

**Due Thursday, December 5, 2024 at 11:59 PM as PDF uploaded on Moodle. Show as much work as possible in order to get FULL credit.** There are 6 problems with a total of 60 points available. **Important:** If you use MATLAB for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables, note this in the respective solution, as well.

## Problem 1 Upper confidence bound (3 points)

The charge-to-tap time (min) for a carbon steel in one type of open hearth furnace was determined for each heat<sup>a</sup> in a sample of size 46, resulting in a sample mean time of 382.1 and a population standard deviation of 31.5. Calculate a 95% upper confidence bound for true average charge-to-tap time.

(RIGHT:) An open hearth furnace being tapped at a Swedish steel mill. Source: [https://en.wikipedia.org/wiki/File:Tapping\\_av\\_martinugn.jpg](https://en.wikipedia.org/wiki/File:Tapping_av_martinugn.jpg)

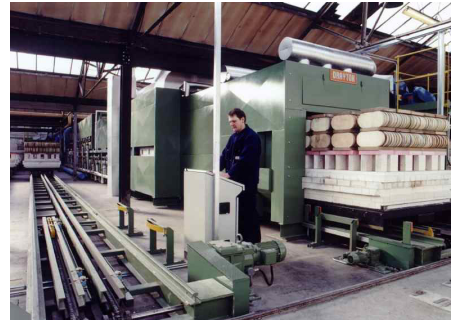
<sup>a</sup>A “heat” describes each batch in an open-hearth process for steel production. Read this article for more information: [https://en.wikipedia.org/wiki/Open\\_hearth\\_furnace](https://en.wikipedia.org/wiki/Open_hearth_furnace)



**Problem 2**    *Confidence Intervals and Sample Size (7 points)*

The article “Evaluating Tunnel Kiln Performance” (*Amer. Ceramic Soc. Bull.*, Aug 1997: 59–63) gave the following summary information for fracture strengths (MPa) of  $n = 169$  ceramic bars fired in a particular kiln:  $\bar{x} = 89.10$ ,  $s = 3.73$ .

(RIGHT:) A tunnel kiln.    Source: <https://blog.therseruk.com/hubfs/Tunnel%20Kiln%20for%20Refractories%20in%20UK%2017-2.jpg>



- [4] (a) Calculate a [two-sided] confidence interval for true average fracture strength using a confidence level of 95%. Does it appear that true average fracture strength has been precisely estimated?
- [3] (b) Suppose the investigators had believed a priori that the population standard deviation was about 4 MPa. Based on this supposition, how large a sample would have been required to estimate  $\mu$  to within 0.5 MPa with 95% confidence?

**Problem 3**    *Confidence intervals (5 points)*

A 95% confidence interval for a population mean,  $\mu$ , is given as (18.985, 21.015). This confidence interval is based on a simple random sample of 36 observations. Assume that all conditions necessary for inference are satisfied. Using the  $t$ -distribution, calculate the

(a) Margin of error [1]

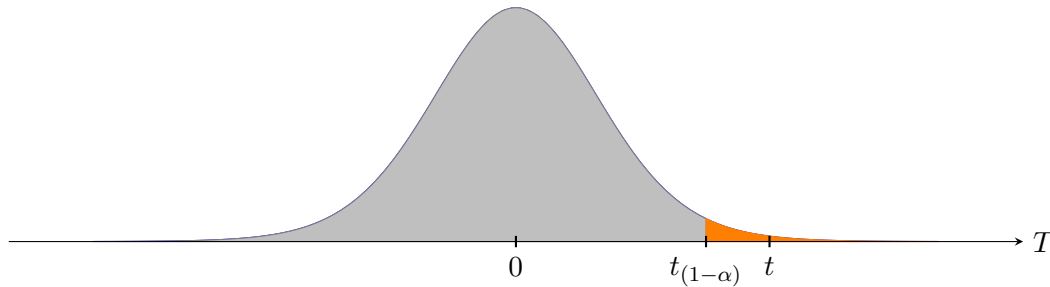
(b) Sample mean [1]

(c) Sample standard deviation [3]

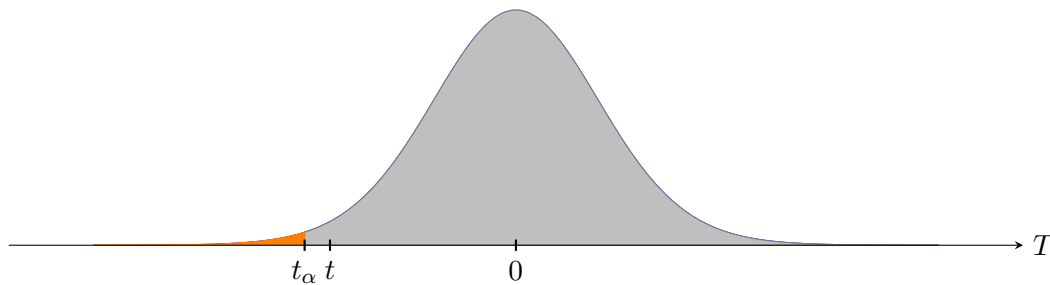
### Problem 4 Hypothesis testing (4 points)

In the following hypothesis tests, decide whether to “Reject  $H_0$ ” or “Fail to reject  $H_0$ ” by comparing the  $Z$  or  $T$  scores ( $z$  or  $t$ , respectively) to the critical values. (Critical regions in orange.)

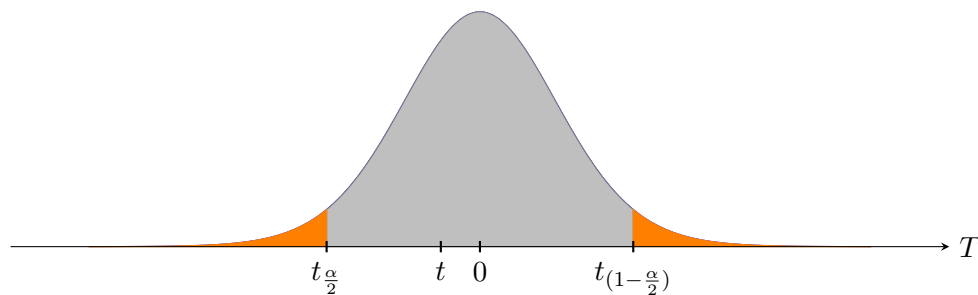
(a)  $H_0 : \mu = \mu_0; H_1 : \mu > \mu_0$



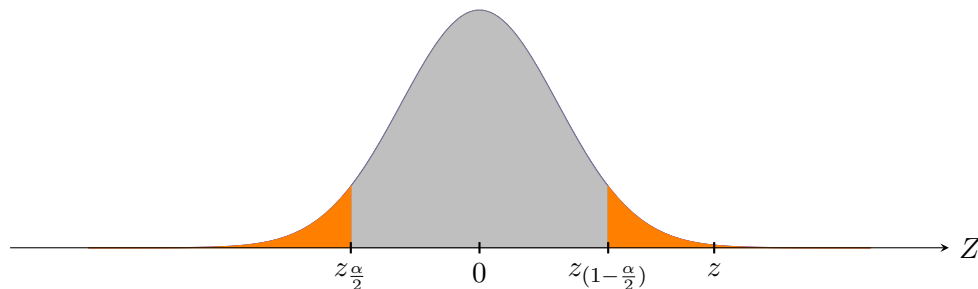
(b)  $H_0 : \mu = \mu_0; H_1 : \mu < \mu_0$



(c)  $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$



(d)  $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$



**Problem 5**    *Identifying significance levels with Z-score (6 points)*

Let the test statistic  $Z$  have a standard normal distribution when  $H_0$  is true. Find the significance level for each of the following situations (show your work and/or calculator/Matlab/Python functions):

(a)  $H_1 : \mu > \mu_0$ , critical region:  $z \geq 1.88$ .

(b)  $H_1 : \mu < \mu_0$ , critical region:  $z \leq -2.75$ .

(c)  $H_1 : \mu \neq \mu_0$ , critical region:  $z \geq 2.88$  or  $z \leq -2.88$ .

**Problem 6**    *Identifying significance levels with  $T$ -score (6 points)*

Let the test statistic  $T$  have a  $t$  distribution when  $H_0$  is true. Find the significance level for each of the following situations:

(a)  $H_1 : \mu > \mu_0$ , d.o.f. = 15, rejection region:  $t \geq 3.733$ .

(b)  $H_1 : \mu < \mu_0$ , d.o.f. = 24, rejection region:  $t \leq -2.500$ .

(c)  $H_1 : \mu \neq \mu_0$ , d.o.f. = 31, rejection region:  $t \geq 1.697$  or  $t \leq -1.697$ .

**Problem 7**    *Two-tailed hypothesis test (6 points)*

The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in  $\bar{x} = 94.32$ . Assume that the distribution of melting point is normal with  $\sigma = 1.20$ . Test  $H_0 : \mu = 95$  versus  $H_1 : \mu \neq 95$  using a two-tailed level 0.01 test.

*NOTE: to receive full credit, make sure you show the following:*

- *How you compute the test statistic and critical value(s) OR p-value* [3]
- *Explicitly compare test statistic to the critical value (or p-value to  $\alpha$ )* [1]
- *State the outcome of the hypothesis test and write a concluding statement* [2]

**Problem 8**    *p-values (7 points)*

- (a) Pairs of  $p$ -values and significance levels  $\alpha$  are given. For each pair, state whether the observed  $p$ -value would lead to rejection of  $H_0$  at the given significance level:

[1]            (i)  $p\text{-value} = 0.084; \alpha = 0.05$

[1]            (ii)  $p\text{-value} = 3.2 \times 10^{-5}; \alpha = 0.001$

[1]            (iii)  $p\text{-value} = 0.039; \alpha = 0.01$

- (b) Let  $\mu$  denote the mean reaction time to a certain stimulus. For a large-sample test of  $H_0 : \mu = 5$ , versus  $H_1 : \mu > 5$ , find the  $p$ -value associated with the following test statistic values

[2]            (i)  $z = 1.42$

[2]            (ii)  $z = -0.11$

**Problem 9**    *One-tailed hypothesis test (8 points)*

The times of first sprinkler activation for a series of tests with fire prevention sprinkler systems using an aqueous film-forming foam were (in sec):

27, 41, 22, 27, 23, 35, 30, 33, 24, 27, 28, 22, 24

The system has been designed so that true average activation time is at most 25 sec under such conditions. Test the relevant hypotheses at significance level 0.05 using the  $p$ -value approach to determine if the data strongly contradict the validity of this design specification.

*NOTE: To get full credit for this problem, make sure you do the following:*

- *State the hypotheses (there are two)* [1]
- *Find the sample mean and standard deviation* [2]
- *Find the test statistic (hint: T-score)* [1]
- *Find the p-value* [1]
- *Compare the appropriate values* [1]
- *Clearly state the outcome from your hypothesis test* [1]
- *Write a final concluding statement in response to the question* [1]

### Problem 10 *Difference of two means; known variances (8 points)*

An article published in 1983 compared various types of batteries. The average lifetimes of Duracell Alkaline AA batteries and Eveready Energizer Alkaline AA batteries were given as 4.1 hours and 4.5 hours, respectively. Suppose these are the population average lifetimes. (*RIGHT:*) Eveready and Duracell batteries. Source: <https://mybroadband.co.za/news/gadgets/413368-duracell-and-eveready-tested-with-surprising-results.html>



- [1] (a) Let  $\bar{x}$  be the sample average lifetime of 100 Duracell batteries and  $\bar{y}$  be the sample average lifetime of 100 Eveready batteries. What is the mean value of  $\bar{x} - \bar{y}$  (i.e. where is the distribution of  $\bar{x} - \bar{y}$  centered)?
- [3] (b) Suppose the population standard deviations of lifetime are 1.8 hours for Duracell batteries and 2.0 hours for Eveready batteries. With the sample sizes given in part (a), what is the **standard error** of the statistic  $\bar{x} - \bar{y}$
- [4] (c) Compute and **interpret** the 99% CI of the difference in lifetime between Duracell and Eveready batteries,  $\bar{x} - \bar{y}$ .

