CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture M3c: Lognormal and Exponential Distributions

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College of Engineering

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Outline

Recap of normal distribution

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- In MATLAB, the `normcdf(x, mu, sigma)` and `norminv(p, mu, sigma)` can be used to compute probabilities and inverse CDFs of the normal distribution, respectively.

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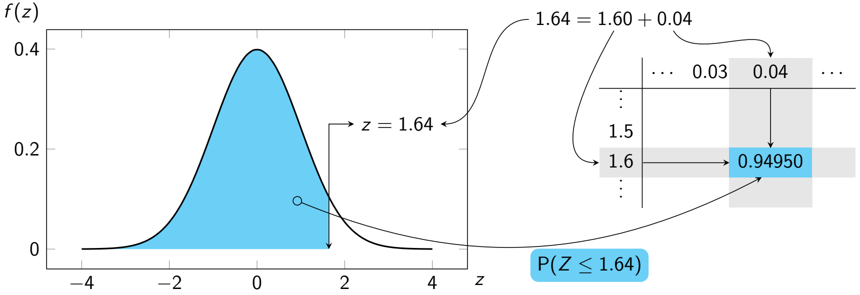

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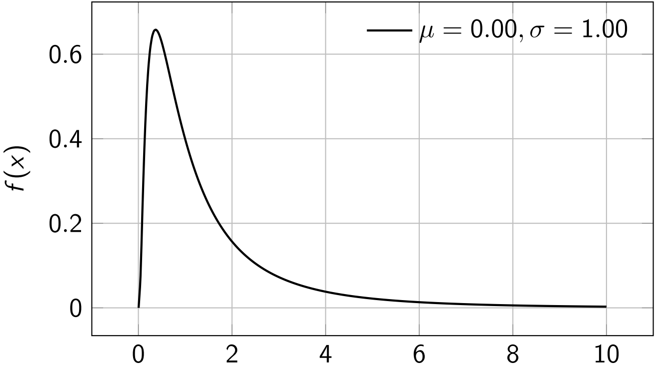

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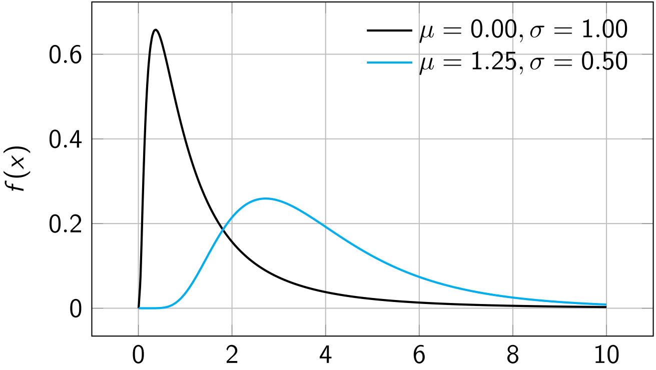

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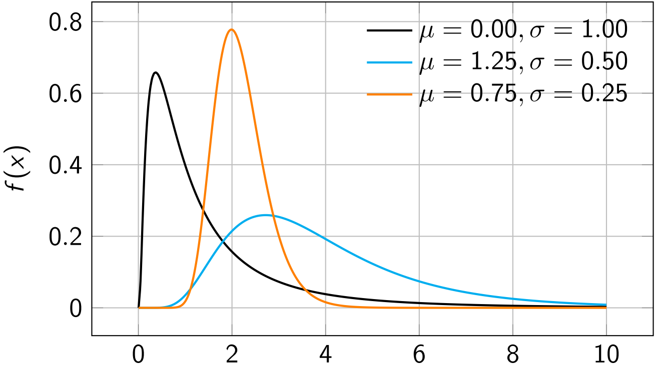

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$$\mathbb{V}(X) = (e^{\sigma^2} - 1)e^{(2\mu + \sigma^2)} \tag{6}$$

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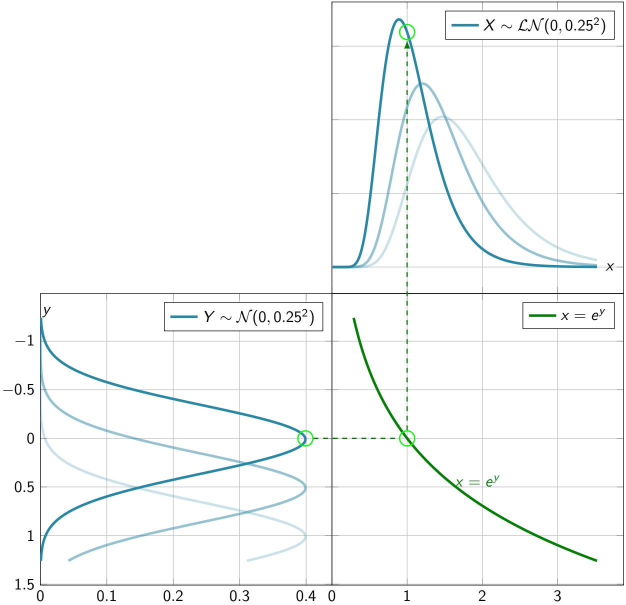

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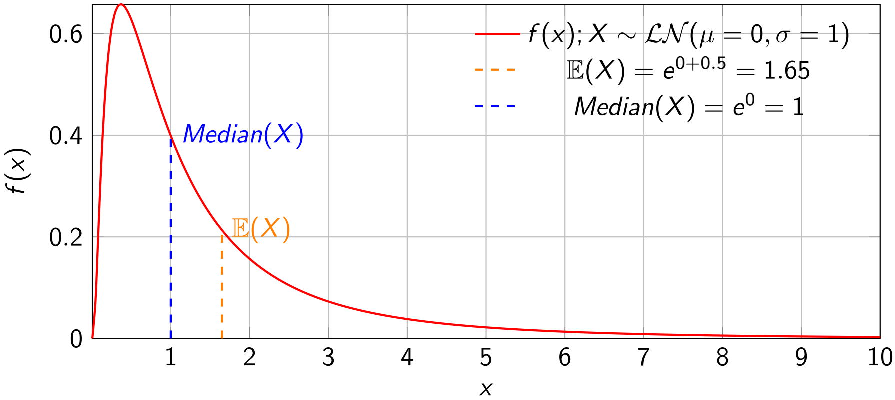

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Probability of a lognormal random variate

Given a r.v. X that is lognormally distributed with parameters μ and σ^2:

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Example 2: Equipment breakdown

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The lifetime X of a major oil platform equipment is lognormally distributed with a $Median(X) = 6$ months and $\sigma = 0.30$. To ensure 95% reliability, determine the desired interval x_0 for maintenance.

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Given: $\mu = \ln 6 = 1.792$ and $\sigma = 0.30$, we want to find x_0 such that:

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Example 2: Equipment breakdown (cont.)

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Solution

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Solution

$$\frac{\ln x_0 - 1.792}{0.30} = \Phi^{-1}(0.05)$$

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&= 1.792 - 0.495 =
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Therefore, the required inspection interval is:

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&= 1.792 - 0.495 = 1.297
\end{aligned}$$

Therefore, the required inspection interval is:

$$x_0 = e^{1.297} = 3.66 \text{ months}$$

Modeling probabilities of elapsed times

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Now consider the variable Y representing the **elapsed time** between successive arrivals.

- What is the probability the time between the third and fourth arrivals is less than y minutes, for instance?
- This is modeled by the **exponential distribution** with parameter λ.

Exponential distribution

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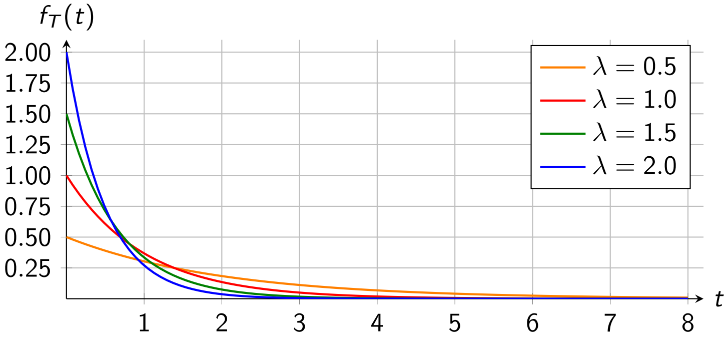

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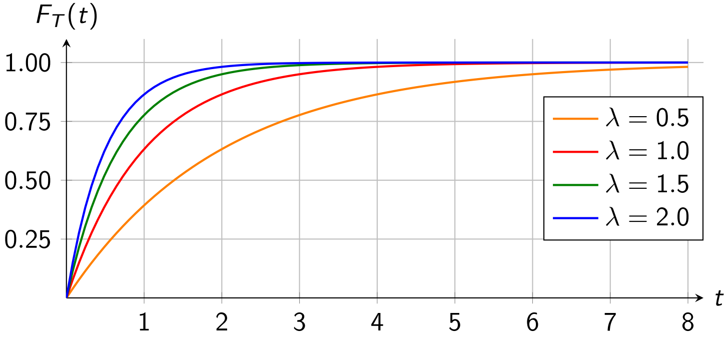

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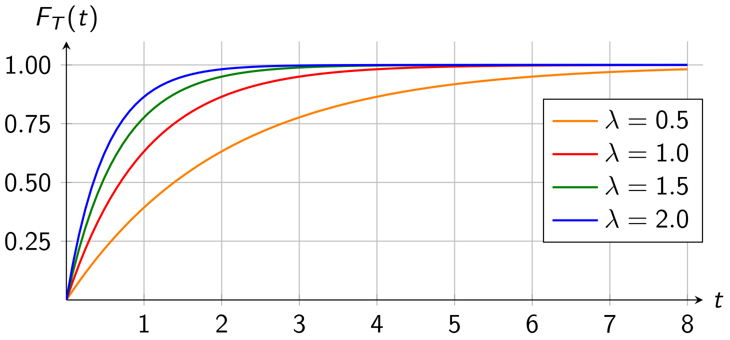

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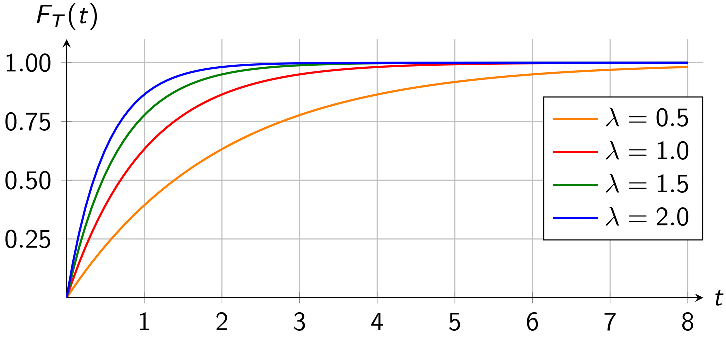

Note that $P(X \le x) = 1 - e^{-\lambda x}$, while $P(X > x) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x}$

Mean and variance of the exponential distribution

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Let $X \sim \text{Exponential}(\lambda)$.

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The mean of X is given by:

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Variance

The variance of X is given by:

$$\mathbb{V}(X) = \frac{1}{\lambda^2} \tag{14}$$

Example 3: Waiting for a flight

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The delay time T of a flight is exponentially distributed wtih $\lambda = 2$ (delays per hour).

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The delay time T of a flight is exponentially distributed wtih $\lambda = 2$ (delays per hour). Answer the following questions:

(a) What is the mean delay (waiting) time, $\mathbb{E}(T)$?

(b) What is the variance of the delay time $\mathbb{V}(T)$?

(c) Find the probability that a flight will be delayed by no more than 10 minutes.

(d) Assuming you have been waiting for a flight for an hour, what is the probability that the flight will be delayed for an additional 30 minutes? (i.e. Find $P(T > 1.5 | T > 1)$).

Example 3: Waiting for a flight (cont.)

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Solution

(a) The mean delay is given by

Example 3: Waiting for a flight (cont.)

Solution

(a) The mean delay is given by

$$E(T) \quad =$$

Example 3: Waiting for a flight (cont.)

Solution

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$$E(T) = \frac{1}{\lambda}$$

Example 3: Waiting for a flight (cont.)

Solution

(a) The mean delay is given by

$$E(T) = \frac{1}{\lambda} = \frac{1}{2}$$

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(a) The mean delay is given by

$$E(T) \quad = \quad \frac{1}{\lambda} = \frac{1}{2} = \boxed{0.5\text{hr}}$$

Example 3: Waiting for a flight (cont.)

Solution

(a) The mean delay is given by

$$E(T) \quad = \quad \frac{1}{\lambda} = \frac{1}{2} = \boxed{0.5\text{hr}}$$

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Example 3: Waiting for a flight (cont.)

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(b) The variance is:

$$\mathbb{V}(T) = \frac{1}{\lambda^2}$$

Example 3: Waiting for a flight (cont.)

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(a) The mean delay is given by

$$E(T) = \frac{1}{\lambda} = \frac{1}{2} = \boxed{0.5\text{hr}}$$

(b) The variance is:

$$\mathbb{V}(T) = \frac{1}{\lambda^2} = \frac{1}{2^2} =$$

Example 3: Waiting for a flight (cont.)

Solution

(a) The mean delay is given by

$$E(T) \quad = \quad \frac{1}{\lambda} = \frac{1}{2} = \boxed{0.5\text{hr}}$$

(b) The variance is:

$$\mathbb{V}(T) \quad = \quad \frac{1}{\lambda^2} = \frac{1}{2^2} = \boxed{0.25\text{hr}^2}$$

Example 3: Waiting for a flight (cont.)

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Solution

(c) The probability the flight will be delayed by no more than 10 minutes ($\frac{1}{6}$ hr) is given by:

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$$P\left(T \leq \frac{1}{6}\right) = 1 - e^{-\lambda \cdot \frac{1}{6}}$$

Example 3: Waiting for a flight (cont.)

Solution

(c) The probability the flight will be delayed by no more than 10 minutes ($\frac{1}{6}$ hr) is given by:

$$P\left(T \leq \frac{1}{6}\right) \;=\; 1 - e^{-\lambda \cdot \frac{1}{6}} = 1 - e^{-2\left(\frac{1}{6}\right)}$$

Example 3: Waiting for a flight (cont.)

Solution

(c) The probability the flight will be delayed by no more than 10 minutes ($\frac{1}{6}$ hr) is given by:

$$\begin{aligned} P\left(T \leq \frac{1}{6}\right) &= 1 - e^{-\lambda \cdot \frac{1}{6}} = 1 - e^{-2\left(\frac{1}{6}\right)} \\ &= 1 - e^{-\frac{1}{3}} \end{aligned}$$

Example 3: Waiting for a flight (cont.)

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(c) The probability the flight will be delayed by no more than 10 minutes ($\frac{1}{6}$ hr) is given by:

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\begin{aligned}
P\left(T \leq \frac{1}{6}\right) &= 1 - e^{-\lambda \cdot \frac{1}{6}} = 1 - e^{-2\left(\frac{1}{6}\right)} \\
&= 1 - e^{-\frac{1}{3}} = \boxed{0.283}
\end{aligned}
$$

Example 3: Waiting for a flight (cont.)

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Solution

(d) The probability that the flight will be delayed by a further 0.5hr after 1hr of waiting is given by:

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$$P(T > (0.5 + 1) | T > 1) = P(T > 1.5 | T > 1)$$

Example 3: Waiting for a flight (cont.)

Solution

(d) The probability that the flight will be delayed by a further 0.5hr after 1hr of waiting is given by:

$$\begin{aligned} P(T > (0.5 + 1) | T > 1) &= P(T > 1.5 | T > 1) \\ &= \frac{P((T > 1.5) \cap (T > 1))}{P(T > 1)} \end{aligned}$$

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&= \frac{P((T > 1.5) \cap (T > 1))}{P(T > 1)} \quad \text{(mult. rule)} \\
&= \frac{P(T > 1.5)}{P(T > 1)}
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&= \frac{P(T > 1.5)}{P(T > 1)} \\
&= \frac{e^{-2(1.5)}}{e^{-2(1)}}
\end{aligned}$$

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&= \frac{P(T > 1.5)}{P(T > 1)} \\
&= \frac{e^{-2(1.5)}}{e^{-2(1)}} = e^{-2[1.5-1.0]}
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&= \frac{e^{-2(1.5)}}{e^{-2(1)}} = e^{-2[1.5-1.0]} \\
&= e^{-2(0.5)} \quad (= P(T > 0.5))
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&= \frac{e^{-2(1.5)}}{e^{-2(1)}} = e^{-2[1.5-1.0]} \\
&= e^{-2(0.5)} \quad (= P(T > 0.5)) \\
&= e^{-1} = \boxed{0.37}
\end{aligned}
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Memorylessness of the exponential distribution

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Memoryless property

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Memorylessness of the exponential distribution

This leads us to an important property of the exponential distribution

Memoryless property

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That is, it does not matter from which time the waiting begins (i.e. conditioning); the probability of an elapsed time remains the same.

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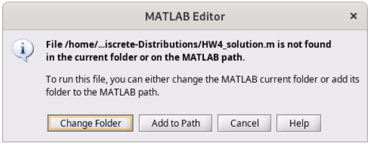

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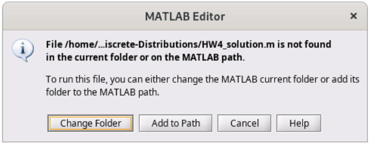

If so, simply click on `Change Folder` or move the file to the current folder you are in. Finally, always make sure the path of a file being read by a script is valid from its location, otherwise you will have to deal with "File not found" errors.

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