Problem Set 5

Prof. Oke

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *9.29.2025*

Due Tuesday, October 7, 2025 at 1:5900 PM as PDF uploaded on Canvas. *Use this document as your template.* ***Show as much work as possible in order to get FULL credit.*** *There are 4 problems with a total of 15 points available.* ***Important:*** *If you use Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables, note this in the respective solution, as well.*

Problem 1 *(4 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) ☐ If the natural logarithm of a random variable is normally distributed with parameters μ and σ, then the variable is lognormally distributed with the same parameters.

(ii) ☐ The mean of a lognormal distribution is equal to or greater than its median.

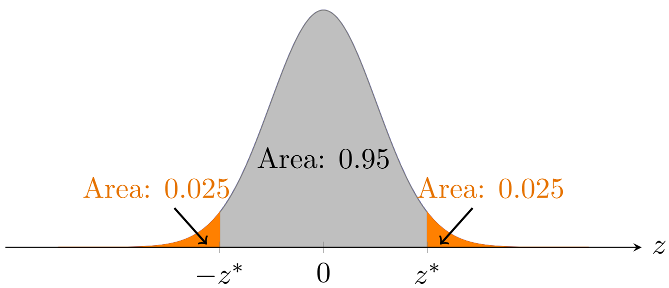
(iii) ☐ The mean of a exponentially-distributed random variable is equal to the inverse of its variance.

(iv) ☐ A binomial distribution with sufficiently large n can be approximated by a normal distribution with $\mu = np$.

Problem 2: Normal and Lognormal Distributions *(4 points)*

Choose the option that best fills in the blank.

[1] **(a)** The figure below depicts the PDF of a standard normal distribution. What is the value of z^* in the figure?



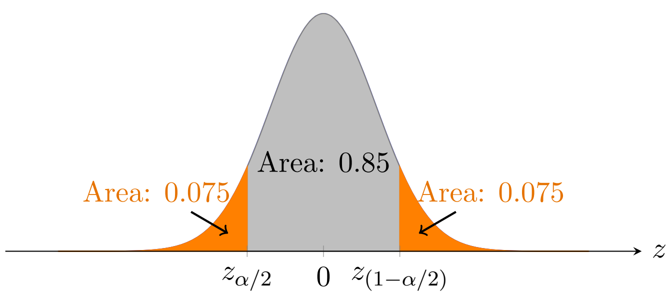
(i) 0

(ii) 1.65

(iii) 1.96

(iv) 2.58

[1] **(b)** The figure below depicts the PDF of a standard normal distribution. What is the value of $z_{\alpha/2}$ in the figure?



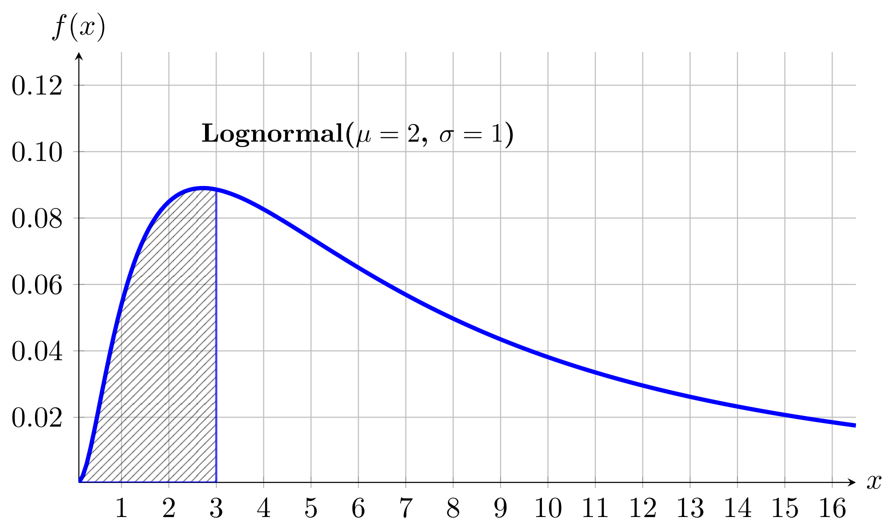
(i) 0

(ii) -1.04

(iii) -1.28

(iv) -1.44

[2] **(c)** Find the area of the shaded portion in the figure below.



Answer:

Problem 3: Lognormal Distribution *(5 points)*

Given that the lifetime in days of an electronic component is lognormally distributed with $\mu = 1.1$ and $\sigma = 0.5$.

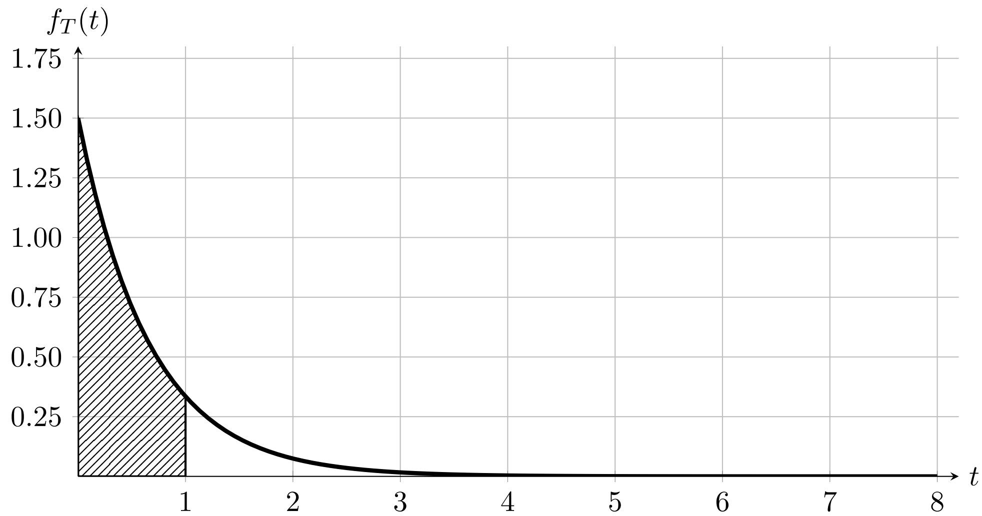
(a) Find the median lifetime of the component. *[1]*

(b) Find the mean lifetime of the component. *[2]*

(c) Find the probability that a component lasts between 3 and 5 days. *[3]*

Problem 4: Exponential Distribution I *(4 points)*

[1] **(a)** The graph below is the PDF of an exponentially distributed random variable T, given by $f_T(t) = \lambda e^{-\lambda t}$. What is the value of the parameter λ?



Answer:

[1] **(b)** What is the mean of T?

Answer:

[2] **(c)** What is the probability represented by the shaded area in the figure in part **(i)**? (A numeric value is expected here, not just a symbolic expression.)

Answer:

Problem 5: Exponential Distribution II *(4 points)*

The delay time T of a flight is exponentially distributed with $\lambda = 4$ (mean rate of occurrence per hour).

(a) What is the expectation of T? *[1]*

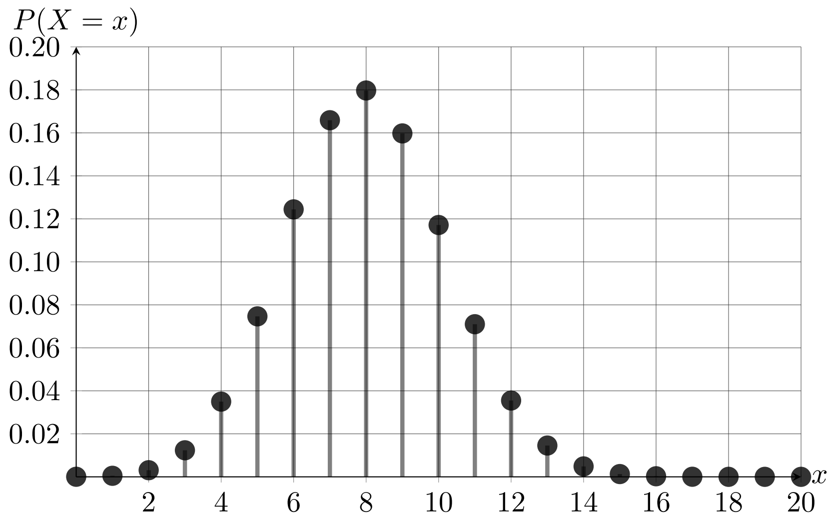
(b) What is the standard deviation of T? *[1]*

(c) What is the probability that a flight is delayed by at least half an hour? *[2]*

Problem 6: Binomial Distribution I *(6 points)*

Show brief amount of work for partial credit if answer is wrong. Not required however for full credit.

The PMF of a random variable X is given in the figure below.



[1] **(a)** Use the figure to estimate the probability $P(X = 8)$.

Answer:

[2] **(b)** Use the figure to estimate the probability $P(X = 8 \cup X = 10)$.

Answer:

[2] **(c)** Use the figure to estimate the probability $P(5 < X \leq 8)$.

Answer:

[1] **(d)** If the PMF in the figure above is that of a Binomial distribution with $p = 0.4$, what is $\mathbb{E}(X)$?

Answer:

Problem 7: Binomial Distribution II *(8 points)*

75% of all vehicles examined at an emissions inspection station pass. Successive vehicles pass or fail independently of one another. Let X be the number of vehicles that pass the inspection out of the next $n = 6$ vehicles inspected.

(a) What is the expectation of X, i.e. $\mathbb{E}[X]$? *[1]*

(b) What is the standard deviation of X? *[1]*

(c) Find the probability that all of the next six vehicles inspected pass, i.e. $P(X = 6)$. *[1]*

(d) Find the probability that only two of the next six vehicles inspected pass, i.e. $P(X = 2)$. *[2]*

(e) Find the probability that at least four of the next six vehicles inspected pass. *[3]*