CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture 6A: Inference for One Sample Means

Jimi Oke

UMass Amherst

College of Engineering

November 18, 2025

Outline

1. Introduction
2. Confidence intervals
3. Confidence bounds
4. Sample size
5. Hypothesis testing
6. *p*-values
7. Outlook

Today's objectives

Today's objectives

- Find confidence intervals and perform hypothesis tests for **one-sample means** with

Today's objectives

- Find confidence intervals and perform hypothesis tests for **one-sample means** with
  - known population variance (normal distribution)

Today's objectives

- Find confidence intervals and perform hypothesis tests for **one-sample means** with
  - known population variance (normal distribution)
  - unknown population variance (t-distribution)

Today's objectives

- Find confidence intervals and perform hypothesis tests for **one-sample means** with
  - known population variance (normal distribution)
  - unknown population variance (t-distribution)
- Compute 2-sided CIs and 1-sided CIs (confidence bounds) for sample means

Today's objectives

- Find confidence intervals and perform hypothesis tests for **one-sample means** with
  - known population variance (normal distribution)
  - unknown population variance (t-distribution)
- Compute 2-sided CIs and 1-sided CIs (confidence bounds) for sample means
- Compute sample size to required confidence level

One-sample means

One-sample means

Inference on the mean of a single sample $\overline{x}$ can be performed using **normal distribution** statistics (by the Central Limit Theorem), which we assume if:

- The observations are **independent**

One-sample means

Inference on the mean of a single sample $\overline{x}$ can be performed using **normal distribution** statistics (by the Central Limit Theorem), which we assume if:

- The observations are **independent**
- The sample size n is large enough (typically $n \geq 30$) and/or there are no outliers in the data

For sample means, we add another requirement:

One-sample means

Inference on the mean of a single sample $\overline{x}$ can be performed using **normal distribution** statistics (by the Central Limit Theorem), which we assume if:

- The observations are **independent**
- The sample size n is large enough (typically $n \geq 30$) and/or there are no outliers in the data

For sample means, we add another requirement:

- If the population variance σ is **known**, then we can assume a normal distribution

One-sample means

Inference on the mean of a single sample $\overline{x}$ can be performed using **normal distribution** statistics (by the Central Limit Theorem), which we assume if:

- The observations are **independent**
- The sample size n is large enough (typically $n \geq 30$) and/or there are no outliers in the data

For sample means, we add another requirement:

- If the population variance σ is **known**, then we can assume a normal distribution
- If the population variance is **unknown** and can only be estimated from a sample as s, then we use the **Student's t distribution**

Normal and t-distributions

Normal and t-distributions

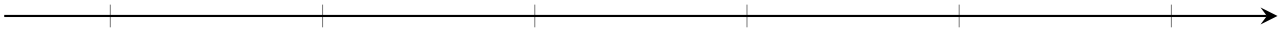
- The t-distribution has thicker tails compared to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)

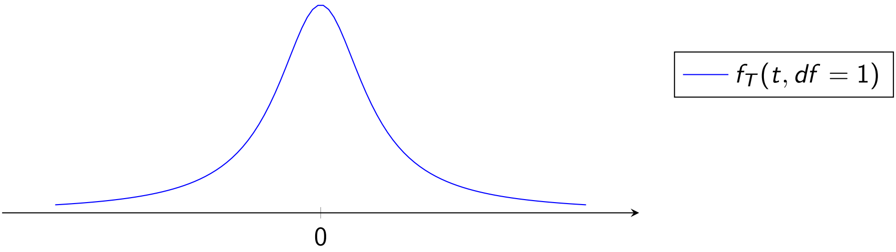
Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution



Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution

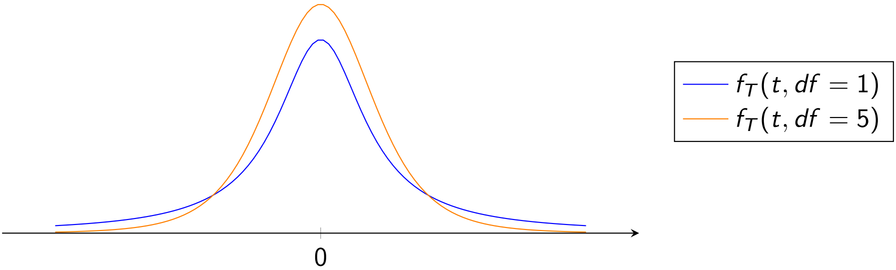


Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution

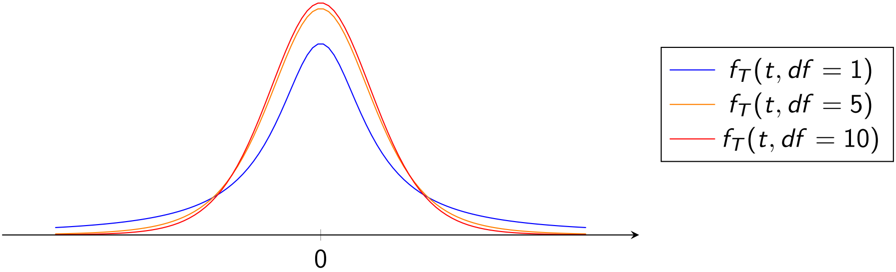


Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution

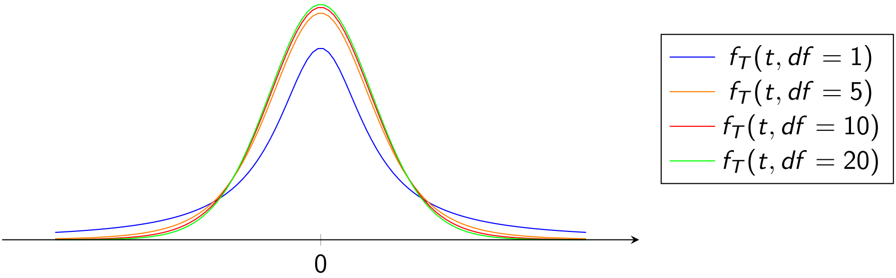


Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution

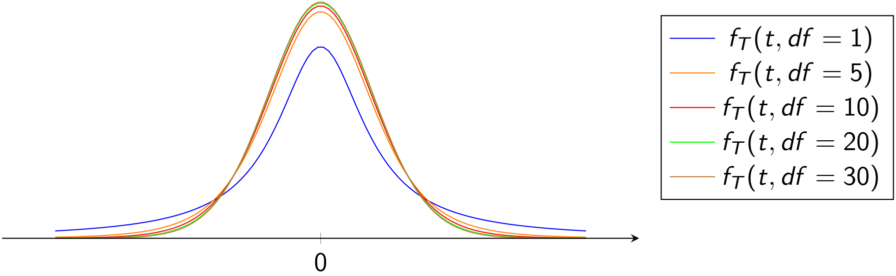


Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution

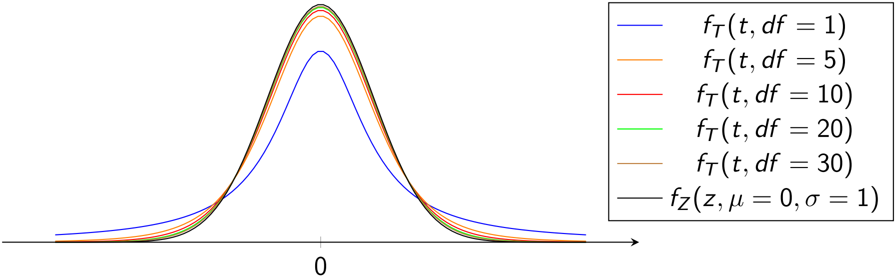


Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Normal and t-distributions

- The t-distribution has thicker tails compared to the normal distribution.
- It is centered at 0 and has a single parameter: df (degrees of freedom)
- As df increases, the t-distribution approaches the normal distribution



Note

In fact, as $df \to \infty$, the t-distribution converges to the normal distribution.

Confidence intervals

Confidence intervals

Definition

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1 - \alpha$)

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1-\alpha$)

Two-sided confidence intervals:

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1 - \alpha$)

Two-sided confidence intervals:

Known population variance: normal distribution used

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1 - \alpha$)

Two-sided confidence intervals:

Known population variance: normal distribution used

$$\langle \mu \rangle_{1-\alpha}$$

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1 - \alpha$)

Two-sided confidence intervals:

Known population variance: normal distribution used

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \right.$$

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1 - \alpha$)

Two-sided confidence intervals:

Known population variance: normal distribution used

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \overline{x} + z_{\left(1-\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} \right) \tag{1}$$

Confidence intervals

Definition

A confidence interval defines the range within which a population parameter lies with a given probability (the confidence level, $1-\alpha$)

Two-sided confidence intervals:

Known population variance: normal distribution used

$$\langle\mu\rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}};\ \overline{x} + z_{\left(1-\frac{\alpha}{2}\right)}\frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

Unknown population variance: *t* distribution used

$$\langle\mu\rangle_{1-\alpha} = \left(\overline{x} + t_{\frac{\alpha}{2},df}\frac{s}{\sqrt{n}};\ \overline{x} + t_{\left(1-\frac{\alpha}{2}\right),df}\frac{s}{\sqrt{n}}\right) \tag{2}$$

Two-sided confidence intervals

Two-sided confidence intervals

We define:

Two-sided confidence intervals

We define:

- Confidence level: $1 - \alpha$; significance level: α

Two-sided confidence intervals

We define:

- Confidence level: $1 - \alpha$; significance level: α
- Find critical *z*-score (standardized) values: $z_{\alpha/2}$ and $z_{(1-\alpha/2)}$

Two-sided confidence intervals

We define:

- Confidence level: $1 - \alpha$; significance level: α
- Find critical z-score (standardized) values: $z_{\alpha/2}$ and $z_{(1-\alpha/2)}$
- Convert these to same scale as original variable X

Two-sided confidence intervals

We define:

- Confidence level: $1 - \alpha$; significance level: α
- Find critical z-score (standardized) values: $z_{\alpha/2}$ and $z_{(1-\alpha/2)}$
- Convert these to same scale as original variable X

Two-sided confidence intervals

We define:

- Confidence level: $1 - \alpha$; significance level: α
- Find critical *z*-score (standardized) values: $z_{\alpha/2}$ and $z_{(1-\alpha/2)}$
- Convert these to same scale as original variable X

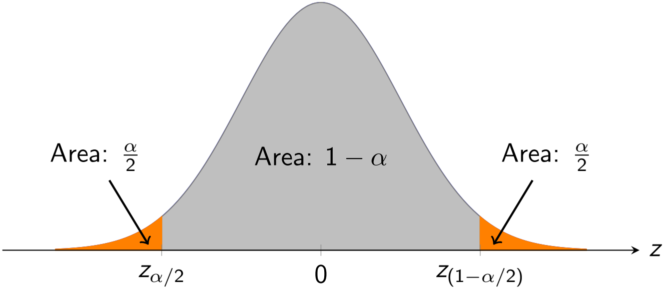


Figure: Standard normal distribution of the mean

Margin of error

Margin of error

The margin of error (ME) of the sample mean is defined as

Margin of error

The margin of error (ME) of the sample mean is defined as

$$ME = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \tag{3}$$

Margin of error

The margin of error (ME) of the sample mean is defined as

$$ME = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \tag{3}$$

when the population variance σ is known.

Margin of error

The margin of error (ME) of the sample mean is defined as

$$ME = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \tag{3}$$

when the population variance σ is known. Otherwise, it is given by:

$$ME = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad \text{(Unknown population variance)} \tag{4}$$

Thus, we can write the confidence interval of a given mean μ as:

Margin of error

The margin of error (ME) of the sample mean is defined as

$$ME = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \tag{3}$$

when the population variance σ is known. Otherwise, it is given by:

$$ME = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad \text{(Unknown population variance)} \tag{4}$$

Thus, we can write the confidence interval of a given mean μ as:

$$\langle \mu \rangle_{1-\alpha} = \overline{X} \pm ME \tag{5}$$

Margin of error

The margin of error (ME) of the sample mean is defined as

$$ME = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \tag{3}$$

when the population variance σ is known. Otherwise, it is given by:

$$ME = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad \text{(Unknown population variance)} \tag{4}$$

Thus, we can write the confidence interval of a given mean μ as:

$$\langle \mu \rangle_{1-\alpha} = \overline{X} \pm ME \tag{5}$$

To use the t-distribution functions in Python, use `from scipy.stats import t`

Working with confidence intervals

Example 1: Identifying confidence levels

Given a normal population distribution with known variance:

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$z_{(1-\alpha/2)} = +2.81$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +2.81 \\
1-\alpha/2 &= \Phi(2.81)
\end{aligned}
$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) = 99.75\% \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) = 99.75\% \\ \alpha/2 &= 0.25\% \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) = 99.75\% \\ \alpha/2 &= 0.25\% \\ \alpha &= 0.5\% \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +2.81 \\
1-\alpha/2 &= \Phi(2.81) = 99.75\% \\
\alpha/2 &= 0.25\% \\
\alpha &= 0.5\%
\end{aligned}
$$

The confidence level is

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +2.81 \\
1-\alpha/2 &= \Phi(2.81) = 99.75\% \\
\alpha/2 &= 0.25\% \\
\alpha &= 0.5\%
\end{aligned}
$$

The confidence level is $= 1 - \alpha$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) = 99.75\% \\ \alpha/2 &= 0.25\% \\ \alpha &= 0.5\% \end{aligned}$$

The confidence level is $= 1-\alpha = \boxed{99.5\%}$.

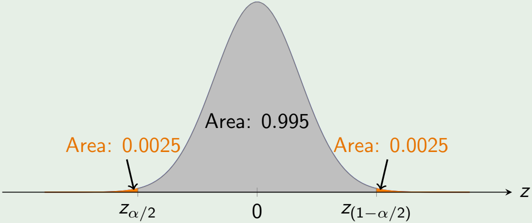
Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(a) What is the confidence level for the interval $\overline{x} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) = 99.75\% \\ \alpha/2 &= 0.25\% \\ \alpha &= 0.5\% \end{aligned}$$

The confidence level is $= 1 - \alpha = \boxed{99.5\%}$.



Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$z_{(1-\alpha/2)} = +1.44$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +1.44 \\
1-\alpha/2 &=
\end{aligned}
$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +1.44 \\
1-\alpha/2 &= \Phi(1.44)
\end{aligned}
$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +1.44 \\ 1-\alpha/2 &= \Phi(1.44) = 92.5\% \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +1.44 \\ 1-\alpha/2 &= \Phi(1.44) = 92.5\% \\ \alpha &= 15\% \end{aligned}$$

The confidence level is

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +1.44 \\ 1-\alpha/2 &= \Phi(1.44) = 92.5\% \\ \alpha &= 15\% \end{aligned}$$

The confidence level is $= 1 - \alpha$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +1.44 \\ 1-\alpha/2 &= \Phi(1.44) = 92.5\% \\ \alpha &= 15\% \end{aligned}$$

The confidence level is $= 1 - \alpha = \boxed{85\%}$.

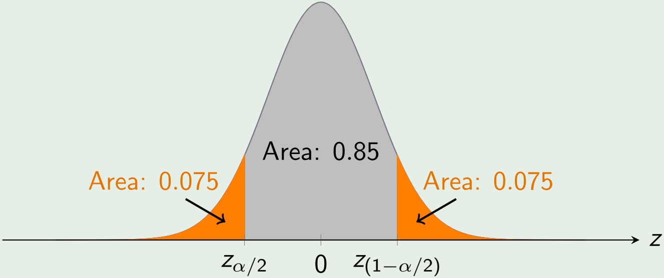
Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(b) What is the confidence level for the interval $\overline{x} \pm 1.44\sigma/\sqrt{n}$?

$$
\begin{aligned}
z_{(1-\alpha/2)} &= +1.44 \\
1 - \alpha/2 &= \Phi(1.44) = 92.5\% \\
\alpha &= 15\%
\end{aligned}
$$

The confidence level is $= 1 - \alpha = \boxed{85\%}$.



Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

$$z_{\alpha/2} = \Phi^{-1}(0.05)$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

$$\begin{aligned} z_{\alpha/2} &= \Phi^{-1}(0.05) \\ &= -\Phi^{-1}(0.95) \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

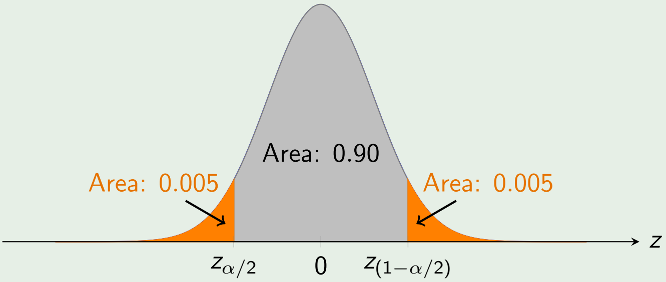
$$\begin{aligned} z_{\alpha/2} &= \Phi^{-1}(0.05) \\ &= -\Phi^{-1}(0.95) \\ &= -1.64 \end{aligned}$$

Working with confidence intervals

Example 1: Identifying confidence levels (cont.)

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

$$\begin{aligned} z_{\alpha/2} &= \Phi^{-1}(0.05) \\ &= -\Phi^{-1}(0.95) \\ &= -1.64 \end{aligned}$$



Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort.

Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (*Human Factors*, 1985:175-187) reports on a study of preferred height for an experimental keyboard with large forearm-wrist support.

Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (*Human Factors*, 1985:175-187) reports on a study of preferred height for an experimental keyboard with large forearm-wrist support.

A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist.

Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (*Human Factors*, 1985:175-187) reports on a study of preferred height for an experimental keyboard with large forearm-wrist support.

A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist.The resulting sample average preferred height was $\bar{x} = 80.0$ cm.

Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (*Human Factors*, 1985:175-187) reports on a study of preferred height for an experimental keyboard with large forearm-wrist support.

A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist.The resulting sample average preferred height was $\overline{x} = 80.0$ cm. Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm (a value suggested by data in the article),

Confidence interval (normal distribution)

Example 1: Keyboard height

Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (*Human Factors*, 1985:175-187) reports on a study of preferred height for an experimental keyboard with large forearm-wrist support.

A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist.The resulting sample average preferred height was $\overline{x} = 80.0$ cm. Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm (a value suggested by data in the article), obtain a 95% CI for μ, the true average preferred height for the population of all experienced typists.

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

First, we find the sample SD of the mean (standard error):

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

First, we find the sample SD of the mean (standard error):

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{31}}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

First, we find the sample SD of the mean (standard error):

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{31}}$$

The z-score is given by:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

First, we find the sample SD of the mean (standard error):

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{31}}$$

The z-score is given by:

$$z = \frac{\overline{x} - \mu}{SE}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

From tables, this implies that:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

From tables, this implies that:

$$z_{\frac{\alpha}{2}} = z_{0.025} = -1.96$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

From tables, this implies that:

$$\begin{aligned} z_{\frac{\alpha}{2}} &= z_{0.025} = -1.96 \\ z_{\left(1-\frac{\alpha}{2}\right)} &= z_{0.975} = +1.96 \end{aligned}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

From tables, this implies that:

$$\begin{aligned} z_{\frac{\alpha}{2}} &= z_{0.025} = -1.96 \\ z_{\left(1-\frac{\alpha}{2}\right)} &= z_{0.975} = +1.96 \end{aligned}$$

Thus, we have:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Given a confidence level of 95%, we write:

$$P\left(z_{\frac{\alpha}{2}} < z < z_{\left(1-\frac{\alpha}{2}\right)}\right) = 0.95$$

From tables, this implies that:

$$\begin{aligned} z_{\frac{\alpha}{2}} &= z_{0.025} = -1.96 \\ z_{\left(1-\frac{\alpha}{2}\right)} &= z_{0.975} = +1.96 \end{aligned}$$

Thus, we have:

$$P\left(-1.96 < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right) = 0.95$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Confidence interval (normal distribution, cont.)

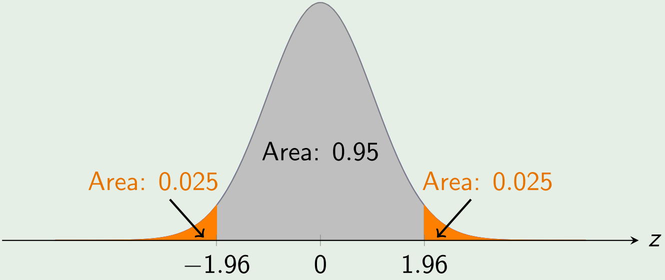
Example 1: Keyboard height (cont.)

We can also plot the standard normal distribution as a visual aid:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

We can also plot the standard normal distribution as a visual aid:



Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right)$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right)$$

We first multiply all terms in the inequality by $\sigma/\sqrt{n}$:

$$-1.96\frac{\sigma}{\sqrt{n}} < \overline{x} - \mu < 1.96\frac{\sigma}{\sqrt{n}}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right)$$

We first multiply all terms in the inequality by $\sigma/\sqrt{n}$:

$$-1.96\frac{\sigma}{\sqrt{n}} < \overline{x} - \mu < 1.96\frac{\sigma}{\sqrt{n}}$$

Then we subtract $\overline{x}$ to all terms to obtain:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right)$$

We first multiply all terms in the inequality by $\sigma/\sqrt{n}$:

$$-1.96\frac{\sigma}{\sqrt{n}} < \overline{x} - \mu < 1.96\frac{\sigma}{\sqrt{n}}$$

Then we subtract $\overline{x}$ to all terms to obtain:

$$-\overline{x} - 1.96\frac{\sigma}{\sqrt{n}} < -\mu < -\overline{x} + 1.96\frac{\sigma}{\sqrt{n}}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} < 1.96\right)$$

We first multiply all terms in the inequality by $\sigma/\sqrt{n}$:

$$-1.96\frac{\sigma}{\sqrt{n}} < \overline{x}-\mu < 1.96\frac{\sigma}{\sqrt{n}}$$

Then we subtract $\overline{x}$ to all terms to obtain:

$$-\overline{x}-1.96\frac{\sigma}{\sqrt{n}} < -\mu < -\overline{x}+1.96\frac{\sigma}{\sqrt{n}}$$

Then we multiply by -1:

$$\overline{x}+1.96\frac{\sigma}{\sqrt{n}} > \mu > \overline{x}-1.96\frac{\sigma}{\sqrt{n}}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

The endpoints of the resulting inequality form the **confidence interval** for μ:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

The endpoints of the resulting inequality form the **confidence interval** for μ:

$$(\overline{x} - 1.96SE, \overline{x} + 1.96SE)$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

The endpoints of the resulting inequality form the **confidence interval** for μ:

$$(\overline{x} - 1.96SE, \overline{x} + 1.96SE)$$

which corresponds to Equation (25) for a confidence level of 95%.

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

The endpoints of the resulting inequality form the **confidence interval** for μ:

$$(\overline{x} - 1.96SE, \overline{x} + 1.96SE)$$

which corresponds to Equation (25) for a confidence level of 95%.
Now, plugging in the numbers we have, we find:

$$\begin{aligned}
\overline{x} \pm 1.96SE &= 80.0 \pm (1.96)\,\frac{2.0}{\sqrt{31}} \\
&= 80.0 \pm 0.7 \\
&= (79.3, 80.7)
\end{aligned}$$

One-sided confidence intervals (confidence bounds)

One-sided confidence intervals (confidence bounds)

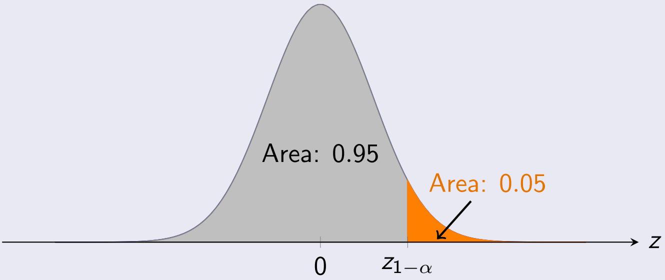
Upper confidence bound

One-sided confidence intervals (confidence bounds)

Upper confidence bound

$$\mu < \bar{x} + z_{(1-\alpha)}\frac{\sigma}{\sqrt{n}} \quad \text{(known variance)} \tag{6}$$

$$\mu < \bar{x} + t_{(1-\alpha)}\frac{s}{\sqrt{n}} \quad \text{(unknown variance; } n-1 \text{ } df\text{)} \tag{7}$$



One-sided confidence intervals (confidence bounds)

One-sided confidence intervals (confidence bounds)

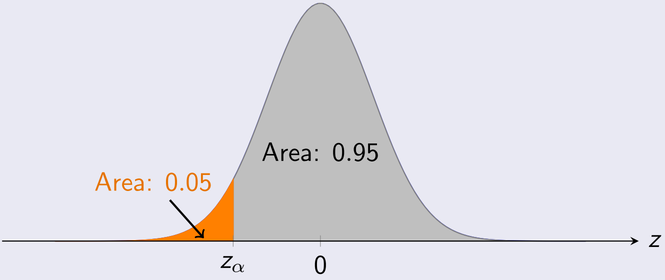
Lower confidence bound

One-sided confidence intervals (confidence bounds)

Lower confidence bound

$$\mu > \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} \quad \text{(known variance)} \tag{8}$$

$$\mu > \bar{x} + t_\alpha \frac{s}{\sqrt{n}} \quad (\text{unknown variance; } n-1 \; df) \tag{9}$$



Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels

Determine the confidence levels for each of the following one-sided confidence bounds:

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$z_{(1-\alpha)} = 0.84$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$\begin{aligned} z_{(1-\alpha)} &= 0.84 \\ 1-\alpha &= \Phi(0.84) \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$\begin{aligned} z_{(1-\alpha)} &= 0.84 \\ 1-\alpha &= \Phi(0.84) \approx 0.80 \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$\begin{aligned} z_{(1-\alpha)} &= 0.84 \\ 1-\alpha &= \Phi(0.84) \approx 0.80 \end{aligned}$$

Confidence level:

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$\begin{aligned} z_{(1-\alpha)} &= 0.84 \\ 1-\alpha &= \Phi(0.84) \approx 0.80 \end{aligned}$$

Confidence level: $\boxed{80\%}$.

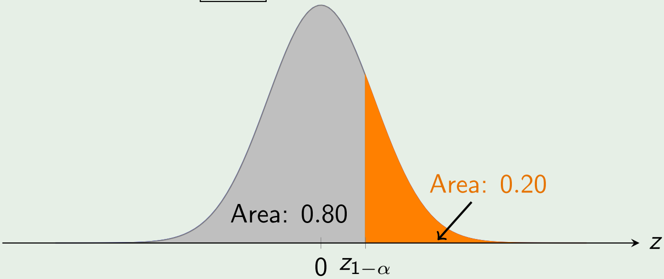
Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\overline{x} + 0.84\sigma/\sqrt{n}$

$$
\begin{aligned}
z_{(1-\alpha)} &= 0.84 \\
1-\alpha &= \Phi(0.84) \approx 0.80
\end{aligned}
$$

Confidence level: $\boxed{80\%}$.



Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$z_\alpha = -2.05$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$\begin{aligned} z_\alpha &= -2.05 \\ \alpha &= \Phi(-2.05) = -\Phi(2.05) \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$
\begin{aligned}
z_\alpha &= -2.05 \\
\alpha &= \Phi(-2.05) = -\Phi(2.05) \approx 0.98
\end{aligned}
$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$\begin{aligned} z_\alpha &= -2.05 \\ \alpha &= \Phi(-2.05) = -\Phi(2.05) \approx 0.98 \end{aligned}$$

Confidence level:

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$\begin{aligned} z_\alpha &= -2.05 \\ \alpha &= \Phi(-2.05) = -\Phi(2.05) \approx 0.98 \end{aligned}$$

Confidence level: $\boxed{98\%}$.

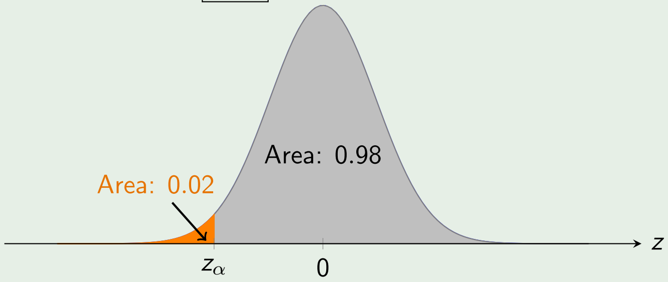
Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\overline{x} - 2.05\sigma/\sqrt{n}$

$$\begin{aligned} z_\alpha &= -2.05 \\ \alpha &= \Phi(-2.05) = -\Phi(2.05) \approx 0.98 \end{aligned}$$

Confidence level: $\boxed{98\%}$.



Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, ($n = 12$)

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

$$t_{(1-\alpha)} = 2.2$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, ($n = 12$)

$$\begin{aligned} t_{(1-\alpha)} &= 2.2 \\ 1 - \alpha &= F_{T,df}(2.2) \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, ($n = 12$)

$$\begin{aligned} t_{(1-\alpha)} &= 2.2 \\ 1-\alpha &= F_{T,df}(2.2) \quad df = 11 \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, ($n = 12$)

$$
\begin{aligned}
t_{(1-\alpha)} &= 2.2 \\
1-\alpha &= F_{T,df}(2.2) \quad df = 11 \\
1-\alpha &= 0.975
\end{aligned}
$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

$$\begin{aligned} t_{(1-\alpha)} &= 2.2 \\ 1-\alpha &= F_{T,df}(2.2) \quad df = 11 \\ 1-\alpha &= 0.975 \quad \texttt{t.cdf(2.2, 11)} \end{aligned}$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

$$
\begin{aligned}
t_{(1-\alpha)} &= 2.2 \\
1-\alpha &= F_{T,df}(2.2) \quad df = 11 \\
1-\alpha &= 0.975 \quad \texttt{t.cdf(2.2, 11)}
\end{aligned}
$$

Confidence level:

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

$$\begin{aligned} t_{(1-\alpha)} &= 2.2 \\ 1-\alpha &= F_{T,df}(2.2) \quad df = 11 \\ 1-\alpha &= 0.975 \quad \texttt{t.cdf(2.2, 11)} \end{aligned}$$

Confidence level: $\boxed{97.5\%}$

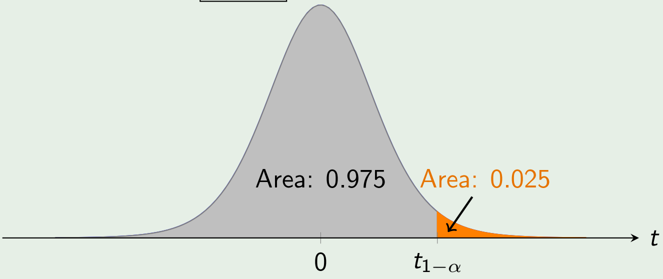
Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\overline{x} + 2.2s/\sqrt{n}$, $(n = 12)$

$$
\begin{aligned}
t_{(1-\alpha)} &= 2.2 \\
1-\alpha &= F_{T,df}(2.2) \quad df = 11 \\
1-\alpha &= 0.975 \quad \texttt{t.cdf(2.2, 11)}
\end{aligned}
$$

Confidence level: $\boxed{97.5\%}$



Confidence bound, t distribution

Confidence bound, t distribution

Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2. Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

Confidence bound, t distribution

Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2. Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

$$\overline{x} + t_\alpha \frac{s}{\sqrt{n}} \quad = \quad 17.17 - 1.6794 \frac{3.28}{\sqrt{48}}$$

Confidence bound, t distribution

Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2. Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

$$\begin{aligned}\bar{x} + t_\alpha \frac{s}{\sqrt{n}} &= 17.17 - 1.6794 \frac{3.28}{\sqrt{48}} \\ &= 17.17 - 0.7951\end{aligned}$$

Confidence bound, t distribution

Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2. Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

$$
\begin{aligned}
\overline{x} + t_\alpha \frac{s}{\sqrt{n}} &= 17.17 - 1.6794 \frac{3.28}{\sqrt{48}} \\
&= 17.17 - 0.7951 \\
&\approx \boxed{16.38}
\end{aligned}
$$

Confidence bound, t distribution

Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2. Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

$$\begin{aligned} \overline{x} + t_\alpha \frac{s}{\sqrt{n}} &= 17.17 - 1.6794 \frac{3.28}{\sqrt{48}} \\ &= 17.17 - 0.7951 \\ &\approx \boxed{16.38} \end{aligned}$$

In other words, with a CI of 95%, μ lies in the random interval $(16.38, \infty)$.

Choice of sample size

Choice of sample size

Reliability-precision tradeoff

Choice of sample size

Reliability-precision tradeoff

Achieving a greater confidence interval (reliability) results in a wider interval

Choice of sample size

Reliability-precision tradeoff

Achieving a greater confidence interval (reliability) results in a wider interval $\implies$ less precision.

Choice of sample size

Reliability-precision tradeoff

Achieving a greater confidence interval (reliability) results in a wider interval $\Longrightarrow$ less precision.

If we fix reliability and precision, then we can determine the sample size n via:

$$n = \left(z_{(1-\alpha/2)} \frac{\sigma}{h} \right)^2 \tag{10}$$

Choice of sample size

Reliability-precision tradeoff

Achieving a greater confidence interval (reliability) results in a wider interval $\implies$ less precision.

If we fix reliability and precision, then we can determine the sample size n via:

$$n = \left(z_{(1-\alpha/2)} \frac{\sigma}{h} \right)^2 \tag{10}$$

where h is the half-width (i.e. the desired margin of error).

Choice of sample size

Reliability-precision tradeoff

Achieving a greater confidence interval (reliability) results in a wider interval $\implies$ less precision.

If we fix reliability and precision, then we can determine the sample size n via:

$$n = \left(z_{(1-\alpha/2)} \frac{\sigma}{h} \right)^2 \tag{10}$$

where h is the half-width (i.e. the desired margin of error).

The greater the sample size, the lower the standard error

Choice of sample size (cont.)

Example 5: Response time of operating system

Choice of sample size (cont.)

Example 5: Response time of operating system

Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with SD 25 millisec.

Choice of sample size (cont.)

Example 5: Response time of operating system

Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with SD 25 millisec. A new operating system has been installed and we wish to estimate the true average response time μ for the new environment.

Choice of sample size (cont.)

Example 5: Response time of operating system

Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with SD 25 millisec. A new operating system has been installed and we wish to estimate the true average response time μ for the new environment. Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% CI has a width of (at most) 10?

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$n = \left(z_{(\alpha/2)} \frac{\sigma}{h}\right)^2$$

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$
\begin{aligned}
n &= \left(z_{(\alpha/2)}\frac{\sigma}{h}\right)^2 \\
&= \left(z_{0.025} \times \frac{25}{5}\right)^2
\end{aligned}
$$

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$
\begin{aligned}
n &= \left(z_{(\alpha/2)}\frac{\sigma}{h}\right)^2 \\
&= \left(z_{0.025} \times \frac{25}{5}\right)^2 \\
&= (1.96 \times 5)^2
\end{aligned}
$$

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$
\begin{aligned}
n &= \left(z_{(\alpha/2)}\frac{\sigma}{h}\right)^2 \\
&= \left(z_{0.025} \times \frac{25}{5}\right)^2 \\
&= (1.96 \times 5)^2 \\
&= 96.04
\end{aligned}
$$

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$\begin{aligned} n &= \left(z_{(\alpha/2)}\frac{\sigma}{h}\right)^2 \\ &= \left(z_{0.025} \times \frac{25}{5}\right)^2 \\ &= (1.96 \times 5)^2 \\ &= 96.04 \end{aligned}$$

Since n must be an integer,

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

Here, $h = 10/2 = 5 = ME$. Thus,

$$\begin{aligned} n &= \left(z_{(\alpha/2)}\frac{\sigma}{h}\right)^2 \\ &= \left(z_{0.025} \times \frac{25}{5}\right)^2 \\ &= (1.96 \times 5)^2 \\ &= 96.04 \end{aligned}$$

Since n must be an integer,a sample size of $\boxed{97}$ is required.

Summary of hypothesis testing approach

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)
5. *Define* the **region of rejection/critical region** of the null hypothesis by choosing the **critical value**.

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)
5. *Define* the **region of rejection/critical region** of the null hypothesis by choosing the **critical value**.
6. *Decide.* If the test statistic is in the critical region, reject H_0. If not, do not reject H_0 (fail to reject it)

One-sided tests

One-sided tests

Case A: upper tail

One-sided tests

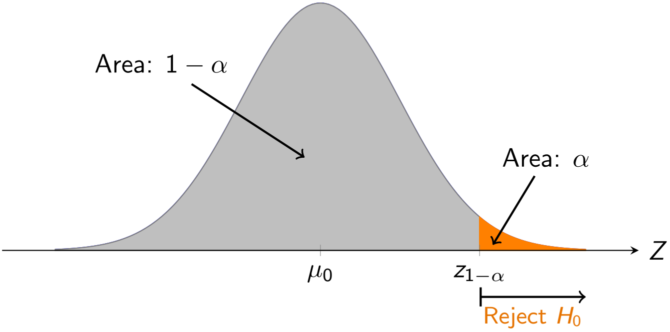
Case A: upper tail

- $H_0 : \mu = \mu_0$
- $H_1 : \mu > \mu_0$

One-sided tests

Case A: upper tail

- $H_0 : \mu = \mu_0$
- $H_1 : \mu > \mu_0$



One-sided tests (cont.)

One-sided tests (cont.)

Case B: lower tail

One-sided tests (cont.)

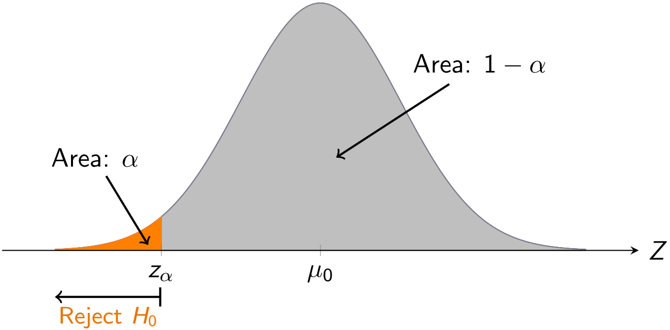
Case B: lower tail

- $H_0 : \mu = \mu_0$
- $H_1 : \mu < \mu_0$

One-sided tests (cont.)

Case B: lower tail

- $H_0 : \mu = \mu_0$
- $H_1 : \mu < \mu_0$



Two-sided tests

Two-sided tests

Case C: both tails

Two-sided tests

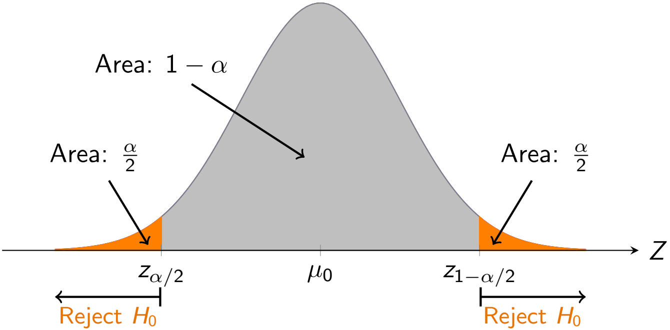
Case C: both tails

- $H_0 : \mu = \mu_0$
- $H_1 : \mu \neq \mu_0$

Two-sided tests

Case C: both tails

- $H_0 : \mu = \mu_0$
- $H_1 : \mu \neq \mu_0$



Distribution of the test statistic

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**.

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**. As mentioned earlier, after choosing the test statistic, we must determine its distribution.

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**.
As mentioned earlier, after choosing the test statistic, we must determine its distribution.
There are 2 cases to consider:

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**. As mentioned earlier, after choosing the test statistic, we must determine its distribution.

There are 2 cases to consider:

Case 1: Sample mean with known population variance

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**.
As mentioned earlier, after choosing the test statistic, we must determine its distribution.
There are 2 cases to consider:

Case 1: Sample mean with known population variance

The sample mean is **normally** distributed and its variance is :

$$Var(\overline{X}) = \frac{\sigma^2}{n} \tag{11}$$

And thus, the standard deviation is $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$.

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**. As mentioned earlier, after choosing the test statistic, we must determine its distribution.

There are 2 cases to consider:

Case 1: Sample mean with known population variance

The sample mean is **normally** distributed and its variance is :

$$Var(\overline{X}) = \frac{\sigma^2}{n} \tag{11}$$

And thus, the standard deviation is $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$.

Thus, to compute the probability (area under curve) of the test statistic,

Distribution of the test statistic

In many hypothesis testing situations, the test statistic is the **sample mean**.
As mentioned earlier, after choosing the test statistic, we must determine its distribution.
There are 2 cases to consider:

Case 1: Sample mean with known population variance

The sample mean is **normally** distributed and its variance is :

$$Var(\overline{X}) = \frac{\sigma^2}{n} \tag{11}$$

And thus, the standard deviation is $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$.
Thus, to compute the probability (area under curve) of the test statistic, we use the standardized variable (Z-statistic or Z-score):

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \tag{12}$$

which is normally distributed.

Distribution of the test statistic (cont.)

Distribution of the test statistic (cont.)

Case 2: Sample mean with unknown population variance

Distribution of the test statistic (cont.)

Case 2: Sample mean with unknown population variance

The estimated sample mean in this case has a Student's *t*-distribution with $n-1$ degrees of freedom (df). Thus, its variance is:

$$Var(\overline{X}) = \frac{s^2}{n} \tag{13}$$

And thus, the standard error is $SE_{\overline{x}} = \frac{s}{\sqrt{n}}$.

Distribution of the test statistic (cont.)

Case 2: Sample mean with unknown population variance

The estimated sample mean in this case has a Student's t-distribution with $n-1$ degrees of freedom (df). Thus, its variance is:

$$Var(\overline{X}) = \frac{s^2}{n} \tag{13}$$

And thus, the standard error is $SE_{\overline{x}} = \frac{s}{\sqrt{n}}$.
Thus, to compute the probability (area under curve) of the test statistic,

Distribution of the test statistic (cont.)

Case 2: Sample mean with unknown population variance

The estimated sample mean in this case has a Student's *t*-distribution with $n-1$ degrees of freedom (df). Thus, its variance is:

$$Var(\overline{X}) = \frac{s^2}{n} \tag{13}$$

And thus, the standard error is $SE_{\overline{X}} = \frac{s}{\sqrt{n}}$.
Thus, to compute the probability (area under curve) of the test statistic, we use the standardized variable (T-statistic or T-score):

$$t = \frac{\overline{X} - \mu}{SE_{\overline{X}}} \tag{14}$$

Distribution of the test statistic (cont.)

Distribution of the test statistic (cont.)

Much of our focus will be on using the **sample mean** as the test statistic:

Distribution of the test statistic (cont.)

Much of our focus will be on using the **sample mean** as the test statistic:

1. If the variance is known, then we use the normal distribution to find the probability of the standardized Z-statistic:

$$z = \frac{\overline{X} - \mu}{\boldsymbol{\sigma}/\sqrt{n}} \tag{15}$$

   and compare it to the appropriate critical value to test our hypotheses

Distribution of the test statistic (cont.)

Much of our focus will be on using the **sample mean** as the test statistic:

1. If the variance is known, then we use the normal distribution to find the probability of the standardized Z-statistic:

$$z = \frac{\overline{X} - \mu}{\boldsymbol{\sigma}/\sqrt{n}} \tag{15}$$

   and compare it to the appropriate critical value to test our hypotheses

2. If the variance is unknown, we use the *t*-distribution to find the probability of the standardized T-statistic:

$$t = \frac{\overline{X} - \mu}{\boldsymbol{s}/\sqrt{n}} \tag{16}$$

   and compare it to the appropriate critical value to test our hypotheses

What is a p-value?

What is a *p*-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.

What is a p-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,

What is a p-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

What is a p-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted.

What is a p-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

- Provides more information about the strength of a test

What is a *p*-value?

Definition

The *p*-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The *p*-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the *p*-value, the more contradictory are the data to H_0.

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant

What is a p-value?

Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

What is a *p*-value?

Definition

The *p*-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The *p*-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the *p*-value, the more contradictory are the data to H_0.

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

What is a *p*-value?

Definition

The *p*-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Alternative definition

The *p*-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the *p*-value, the more contradictory are the data to H_0.

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

Hypothesis testing with the *p*-value

Hypothesis testing with the p-value

Step 1. Formulate your hypotheses

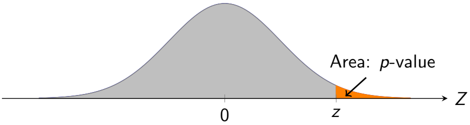
Step 2. Determine the p-value from the test statistic

Step 3. Conclude the test based on a chosen level of significance:

1. p-value $\leq \alpha \implies$ reject H_0 at level α.
2. p-value $> \alpha \implies$ do not reject H_0 at level α.

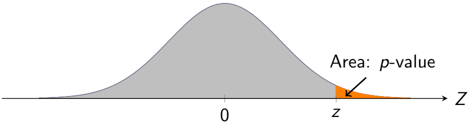
p-value for z tests

p-value for *z* tests



p-value: area upper tail

p-value for *z* tests

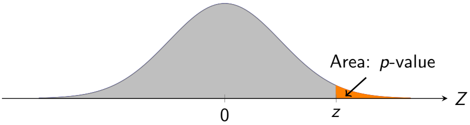


p-value: area upper tail

$$p = 1 - \Phi(z) \qquad (17)$$

`norm.sf(z)`

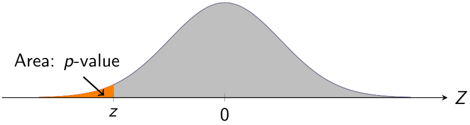
p-value for z tests



p-value: area upper tail

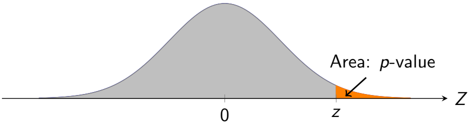
$$p = 1 - \Phi(z) \qquad (17)$$

```
norm.sf(z)
```



p-value: area lower tail

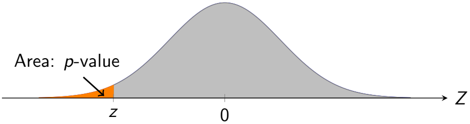
p-value for z tests



p-value: area upper tail

$$p = 1 - \Phi(z) \qquad (17)$$

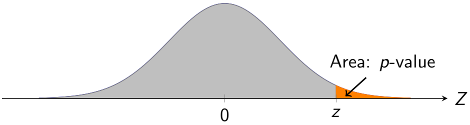
```
norm.sf(z)
```



p-value: area lower tail

$$p = \Phi(z) \qquad (18)$$

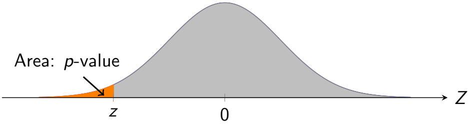
p-value for z tests



p-value: area upper tail

$$p = 1 - \Phi(z) \tag{17}$$

```
norm.sf(z)
```

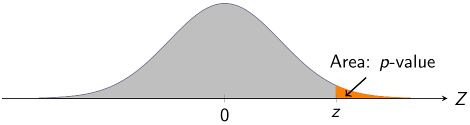


p-value: area lower tail

$$p = \Phi(z) \tag{18}$$

```
norm.cdf(z)
```

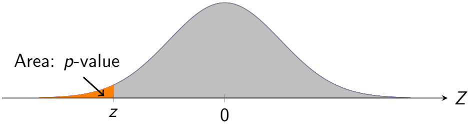
p-value for z tests



p-value: area upper tail

$$p = 1 - \Phi(z) \tag{17}$$

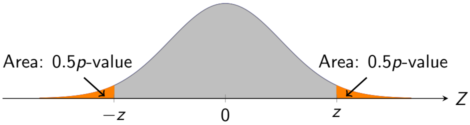
`norm.sf(z)`



p-value: area lower tail

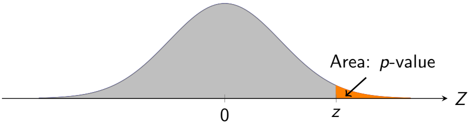
$$p = \Phi(z) \tag{18}$$

`norm.cdf(z)`



p-value: area both tails

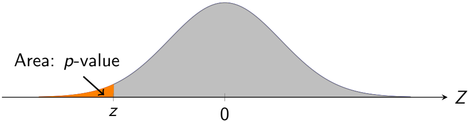
p-value for *z* tests



p-value: area upper tail

$$p = 1 - \Phi(z) \qquad (17)$$

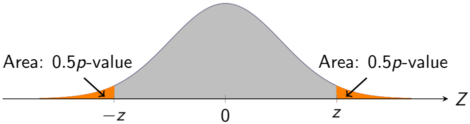
```
norm.sf(z)
```



p-value: area lower tail

$$p = \Phi(z) \qquad (18)$$

```
norm.cdf(z)
```



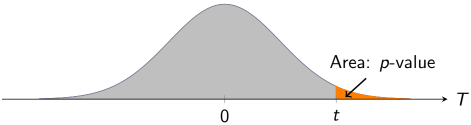
p-value: area both tails

$$p = 2(1 - \Phi(|z|)) \qquad (19)$$

```
2*norm.cdf(z)
```

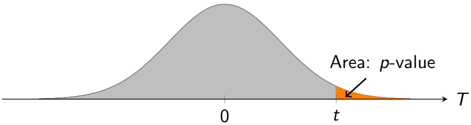
p-value for z tests

p-value for z tests



p-value: area upper tail

p-value for z tests

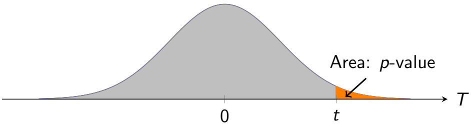


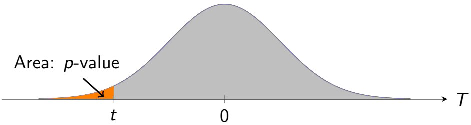
p-value: area upper tail

$$p = 1 - F_{T,n-1}(t) \quad (20)$$

```
t.sf(t, n-1)
```

p-value for *z* tests





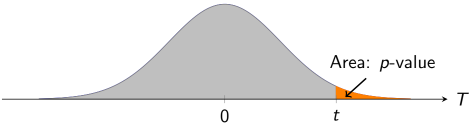
p-value: area upper tail

$$p = 1 - F_{T,n-1}(t) \tag{20}$$

```
t.sf(t, n-1)
```

p-value: area lower tail

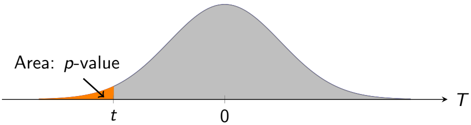
p-value for z tests



p-value: area upper tail

$$p = 1 - F_{T,n-1}(t) \quad (20)$$

```
t.sf(t, n-1)
```

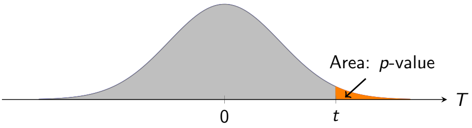


p-value: area lower tail

$$p = F_{T,n-1}(t) \quad (21)$$

```
t.cdf(t, n-1)
```

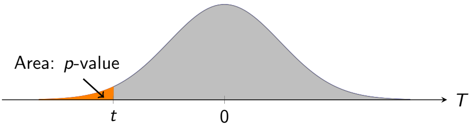
p-value for z tests



p-value: area upper tail

$$p = 1 - F_{T,n-1}(t) \quad (20)$$

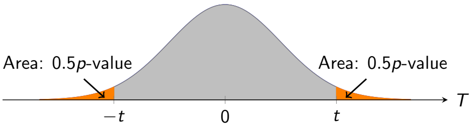
```
t.sf(t, n-1)
```



p-value: area lower tail

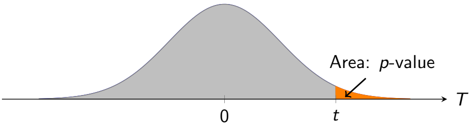
$$p = F_{T,n-1}(t) \quad (21)$$

```
t.cdf(t, n-1)
```



p-value: area both tails

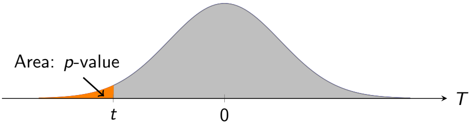
p-value for z tests



p-value: area upper tail

$$p = 1 - F_{T,n-1}(t) \quad (20)$$

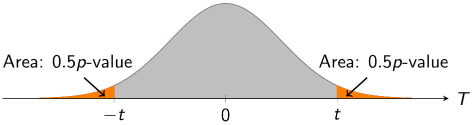
```
t.sf(t, n-1)
```



p-value: area lower tail

$$p = F_{T,n-1}(t) \quad (21)$$

```
t.cdf(t, n-1)
```



p-value: area both tails

$$p = 2(1 - F_{T,n-1}(|t|)) \quad (22)$$

```
2*t.cdf(t, n-1)
```

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest:

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis:

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Step 3. Alternative hypothesis:

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Step 3. Alternative hypothesis: $H_1 : \mu \neq 245$.

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Step 3. Alternative hypothesis: $H_1 : \mu \neq 245$.

Step 4. Formula for test statistic value:

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Step 3. Alternative hypothesis: $H_1 : \mu \neq 245$.

Step 4. Formula for test statistic value: $z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness 246.18 μm. The population standard deviation of 3.60 μm. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

Step 1. Parameter of interest: μ (true average wafer thickness)

Step 2. Null hypothesis: $H_0 : \mu = 245$.

Step 3. Alternative hypothesis: $H_1 : \mu \neq 245$.

Step 4. Formula for test statistic value: $z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

Hypothesis testing using *p*-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}}$$

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p-value

Hypothesis testing using *p*-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine *p*-value (two-tailed test):

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32))$$

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32)) = 0.0204$$

(In Python: `2*norm.sf(2.32)`)

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32)) = 0.0204$$

(In Python: `2*norm.sf(2.32)`)

Step 7. Conclude:

Hypothesis testing using p-value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32)) = 0.0204$$

(In Python: `2*norm.sf(2.32)`)

Step 7. Conclude:
Using a significance level of 0.01, we fail to reject H_0 since $0.0204 > 0.01$. Thus, at the 1% significance level, there is insufficient evidence to conclude that true average thickness differs from the target value.

Two-tailed tests: unknown variance

Two-tailed tests: unknown variance

Example 7: Golf ball production

A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down andrepaired. This hour's sample of 18 balls has a mean of 1.611 oz and a standard deviation of 0.065 oz. Do you shut down the line?

Two-tailed tests: unknown variance

Example 7: Golf ball production

A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down andrepaired. This hour's sample of 18 balls has a mean of 1.611 oz and a standard deviation of 0.065 oz. Do you shut down the line?

Step 1. Formulate hypotheses:

Two-tailed tests: unknown variance

Example 7: Golf ball production

A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down andrepaired. This hour's sample of 18 balls has a mean of 1.611 oz and a standard deviation of 0.065 oz. Do you shut down the line?

Step 1. Formulate hypotheses:

$$
\begin{aligned}
H_0 : \mu &= 1.615 \\
H_1 : \mu &\neq 1.615
\end{aligned}
$$

Two-tailed tests: unknown variance

Example 7: Golf ball production

A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down andrepaired. This hour's sample of 18 balls has a mean of 1.611 oz and a standard deviation of 0.065 oz. Do you shut down the line?

Step 1. Formulate hypotheses:

$$\begin{aligned} H_0 : \mu &= 1.615 \\ H_1 : \mu &\neq 1.615 \end{aligned}$$

Two-tailed tests: unknown variance

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$
\begin{aligned}
t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{1.611 - 1.615}{0.065/\sqrt{18}} =
\end{aligned}
$$

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$
\begin{aligned}
t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{1.611 - 1.615}{0.065/\sqrt{18}} = -0.261
\end{aligned}
$$

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{1.611 - 1.615}{0.065/\sqrt{18}} = -0.261$$

Step 3. $\alpha = 1\% = 0.01$.

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$
\begin{aligned}
t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{1.611 - 1.615}{0.065/\sqrt{18}} = -0.261
\end{aligned}
$$

Step 3. $\alpha = 1\% = 0.01$.
Given that this is a two-tailed test, we have two critical regions with areas: $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$.

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T-statistic:

$$\begin{aligned} t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{1.611 - 1.615}{0.065/\sqrt{18}} = -0.261 \end{aligned}$$

Step 3. $\alpha = 1\% = 0.01$.
Given that this is a two-tailed test, we have two critical regions with areas: $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$.
The lower tail is bounded by $t_{0.005}$ and the upper tail by $t_{1-0.005} = t_{0.995}$.

Two-tailed tests: unknown variance

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$).

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$).
(In Python: `t.ppf(0.005,17)` and `t.ppf(0.995,17)`)

Two-tailed tests: unknown variance

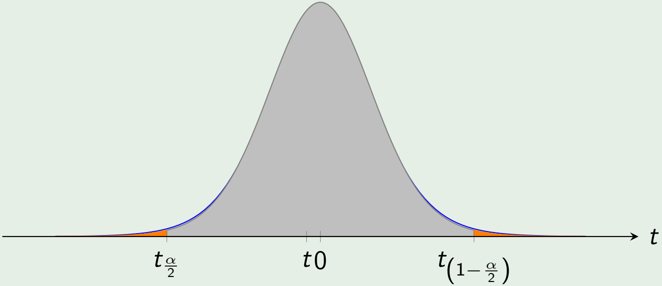
Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$). (In Python: `t.ppf(0.005,17)` and `t.ppf(0.995,17)`)
Note that in two-sided tests, the critical regions on either side have the same area.

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$). (In Python: `t.ppf(0.005,17)` and `t.ppf(0.995,17)`)
Note that in two-sided tests, the critical regions on either side have the same area.

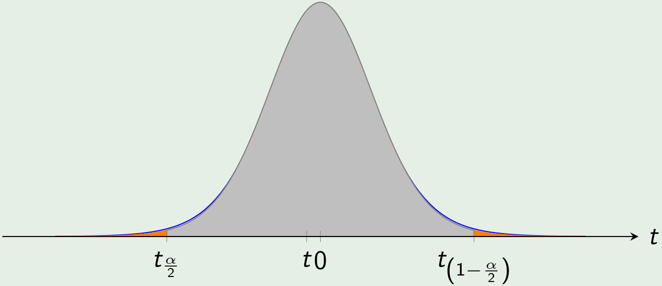


Step 5. We that the test statistic is within the region of nonrejection:

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$). (In Python: `t.ppf(0.005,17)` and `t.ppf(0.995,17)`)
Note that in two-sided tests, the critical regions on either side have the same area.



Step 5. We that the test statistic is within the region of nonrejection:

$$t_{\frac{\alpha}{2}} = -2.8982 < t = -0.261 < t_{\left(1-\frac{\alpha}{2}\right)} = 2.8982$$

Example 7: Golf ball production

Step 6. Thus, we **fail to reject** the null hypothesis.

Example 7: Golf ball production

Step 6. Thus, we **fail to reject** the null hypothesis.
In real terms, this means that the sample was within the bounds of what would be acceptable if the population mean were 1.615 oz. Therefore, we would not stop the production line.

One-sided test: known variance

Example 8: Light bulbs

One-sided test: known variance

Example 8: Light bulbs

A quality control (QC) engineer finds that a sample of 100 light bulbs had an average lifetime of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hrs, test the null hypothesis that the population mean is 480 hrs against the alternative hypothesis it is less than 480 hrs at a significance level of $\alpha = 0.05$.

One-sided test: known variance

Example 8: Light bulbs

A quality control (QC) engineer finds that a sample of 100 light bulbs had an average lifetime of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hrs, test the null hypothesis that the population mean is 480 hrs against the alternative hypothesis it is less than 480 hrs at a significance level of $\alpha = 0.05$.

Step 1. Formulate the hypotheses:

$$H_0 : \mu = 480$$

One-sided test: known variance

Example 8: Light bulbs

A quality control (QC) engineer finds that a sample of 100 light bulbs had an average lifetime of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hrs, test the null hypothesis that the population mean is 480 hrs against the alternative hypothesis it is less than 480 hrs at a significance level of $\alpha = 0.05$.

Step 1. Formulate the hypotheses:

$$\begin{aligned} H_0 : \mu &= 480 \\ H_1 : \mu &< 480 \end{aligned}$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}}$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}} = -4.0$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}} = -4.0$$

Recall that the Z-statistic is normally distributed: $\mathcal{N}(0, 1)$.

One-sided test: known variance

Example 8: Light bulbs (cont.)

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05)$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

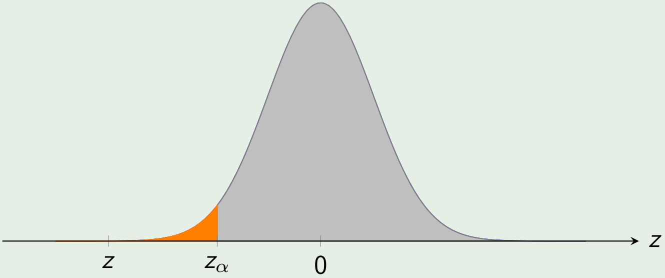
Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)

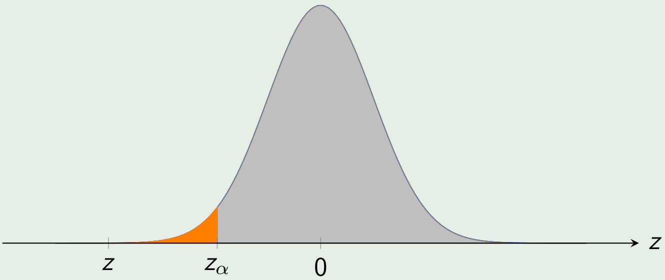


One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)



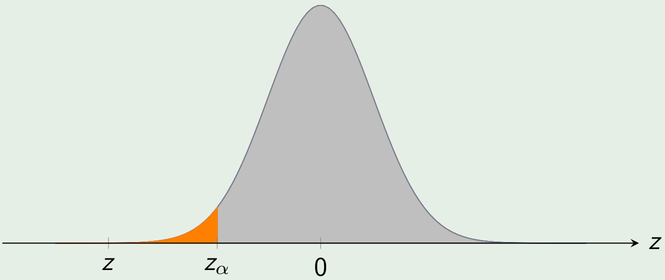
Step 5. We see that $z < z_\alpha$,

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)



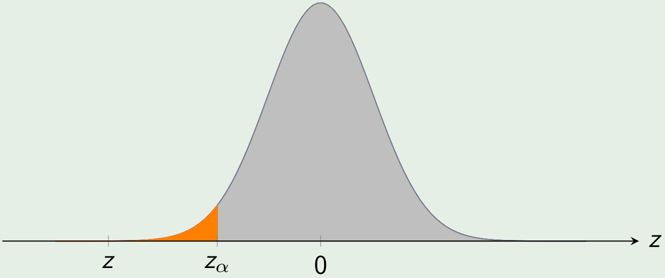
Step 5. We see that $z < z_\alpha$, i.e. z lies inside the region of rejection.

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
$z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)



Step 5. We see that $z < z_\alpha$, i.e. z lies inside the region of rejection. Thus, we **reject the null hypothesis**.

One-sided test: unknown variance

One-sided test: unknown variance

Example 9: Vacuum cleaner

One-sided test: unknown variance

Example 9: Vacuum cleaner

A vacuum cleaner is claimed to expend 46 kWh per year.

One-sided test: unknown variance

Example 9: Vacuum cleaner

A vacuum cleaner is claimed to expend 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with sample SD $s = 11.9$ kWh. At a 0.05 level of significance, does this suggest that on average, vacuum cleaners expend less than 46 kWh per year? Assume the population is normally distributed.

One-sided test: unknown variance

Example 9: Vacuum cleaner

A vacuum cleaner is claimed to expend 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with sample SD $s = 11.9$ kWh. At a 0.05 level of significance, does this suggest that on average, vacuum cleaners expend less than 46 kWh per year? Assume the population is normally distributed.

Step 1. Formulate hypotheses:

One-sided test: unknown variance

Example 9: Vacuum cleaner

A vacuum cleaner is claimed to expend 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with sample SD $s = 11.9$ kWh. At a 0.05 level of significance, does this suggest that on average, vacuum cleaners expend less than 46 kWh per year? Assume the population is normally distributed.

Step 1. Formulate hypotheses:

$$H_0 : \mu = 46$$

One-sided test: unknown variance

Example 9: Vacuum cleaner

A vacuum cleaner is claimed to expend 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with sample SD $s = 11.9$ kWh. At a 0.05 level of significance, does this suggest that on average, vacuum cleaners expend less than 46 kWh per year? Assume the population is normally distributed.

Step 1. Formulate hypotheses:

$$\begin{aligned} H_0 : \mu &= 46 \\ H_1 : \mu &< 46 \end{aligned}$$

One-sided test: unknown variance

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T-statistic:

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T-statistic:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T-statistic:

$$\begin{aligned} t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{42 - 46}{11.9/\sqrt{12}} = -1.16 \end{aligned}$$

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T-statistic:

$$
\begin{aligned}
t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{42 - 46}{11.9/\sqrt{12}} = -1.16
\end{aligned}
$$

Step 3. At $\alpha = 0.05$, the critical value[a] is:

$$t_{\alpha,df} = F_T^{-1}(0.05);$$

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T-statistic:

$$
\begin{aligned}
t &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{42 - 46}{11.9/\sqrt{12}} = -1.16
\end{aligned}
$$

Step 3. At $\alpha = 0.05$, the critical value[a] is:

$$
\begin{aligned}
t_{\alpha,df} &= F_T^{-1}(0.05); \quad df = 12 - 1 = 11 \\
&= -F_T^{-1}(1 - 0.05) \quad \text{(standardized CDF symmetric about 0)} \\
&= -F_T^{-1}(0.95) \\
&= -1.7959
\end{aligned}
$$

[a] Alternately, `from scipy.stats import t` followed by `t.ppf(0.05,11)` will give the answer in Python

One-sided test: unknown variance

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 5. Since $t = -1.16 > t_\alpha = -1.7959$, we

One-sided test: unknown variance

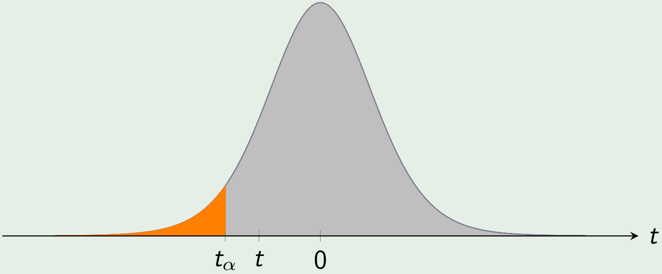
Example 9: Vacuum cleaner (cont.)

Step 5. Since $t = -1.16 > t_\alpha = -1.7959$, we **fail to reject** the null hypothesis.

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 5. Since $t = -1.16 > t_\alpha = -1.7959$, we **fail to reject** the null hypothesis.



Thus, to answer the question, vacuum cleaners do not expend less than 46 kWh per year (with 95% confidence).

Standard error of the mean (SEM)

Standard error (deviation) of sample mean (with **known** population variance):

$$\boxed{SE = \frac{\sigma}{\sqrt{n}}} \tag{23}$$

Standard error of the mean (SEM)

Standard error (deviation) of sample mean (with **known** population variance):

$$\boxed{SE = \frac{\sigma}{\sqrt{n}}} \tag{23}$$

Standard error (deviation) of sample mean (**unknown** population variance):

$$\boxed{SE \approx \frac{s}{\sqrt{n}}} \tag{24}$$

Equation (24) is also called the **standard error** of the mean

Confidence intervals: Recap

Confidence intervals: Recap

Definition

A confidence interval defines the range within which a population parameter lies with a given probability.

Two-sided confidence intervals:

Known population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \, \overline{x} + z_{\left(1-\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} \right)$$

Confidence intervals: Recap

Definition

A confidence interval defines the range within which a population parameter lies with a given probability.

Two-sided confidence intervals:

Known population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \overline{x} + z_{\left(1-\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} \right) = \overline{x} \pm z^* SE \tag{25}$$

Unknown population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}; \overline{x} + t_{\left(1-\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} \right),$$

Confidence intervals: Recap

Definition

A confidence interval defines the range within which a population parameter lies with a given probability.

Two-sided confidence intervals:

Known population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} ; \overline{x} + z_{\left(1-\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} \right) = \overline{x} \pm z^* SE \tag{25}$$

Unknown population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\overline{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} ; \overline{x} + t_{\left(1-\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} \right), = \overline{x} \pm t^*_{df} SE \quad (df = n-1) \tag{26}$$

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution);

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution);Z-statistic:

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution);Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution);

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic:

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error. For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1 - \Phi(z)$;

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1-\Phi(z)$; Python: `norm.sf(z)`

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p-\text{value} = \Phi(z)$;

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1-\Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p-\text{value} = \Phi(z)$; Python: `norm.cdf(z)`

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1-\Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p-\text{value} = \Phi(z)$; Python: `norm.cdf(z)`
  - Two-tailed test: $p-\text{value} = 2(1-\Phi(|z|))$;

Hypothesis testing: Recap

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome)
  - Alternate hypothesis (what we want to test/support; research hypothesis)
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic cases:
  - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
  - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error. For known variance (assume normal distribution):
  - Upper-tailed test: $p-\text{value} = 1-\Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p-\text{value} = \Phi(z)$; Python: `norm.cdf(z)`
  - Two-tailed test: $p-\text{value} = 2(1-\Phi(|z|))$; Python: `2 * norm.cdf(np.abs(z))`

Hypothesis testing: Recap

- Definition of hypothesis testing
    - Null hypothesis (default/expected outcome)
    - Alternate hypothesis (what we want to test/support; research hypothesis)
    - One-tailed or two-tailed
- Types of errors:
    - Type I: false positive
    - Type II: false negative
- Test statistic cases:
    - Sample mean with known variance (normal distribution); Z-statistic: $\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
    - Sample mean with unknown variance (t-distribution); T-statistic: $\frac{\overline{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (df $n-1$)
- The p-value is the minimum probability of a Type I error.For known variance (assume normal distribution):
    - Upper-tailed test: $p-\text{value} = 1-\Phi(z)$; Python: `norm.sf(z)`
    - Lower-tailed test: $p-\text{value} = \Phi(z)$; Python: `norm.cdf(z)`
    - Two-tailed test: $p-\text{value} = 2(1-\Phi(|z|))$; Python: `2 * norm.cdf(np.abs(z))`

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$;

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`
  - Lower-tailed test: $p - \text{value} = F_{df}(t)$;

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`
  - Lower-tailed test: $p - \text{value} = F_{df}(t)$; Python: `t.cdf(t)`

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`
  - Lower-tailed test: $p - \text{value} = F_{df}(t)$; Python: `t.cdf(t)`
  - Two-tailed test: $p - \text{value} = 2(1 - F_{df}(|t|))$;

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
    - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`
    - Lower-tailed test: $p - \text{value} = F_{df}(t)$; Python: `t.cdf(t)`
    - Two-tailed test: $p - \text{value} = 2(1 - F_{df}(|t|))$; Python: `2 * t.cdf(np.abs(t))`

Hypothesis testing: Recap (cont.)

- p-values for unknown variance (assume t distribution):
  - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`
  - Lower-tailed test: $p - \text{value} = F_{df}(t)$; Python: `t.cdf(t)`
  - Two-tailed test: $p - \text{value} = 2(1 - F_{df}(|t|))$; Python: `2 * t.cdf(np.abs(t))`