Q1 States $= s_t = (y_t, z_t, c_t)$ where y_t represents the debt we have (to the other bank) z_t is our investment in the risky asset. c_t is the cash our bank has.

suppose we have $(y_{t-1}, z_{t-1}, c_{t-1})$ at the end of the $(t-1)$th day. We will show how the following day can be shaped. Suppose we have d_t deposit, w_t withdrawal in the day t. Also, suppose we borrow y_t' from the other bank in day t. Also, suppose we invest z_t' in the risky asset in day t. (y_t' and z_t' can be negative, meaning we make a payment to to other bank and take cash from the risky asset, respectively.

Then the next day state become:

$$\Big(\underbrace{y_{t-1}(1+R)+y_t'}_{y_t},\ \underbrace{\phi_t(z_{t-1})+z_t'}_{z_t},\ \underbrace{c_{t-1}+d_t-w_t+y_t'-z_t'}_{c_t}\Big)$$

$\phi_t(z_{t-1})$ is the value of the risky asset in day t where we had z_{t-1} in day $t-1$.

So our actions become $\rightarrow a_{t-1} = (y_t', z_t')$ adjusting the debt and the investment.

When it comes to the reward; $R(s_{t-1}, a_{t-1}, s_t) = -K \cdot \cot\left(\frac{\pi c_t}{2C}\right)$ if $c_t \leq C$.

If $c_t > C \Rightarrow R(s_{t-1}, a_{t-1}, s_t) = 0$. Also; for the last time step;

$R(s_{T-1}, a_{T-1}, s_T) = c_T - y_T + z_T$. (All money we have).

When it comes to the transition probabilities we need to say it depends on the d_t, w_t, and $\phi_{t-1}(z_{t-1})$. $\Rightarrow$

$P(s_{t-1}, a_{t-1}, s_t) = \mathrm{Prob}(d_t)\,\mathrm{Prob}(w_t)\,\mathrm{Prob}(\phi_t(z_{t-1}))$

To solve this MDP, as the state function space is huge we can use ADP. Also as every variable is continous, I would use function approximation, to solve the MDP. (Neural Network or a linear function)

Q21 $g(s) = p \int_s^{\infty} (x-s) f(x) dx + h \int_{-\infty}^{s} (s-x) f(x) dx = p \int_s^{\infty} x f(x) dx - ps \int_s^{\infty} f(x) dx +$

$hs \int_{-\infty}^{s} f(x) dx - h \int_{-\infty}^{s} x f(x) dx$

$\Rightarrow$ We will take $\frac{\partial}{\partial s}$ and equate to 0.

$$\frac{\partial}{\partial s}\left(p \int_s^{\infty} x f(x) dx\right) = \frac{\partial}{\partial s}\left(p \int_{-s}^{-\infty} x f(-x) dx\right) = \frac{\partial}{\partial s}\left(-p \int_{-\infty}^{-s} x f(-x) dx\right) \Rightarrow \underbrace{\frac{\partial (-s)}{\partial s}}_{-1}\left(\frac{\partial}{\partial (-s)} (-p) \int_{-\infty}^{-s} x f(-x) dx\right)$$

$$(-1)(-p).(-s) f(s) = \boxed{-sp f(s)}$$

$$\frac{\partial}{\partial s}\left(-ps \underbrace{\int_s^{\infty} f(x) dx}_{K(S).}\right) = -p K(s) + (-ps). \frac{\partial K}{\partial s} = \boxed{-p K(s) + ps. f(s)}$$

$$\frac{\partial K(s)}{\partial s} \Rightarrow K(s) = -\int_{-s}^{-\infty} f(-x) dx = \int_{-\infty}^{-s} f(-x) dx \Rightarrow \frac{\partial K}{\partial -s} \frac{\partial -s}{\partial s}$$

$$= f(s).(-1)$$

$$\frac{\partial}{\partial s}\left(hs \int_{-\infty}^{s} f(x) dx\right) = \boxed{h. \int_{-\infty}^{s} f(x) dx + hS. f(S)}$$

$$\frac{\partial}{\partial s}\left(-h \int_{-\infty}^{s} x f(x) dx\right) = \boxed{-h. S f(S)}$$

Sum all them up: $-sp\cancel{f(s)} + -pK(s) + sp\cancel{f(s)} + h \int_{-\infty}^{s} f(x) dx + hs\cancel{f(s)} - hs\cancel{f(s)}$

$= -p.(1 - F(S)) + h.F(S) = -p + F(S)(h+p)$ where $F(s)$ is CMF of the distribution $f(x)$

$= 0$

$\Rightarrow F(s^*) = p/h+p$

$$\Rightarrow \boxed{s^* = F^{-1}(p/h+p).}$$