

CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture 6A: Inference for One Sample Means

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Outline

- 1 Introduction
- 2 Confidence intervals
- 3 Confidence bounds
- 4 Sample size
- 5 Hypothesis testing
- 6 p -values
- 7 Outlook

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- Compute sample size to required confidence level

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For sample means, we add another requirement:

- If the population variance σ is **known**, then we can assume a normal distribution
- If the population variance is **unknown** and can only be estimated from a sample as s , then we use the **Student's t distribution**

Normal and t -distributions

Normal and t -distributions

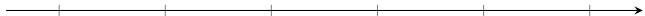
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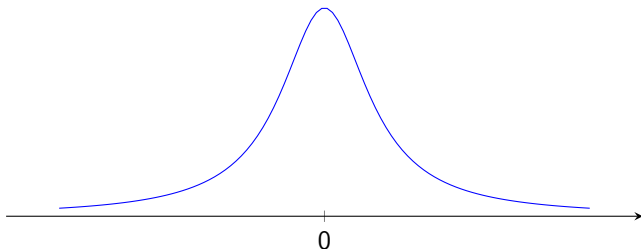
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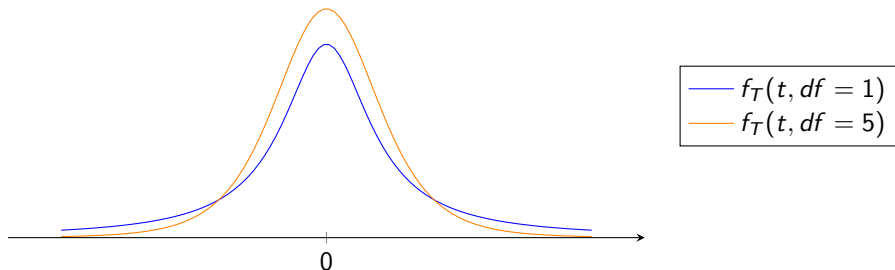
$$f_T(t, df = 1)$$

Note

In fact, as $df \rightarrow \infty$, the t -distribution converges to the normal distribution.

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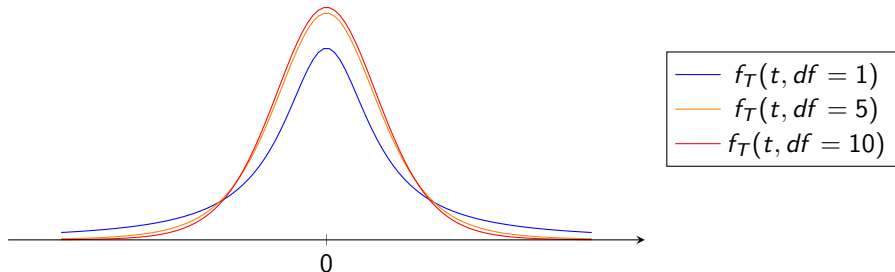


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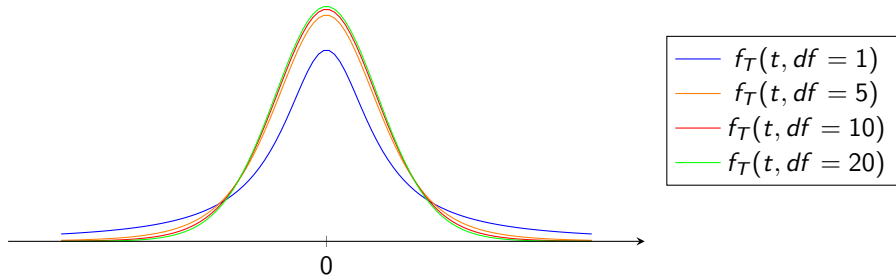


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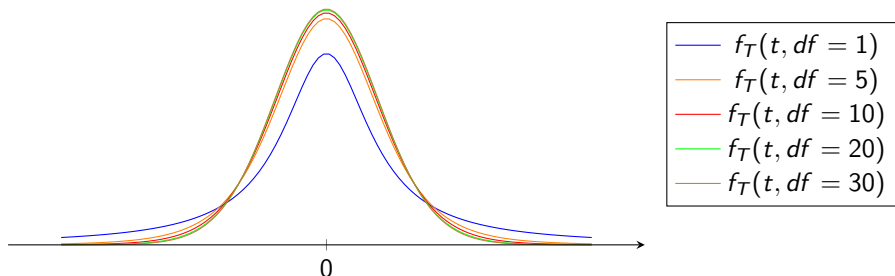


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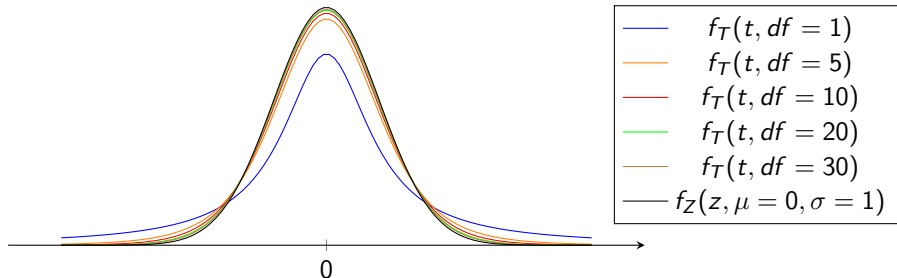


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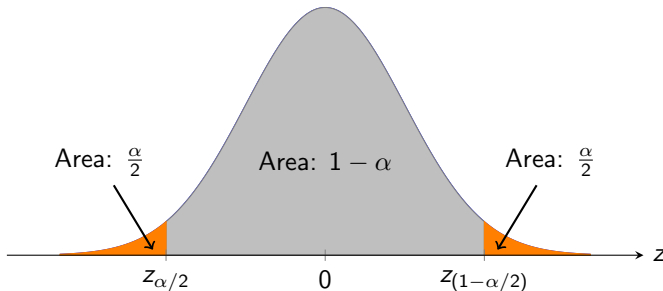


Figure: Standard normal distribution of the mean

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To use the t -distribution functions in Python, use `from scipy.stats import t`

Working with confidence intervals

Example 1: Identifying confidence levels

Given a normal population distribution with known variance:

- (a) What is the confidence level for the interval $\bar{x} \pm 2.81\sigma/\sqrt{n}$?
- (b) What is the confidence level for the interval $\bar{x} \pm 1.44\sigma/\sqrt{n}$?
- (c) What value of $z_{\alpha/2}$ results in a confidence level of 90%?

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$$z_{(1-\alpha/2)} = +2.81$$

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The confidence level is $= 1 - \alpha = \boxed{99.5\%}$.

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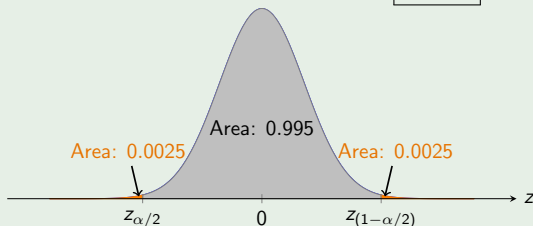
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The confidence level is $= 1 - \alpha = \boxed{85\%}$.

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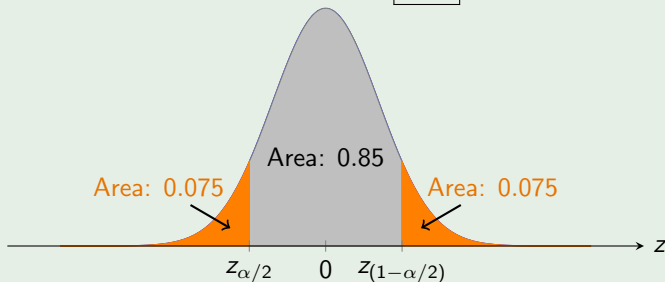
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$$\alpha = 15\%$$

The confidence level is $= 1 - \alpha =$ 85%.



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$$\begin{aligned} z_{\alpha/2} &= \Phi^{-1}(0.05) \\ &= -\Phi^{-1}(0.95) \end{aligned}$$

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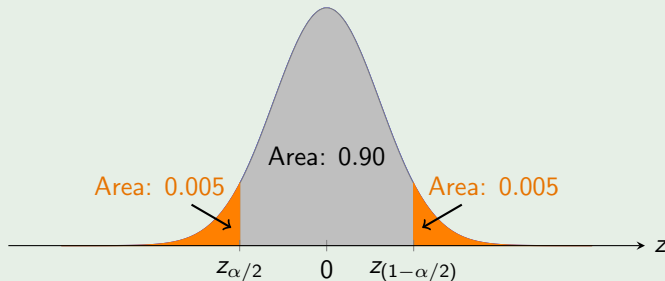
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A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist. The resulting sample average preferred height was $\bar{x} = 80.0$ cm. Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm (a value suggested by data in the article),

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A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist. The resulting sample average preferred height was $\bar{x} = 80.0$ cm. Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm (a value suggested by data in the article), obtain a 95% CI for μ , the true average preferred height for the population of all experienced typists.

Confidence interval (normal distribution, cont.)

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Thus, we have:

$$P\left(-1.96 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 1.96\right) = 0.95$$

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Confidence interval (normal distribution, cont.)

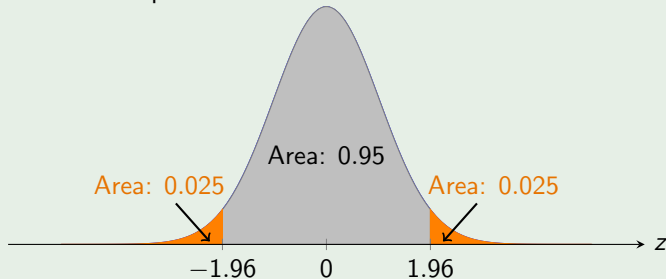
Example 1: Keyboard height (cont.)

We can also plot the standard normal distribution as a visual aid:

Confidence interval (normal distribution, cont.)

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Rearranging the inequality:

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 1.96 \right)$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

Rearranging the inequality:

$$\left(-1.96 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 1.96 \right)$$

We first multiply all terms in the inequality by σ/\sqrt{n} :

$$-1.96 \frac{\sigma}{\sqrt{n}} < \bar{x} - \mu < 1.96 \frac{\sigma}{\sqrt{n}}$$

Confidence interval (normal distribution, cont.)

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$$-\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < -\mu < -\bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

Then we multiply by -1 :

$$\bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} > \mu > \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}$$

Confidence interval (normal distribution, cont.)

Example 1: Keyboard height (cont.)

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The endpoints of the resulting inequality form the **confidence interval** for μ :

Confidence interval (normal distribution, cont.)

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which corresponds to Equation (25) for a confidence level of 95%.

Confidence interval (normal distribution, cont.)

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The endpoints of the resulting inequality form the **confidence interval** for μ :

$$(\bar{x} - 1.96SE, \bar{x} + 1.96SE)$$

which corresponds to Equation (25) for a confidence level of 95%.

Now, plugging in the numbers we have, we find:

$$\begin{aligned}\bar{x} \pm 1.96SE &= 80.0 \pm (1.96) \frac{2.0}{\sqrt{31}} \\ &= 80.0 \pm 0.7 \\ &= (79.3, 80.7)\end{aligned}$$

One-sided confidence intervals (confidence bounds)

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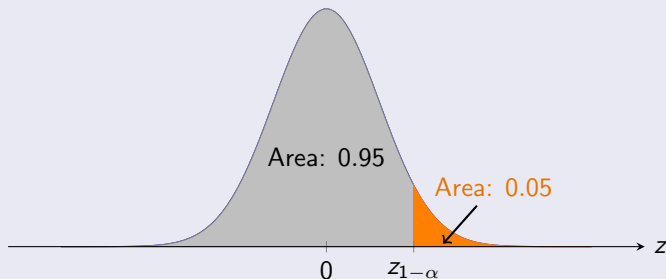
Upper confidence bound

One-sided confidence intervals (confidence bounds)

Upper confidence bound

$$\mu < \bar{x} + z_{(1-\alpha)} \frac{\sigma}{\sqrt{n}} \quad (\text{known variance}) \quad (6)$$

$$\mu < \bar{x} + t_{(1-\alpha)} \frac{s}{\sqrt{n}} \quad (\text{unknown variance; } n - 1 \text{ df}) \quad (7)$$



One-sided confidence intervals (confidence bounds)

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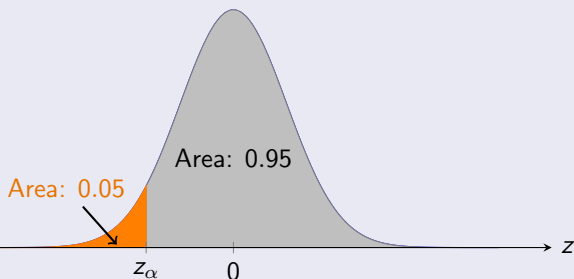
Lower confidence bound

One-sided confidence intervals (confidence bounds)

Lower confidence bound

$$\mu > \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} \quad (\text{known variance}) \quad (8)$$

$$\mu > \bar{x} + t_{\alpha} \frac{s}{\sqrt{n}} \quad (\text{unknown variance; } n - 1 \text{ df}) \quad (9)$$



Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels

Determine the confidence levels for each of the following one-sided confidence bounds:

- (a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$
- (b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$
- (c) Upper bound: $\bar{x} + 2.2s/\sqrt{n}$, ($n = 12$)

Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

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(a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$

Confidence bounds in practice

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$$z_{(1-\alpha)} = 0.84$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$

$$z_{(1-\alpha)} = 0.84$$

$$1 - \alpha = \Phi(0.84)$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$

$$z_{(1-\alpha)} = 0.84$$

$$1 - \alpha = \Phi(0.84) \approx 0.80$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$

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Confidence level:

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(a) Upper bound: $\bar{x} + 0.84\sigma/\sqrt{n}$

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$$1 - \alpha = \Phi(0.84) \approx 0.80$$

Confidence level: 80%.

Confidence bounds in practice

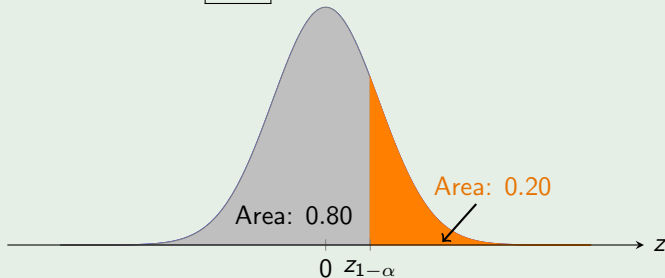
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Confidence bounds in practice

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Example 3: Identifying one-sided confidence levels (cont.)

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(b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$

Confidence bounds in practice

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$$z_{\alpha} = -2.05$$

Confidence bounds in practice

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Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$

$$z_{\alpha} = -2.05$$

$$\alpha = \Phi(-2.05) = -\Phi(2.05) \approx 0.98$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$

$$z_{\alpha} = -2.05$$

$$\alpha = \Phi(-2.05) = -\Phi(2.05) \approx 0.98$$

Confidence level:

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$

$$z_{\alpha} = -2.05$$

$$\alpha = \Phi(-2.05) = -\Phi(2.05) \approx 0.98$$

Confidence level: 98%.

Confidence bounds in practice

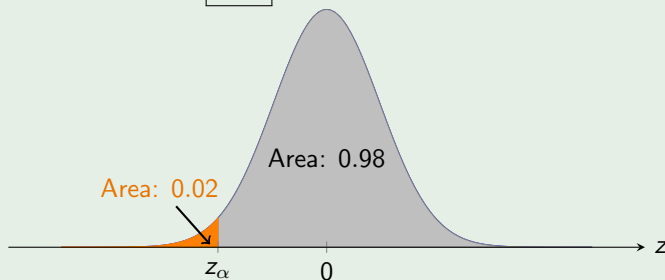
Example 3: Identifying one-sided confidence levels (cont.)

(b) Lower bound: $\bar{x} - 2.05\sigma/\sqrt{n}$

$$z_{\alpha} = -2.05$$

$$\alpha = \Phi(-2.05) = -\Phi(2.05) \approx 0.02$$

Confidence level: 98%.



Confidence bounds in practice

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

Confidence bounds in practice

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(c) Upper bound: $\bar{x} + 2.2s/\sqrt{n}$, ($n = 12$)

Confidence bounds in practice

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$$1 - \alpha = F_{T,df}(2.2)$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

(c) Upper bound: $\bar{x} + 2.2s/\sqrt{n}$, ($n = 12$)

$$t_{(1-\alpha)} = 2.2$$

$$1 - \alpha = F_{T,df}(2.2) \quad df = 11$$

Confidence bounds in practice

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(c) Upper bound: $\bar{x} + 2.2s/\sqrt{n}$, ($n = 12$)

$$t_{(1-\alpha)} = 2.2$$

$$1 - \alpha = F_{T,df}(2.2) \quad df = 11$$

$$1 - \alpha = 0.975$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

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$$1 - \alpha = 0.975 \quad \text{t.cdf}(2.2, 11)$$

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

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Confidence level:

Confidence bounds in practice

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Confidence level: 97.5%

Confidence bounds in practice

Example 3: Identifying one-sided confidence levels (cont.)

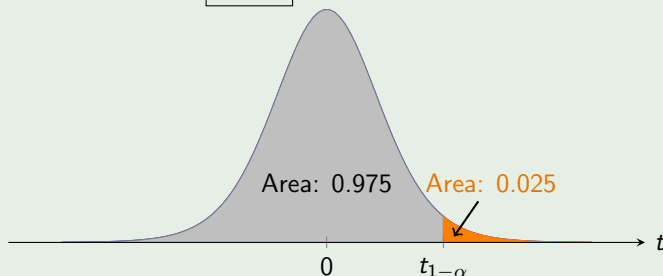
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Confidence level: 97.5%



Confidence bound, t distribution

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Example 4: Shear strength

In a certain investigation, a sample of 46 shear strength observations gave a sample mean strength of 17.17 N/mm^2 and a **sample standard deviation** of 3.28 N/mm^2 . Find the lower confidence bound for the true average shear strength μ with confidence level 95%.

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$$\bar{x} + t_{\alpha} \frac{s}{\sqrt{n}} = 17.17 - 1.6794 \frac{3.28}{\sqrt{46}}$$

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$$\begin{aligned}\bar{x} + t_{\alpha} \frac{s}{\sqrt{n}} &= 17.17 - 1.6794 \frac{3.28}{\sqrt{46}} \\ &= 17.17 - 0.7951\end{aligned}$$

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 &\approx \boxed{16.38}
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In other words, with a CI of 95%, μ lies in the random interval $(16.38, \infty)$.

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The greater the sample size, the lower the standard error

Choice of sample size (cont.)

Example 5: Response time of operating system

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Choice of sample size (cont.)

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Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with SD 25 millisc. A new operating system has been installed and we wish to estimate the true average response time μ for the new environment. Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% CI has a width of (at most) 10?

Choice of sample size (cont.)

Example 5: Response time of operating system (cont.)

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Choice of sample size (cont.)

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Here, $h = 10/2 = 5 = ME$. Thus,

$$\begin{aligned} n &= \left(z_{(\alpha/2)} \frac{\sigma}{h} \right)^2 \\ &= \left(z_{0.025} \times \frac{25}{5} \right)^2 \end{aligned}$$

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Since n must be an integer, a sample size of 97 is required.

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- ⑥ *Decide*. If the test statistic is in the critical region, reject H_0 . If not, do not reject H_0 (fail to reject it)

One-sided tests

One-sided tests

Case A: upper tail

One-sided tests

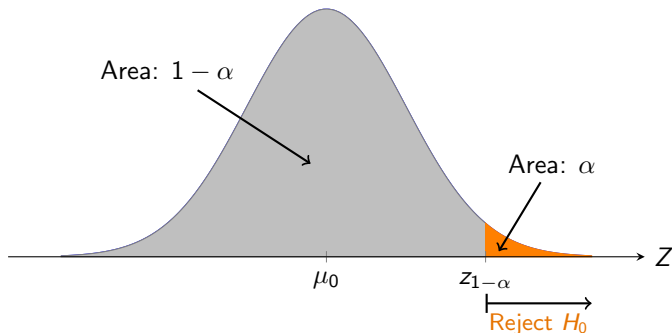
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One-sided tests (cont.)

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Case B: lower tail

One-sided tests (cont.)

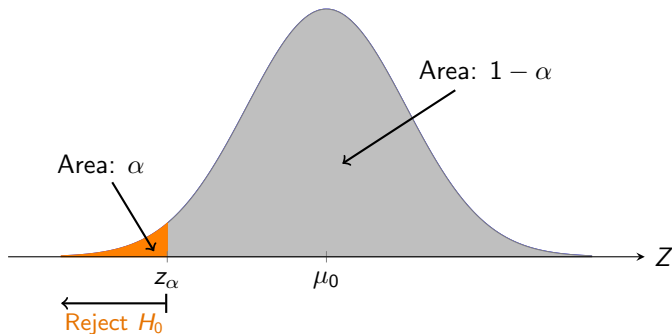
Case B: lower tail

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One-sided tests (cont.)

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Two-sided tests

Two-sided tests

Case C: both tails

Two-sided tests

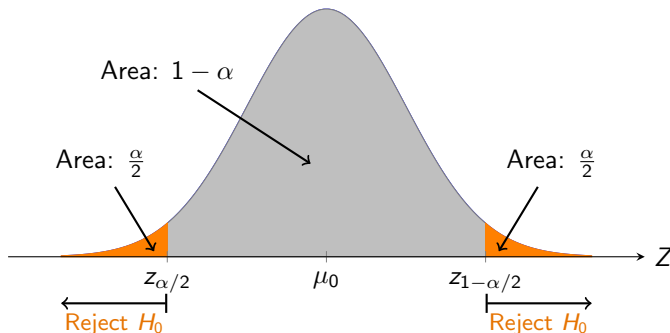
Case C: both tails

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- $H_1 : \mu \neq \mu_0$

Two-sided tests

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The sample mean is **normally** distributed and its variance is :

$$\text{Var}(\bar{X}) = \frac{\sigma^2}{n} \quad (11)$$

And thus, the standard deviation is $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$.

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And thus, the standard deviation is $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$.

Thus, to compute the probability (area under curve) of the test statistic, we use the standardized variable (Z-statistic or Z-score):

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \quad (12)$$

which is **normally** distributed.

Distribution of the test statistic (cont.)

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Case 2: Sample mean with unknown population variance

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The estimated sample mean in this case has a Student's *t-distribution* with $n - 1$ degrees of freedom (df). Thus, its variance is:

$$\text{Var}(\bar{X}) = \frac{s^2}{n} \quad (13)$$

And thus, the standard error is $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$.

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The estimated sample mean in this case has a Student's *t*-distribution with $n - 1$ degrees of freedom (*df*). Thus, its variance is:

$$\text{Var}(\bar{X}) = \frac{s^2}{n} \quad (13)$$

And thus, the standard error is $SE_{\bar{X}} = \frac{s}{\sqrt{n}}$.

Thus, to compute the probability (area under curve) of the test statistic, we use the standardized variable (*T*-statistic or *T*-score):

$$t = \frac{\bar{X} - \mu}{SE_{\bar{X}}} \quad (14)$$

Distribution of the test statistic (cont.)

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Hypothesis testing with the p -value

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Step 1. Formulate your hypotheses

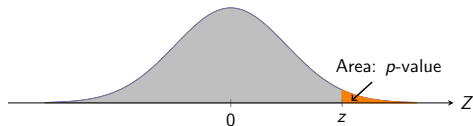
Step 2. Determine the p -value from the test statistic

Step 3. Conclude the test based on a chosen level of significance:

- ① $p\text{-value} \leq \alpha \implies$ reject H_0 at level α .
- ② $p\text{-value} > \alpha \implies$ do not reject H_0 at level α .

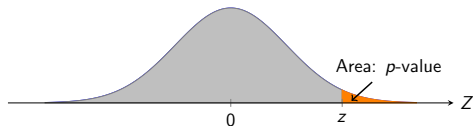
p-value for *z* tests

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p-value: area upper tail

p-value for z tests



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$$p = 1 - \Phi(z) \quad (17)$$

`norm.sf(z)`

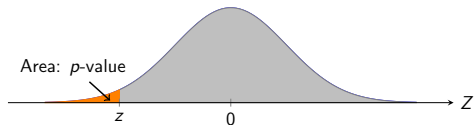
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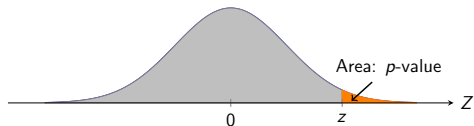
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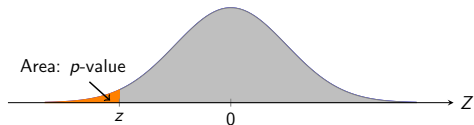
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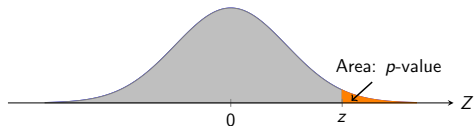
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$$p = \Phi(z) \quad (18)$$

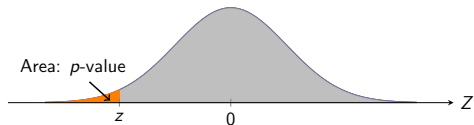
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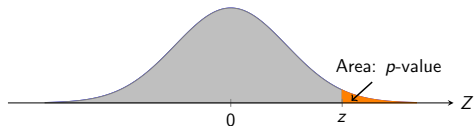


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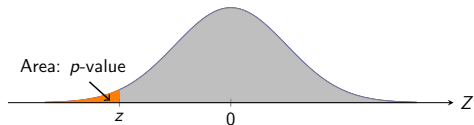
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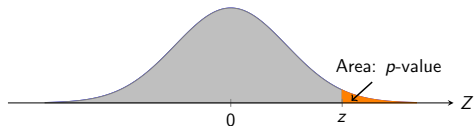
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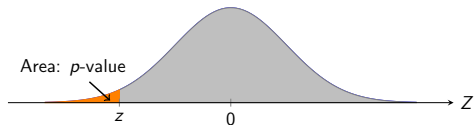
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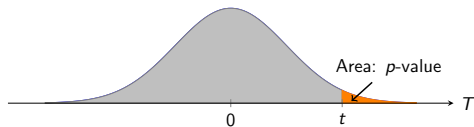
p-value: area both tails

$$p = 2(1 - \Phi(|z|)) \quad (19)$$

`2*norm.cdf(z)`

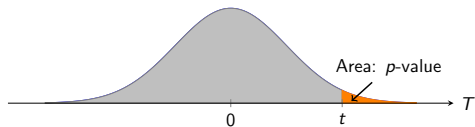
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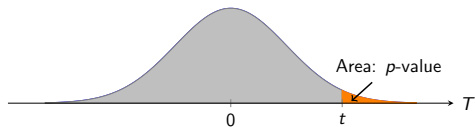


p-value: area upper tail

$$p = 1 - F_{T, n-1}(t) \quad (20)$$

`t.sf(t, n-1)`

p-value for z tests



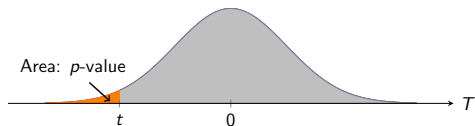
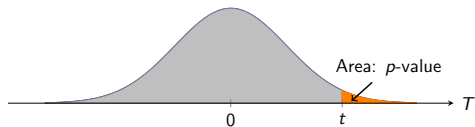
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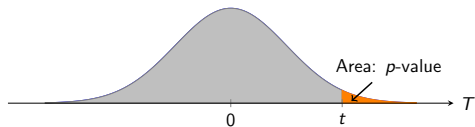
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p-value: area lower tail

$$p = F_{T,n-1}(t) \quad (21)$$

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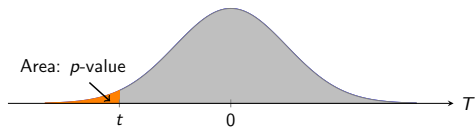
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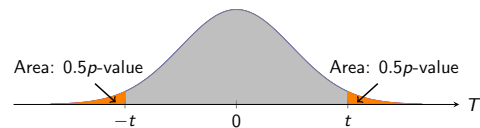
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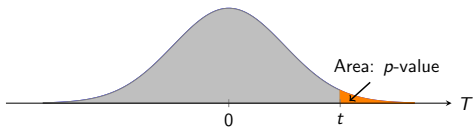
$$p = F_{T,n-1}(t) \quad (21)$$

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p-value: area both tails

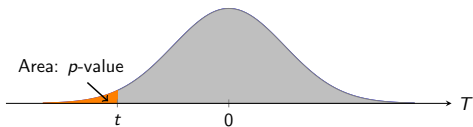
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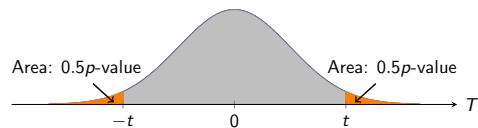
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$$p = F_{T,n-1}(t) \quad (21)$$

`t.cdf(t, n-1)`



p-value: area both tails

$$p = 2(1 - F_{T,n-1}(|t|)) \quad (22)$$

`2*t.cdf(t, n-1)`

Two-tailed test (known variance)

Example 6: Silicon wafer thickness

The target thickness for silicon wafers used in a certain type of integrated circuit is $245 \mu\text{m}$. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean of thickness $246.18 \mu\text{m}$. The population standard deviation of $3.60 \mu\text{m}$. Does this data suggest that true average wafer thickness is something other than the target value ($\alpha = 0.01$)?

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Step 3. Alternative hypothesis: $H_1 : \mu \neq 245$.

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Step 4. Formula for test statistic value:

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Step 2. Null hypothesis: $H_0 : \mu = 245$.

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Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

Hypothesis testing using p -value approach

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Step 5. Calculate test statistic value:

Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}}$$

Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Hypothesis testing using p -value approach

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Step 6. Determine p -value

Hypothesis testing using p -value approach

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Step 6. Determine p -value (two-tailed test):

Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p -value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32))$$

Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

Step 5. Calculate test statistic value:

$$z = \frac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$$

Step 6. Determine p -value (two-tailed test):

$$p\text{-value} = 2(1 - \Phi(2.32)) = 0.0204$$

(In Python: `2*norm.sf(2.32)`)

Hypothesis testing using p -value approach

Example 6: Silicon wafer thickness (cont.)

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Step 7. Conclude:

Hypothesis testing using p -value approach

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(In Python: `2*norm.sf(2.32)`)

Step 7. Conclude:

Using a significance level of 0.01, we fail to reject H_0 since $0.0204 > 0.01$. Thus, at the 1% significance level, there is insufficient evidence to conclude that true average thickness differs from the target value.

Two-tailed tests: unknown variance

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Example 7: Golf ball production

A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down and repaired. This hour's sample of 18 balls has a mean of 1.611 oz and a standard deviation of 0.065 oz. Do you shut down the line?

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$$H_0 : \mu = 1.615$$

$$H_1 : \mu \neq 1.615$$

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Two-tailed tests: unknown variance

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Example 7: Golf ball production

Step 2. Compute T -statistic:

Two-tailed tests: unknown variance

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Step 2. Compute T -statistic:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Two-tailed tests: unknown variance

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Step 2. Compute T -statistic:

$$\begin{aligned} t &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\ &= \frac{1.611 - 1.615}{0.065/\sqrt{18}} = \end{aligned}$$

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 2. Compute T -statistic:

$$\begin{aligned}
 t &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\
 &= \frac{1.611 - 1.615}{0.065/\sqrt{18}} = -0.261
 \end{aligned}$$

Two-tailed tests: unknown variance

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Step 3. $\alpha = 1\% = 0.01$.

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Step 3. $\alpha = 1\% = 0.01$.

Given that this is a two-tailed test, we have two critical regions with areas: $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$.

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Given that this is a two-tailed test, we have two critical regions with areas: $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$.

The lower tail is bounded by $t_{0.005}$ and the upper tail by $t_{1-0.005} = t_{0.995}$.

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Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$).

Two-tailed tests: unknown variance

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(In Python: `t.ppf(0.005,17)` and `t.ppf(0.995,17)`)

Two-tailed tests: unknown variance

Example 7: Golf ball production

Step 4. The critical values are $t_{0.005} = -2.8982$ and $t_{0.95} = 2.8982$ ($df = 17$).
(In Python: `t.ppf(0.005, 17)` and `t.ppf(0.995, 17)`)

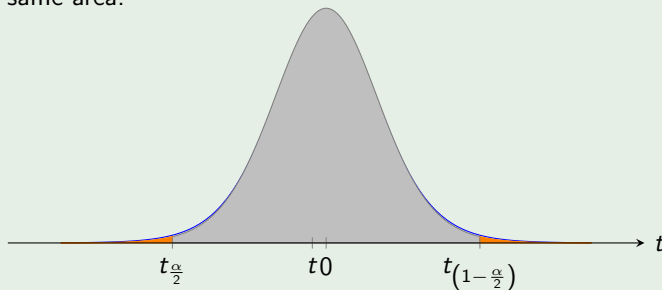
Note that in two-sided tests, the critical regions on either side have the same area.

Two-tailed tests: unknown variance

Example 7: Golf ball production

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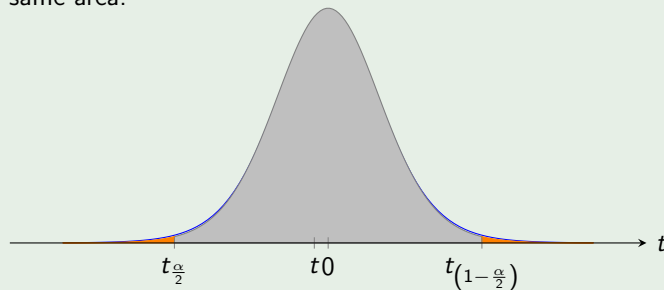
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Note that in two-sided tests, the critical regions on either side have the same area.



Step 5. We that the test statistic is within the region of nonrejection:

$$t_{\frac{\alpha}{2}} = -2.8982 < t = -0.261 < t_{(1-\frac{\alpha}{2})} = 2.8982$$

Example 7: Golf ball production

Step 6. Thus, we **fail to reject** the null hypothesis.

Example 7: Golf ball production

Step 6. Thus, we **fail to reject** the null hypothesis.

In real terms, this means that the sample was within the bounds of what would be acceptable if the population mean were 1.615 oz. Therefore, we would not stop the production line.

One-sided test: known variance

Example 8: Light bulbs

One-sided test: known variance

Example 8: Light bulbs

A quality control (QC) engineer finds that a sample of 100 light bulbs had an average lifetime of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hrs, test the null hypothesis that the population mean is 480 hrs against the alternative hypothesis it is less than 480 hrs at a significance level of $\alpha = 0.05$.

One-sided test: known variance

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Step 1. Formulate the hypotheses:

$$H_0 : \mu = 480$$

One-sided test: known variance

Example 8: Light bulbs

A quality control (QC) engineer finds that a sample of 100 light bulbs had an average lifetime of 470 hours. Assuming a population standard deviation of $\sigma = 25$ hrs, test the null hypothesis that the population mean is 480 hrs against the alternative hypothesis it is less than 480 hrs at a significance level of $\alpha = 0.05$.

Step 1. Formulate the hypotheses:

$$H_0 : \mu = 480$$

$$H_1 : \mu < 480$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}}$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z-statistic:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}} = -4.0$$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 2. The population variance is known, so we use the Z -statistic:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}} = -4.0$$

Recall that the Z -statistic is normally distributed: $\mathcal{N}(0, 1)$.

One-sided test: known variance

Example 8: Light bulbs (cont.)

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

One-sided test: known variance

Example 8: Light bulbs (cont.)

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Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
 $z_{\alpha} = \Phi^{-1}(0.05)$

One-sided test: known variance

Example 8: Light bulbs (cont.)

Step 3. The level of significance, $\alpha = 0.05$.

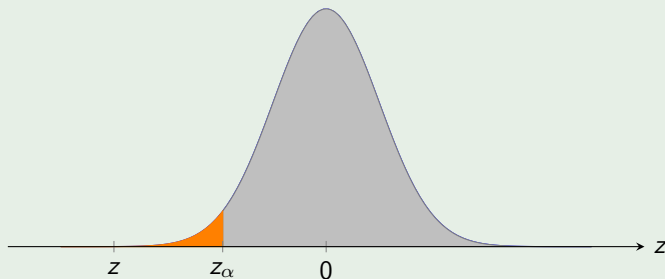
Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by
 $z_{\alpha} = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)

One-sided test: known variance

Example 8: Light bulbs (cont.)

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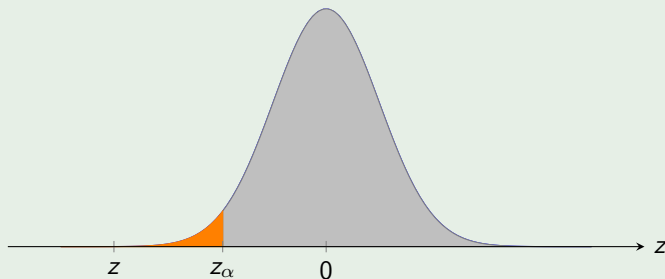


One-sided test: known variance

Example 8: Light bulbs (cont.)

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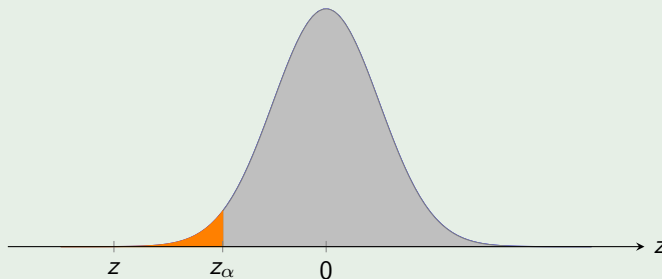
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Example 8: Light bulbs (cont.)

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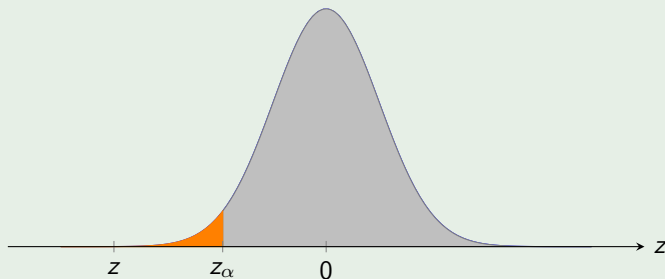
Step 5. We see that $z < z_\alpha$, i.e. z lies inside the region of rejection.

One-sided test: known variance

Example 8: Light bulbs (cont.)

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Step 4. This is a lower-tailed test and the critical region is defined by the area under the normal curve, bounded by $z_\alpha = \Phi^{-1}(0.05) = -\Phi^{-1}(0.95) = -1.645$ (Python: `norm.ppf(0.05)`)



Step 5. We see that $z < z_\alpha$, i.e. z lies inside the region of rejection. Thus, we **reject the null hypothesis**.

One-sided test: unknown variance

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Example 9: Vacuum cleaner

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A vacuum cleaner is claimed to expend 46 kWh per year.

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A vacuum cleaner is claimed to expend 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with sample SD $s = 11.9$ kWh. At a 0.05 level of significance, does this suggest that on average, vacuum cleaners expend less than 46 kWh per year? Assume the population is normally distributed.

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Step 1. Formulate hypotheses:

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Step 1. Formulate hypotheses:

$$H_0 : \mu = 46$$

One-sided test: unknown variance

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Step 1. Formulate hypotheses:

$$H_0 : \mu = 46$$

$$H_1 : \mu < 46$$

One-sided test: unknown variance

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Example 9: Vacuum cleaner (cont.)

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T -statistic:

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$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

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Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T -statistic:

$$\begin{aligned} t &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\ &= \frac{42 - 46}{11.9/\sqrt{12}} = -1.16 \end{aligned}$$

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 2. The population variance is unknown, so we compute the T -statistic:

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Step 3. At $\alpha = 0.05$, the critical value^a is:

$$t_{\alpha, df} = F_T^{-1}(0.05);$$

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

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Step 3. At $\alpha = 0.05$, the critical value^a is:

$$\begin{aligned} t_{\alpha, df} &= F_T^{-1}(0.05); \quad df = 12 - 1 = 11 \\ &= -F_T^{-1}(1 - 0.05) \quad (\text{standardized CDF symmetric about 0}) \\ &= -F_T^{-1}(0.95) \\ &= -1.7959 \end{aligned}$$

^aAlternately, `from scipy.stats import t` followed by `t.ppf(0.05,11)` will give the answer in Python

One-sided test: unknown variance

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Example 9: Vacuum cleaner (cont.)

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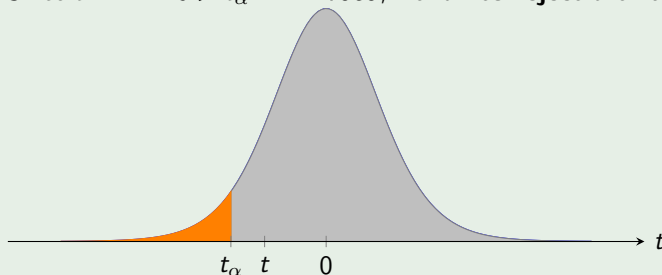
Example 9: Vacuum cleaner (cont.)

Step 5. Since $t = -1.16 > t_{\alpha} = -1.7959$, we **fail to reject** the null hypothesis.

One-sided test: unknown variance

Example 9: Vacuum cleaner (cont.)

Step 5. Since $t = -1.16 > t_\alpha = -1.7959$, we **fail to reject** the null hypothesis.



Thus, to answer the question, vacuum cleaners do not expend less than 46 kWh per year (with 95% confidence).

Standard error of the mean (SEM)

Standard error (deviation) of sample mean (with **known** population variance):

$$SE = \frac{\sigma}{\sqrt{n}} \quad (23)$$

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$$SE = \frac{\sigma}{\sqrt{n}} \quad (23)$$

Standard error (deviation) of sample mean (**unknown** population variance):

$$SE \approx \frac{s}{\sqrt{n}} \quad (24)$$

Equation (24) is also called the **standard error** of the mean

Confidence intervals: Recap

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Definition

A confidence interval defines the range within which a population parameter lies with a given probability.

Two-sided confidence intervals:

Known population variance

$$\langle \mu \rangle_{1-\alpha} = \left(\bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \bar{x} + z_{(1-\frac{\alpha}{2})} \frac{\sigma}{\sqrt{n}} \right)$$

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Unknown population variance

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Unknown population variance

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Hypothesis testing: Recap

- Definition of hypothesis testing
 - Null hypothesis (default/expected outcome)
 - Alternate hypothesis (what we want to test/support; research hypothesis)
 - One-tailed or two-tailed
- Types of errors:
 - Type I: false positive
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- The p -value is the minimum probability of a Type I error.

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- The p -value is the minimum probability of a Type I error. For known variance (assume normal distribution):
 - Upper-tailed test: $p\text{-value} = 1 - \Phi(z)$;

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Hypothesis testing: Recap (cont.)

- *p*-values for unknown variance (assume *t* distribution):
 - Upper-tailed test: $p\text{-value} = 1 - F_{df}(t)$;

Hypothesis testing: Recap (cont.)

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 - Upper-tailed test: $p - \text{value} = 1 - F_{df}(t)$; Python: `t.sf(t)`

Hypothesis testing: Recap (cont.)

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