CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture M4b: Confidence Intervals for a Proportion

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College of Engineering

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Outline

Recap: parameter estimation

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$$\text{Bias}_{\hat{\theta}} = \mathbb{E}(\hat{\theta}) - \theta \tag{1}$$

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- Learn how to use the success-failure condition to determine whether CLT holds for a proportion

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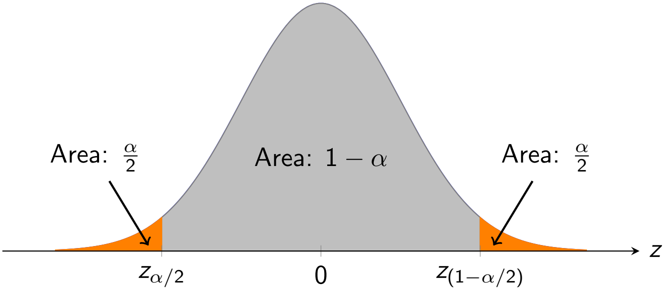

Figure: Standard normal distribution of the mean

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Must check that success-failure condition holds: $np \geq 10$ and $n(1-p) \geq 10$

Working with confidence intervals

Example 1: Identifying confidence levels

Given a normal population distribution with known variance:

(a) What is the confidence level for the interval $\hat{p} \pm 2.81\sigma/\sqrt{n}$?

(b) What is the confidence level for the interval $\hat{p} \pm 1.44\sigma/\sqrt{n}$?

(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

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$$z_{(1-\alpha/2)} = +2.81$$

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(a) What is the confidence level for the interval $\hat{p} \pm 2.81\sigma/\sqrt{n}$?

$$\begin{aligned} z_{(1-\alpha/2)} &= +2.81 \\ 1-\alpha/2 &= \Phi(2.81) \end{aligned}$$

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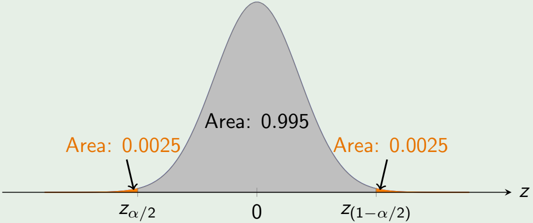

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$$\begin{aligned} z_{(1-\alpha/2)} &= +1.44 \\ 1-\alpha/2 &= \Phi(1.44) = 92.5\% \end{aligned}$$

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The confidence level is $= 1 - \alpha = \boxed{85\%}$.

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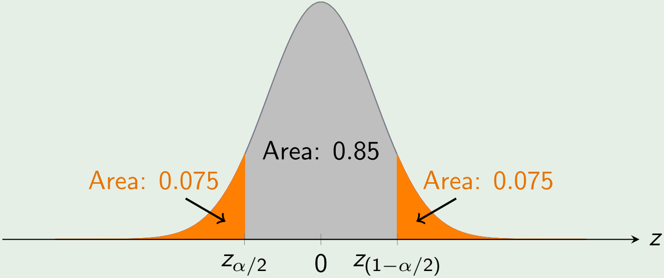

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(c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

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$$z_{\alpha/2} = \Phi^{-1}(0.05)$$

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$$\begin{aligned} z_{\alpha/2} &= \Phi^{-1}(0.05) \\ &= -\Phi^{-1}(0.95) \end{aligned}$$

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z_{\alpha/2} &= \Phi^{-1}(0.05) \\
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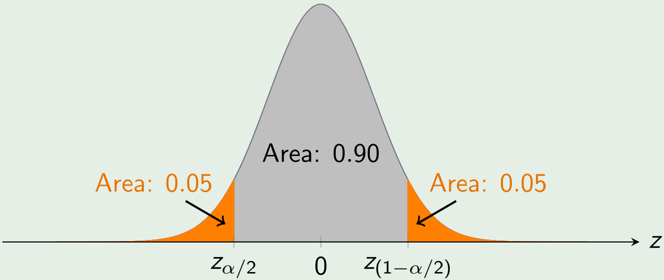

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$$\langle p \rangle_{90\%} \quad = \quad \hat{p} \pm z^* \times SE_{\hat{p}}$$

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$$\begin{aligned} \langle p \rangle_{90\%} &= \hat{p} \pm z^* \times SE_{\hat{p}} \\ &= .887 \pm 1.96(.01) \\ &= (0.8674, .9066) \end{aligned}$$

Example 3: Mental health

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The General Social Survey asked the question: "For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?" Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010.

(a) Interpret this interval in context of the data.

(b) What does "95% confident" mean? Explain in the context of the application.

(c) Suppose the researchers think a 99% confidence level would be more appropriate for this interval. Will this new interval be smaller or wider than the 95% confidence interval?

(d) If a new survey were to be done with 500 Americans, do you think the standard error of the estimate be larger, smaller, or about the same.

Recap

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Steps to find the CI of a proportion

- Compute n and $\hat{p}$

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- Check that the success-failure condition holds

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- Find the critical z-score for the confidence level $1 - \alpha$

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- Find the ME ($z^* \times SE$)

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- Compute n and $\hat{p}$
- Check that the success-failure condition holds
- Find the critical z-score for the confidence level $1 - \alpha$
- Compute the $SE = \sqrt{p(1-p)/n}$
- Find the ME ($z^* \times SE$)
- Find the CI as $\hat{p} \pm ME$