Q11 $dW_t = \left(\left(\pi_t(\mu-r)+r\right)W_t - c_t\right)dt + \pi_t \sigma W_t dz_t$

$$\max \mathbb{E}\left\{ \int_t^T e^{-\rho(s-t)} \log(c_s)ds + e^{-\rho(T-t)} \cdot B(T)\log(W_t) \right]$$

$$\Rightarrow V^*(t,W_t) = \max_{\pi,c} \mathbb{E}\left\{ \int_t^{t_1} e^{-\rho(s-t)}\log(c_s)ds + e^{-\rho(t_1-t)} V^*(t_1, W_{t_1}) \right\}$$

$$\Rightarrow e^{-\rho t}V^*(t,W_t) = \max_{\pi,c} \mathbb{E}\left\{ \int_t^{t_1} e^{-\rho s}\log(c_s)ds + e^{-\rho t_1} V^*(t_1, W_{t_1}) \right\}$$

$$\Rightarrow \max_{\pi,c} \mathbb{E}\left\{ d\left(e^{-\rho t}V^*(t,W_t)\right) + e^{-\rho t}\log c_t dt \right\} = 0$$

$$\Rightarrow \max_{\pi,c} \mathbb{E}\left\{ d\left(V^*(t,W_t)\right) + \log c_t d_t = \rho V^*(t,W_t)dt \Rightarrow\right.$$

$$\max_{\pi,c} \frac{\partial V^*}{\partial t} + \frac{\partial V^*}{\partial W}\left(\left(\pi_t(\mu-r)+r\right)W_t - c_t\right) + \frac{\partial^2 V^*}{\partial W^2}\frac{\pi_t^2\sigma^2W_t^2}{2} + \log(c_t) = \rho V^*(t,W_t)$$

Take partial derivatives wrt π and c and equate to $0 \Rightarrow$

$$\frac{\partial V^*}{\partial W_t}(\mu-r) + \frac{\partial^2 V^*}{\partial W_t^2}\pi_t\sigma^2 W_t = 0 \Rightarrow \boxed{\pi_t^* = \frac{-\frac{\partial V^*}{\partial W_t}(\mu-r)}{\frac{\partial^2 V^*}{\partial W_t^2}\sigma^2 W_t}}$$

$$\frac{-\partial V^*}{\partial W_t} + \frac{1}{c_t} = 0 \Rightarrow \boxed{c_t = \left(\frac{\partial V^*}{\partial W_t}\right)^{-1}}$$

Q3 States $S_t = (e_t, V_t)$ $e_t \in \{0,1\}$ $e_t = 0$ means unemployed, $e_t = 1$ means employment

V_t is the skill level at time (day) $= t$.

action $= a_t \Rightarrow$ $a_t = 0$ if we are unemployed at time t, namely $e_t = 0$.

$a_t = \alpha_t$ if we work α_t portion of the time, so $1-\alpha_t$ becomes the portion for learning.

reward $\Rightarrow R(s_t, a_t) = f(V_t) \cdot \alpha_t$; $R(s_t, a_t) = 0$ if $a_t = 0$ (unemployed)

State trasitions: $P(s_t, a_t, s_{t+1})$:

If $s_t = (0, V_t) \Rightarrow a_t = 0$ $\Rightarrow$ for $s_{t+1} = (1, V_t e^{-\lambda \cdot \rho})$ where $e^{-\lambda\rho}$ is the loosing the skill in one day.

$\Rightarrow$ probability $h(V_t)$.

$$P((0, V_t), 0, (1, V_t e^{-\lambda\rho})) = h(V_t).$$

$$P((0, V_t), 0, (0, V_t e^{-\lambda\rho})) = 1 - h(V_t).$$

for $s_t = (1, V_t) \Rightarrow$

$$P\Big((1, V_t), \alpha_t, (0, V_t \cdot g(V_t)(1-\alpha_t))\Big) = p \quad \text{for all } \alpha_t.$$

$$P\Big((1, V_t), \alpha_t, (1, V_t \cdot g(V_t)(1-\alpha_t))\Big) = 1-p \quad \text{for all } \alpha_t.$$

our goal is to maximize $\Rightarrow$ $\mathbb{E}(G_t) = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots$

for infinite horizon.

For finite horizon $\Rightarrow$ $\max R_t + \gamma R_{t+1} + \cdots + \gamma^{T-t} R_T$.