1) First note that $E(X) = \int x f(x)dx = \mu$ $\quad \sigma^2 = Var(X) = E(X^2) - E(X)^2 = \int x^2 f(x)dx - \mu^2 \Rightarrow \underline{E(X^2) = \mu^2 + \sigma^2}$.

$$E(u(x)) = \int \left(x - \frac{\alpha}{2}x^2\right) f(x)dx = \int x f(x)dx - \frac{\alpha}{2}\int x^2 f(x)dx = E(X) - \frac{\alpha}{2}E(X^2) = \boxed{\mu - \frac{\alpha}{2}(\mu^2+\sigma^2)}$$

$$U(X_{CE}) = E(u(x)) = \mu - \frac{\alpha}{2}(\mu^2+\sigma^2) = X_{CE} - \frac{\alpha}{2}X_{CE}^2 \Rightarrow \left(X_{CE} - \frac{1}{\alpha}\right)^2 = \mu^2 + \sigma^2 - \frac{2\mu}{\alpha} + \left(\frac{1}{\alpha}\right)^2 = \left(\mu - \frac{1}{\alpha}\right)^2 + \sigma^2$$

$$X_{CE} = \frac{1}{\alpha} \pm \sqrt{\sigma^2 + \left(\mu - \frac{1}{\alpha}\right)^2}$$

if $X_{CE} = \frac{1}{\alpha} + \sqrt{\sigma^2 + (\mu - 1/\alpha)^2} > (\mu - \frac{1}{\alpha}) + \frac{1}{\alpha} = \mu$

$X_{CE} > \mu$ imposible.

$$\Rightarrow X_{CE} = \frac{1}{\alpha} - \sqrt{\sigma^2 + (\mu - 1/\alpha)^2}$$

$$\Rightarrow \pi_A = \mu - X_{CE} = \mu - \frac{1}{\alpha} + \sqrt{\sigma^2 + \left(\mu - \frac{1}{\alpha}\right)^2}$$

Instead of 1 million, lets do this for 1 dollar (we can multiply with 1 million at the end)

$\Rightarrow$ At the end of the year $N(1 + r + z(\mu - r), z^2\sigma^2)$. $\Rightarrow$ expectation

$$\Rightarrow \left(1 + r + z(\mu - r)\right) - \frac{\alpha}{2}\left((1 + r + z(\mu - r))^2 + z^2\sigma^2\right) =$$

$$1 + r + z(\mu - r) - \frac{\alpha}{2}\left[(1+r)^2 + 2(1+r)z(\mu - r) + z^2(\mu - r)^2 + z^2\sigma^2\right] \Rightarrow \frac{\partial}{\partial z} \Rightarrow$$

$$(\mu - r) - \frac{\alpha}{2}\left[2(1+r)(\mu - r) + 2z(\mu - r) + 2z\sigma^2\right] = 0 \rightarrow (\mu - r) - \alpha z(\mu - r) - \alpha z\sigma^2 - \alpha(1+r)(\mu - r) = 0$$

$$z = \frac{(\mu - r)(1 - \alpha(1+r))}{\alpha(\mu - r) + \alpha\sigma^2} = \frac{(\mu - r)\left[\frac{1}{\alpha} - (1+r)\right]}{\mu - r + \sigma^2}$$

So as $\mu > r$. $\Rightarrow$ As α decreases

$\Rightarrow \frac{1}{\alpha}$ increases $\Rightarrow z$ increases.

$\Rightarrow$ As α increases $\Rightarrow z$ decreases.

Q3

- with probability $p \Rightarrow (1+\alpha)fW_0 + (1-f)W_0$

  with probability $1-p \Rightarrow (1-\beta)fW_0 + (1-f)W_0$

- wp (with probability) p $U(W) = \log\left((1+\alpha)fW + (1-f)W\right)$

  wp $1-p \Rightarrow U(W) = \log\left((1-\beta)fW + (1-f)W\right)$

- $\mathbb{E}[\log(W)] = p \cdot \log\left((1+\alpha)fW + (1-f)W\right) + (1-p)\log\left((1-\beta)fW + (1-f)W\right)$

- $\dfrac{\partial E[\log(W)]}{\partial f} = p \cdot \dfrac{\alpha W}{\alpha f W + W} + (1-p) \cdot \dfrac{-\beta W}{W - \beta f W}$

$$= \frac{p\alpha}{\alpha f + 1} - \frac{(1-p)\beta}{1-\beta f} \Rightarrow p\alpha(1-f\beta) = \beta(1-p)(1+\alpha f)$$

$$p\alpha - f[p\alpha\beta] = \beta(1-p) + f[\beta\alpha(1-p)]$$

$$\Rightarrow p\alpha - \beta(1-p) = f\left[\alpha\beta - \cancel{\alpha\beta p} + \cancel{\alpha\beta p}\right]$$

$$\boxed{f^* = \frac{\alpha p - \beta(1-p)}{\alpha\beta}} = \frac{p}{\beta} - \frac{1-p}{\alpha}$$

It is a maxima as $\dfrac{\partial^2}{\partial f^2} =$

$$\underbrace{p\alpha(1+\alpha f)^{-2} \cdot (-\alpha)}_{\text{negative}} - \underbrace{(1-p)\beta \frac{1}{(1-\beta f)^2}(-1)(-\beta)}_{\text{positive}} \leq 0 \Rightarrow \text{Concave} \Rightarrow f^* \text{ is a local max.}$$

- It makes sense for me because f^* goes up if α and/or p goes up. Similarly it decreases if we increase $(1-p)$ and/or β increase.