

## 1 Set theory

Properties:

$$A \cup B = B \cup A \quad (1)$$

$$A \cap B = B \cap A \quad (2)$$

$$(A \cup B) \cup C = A \cup (B \cup C) \quad (3)$$

$$(AB)C = A(BC) \quad (4)$$

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C) \quad (5)$$

$$(AB) \cup C = (A \cup C) \cap (B \cup C) \quad (6)$$

De Morgan's rule:

$$\overline{E_1 \cup E_2 \cup \dots \cup E_n} = \overline{E_1} \cap \overline{E_2} \cap \dots \cap \overline{E_n} \quad (7)$$

$$\overline{E_1 \cap E_2 \cap \dots \cap E_n} = \overline{E_1} \cup \overline{E_2} \cup \dots \cup \overline{E_n} \quad (8)$$

## 2 Mathematics of probability

General rules:

$$P(\overline{E}) = 1 - P(E) \quad (9)$$

$$P(A \cup B) = P(A) + P(B) - P(AB) \quad (10)$$

$$P(A \cap B) = P(AB) = P(A|B)P(B) \quad (11)$$

$$P(A|B) = \frac{P(AB)}{P(B)} \quad (12)$$

$$P(A|B) = 1 - P(\overline{A}|B) \quad (13)$$

Mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) \quad (14)$$

Statistically independent events:

$$P(AB) = P(A)P(B) \quad (15)$$

$$P(A|B) = P(A) \quad (16)$$

$$P(B|A) = P(B) \quad (17)$$

Total probability theorem:

$$P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \dots + P(A|E_n)P(E_n) = \sum_{i=1}^n P(A|E_i)P(E_i) \quad (18)$$

Bayes' theorem:

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{j=1}^n P(A|E_j)P(E_j)} \quad (19)$$

### 3 Counting

Number of ways to arrange  $n$  objects:

$$n! = n(n-1)(n-2) \cdots 3 \times 2 \times 1 \quad (20)$$

Permutations (arrangements) of  $n$  objects taken  $k$  at a time:

$${}_nP_k = \frac{n!}{(n-k)!} \quad (21)$$

Combinations of  $n$  objects taken  $k$  at a time:

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (22)$$

### 4 Probability distributions

Distribution	Mean	Variance	Median	$P(X \leq x)$
Uniform( $a, b$ )	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{a+b}{2}$	<code>uniform.cdf(x, a, b)</code>
$\mathcal{N}(\mu, \sigma)$	$\mu$	$\sigma^2$	$\mu$	<code>norm.cdf(x, mu, sigma)</code>
Lognormal( $\mu, \sigma$ )	$[\exp(\mu + \frac{1}{2}\sigma^2)] \exp(2\mu + \sigma^2)$	$\exp(\sigma^2) - 1$	$\exp(\mu)$	<code>lognorm.cdf(x, sigma, np.exp(mu))</code>
Binomial( $n, p$ )	$np$	$np(1-p)$	$\lfloor (n+1)p \rfloor$	<code>binom.cdf(x, n, p)</code>
Poisson( $\lambda$ )	$\lambda$	$\lambda$	$\lfloor \lambda \rfloor$	<code>poisson.cdf(x, lambda)</code>
Exponential( $\lambda$ )	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\ln 2}{\lambda}$	<code>expon.cdf(x, 1/lambda)</code>

Table 1: Common probability distributions.

Note: the  $\lfloor \cdot \rfloor$  symbol means to round down to the nearest integer.