E1 Formula Sheet **Prof. Oke**

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *10.10.2025*

1 Set theory

Properties:

$$A \cup B = B \cup A \tag{1}$$
$$A \cap B = B \cap A \tag{2}$$
$$(A \cup B) \cup C = A \cup (B \cup C) \tag{3}$$
$$(AB)C = A(BC) \tag{4}$$
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C) \tag{5}$$
$$(AB) \cup C = (A \cup C) \cap (B \cup C) \tag{6}$$

De Morgan's rule:

$$\overline{E_1 \cup E_2 \cup \cdots \cup E_n} = \overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_n} \tag{7}$$
$$\overline{E_1 \cap E_2 \cap \cdots \cap E_n} = \overline{E_1} \cup \overline{E_2} \cup \cdots \cup \overline{E_n} \tag{8}$$

2 Mathematics of probability

General rules:

$$P(\overline{E}) = 1 - P(E) \tag{9}$$
$$P(A \cup B) = P(A) + P(B) - P(AB) \tag{10}$$
$$P(A \cap B) = P(AB) = P(A|B)P(B) \tag{11}$$
$$P(A|B) = \frac{P(AB)}{P(B)} \tag{12}$$
$$P(A|B) = 1 - P(\overline{A}|B) \tag{13}$$

Mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) \tag{14}$$

Statistically independent events:

$$P(AB) = P(A)P(B) \tag{15}$$
$$P(A|B) = P(A) \tag{16}$$
$$P(B|A) = P(B) \tag{17}$$

Total probability theorem:

$$P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \cdots + P(A|E_n)P(E_n) = \sum_{i=1}^{n} P(A|E_i)P(E_i) \tag{18}$$

Bayes' theorem:

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{j=1}^{n} P(A|E_j)P(E_j)} \tag{19}$$

3 Probability distributions

Basic definitions:

$$\text{Mean: } E(X) = \mu_X = \begin{cases} \sum_i x_i p_X(x_i) & \text{(discrete)} \\ \int_{-\infty}^{\infty} x f_X(x) dx & \text{(continuous)} \end{cases} \tag{20}$$

$$\text{Variance: } Var(X) = \begin{cases} \sum_i (x_i - E(X))^2 p_X(x_i) & \text{(discrete)} \\ \int_{-\infty}^{\infty} (x - E(X))^2 f_X(x) dx & \text{(continuous)} \end{cases} \tag{21}$$

$$= E(X^2) - E(X)^2 \tag{22}$$

$$\text{Coefficient of variation: } \delta_X = \frac{\sigma_X}{\mu_X} \tag{23}$$

$X \sim \text{Uniform}(a, b)$:

$$P(a < X \leq b) = \frac{x - a}{b - a} \tag{24}$$

$$\mu_X = \frac{a + b}{2} \tag{25}$$

$$Var(X) = \frac{(b - a)^2}{12} \tag{26}$$

$X \sim \mathcal{N}(\mu, \sigma)$:

$$\text{(CDF)} \quad P(X \leq b) = \Phi\left(\frac{b - \mu}{\sigma}\right) \tag{27}$$

$$P(a < X \leq b) = \Phi\left(\frac{b - \mu}{\sigma}\right) - \Phi\left(\frac{a - \mu}{\sigma}\right) \tag{28}$$

$X \sim \text{Lognormal}(\lambda, \zeta)$:

$$P(a < X \leq b) = \Phi\left(\frac{\ln b - \lambda}{\zeta}\right) - \Phi\left(\frac{\ln a - \lambda}{\zeta}\right) \tag{29}$$

$$\lambda = \ln x_m \tag{30}$$

$$\zeta = \sqrt{\ln(1 + \delta_X^2)} \approx \delta_X \tag{31}$$

$$\mu_X = e^{\left(\lambda + \frac{1}{2}\zeta^2\right)} \tag{32}$$

$$Var(X) = \mu_X^2 (e^{\zeta^2} - 1) \tag{33}$$

$X \sim \text{Binomial}(n, p)$:

$$\text{(PDF)} \quad P(X = x) = \binom{n}{x} p^x (1 - p)^{n - x} \tag{34}$$

$$\text{(CDF)} \quad P(X \leq x) = \sum_{k=0}^{x} \binom{n}{k} p^k (1 - p)^{n - k} \tag{35}$$

$$= \binom{n}{0} p^0 (1 - p)^{n - 0} + \binom{n}{1} p^1 (1 - p)^{n - 1} + \cdots + \binom{n}{x} p^x (1 - p)^{n - x} \tag{36}$$

$$\mu_X = np \tag{37}$$

$$Var(X) = np(1 - p) \tag{38}$$

$X \sim$ Poisson(λ), where $\lambda = vt$:

$$\text{(PDF)} \quad P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{39}$$

$$\text{(CDF)} \quad P(X \leq x) = \sum_{k=0}^{x} \frac{\lambda^k e^{-\lambda}}{k!} = \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \cdots + \frac{\lambda^x e^{-\lambda}}{x!} \tag{40}$$

$$\mu_X = Var(X) = \lambda \tag{41}$$

$X \sim$ Exponential(v):

$$\text{(CDF)} \quad P(T \leq t) = 1 - e^{-vt} \tag{42}$$

$$\mu_T = \frac{1}{v} \tag{43}$$

$$Var(T) = \frac{1}{v^2} \tag{44}$$

$$\text{Memorylessness: } P(T > t + t_0 | T > t_0) = P(T > t) \tag{45}$$

$X \sim$ Weibull(k, η, ε), where $\eta = w_1 - \varepsilon$:

$$F_Y(y) = 1 - e^{-\left(\frac{y-\varepsilon}{w_1-\varepsilon}\right)^k}; y \geq \varepsilon \tag{46}$$

$$\mu_Y = \varepsilon + (w_1 - \varepsilon)\Gamma\left(1 + \frac{1}{k}\right) \tag{47}$$

$$\sigma_Y^2 = (w_1 - \varepsilon)^2 \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right] \tag{48}$$

4 Joint distributions

For two **discrete** r.v.'s X and Y:

$$p_{X,Y}(x_i, y_j) = p_{X|Y}(x_i|y_j)p_Y(y_j) = p_{Y|X}(y_j|y_i)p_X(x_i) \tag{49}$$

$$p_X(x_i) = \sum_{y_j} p_{X,Y}(x_i, y_j) \tag{50}$$

$$p_Y(y_j) = \sum_{x_i} p_{X,Y}(x_i, y_j) \tag{51}$$

If X and Y are statistically independent:

$$p_{X,Y}(x_i, y_j) = p_X(x_i)p_Y(y_j) \tag{52}$$

For two **continuous** r.v.'s X and Y:

$$f_{X,Y}(x, y) = f_{X|Y}(x|y)f_Y(y) = f_{Y|X}(y|x)f_X(x) \tag{53}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)dy \tag{54}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)dx \tag{55}$$

If X and Y are statistically independent:

$$f_{X,Y}(x, y) = f_X(x)f_Y(y) \tag{56}$$

Covariance and correlation:

$$Cov(X, Y) = E(XY) - E(X)E(Y) = \rho_{XY}\sigma_X\sigma_Y \tag{57}$$

$$\rho_{XY} = \frac{Cov(X, Y)}{\sigma_X\sigma_Y} \tag{58}$$

5 Functions of random variables

If $Z = aX \pm bY$, where X and Y are normal r.v.'s:

$$E(Z) = aE(X) \pm bE(Y) \tag{59}$$

$$Var(Z) = a^2 Var(X) + b^2 Var(Y) \pm 2ab\, Cov(X,Y) \tag{60}$$

For **statistically independent** random variables:

If $Z = \sum_{i=1}^{n} X_i$, where $X_i \sim \text{Poisson}(v_i)$, then:

$$v_Z = \sum_{i=1}^{n} v_i \tag{61}$$

If $Z = \sum_{i=1}^{n} a_i X_i$, where $X_i \sim \mathcal{N}(\mu_{X_i}, \sigma_{X_i})$, then:

$$\mu_Z = \sum_{i=1}^{n} a_i \mu_{X_i} \quad \sigma_Z^2 = \sum_{i=1}^{n} a_i^2 \sigma_{X_i}^2 \tag{62}$$

If $Z = \prod_{i=1}^{n} X_i$, where $X_i \sim \text{Lognormal}(\lambda_{X_i}, \zeta_{X_i})$, then:

$$\lambda_Z = \sum_{i=1}^{n} \lambda_{X_i} \quad \zeta_Z^2 = \sum_{i=1}^{n} \zeta_{X_i}^2 \tag{63}$$