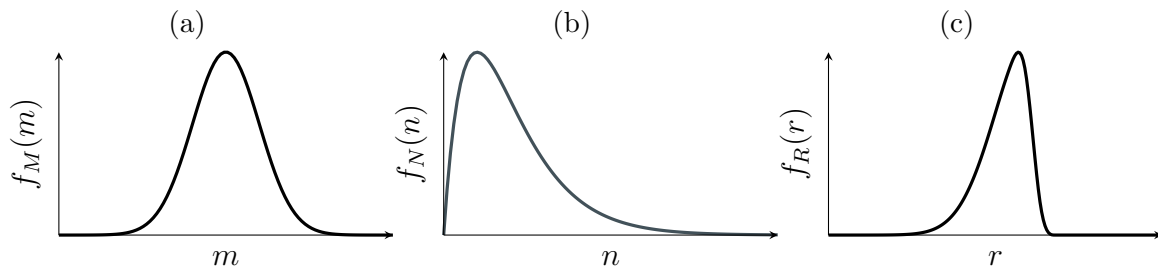
Answer:

- [1] (j) In which of the distributions does the mode appear to be equal to the mean? Circle (a), (b) or (c).

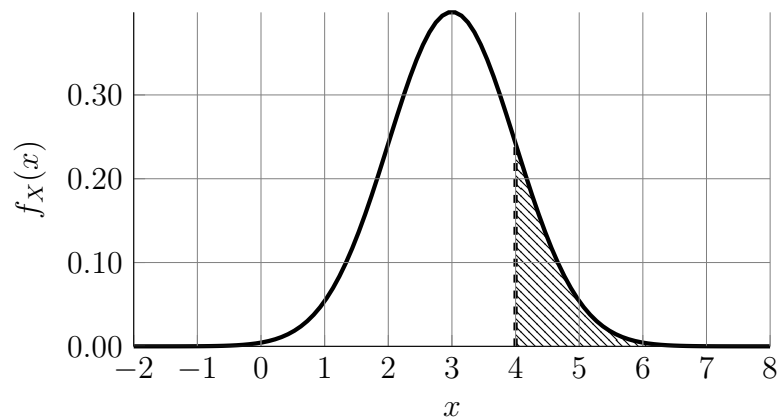


(k) Which of the distributions is right-skewed? Circle (a), (b) or (c).

[1]



(l) The figure below shows the PDF of a normal distribution with a variance of 1. Compute the probability indicated by the shaded portion of the PDF below. [3]



Answer:

Problem 4 *Counting (7 points)*

- [3] (a) License plates in a certain state consist of 1 digit, followed by 3 letters and then 2 digits.
How many different license plates can be manufactured?

- [2] (b) In how many ways can you arrange 6 items?

- [2] (c) How many distinct groups of 3 can you obtain from a larger group of 5 items?

Problem 5 *Bayes' and total probability (9 points)*

Given that $P(A) = 0.6$, $P(B) = 0.3$ and $P(C) = 0.1$ represent the production of machines in a factory. The conditional probabilities of damaged items are $P(D|A) = 0.02$, $P(D|B) = 0.03$ and $P(D|C) = 0.04$.

(a) Find the total probability $P(D)$. [3]

(b) Find the probability that an item was produced by machine B , given that it is damaged. [3]

(c) Draw a Venn diagram depicting the events A , B , C and D in sample space S . [3]

Problem 6 *Binomial distribution (9 points; 5 points EC)*

80% of all vehicles inspected at a certain facility station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, find the following:

- [2] **(a)** The standard deviation of the binomial distribution governing the probability of the next 10 vehicles passing inspection.
- [3] **(b)** The probability that half of the next 10 vehicles inspected pass.
- [4] **(c)** The probability that at least 4 of the next 10 vehicles inspected fail.

- (d) (Extra Credit) Use the normal distribution to estimate the probability that 95 of the next 100 vehicles inspected will pass inspection. (Show all your work to earn all the extra points.) [5]

Problem 7 *Poisson distribution (8 points)*

Hourly arrivals at a small coffee shop can be model using a Poisson distribution with a variance of 4.

[2] (a) What is the mean of the Poisson distribution?

[2] (b) What is the probability that 5 customers will arrive in a given hour?

[4] (c) Find the probability that no more than 8 customers will arrive over a duration of 2 consecutive hours.

Problem 8 *Normal distribution (7 points)*

The mean daily high temperature in June in LA is 77°F with a standard deviation of 5°F . Suppose that the temperatures in June closely follow a normal distribution.

- (a) What is the 50th percentile of daily high temperatures in June in LA? [1]
- (b) What is the probability that the high temperature on a random day in June in LA is lower than 60°F ? [3]
- (c) What is the probability that the high temperature on a random day in June in LA is between 60° and 80°F ? [3]

Problem 9 *Logormal distribution (10 points)*

The lifetime (in years) of a machine is lognormally distributed with $\mu = 2$ and $\sigma = 0.5$.

- [2] (a) What is the median lifetime of the machine?
- [2] (b) Find the mean lifetime.
- [3] (c) What is the probability that the lifetime is at least 5 years?
- [3] (d) What is the probability that the lifetime is less than 2 years?

Problem 10 *Exponential distribution (8 points)*

The delay time T of a bus is exponentially distributed with $\lambda = 1$ (mean rate of occurrence per hour).

(a) What is the mean of T ? [1]

(b) What is the variance of T ? [1]

(c) What is the probability that a train is delayed by no more than 15 minutes? [3]

(d) Given that you have already waited for 30 minutes, what is the probability that a certain bus will be further delayed by more than 15 minutes? [3]



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DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

UNIVERSITY OF MASSACHUSETTS AMHERST