CEE 260/MIE 273: Probability and Statistics in Civil Engineering
Lecture 3E: The Poisson Distribution

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Outline

Recap of Lecture 3b: Binomial distribution

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The above are all **Poisson processes** and the respective probabilities can be modeled by the **Poisson distribution**

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Further examples of Poisson processes

- Breakdowns of a computer system over time
- Number of accidents in an industrial facility over time
- Customer arrivals at a bike store in a given morning

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$$P(X \leq x) = \sum_{k=0}^{x} \frac{\lambda^k}{k!} e^{-\lambda} \tag{4}$$

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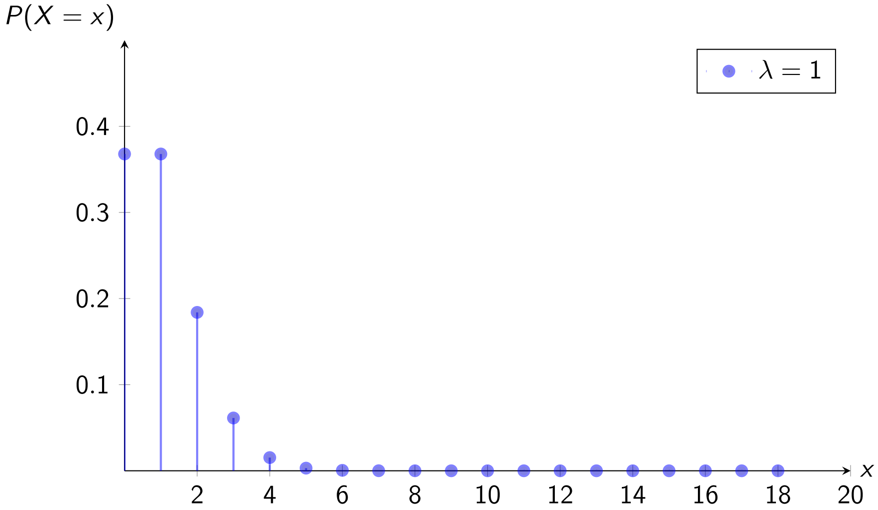
PMF of $X \sim \text{Poisson}(\lambda)$



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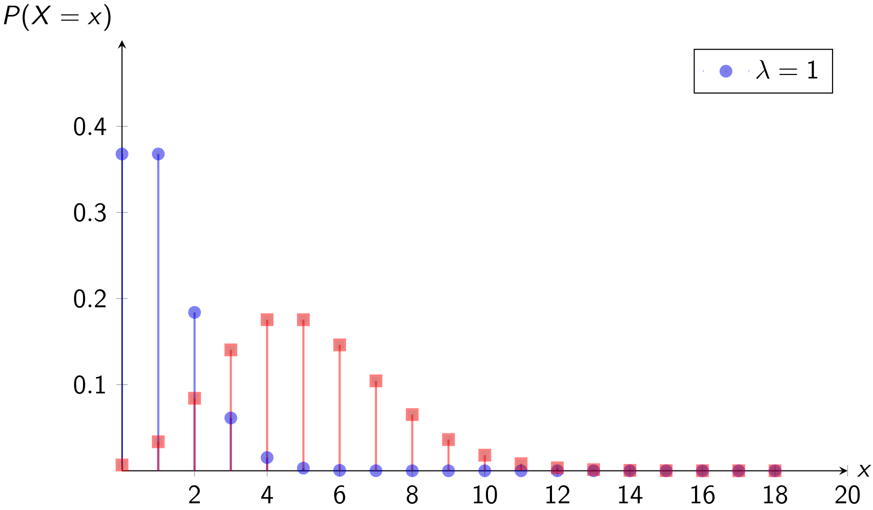

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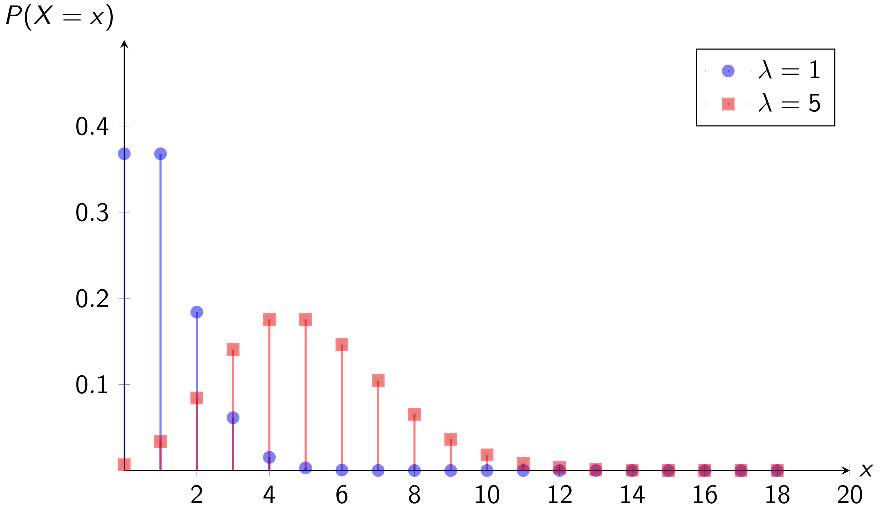

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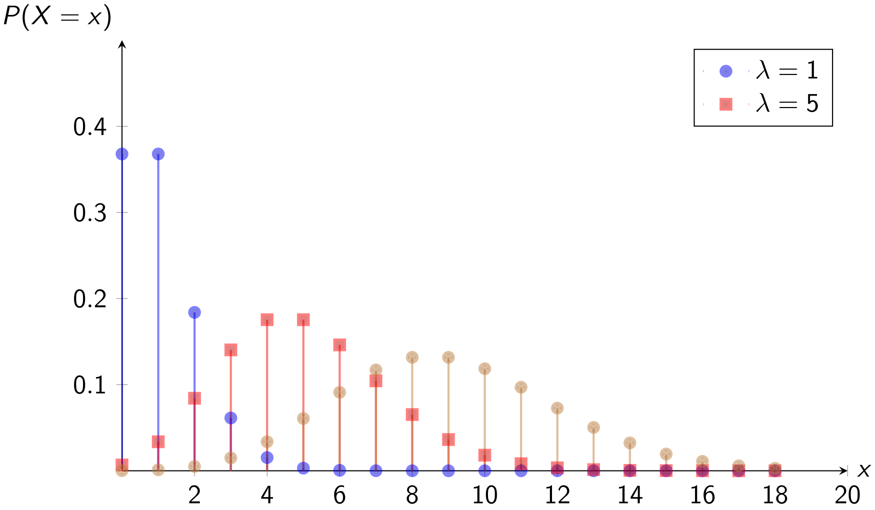

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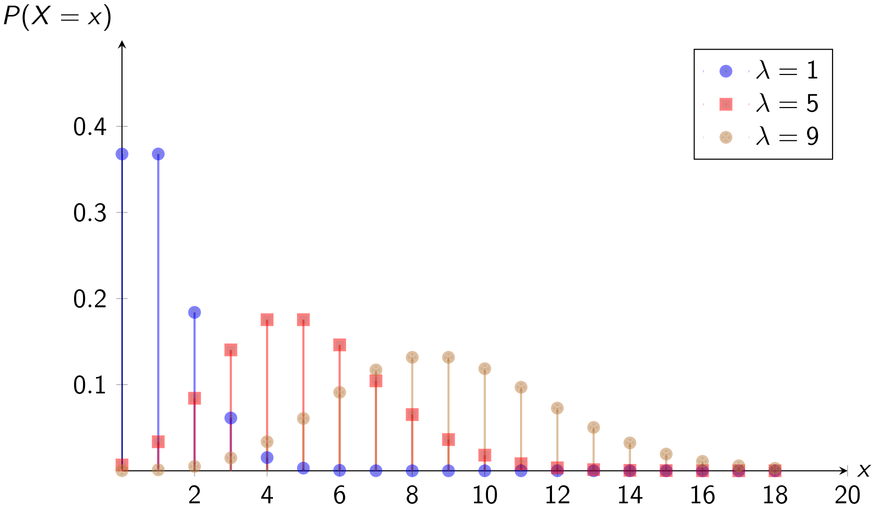

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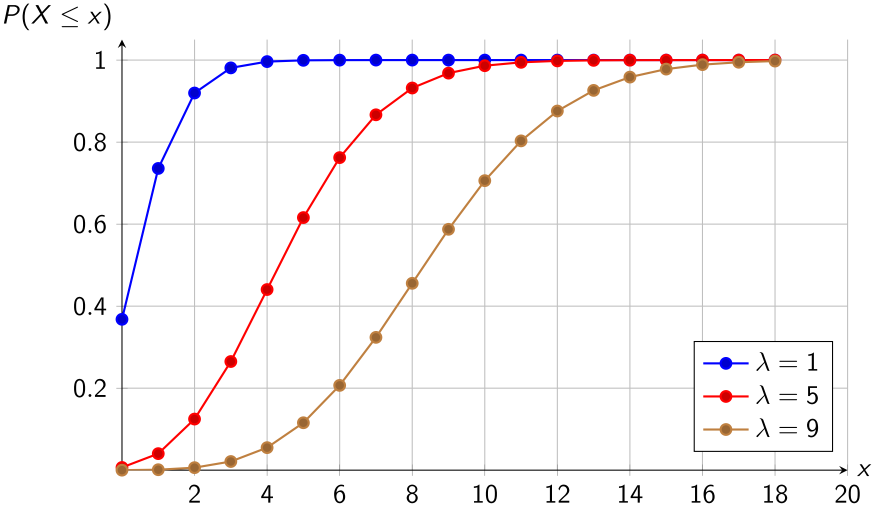

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(a) What is the probability that exactly 6 small aircraft arrive during a 1-hr period?

(b) What is the probability that at least 10 small aircraft arrive during a 1-hr period?

(c) How many aircraft do you expect to arrive during a 90-minute period?

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$$P(X_{1hr} = 6) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{8^6 e^{-8}}{6!} = \texttt{poisson.pmf(6,8)} = \boxed{0.1221}$$

(b) What is the probability that at least 10 small aircraft arrive during a 1-hr period?

Example 2: Aircraft arrivals at airport (cont.)

(a) What is the probability that exactly 6 small aircraft arrive during a 1-hr period?

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(b) What is the probability that at least 10 small aircraft arrive during a 1-hr period?

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- And thus:

$$P(X_t = x) = \frac{(\lambda t)^x}{x!} e^{-(\lambda t)} = \frac{(\lambda')^x}{x!} e^{-(\lambda')} \tag{8}$$

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(c) How many aircraft do you expect to arrive during a 90-minute period?

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Expected arrivals within 90 minutes $= 8/\text{hr} \times 1.5$ hrs $= 12$.

Example 3: Amherst Coffee customer arrivals (revisited)

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$$\lambda^* = 30 \times 2 = 60 \text{ (per 2 hrs)}$$

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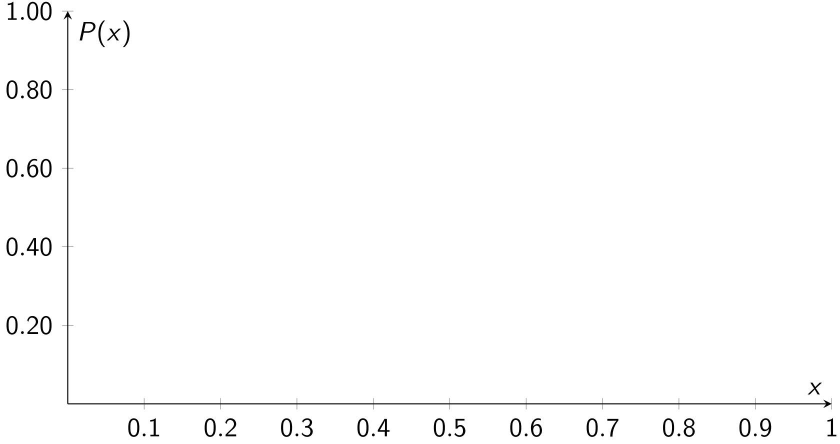

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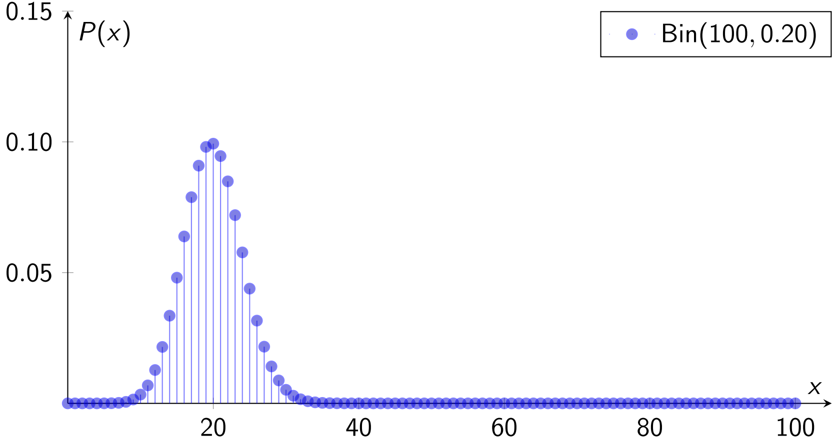

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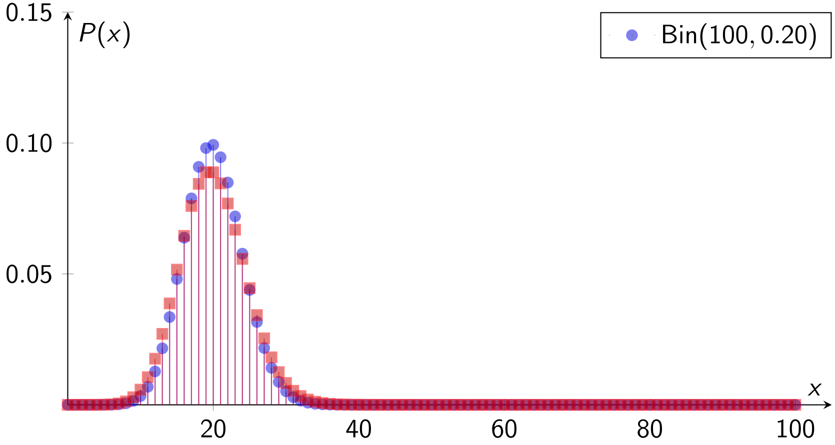

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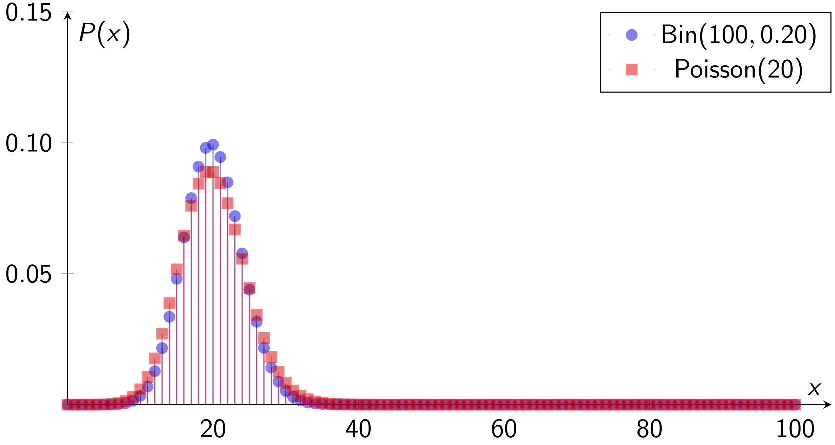

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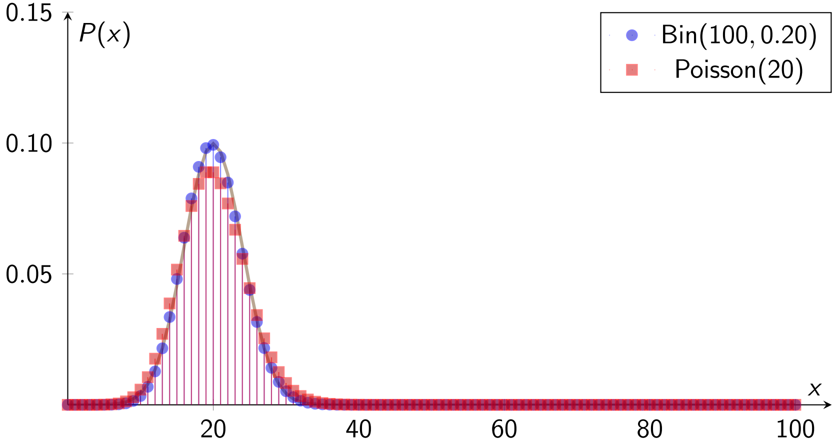

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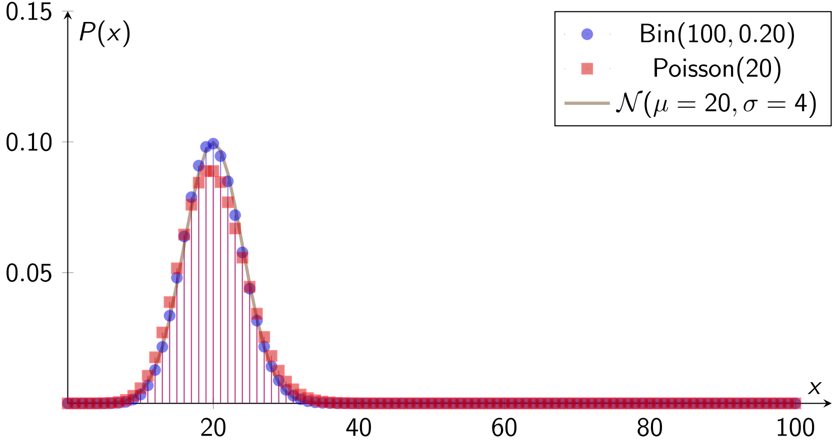

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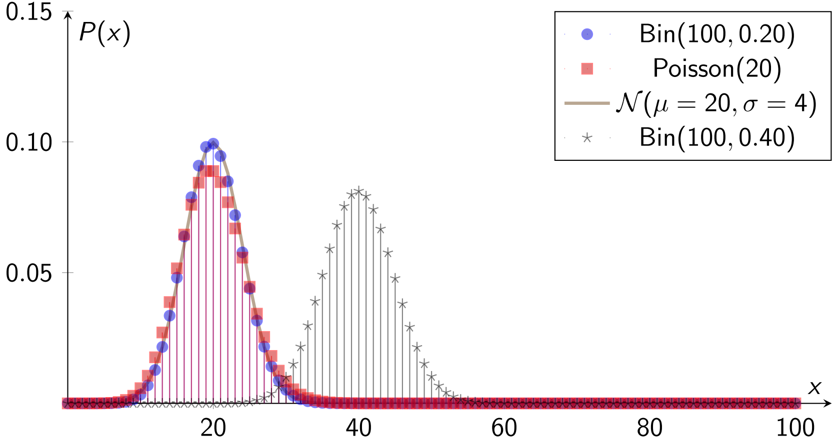

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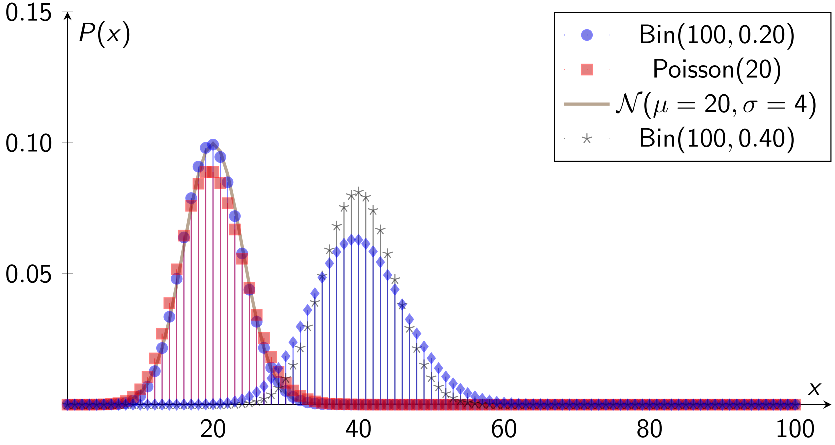

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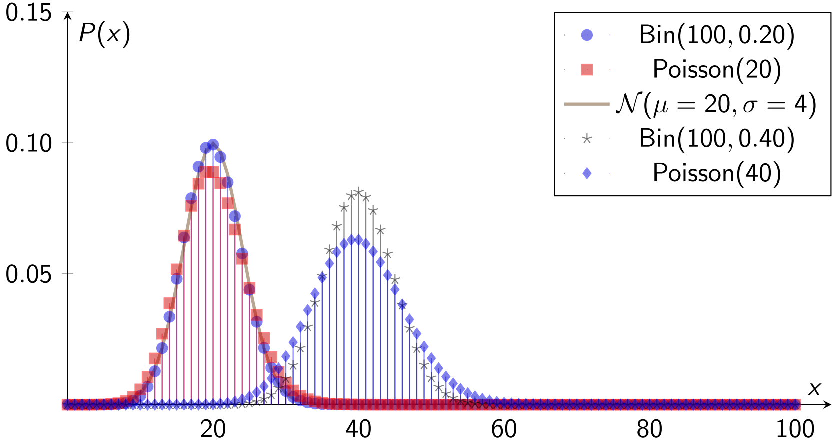

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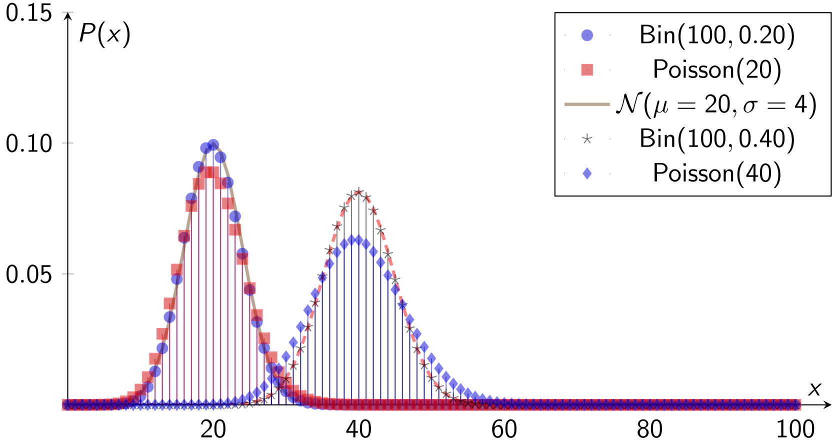

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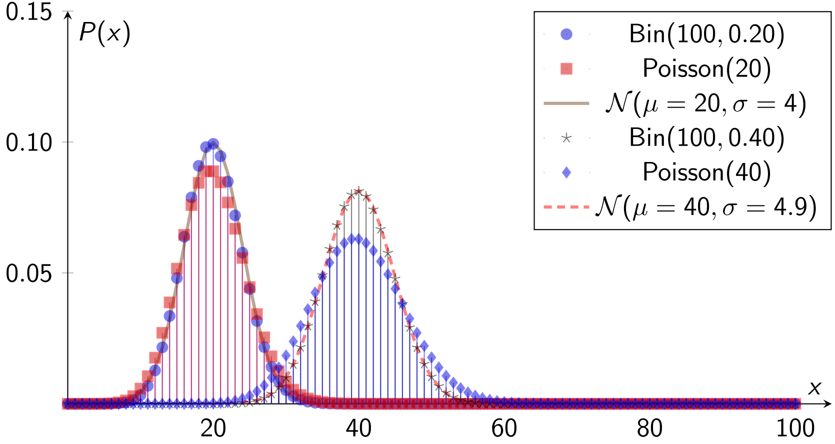

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- Such events are described as **Poisson processes**
- The **PMF** of a Poisson random variable with rate parameter λ is given by:

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda} \tag{10}$$

  where $x \geq 0$
- The **mean** and **variance** of a Poisson random variable are equal:

$$\mathbb{E}(X) = \mathbb{V}(X) = \lambda \tag{11}$$

- Recall that the **standard deviation** is simply the square root of the variance:

$$\sigma(X) = \sqrt{\lambda} \tag{12}$$

Reading

- Open Intro Statistics Section 4.5