CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture M5a: Inference for Single Proportion

Jimi Oke

UMass Amherst

College of Engineering

October 30, 2025

Outline

Module objectives

Module objectives

Inference refers to various ways in which we obtain information and make decisions based on data.

Module objectives

Inference refers to various ways in which we obtain information and make decisions based on data. In this module, we consider: (a) **point estimates**, (b) **confidence intervals** and (c) **hypothesis tests** for

- Single proportions

Module objectives

Inference refers to various ways in which we obtain information and make decisions based on data. In this module, we consider: (a) **point estimates**, (b) **confidence intervals** and (c) **hypothesis tests** for

- Single proportions
- Difference of two proportions

Module objectives

Inference refers to various ways in which we obtain information and make decisions based on data. In this module, we consider: (a) **point estimates**, (b) **confidence intervals** and (c) **hypothesis tests** for

- Single proportions
- Difference of two proportions
- Contigency tables

Today's objectives

- Test for normality

Today's objectives

- Test for normality
- Compute CIs (confidence intervals)

Today's objectives

- Test for normality
- Compute CIs (confidence intervals)
- Conduct hypothesis tests
- Calculate sample size

Success-failure condition

Success-failure condition

Given a sample proportion $\hat{p}$ of size n whose observations are independent.

Success-failure condition

Given a sample proportion $\hat{p}$ of size n whose observations are independent. We can assume the distribution for p is normal if

- $np \geq 10$ and
- $n(1-p) \geq 10$

Example 1

Example 1

A simple random sample of 826 payday loan borrowers was surveyed to better understand their interests around regulation and costs. 70% of the responses supported new regulations on payday lenders. Is it reasonable to model $\hat{p} = 0.70$ using a normal distribution?

CI for proportion

CI for proportion

The confidence interval (CI) for a proportion is given by:

CI for proportion

The confidence interval (CI) for a proportion is given by:

$$\hat{p} \pm z^* \times SE \equiv \hat{p} \pm ME \qquad (1)$$

CI for proportion

The confidence interval (CI) for a proportion is given by:

$$\hat{p} \pm z^* \times SE \equiv \hat{p} \pm ME \tag{1}$$

where:

$$z^* = \Phi\left(1 - \frac{\alpha}{2}\right) \quad \textbf{critical value} \tag{2}$$

CI for proportion

The confidence interval (CI) for a proportion is given by:

$$\hat{p} \pm z^* \times SE \equiv \hat{p} \pm ME \tag{1}$$

where:

$$z^* = \Phi\left(1 - \frac{\alpha}{2}\right) \quad \textbf{critical value} \tag{2}$$

$$SE = \sqrt{\frac{p(1-p)}{n}} \quad \textbf{standard error} \tag{3}$$

CI for proportion

The confidence interval (CI) for a proportion is given by:

$$\hat{p} \pm z^* \times SE \equiv \hat{p} \pm ME \tag{1}$$

where:

$$z^* = \Phi\left(1 - \frac{\alpha}{2}\right) \quad \textbf{critical value} \tag{2}$$

$$SE = \sqrt{\frac{p(1-p)}{n}} \quad \textbf{standard error} \tag{3}$$

$$ME = z^* \times SE \quad \textbf{margin of error} \tag{4}$$

CI for proportion

The confidence interval (CI) for a proportion is given by:

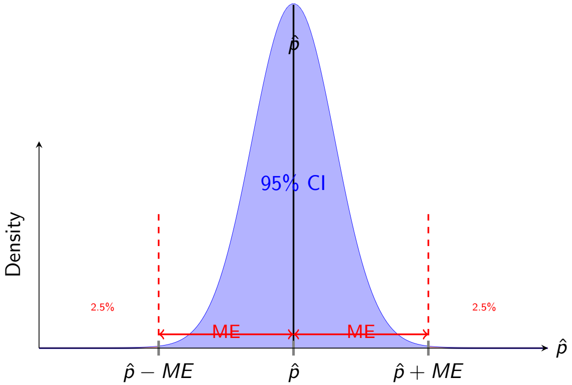
$$\hat{p} \pm z^* \times SE \equiv \hat{p} \pm ME \tag{1}$$

where:

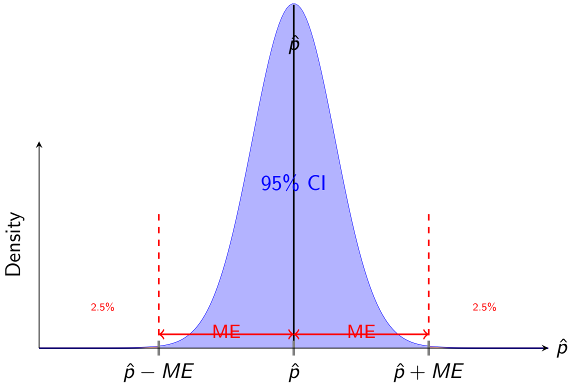
$$\begin{aligned} z^* &= \Phi\left(1 - \frac{\alpha}{2}\right) \quad \textbf{critical value} && (2) \\ SE &= \sqrt{\frac{p(1-p)}{n}} \quad \textbf{standard error} && (3) \\ ME &= z^* \times SE \quad \textbf{margin of error} && (4) \end{aligned}$$

Note that p is unknown, so we use $\hat{p}$ in its place when computing the standard error.

Confidence interval (visualization)



Confidence interval (visualization)



- The confidence interval is symmetric around $\hat{p}$
- Margin of error (ME) extends equally in both directions
- CI $= [\hat{p} - ME, \hat{p} + ME]$

Example 1: Elderly drivers

Example 1: Elderly drivers

The Marist Poll published a report stating that 66% of adults nationally think licensed drivers should be required to retake their road test once they reach 65 years of age. It was also reported that interviews were conducted on 1,018 American adults.

(a) Verify that the success-failure condition is met.

Example 1: Elderly drivers

The Marist Poll published a report stating that 66% of adults nationally think licensed drivers should be required to retake their road test once they reach 65 years of age. It was also reported that interviews were conducted on 1,018 American adults.

(a) Verify that the success-failure condition is met.

(b) Find the standard error for the sample proportion.

Example 1: Elderly drivers

The Marist Poll published a report stating that 66% of adults nationally think licensed drivers should be required to retake their road test once they reach 65 years of age. It was also reported that interviews were conducted on 1,018 American adults.

(a) Verify that the success-failure condition is met.

(b) Find the standard error for the sample proportion.

(c) Find the critical value for a 95% confidence interval.

Example 1: Elderly drivers

The Marist Poll published a report stating that 66% of adults nationally think licensed drivers should be required to retake their road test once they reach 65 years of age. It was also reported that interviews were conducted on 1,018 American adults.

- **(a)** Verify that the success-failure condition is met.
- **(b)** Find the standard error for the sample proportion.
- **(c)** Find the critical value for a 95% confidence interval.
- **(d)** Find the margin of error for the confidence interval.

Example 1: Elderly drivers

The Marist Poll published a report stating that 66% of adults nationally think licensed drivers should be required to retake their road test once they reach 65 years of age. It was also reported that interviews were conducted on 1,018 American adults.

(a) Verify that the success-failure condition is met.

(b) Find the standard error for the sample proportion.

(c) Find the critical value for a 95% confidence interval.

(d) Find the margin of error for the confidence interval.

(e) Construct a 95% confidence interval for p, the proportion of adults

Example 1 (cont.)

(a) Success-failure condition:

$$np = 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark$$

Example 1 (cont.)

(a) Success-failure condition:

$$
\begin{aligned}
np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\
n(1-p) &= 1018 \times (1 - 0.66) = 346.12 \geq 10 \quad \checkmark
\end{aligned}
$$

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

Example 1 (cont.)

(a) Success-failure condition:

$$
\begin{aligned}
np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\
n(1-p) &= 1018 \times (1 - 0.66) = 346.12 \geq 10 \quad \checkmark
\end{aligned}
$$

(b) Standard error:

$$
SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149
$$

(c) Critical value for 95% CI:

$$
z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}
$$

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

$$z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}$$

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

$$z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}$$

(d) Margin of error:

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

$$z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}$$

(d) Margin of error:

$$ME = z^* \times SE = 1.96 \times 0.0149 = 0.0292 \approx 3\%$$

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1-0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

$$z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}$$

(d) Margin of error:

$$ME = z^* \times SE = 1.96 \times 0.0149 = 0.0292 \approx 3\%$$

(e) Confidence interval:

Example 1 (cont.)

(a) Success-failure condition:

$$\begin{aligned} np &= 1018 \times 0.66 = 671.88 \geq 10 \quad \checkmark \\ n(1-p) &= 1018 \times (1 - 0.66) = 346.12 \geq 10 \quad \checkmark \end{aligned}$$

(b) Standard error:

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.66(1-0.66)}{1018}} = 0.0149$$

(c) Critical value for 95% CI:

$$z^* = \Phi\left(1 - \frac{0.05}{2}\right) = \Phi(0.975) = 1.96 \quad \texttt{norm.cdf(0.975)}$$

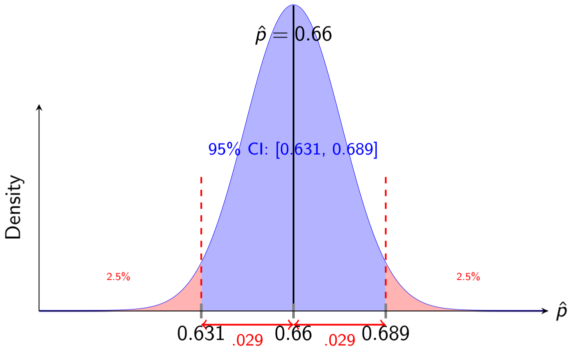
(d) Margin of error:

$$ME = z^* \times SE = 1.96 \times 0.0149 = 0.0292 \approx 3\%$$

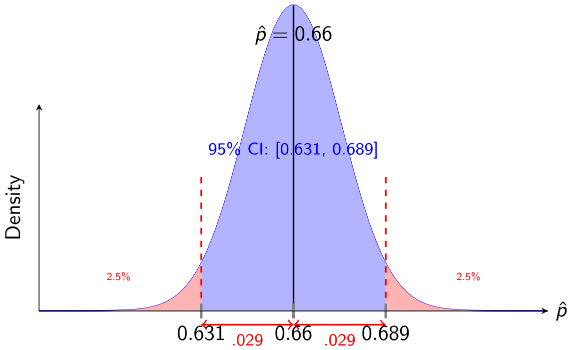
(e) Confidence interval:

$$CI = [\hat{p} - ME, \hat{p} + ME] = [0.66 - 0.0292, 0.66 + 0.0292] = [0.6308, 0.6892]$$

Example 1 (cont.)



Example 1 (cont.)



Conclusion: We are 95% confident that between 63.1% and 68.9% of American adults think licensed drivers should be required to retake their road test once they reach 65 years of age.

Summary of hypothesis testing approach

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)
5. *Define* the **region of rejection/critical region** of the null hypothesis by choosing the **critical value**.

Summary of hypothesis testing approach

1. *Define* the **null** (H_0) and **alternative** (H_1) hypotheses
2. *Determine* the appropriate **test statistic** (and distribution)
3. *Estimate* the test statistic from the sample data
4. *Specify* or *identify* the **level of significance** (α)
5. *Define* the **region of rejection/critical region** of the null hypothesis by choosing the **critical value**.
6. *Decide.* If the test statistic is in the critical region, reject H_0. If not, do not reject H_0 (fail to reject it)

Distribution of the test statistic

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.
We will assume the normal distribution is the success-failure condition holds.

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.
We will assume the normal distribution is the success-failure condition holds.

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.
We will assume the normal distribution is the success-failure condition holds.

The sample proportion is **normally** distributed and its variance is :

$$\mathbb{V}(p) = \frac{p(1-p)}{n} \tag{5}$$

And thus, the standard error is:

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.
We will assume the normal distribution is the success-failure condition holds.

The sample proportion is **normally** distributed and its variance is :

$$\mathbb{V}(p) = \frac{p(1-p)}{n} \tag{5}$$

And thus, the standard error is:

$$SE_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} \tag{6}$$

Thus, to compute the probability (area under curve) of the test statistic,

Distribution of the test statistic

In this lecture, the test statistic is the **sample proportion**.
We will assume the normal distribution is the success-failure condition holds.

The sample proportion is **normally** distributed and its variance is :

$$\mathbb{V}(p) = \frac{p(1-p)}{n} \tag{5}$$

And thus, the standard error is:

$$SE_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} \tag{6}$$

Thus, to compute the probability (area under curve) of the test statistic, we use the z-score:

$$z = \frac{p - p_0}{SE_p} \tag{7}$$

which is normally distributed.

Two-sided tests

Two-sided tests

Case A: both tails

Two-sided tests

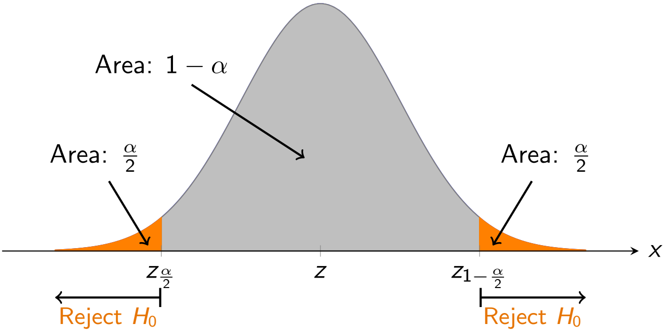
Case A: both tails

- $H_0 : p = p_0$
- $H_1 : p \neq p_0$

Two-sided tests

Case A: both tails

- $H_0 : p = p_0$
- $H_1 : p \neq p_0$



One-sided tests

One-sided tests

Case B: upper tail

One-sided tests

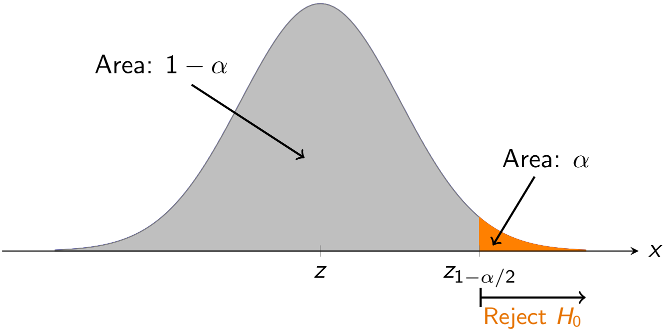
Case B: upper tail

- $H_0 : p = p_0$
- $H_1 : p > p_0$

One-sided tests

Case B: upper tail

- $H_0 : p = p_0$
- $H_1 : p > p_0$



One-sided tests (cont.)

One-sided tests (cont.)

Case C: lower tail

One-sided tests (cont.)

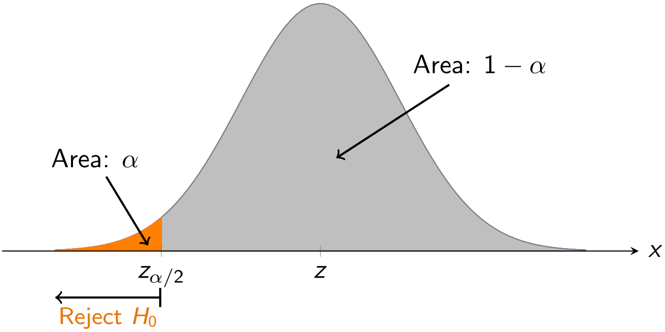
Case C: lower tail

- $H_0 : p = p_0$
- $H_1 : p < p_0$

One-sided tests (cont.)

Case C: lower tail

- $H_0 : p = p_0$
- $H_1 : p < p_0$



Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest:

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis:

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723$.

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723$.

Step 3. Alternative hypothesis:

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Step 4. Formula for test statistic value:

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723.$

Step 3. Alternative hypothesis: $H_1 : p \neq .723.$

Step 4. Formula for test statistic value: $z = \frac{p - p_0}{SE_p}$

Example 2: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : p = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Step 4. Formula for test statistic value: $z = \frac{p - p_0}{SE_p}$

Example 2: Getting enough sleep (cont.)

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}}$$

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

$$z_{\alpha/2} = \Phi^{-1}(1 - \alpha/2) = \Phi^{-1}(0.995) = 2.576 \quad \texttt{norm.ppf(0.995)}$$

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

$$z_{\alpha/2} = \Phi^{-1}(1 - \alpha/2) = \Phi^{-1}(0.995) = 2.576 \quad \texttt{norm.ppf(0.995)}$$

Step 7. Compare test statistic to critical value:

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

$$z_{\alpha/2} = \Phi^{-1}(1 - \alpha/2) = \Phi^{-1}(0.995) = 2.576$$ `norm.ppf(0.995)`

Step 7. Compare test statistic to critical value:

$$|z| = 8.92 > 2.576 = z_{\alpha/2}$$

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

$$z_{\alpha/2} = \Phi^{-1}(1 - \alpha/2) = \Phi^{-1}(0.995) = 2.576$$ `norm.ppf(0.995)`

Step 7. Compare test statistic to critical value:

$$|z| = 8.92 > 2.576 = z_{\alpha/2}$$

Step 8. Conclude:

Example 2: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Compute critical value at $\alpha = 0.01$:

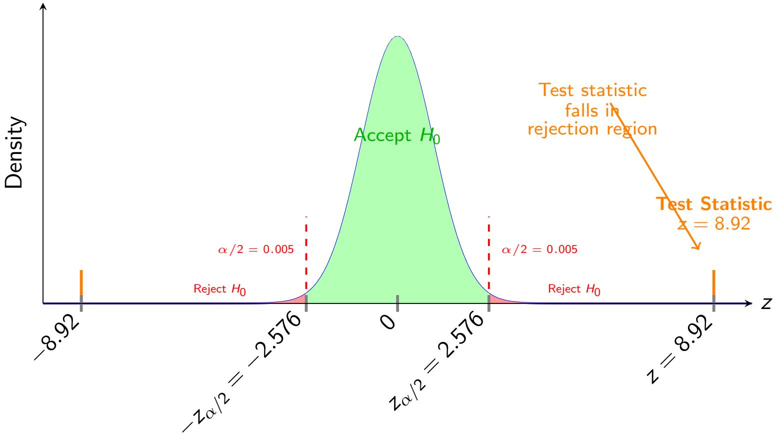
$$z_{\alpha/2} = \Phi^{-1}(1 - \alpha/2) = \Phi^{-1}(0.995) = 2.576 \quad \texttt{norm.ppf(0.995)}$$

Step 7. Compare test statistic to critical value:

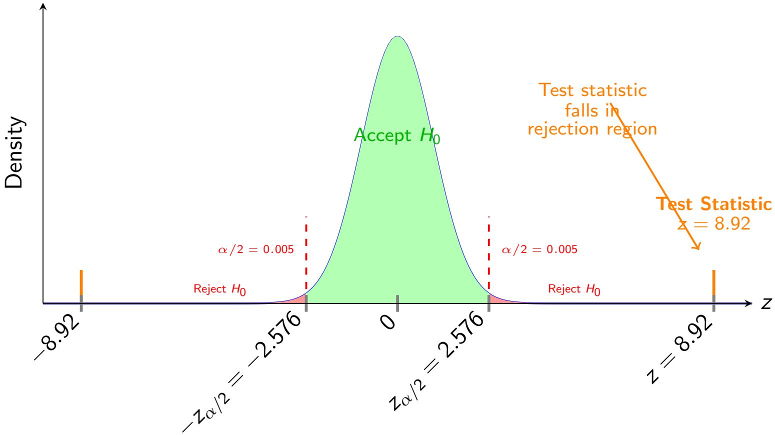
$$|z| = 8.92 > 2.576 = z_{\alpha/2}$$

Step 8. Conclude:
Using a significance level of 0.01, we reject H_0 since $8.92 > 2.576$. Thus, at the 1% significance level, there is sufficient evidence to conclude that true proportion differs from the target value of 0.5.

Example 2: Getting enough sleep (cont.)



Example 2: Getting enough sleep (cont.)



Decision: Since $|z| = 8.92 > 2.576 = z_{\alpha/2}$, we **reject** H_0 at $\alpha = 0.01$ significance level.

Example 3: Equipment quality

Example 3: Equipment quality

A car manufacturer is considering switching to a new, higher quality piece of equipment that con- structs vehicle door hinges. They figure that they will save money in the long run if this new machine produces hinges that have flaws no more than 0.2% of the time. A random sample of 800 hinges produced by the new machine shows that 4 are flawed. At the 0.05 significance level, is there enough evidence to conclude that the new machine produces hinges with a flaw rate greater than 0.2%?

Example 3 (cont.)

Example 3 (cont.)

This is a one-sided test

Example 3 (cont.)

This is a one-sided test

Example 3 (cont.)

This is a one-sided test
Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Example 3 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\hat{p} = \frac{4}{800} = 0.005$$

Example 3 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\hat{p} = \frac{4}{800} = 0.005$$

$$SE = \sqrt{\frac{0.002(1-0.002)}{800}} \approx 0.0016$$

Example 3 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\begin{aligned}
\hat{p} &= \frac{4}{800} = 0.005 \\
SE &= \sqrt{\frac{0.002(1 - 0.002)}{800}} \approx 0.0016 \\
z &= \frac{\hat{p} - 0.002}{SE} \approx 1.875
\end{aligned}$$

Example 3 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

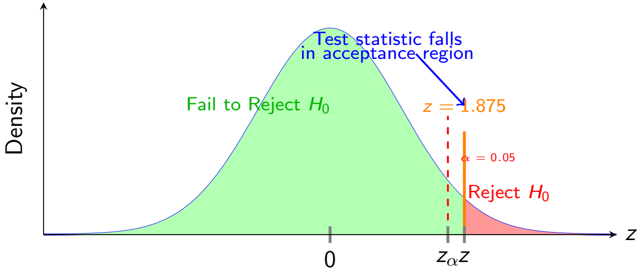
$$\begin{aligned}
\hat{p} &= \frac{4}{800} = 0.005 \\
SE &= \sqrt{\frac{0.002(1-0.002)}{800}} \approx 0.0016 \\
z &= \frac{\hat{p} - 0.002}{SE} \approx 1.875
\end{aligned}$$

Critical Value: For $\alpha = 0.05$, the critical value is:

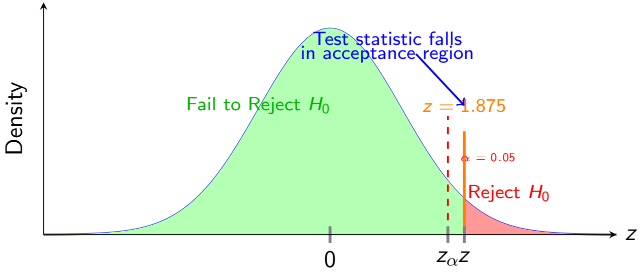
$$z_\alpha = \Phi^{-1}(1-\alpha) = \Phi^{-1}(0.95) = 1.645 \quad \texttt{norm.ppf(0.95)}$$

Example 3 (cont.)

Example 3 (cont.)



Example 3 (cont.)



Decision: Since $z = 1.875 < 1.645 = z_\alpha$, we **fail to reject** H_0 at $\alpha = 0.05$ significance level.

p-value

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted.

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

- Provides more information about the strength of a test

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

p-value

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Usefulness of p-value

- Provides more information about the strength of a test
- Indicates the smallest level at which the data is significant
- Can be compared with α irrespective of which type of test was used

Step 1. Formulate your hypotheses

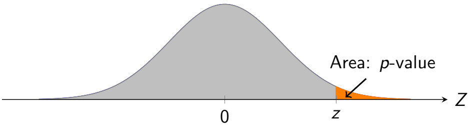
Step 2. Determine the p-value from the test statistic

Step 3. Conclude the test based on a chosen level of significance:

1. p-value $\leq \alpha \implies$ reject H_0 at level α.
2. p-value $> \alpha \implies$ do not reject H_0 at level α.

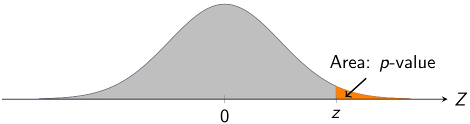
p-value for z tests

p-value for z tests



p-value: area in upper tail

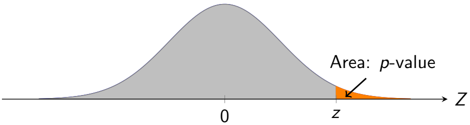
p-value for z tests

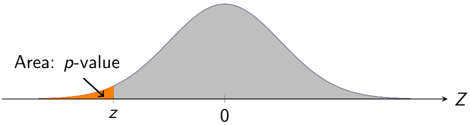


p-value: area in upper tail

$$p = 1 - \Phi(z) \tag{8}$$

p-value for z tests



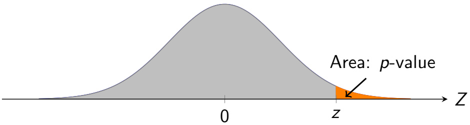


p-value: area in upper tail

$$p = 1 - \Phi(z) \tag{8}$$

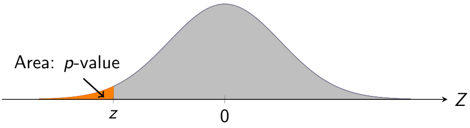
p-value: area in lower tail

p-value for z tests



p-value: area in upper tail

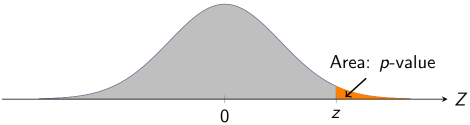
$$p = 1 - \Phi(z) \qquad (8)$$



p-value: area in lower tail

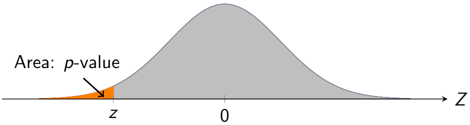
$$p = \Phi(z) \qquad (9)$$

p-value for z tests



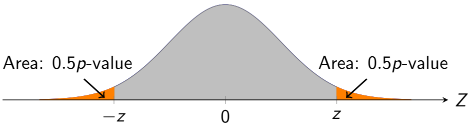
p-value: area in upper tail

$$p = 1 - \Phi(z) \tag{8}$$



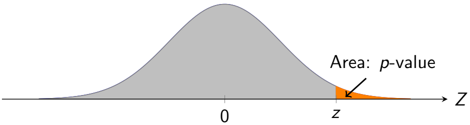
p-value: area in lower tail

$$p = \Phi(z) \tag{9}$$



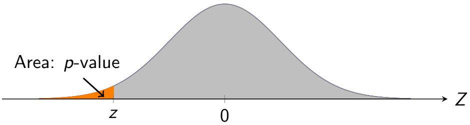
p-value: sum of area in both tails

p-value for z tests



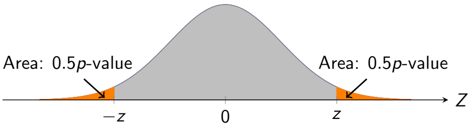
p-value: area in upper tail

$$p = 1 - \Phi(z) \tag{8}$$



p-value: area in lower tail

$$p = \Phi(z) \tag{9}$$



p-value: sum of area in both tails

$$p = 2(1 - \Phi(|z|)) \tag{10}$$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723.$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723$.

Step 3. Alternative hypothesis:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Step 4. Formula for test statistic value:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Step 4. Formula for test statistic value: $z = \frac{p - p_0}{SE_p}$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest: p (proportion of students not getting enough sleep)

Step 2. Null hypothesis: $H_0 : \hat{p} = 289/400 = .723$.

Step 3. Alternative hypothesis: $H_1 : p \neq .723$.

Step 4. Formula for test statistic value: $z = \frac{p - p_0}{SE_p}$

Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (cont.)

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}}$$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine p-value

Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine *p*-value (two-tailed test):

Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine *p*-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92))$$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92)) = 0.0$$

Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine *p*-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92)) = 0.0$$

Step 7. Compare *p*-value to $\alpha = 0.01$:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92)) = 0.0$$

Step 7. Compare p-value to $\alpha = 0.01$:

$$0.0 \leq 0.01$$

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92)) = 0.0$$

Step 7. Compare p-value to $\alpha = 0.01$:

$$0.0 \leq 0.01$$

Step 8. Conclude:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}} = 8.92$$

Step 6. Determine p-value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(8.92)) = 0.0$$

Step 7. Compare p-value to $\alpha = 0.01$:

$$0.0 \leq 0.01$$

Step 8. Conclude:
Using a significance level of 0.01, we reject H_0 since $0.0 < 0.01$.

Example 5: Equipment quality (p-value approach)

Example 5: Equipment quality (p-value approach)

A car manufacturer is considering switching to a new, higher quality piece of equipment that con- structs vehicle door hinges. They figure that they will save money in the long run if this new machine produces hinges that have flaws no more than 0.2% of the time. A random sample of 800 hinges produced by the new machine shows that 4 are flawed. At the 0.05 significance level, is there enough evidence to conclude that the new machine produces hinges with a flaw rate greater than 0.2%?

Example 5 (cont.)

Example 5 (cont.)

This is a one-sided test

Example 5 (cont.)

This is a one-sided test

Example 5 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Example 5 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\hat{p} = \frac{4}{800} = 0.005$$

Example 5 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\hat{p} = \frac{4}{800} = 0.005$$

$$SE = \sqrt{\frac{0.002(1-0.002)}{800}} \approx 0.0016$$

Example 5 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

$$\begin{aligned}
\hat{p} &= \frac{4}{800} = 0.005 \\
SE &= \sqrt{\frac{0.002(1-0.002)}{800}} \approx 0.0016 \\
z &= \frac{\hat{p} - 0.002}{SE} \approx 1.875
\end{aligned}$$

Example 5 (cont.)

This is a one-sided test

Hypotheses:

- $H_0 : p = 0.002$
- $H_a : p < 0.002$

Test Statistic:

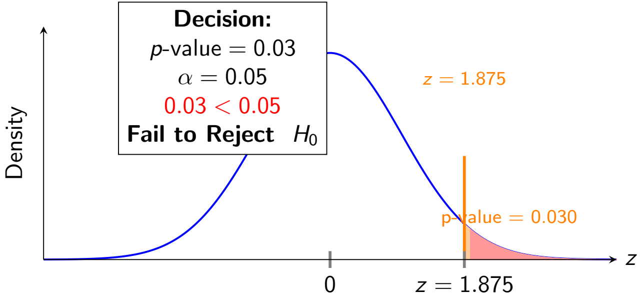
$$\begin{aligned}
\hat{p} &= \frac{4}{800} = 0.005 \\
SE &= \sqrt{\frac{0.002(1-0.002)}{800}} \approx 0.0016 \\
z &= \frac{\hat{p} - 0.002}{SE} \approx 1.875
\end{aligned}$$

p-value: For $z = 1.875$, the p-value is:

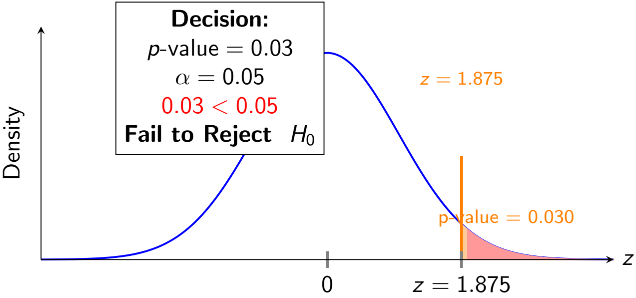
$$p\text{-value} = 1 - \Phi(1.875) \approx 0.0304$$ `1 - norm.cdf(1.875)`

Example 5 (cont.)

Example 5 (cont.)

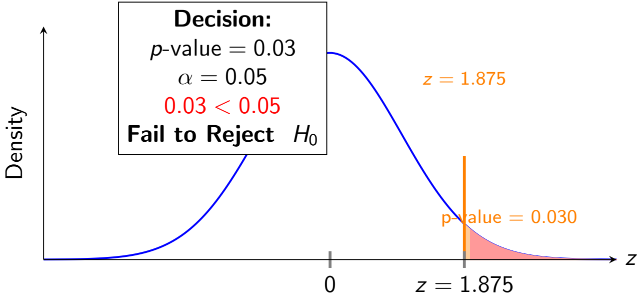


Example 5 (cont.)



Decision: Since the p-value $= 0.03 < \alpha = 0.05$, we **fail to reject** H_0 at the 5% significance level.

Example 5 (cont.)



Decision: Since the p-value $= 0.03 < \alpha = 0.05$, we **fail to reject** H_0 at the 5% significance level.
Conclusion: There is insufficient evidence to conclude that the new machine produces hinges with a flaw rate greater than 0.2%.

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution);

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$;

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p - \text{value} = \Phi(z)$;

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p - \text{value} = \Phi(z)$; Python: `norm.cdf(z)`

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p - \text{value} = \Phi(z)$; Python: `norm.cdf(z)`
  - Two-tailed test: $p - \text{value} = 2(1 - \Phi(|z|))$;

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p - \text{value} = \Phi(z)$; Python: `norm.cdf(z)`
  - Two-tailed test: $p - \text{value} = 2(1 - \Phi(|z|))$;
    Python: `2 * norm.cdf(np.abs(z))`

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0
  - Alternate hypothesis (what we want to test/support; research hypothesis) H_1 or H_A
  - One-tailed or two-tailed
- Types of errors:
  - Type I: false positive
  - Type II: false negative
- Test statistic:
  - Sample proportion with independent observations and large enough sample size (normal distribution); Z-statistic:

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{11}$$

- The p-value is the minimum probability of a Type I error.
  - Upper-tailed test: $p - \text{value} = 1 - \Phi(z)$; Python: `norm.sf(z)`
  - Lower-tailed test: $p - \text{value} = \Phi(z)$; Python: `norm.cdf(z)`
  - Two-tailed test: $p - \text{value} = 2(1 - \Phi(|z|))$;
    Python: `2 * norm.cdf(np.abs(z))`