CME 241 Assignment-2

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Snakes and Ladders: In this problem, we model snakes and ladders game. States can be modeled as the cell that our player sits on. Namely, $s_t = k$, where t stands for the time step and k is the cell number.

For each $k = 0, 1, 2, .., 100$, transition probabilities are as follows:

$$\mathcal{P}(k, k+i) = 1/6 \qquad \text{for} \quad i = 1, 2, 3, 4, 5, 6 \tag{1}$$

Note that for states $s_t \geq 100$, we can say that it is equivalent to the state $s_t = 100$.Namely, states $s_t \geq 100$ stands for the state $s_t = 100$, which is the terminal state.

Note that in the game, there is no states like $s_t = 1$ or $s_t = 4$, because when the player arrives to the cell-1 or cell-4 it actually goes to the cell-38 or cell-14 directly. So, we need to say that state $s_t = 1$ stands for the state $s_t = 38$; or state $s_t = 4$ stands for the state $s_t = 14$.

There are multiple of these situations which is described in the code.

We initialize the game for $n = 10000$ time and get a probability distribution of the time steps required to finish the game, which is provided below.

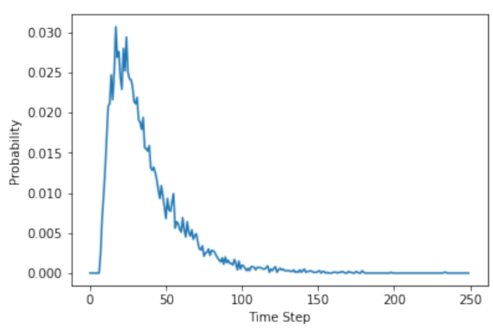

Figure 1

Frog Puzzle Suppose we have n steps. And the expected number of the frog steps is a_n. We know that $a_0 = 0, a_1 = 1$ and $a_2 = \frac{3}{2}$. Let's consider a_{n+1}, we have $n + 1$ different first step to $0, 1, 2, .., n$ with equal probabilities. Then,

$$a_{n+1} = \frac{1}{n+1}(1 + a_0) + \frac{1}{n+1}(1 + a_1) + .. + \frac{1}{n+1}(1 + a_n) = 1 + \frac{1}{n+1}(a_0 + a_1 + .. + a_n) \tag{2}$$

$$(n+1)a_{n+1} - (n+1) = a_0 + ..a_n \tag{3}$$

If we write $n \longrightarrow n-1$ in the last equation we get,

$$na_n - n = a_0 + ..a_{n-1} \tag{4}$$

If we subtratct the last two equations we get

$$a_{n+1} - a_n = \frac{1}{n+1} \Longrightarrow a_n = 1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{n} \tag{5}$$

Reward Process: In order to calculate the expected dice rolls, we can say that we gain a reward $r_t = 1$ if our dice is in the set $\{1, 2, 3, 4, 5\}$, and $r_t = 0$ if it is 6. It is because in the game, if we roll 6, then we do not roll it again, it rolls automatically, so no need to count the first roll. I implemented it in the code.