CEE 260/MIE 273: Probability and Statistics in Civil Engineering

Lecture M4c: Hypothesis Testing and p-values

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Outline

Today's objectives

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- Know when to use a lower-tailed, upper-tailed or two-tailed test
- Understand Type I errors and their relationship to p-values
- Learn how to use p-values to conduct a hypothesis test

Hypothesis testing

Hypothesis testing

- Hypothesis testing provides a framework for evaluating parameter(s) of a population with respect to a desired or known outcome.
- Given that the in most cases, we can only estimate these parameters, hypothesis testing allows us to determine if the estimate supports a **research hypothesis**.
- The results of this testing is useful for **decision-making**.

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$$H_0 : p = p_0 \tag{1}$$

where p_0 is the specified standard.
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$$H_1 : p \neq p_0 \tag{2}$$

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No truth test for the null hypothesis

The failure to reject H_0 does not mean that H_0 is true.

Further explanation of hypothesis test outcomes

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Further explanation of hypothesis test outcomes

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In a jury trial, the hypotheses are:

- H_0: defendant is innocent
- H_1: defendant is guilty (not innocent)

The null hypothesis H_0 is **rejected** if there evidence beyond reasonable doubt that the defendant is guilty.

However, **failure to reject** H_0 does not imply the defendant is innocent, only that there is **insufficient evidence to prove otherwise**.

Hypothesis testing in practice

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Note: this is a one-sided hypothesis test (testing in one direction only)

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We note that this is a Bernoulli process.

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Say the critical value were $p^* = .1$, then $p \geq .1$ could then be considered as strong evidence that $p > 0.05$.

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- Convicting a defendant of a crime when they are innocent (Example 1)
- Diagnosing a patient with a disease when in fact they do not have it (i.e. the null hypothesis is that the disease is NOT present)

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Much of our focus will be on dealing with the level of significance, α.

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4. *Specify* or *identify* the **level of significance** (α)
5. *Define* the **region of rejection/critical region** of the null hypothesis by choosing the **critical value**.
6. *Decide.* If the test statistic is in the critical region, reject H_0. If not, do not reject H_0 (fail to reject it)

One-sided tests

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Case A: upper tail

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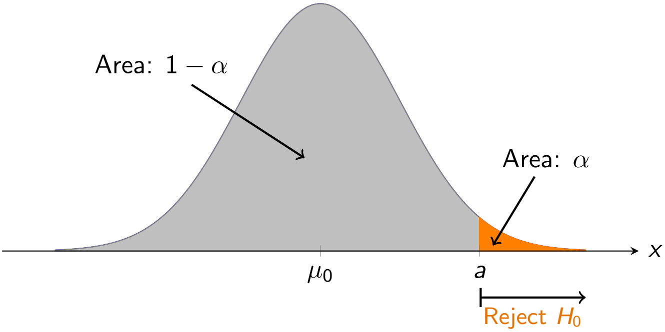

One-sided tests (cont.)

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Case B: lower tail

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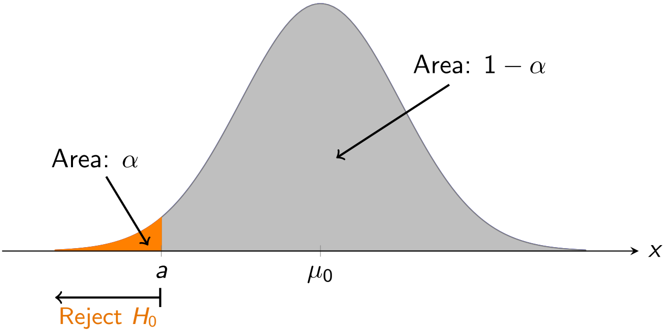

Two-sided tests

Two-sided tests

Case C: both tails

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Case C: both tails

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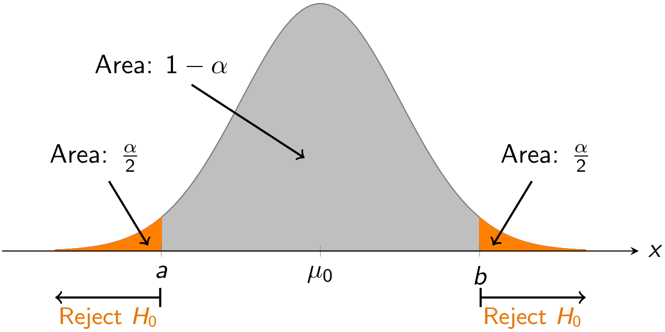

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And thus, the standard error is:

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Thus, to compute the probability (area under curve) of the test statistic,

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And thus, the standard error is:

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Thus, to compute the probability (area under curve) of the test statistic, we use the z-score:

$$z^* = \frac{p - p_0}{SE_p} \tag{9}$$

which is normally distributed.

What is a p-value?

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Definition

The p-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.

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Definition

The *p*-value is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given dataset.
Equivalently,this is the minimum probability of a Type I error.

Motivating the usage of *p*-values

Motivating the usage of p-values

Example 3: Nicotine content

Based on data from a sample of cigarettes, the Z statistic is $z = 2.10$. You want to verify if the true nicotine content (measured in proportion of tobacco weight) is $p = .015$ (H_0) versus the alternative hypothesis that is greater: $H_1 : p > .015$).

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What are your conclusions from testing at the following significance levels:

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This is an **upper-tailed** hypothesis test.
What are your conclusions from testing at the following significance levels:

- $\alpha_1 = 0.05$
- $\alpha_1 = 0.025$
- $\alpha_1 = 0.01$
- $\alpha_1 = 0.005$

Motivating the usage of *p*-values

Example 3: Nicotine content (cont.)

Motivating the usage of p-values

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The *p*-value

Motivating the usage of p-values

Example 3: Nicotine content (cont.)

The p-value $1 - \Phi(2.10)$

Motivating the usage of p-values

Example 3: Nicotine content (cont.)

The p-value $1 - \Phi(2.10)$ (area to the right of z)

Motivating the usage of p-values

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The p-value $1 - \Phi(2.10)$ (area to the right of z) $\therefore p = 1 - 0.9821$

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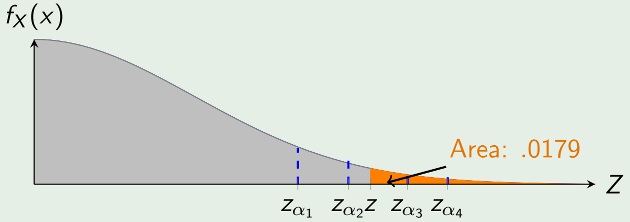

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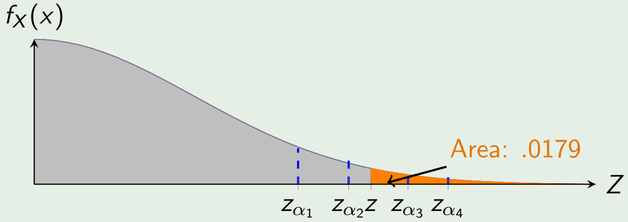

Your conclusions are as follows:

Motivating the usage of *p*-values

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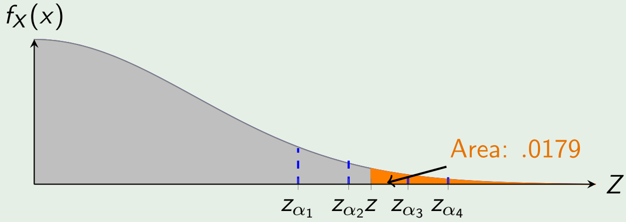

Your conclusions are as follows:

Level of significance α	Rejection Region	Conclusion
$\alpha_1 = 0.05$	$z \geq 1.645$	Reject H_0

Motivating the usage of p-values

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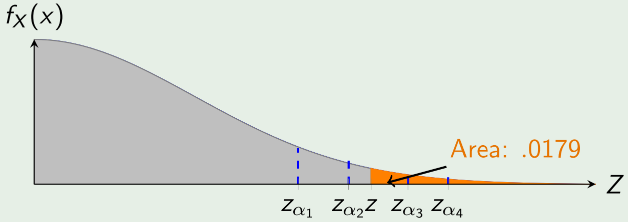

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Motivating the usage of p-values

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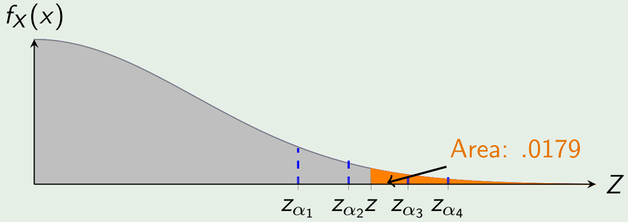

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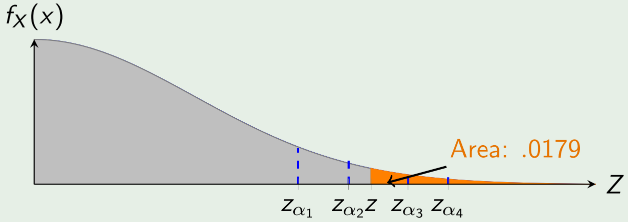

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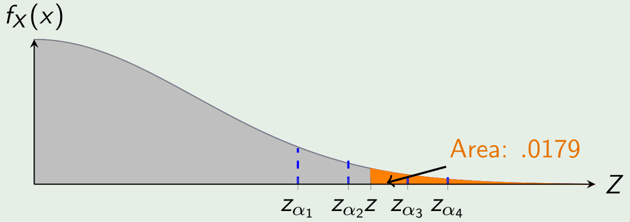

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Motivating the usage of p-values

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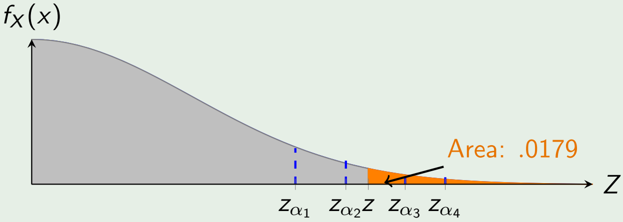

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$\alpha_4 = 0.005$	$z \geq 2.58$	Fail to reject H_0

Usefulness of p-value

Usefulness of *p*-value

- Provides more information about the strength of a test

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Alternative definition

The p-value is the probability of obtaining a test statistic value at least as contradictory to H_0 as the value that actually resulted. The smaller the p-value, the more contradictory are the data to H_0.

Hypothesis testing with the p-value

Hypothesis testing with the *p*-value

Step 1. Formulate your hypotheses

Step 2. Determine the *p*-value from the test statistic

Step 3. Conclude the test based on a chosen level of significance:

1. *p*-value $\leq \alpha \implies$ reject H_0 at level α.
2. *p*-value $> \alpha \implies$ do not reject H_0 at level α.

p-value for z tests

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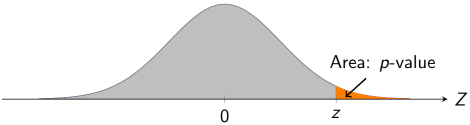

p-value: area in upper tail

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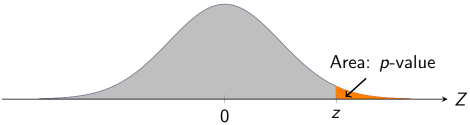

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$$p = 1 - \Phi(z) \tag{10}$$

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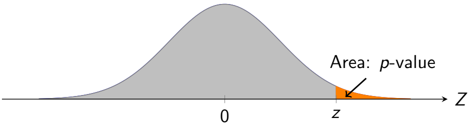

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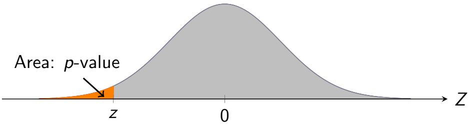

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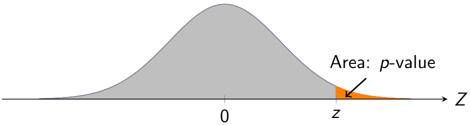

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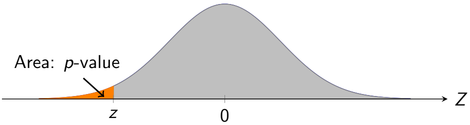

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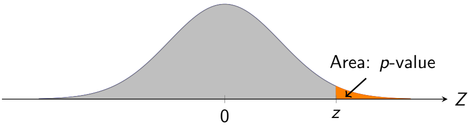

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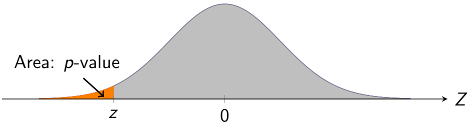

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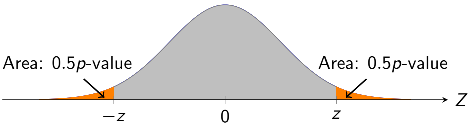
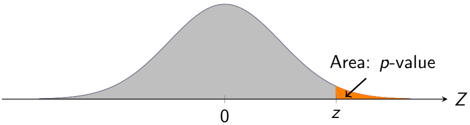


p-value: sum of area in both tails

p-value for *z* tests



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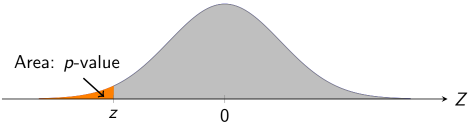

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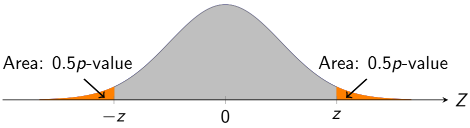

p-value: sum of area in both tails

$$p = 2(1 - \Phi(|z|)) \qquad (12)$$

Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (OS 5.21)

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.01

Step 1. Parameter of interest:

Hypothesis testing using p-value approach

Example 4: Getting enough sleep (OS 5.21)

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Hypothesis testing using *p*-value approach

Example 4: Getting enough sleep (cont.)

Hypothesis testing using p-value approach

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Step 5. Calculate test statistic value:

Hypothesis testing using *p*-value approach

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$$z = \frac{.723 - .5}{\sqrt{.5(.5)/400}}$$

Hypothesis testing using p-value approach

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Step 6. Determine *p*-value

Hypothesis testing using *p*-value approach

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Hypothesis testing using p-value approach

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Step 7. Conclude:

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Step 7. Conclude:
Using a significance level of 0.01, we reject H_0 since $0.0204 > 0.01$. Thus, at the 1% significance level, there is sufficient evidence to conclude that true proportion differs from the target value of 0.5.

Recap of this lecture

- Definition of hypothesis testing
  - Null hypothesis (default/expected outcome) H_0

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