

Exercises

Chapter 4 - Probability and Probability distributions

- Exercise 1** Find all outcomes of the following experiments.
- 1.1 Toss a die and observe the number that appears on the upper face.
 - 1.2 A coin is tossed three times, and the sequence of heads and tails is observed.
 - 1.3 A rat is selected at random from a cage, and its sex is determined.
- Exercise 2** A sample space S consists of 4 simple events with probabilities:
 $P(E_1) = 0.1$, $P(E_2) = 0.3$, $P(E_3) = 0.5$ and $P(E_4) = k$.
- 2.1 Find the value of k .
 - 2.2 Let $A = \{E_1, E_2\}$ and $B = \{E_1, E_3\}$, find $P(A)$, $P(B)$ and $P(A \cup B)$.
- Exercise 3** Toss a fair coin twice. Let A be an event that head appears on the first toss, and B be an event that 2 heads result.
- 3.1 Are A and B mutually exclusive (disjoint)?
 - 3.2 Find $P(A)$, $P(B)$, $P(A \cup B)$, and $P(A \cap B)$.
 - 3.3 If the coin is tossed three times, what is $P(A)$?
- Exercise 4** A man has 2 vests and 4 shirts. In how many outfits can he dress if he wants to wear a vest and a shirt?
- Exercise 5** Herb Garden restaurant offers a special menu, for a fixed lunch cost, you can choose from one of two salads, one of three entrees, and one of four desserts. How many different lunches are available?
- Exercise 6** In how many ways can 5 students be seated in a row that has only 2 chairs? 3 chairs?
- Exercise 7** Find the number of possible four-letter code words using 26 English letters,
- 7.1 if letters can be repeated,
 - 7.2 if letters cannot be repeated.
- Exercise 8** A committee consists of three members selected from four candidates. List all different ways that you can form such committee. In how many ways can the committee be formed?
- Exercise 9** An instructor has a test bank of 20 multiple-choice questions. If she randomly selects 5 questions for a quiz, how many selections can she make?
- Exercise 10** Roll a fair die three times. Find
- a) the probability that the all 3 faces show the same numbers,
 - b) the probability that the first 2 faces show the same number.

Exercise 11 A large number of shoppers at a local shopping mall were categorized by age and gender as shown in the probability table below.

Gender	Under 25 years A_1	25-40 years A_2	Over 40 years A_3	Total
Male (M)	0.15	0.13	0.12	0.4
Female (F)	0.24	0.18	0.18	0.6
Total	0.39	0.31	0.30	1

- 11.1 What is the probability that the randomly selected shopper is under 25 years of age?
- 11.2 What is the probability that the randomly selected shopper is male and under 25 years of age?
- 11.3 What is the probability that the randomly selected shopper is male or under 25 years of age?
- 11.4 If the randomly selected shopper is male, what is the probability that he is under 25 years of age?
- 11.5 For any given shopper who is over 40 years old what is the probability that it is a female?
- 11.6 Are A_1 and M independent events?

Exercise 12 At a certain gas station 40% of the customers request regular gas (S_1), 35% request unleaded gas (S_2), and 25% request premium gas (S_3). Of those customers requesting regular gas, only 30% fill their tanks. Of those customers requesting unleaded gas, 60% fill their tanks, while of those requesting premium, 50% fill their tanks.

- a) If the next customer fills his tank, what is the probability that he requested regular gas? Unleaded gas?
- b) What is the probability that the next customer fills his tank?

Exercise 13 A record of a MUIC student shows that he drives to Salaya 60% of the time. The probability that he arrives on time given that he drives is 0.3; while this probability is 0.9 if he does not drive. If he arrives Salaya on time, what is the probability that he drives his own car?

Exercise 14 A manufacturer of computer chip has 2 assembly lines: A and B. The percentages of output produced by the lines are 30% and 70%, respectively. The probability that a computer chip from line A is defective is 0.02 and 0.05 if from line B.

- a) If a randomly selected computer chip is defective, what is the probability that it is from line A?
- b) What is the probability that a randomly selected computer chip is defective?

Exercise 15 Identify each random variable as discrete or continuous.

- The number of heads observed when a coin is tossed 10 times.
- The number of customers per day in Cup-pa Coffee & Snack.
- The length of time you can run in 100 meters.
- The diastolic blood pressure of any person.
- The speed of a car in kilometers on an express lane.

Exercise 16 Verify whether or not each of the following is a probability distribution. State your conclusion and explain.

- $p(x) = \frac{1}{5}$ for $x = -2, -1, 0, 5, 6$
- $p(x) = \frac{3/4}{x!(3-x)!}$ for $x = 0, 1, 2, 3$
- $p(x) = \frac{3-x}{2}$ for $x = 1, 2, 3, 4$

Exercise 17 A random variable x can assume the values -1, 0, 1, 3 and 5. The probability distribution is

x	-1	0	1	3	5
$p(x)$	0.2	0.1	0.3	k	0.1

- Find the value of k .
- Find the probability that x is exactly 5? Exactly 4?
- Find the probability that x is less than 3.
- Find the expected value (or mean) of x .
- Find the variance of x .

Exercise 18 The arrival of x customers during randomly chosen 10-min intervals at a drive-in service has been found to follow the probability distribution shown below:

Number of arrivals x	0	1	4	5
Probability $p(x)$	0.2	0.1	0.6	0.1

- Find the mean number of arrivals for 10-min intervals.
- Find the standard deviation of the arrivals.

Exercise 19 A construction company is considering submitting bids for two contracts. It will cost the company 10000 baht to prepare and submit the bids, and if won, each bid would produce 500,000 baht of income to the company. The company estimates that it has a 10% chance of winning any given bid.

Here is the probability distribution of x = the number of bids the company wins, and M = the amount of money the company profits from the bids.

x = # of bids won	0	1	2
M = profit	-10,000	490,000	990,000
Probability $p(x)$	0.81	0.18	0.01

Find the mean of M .

ANSWERS: Chapter 4 - Probability and Probability distributions

2.1) $k=0.1$

2.2) 0.4, 0.6 and 0.9

3.1) No, because $P(A \cap B)$ is not equal to 0.

3.2) $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

3.3) $\frac{1}{2}$

4) $2(4) = 8$

5) 24

6) $5(4)=20, \quad 5(4)3=60$

7.1) $26^4 = 456,976$

7.2) $26(25)(24)23 = 358,800$ or P_4^{26}

8) $C_3^4 = 4$

9) 15,504

10.1a-b) $\frac{1}{36}, \frac{1}{6}$

11.1-11.5) 0.39, 0.15, 0.64, 0.375, 0.6

11.6) Not independent. One of reasons is

$$0.15 = P(A_1 \cap M) \neq P(A_1)P(M) = 0.39(0.4)$$

12a) $P(S_1|F) = \frac{0.12}{0.455} = \frac{120}{455}, \quad P(S_2|F) = \frac{0.21}{0.455} = \frac{210}{455}$

12b) 0.455

12) Solutions:

$$P(S_1) = 0.4 \quad P(S_2) = 0.35 \quad P(S_3) = 0.25$$

$$P(F|S_1) = 0.3, \quad P(F|S_2) = 0.6, \quad P(F|S_3) = 0.5$$

$$P(F) = P(S_1)P(F|S_1) + \dots + P(S_3)P(F|S_3)$$

$$= 0.12 + 0.210 + 0.125$$

$$= 0.455$$

$$P(S_1|F) = \frac{0.12}{0.455} = \frac{120}{455}, \quad P(S_2|F) = \frac{0.21}{0.455} = \frac{210}{455}$$

13) $\frac{1}{3}$

13) Solutions:

$$P(D) = 0.6 \quad P(O|D) = 0.3, \quad P(O|\bar{D}) = 0.9$$

$$P(D/O) = \frac{P(D \cap O)}{P(O)} = \frac{0.6(0.3)}{0.6(0.3) + 0.4(0.9)} = \frac{0.18}{0.54} = \frac{1}{3}$$

14a-b) 6/41, 0.041

14) Solutions:

$$P(A) = 0.3, \quad P(B) = 0.7$$

$$P(D|A) = 0.02, \quad P(D|B) = 0.05$$

$$P(A|D) = \frac{P(A)P(D|A)}{P(A)P(D|A) + P(B)P(D|B)} = \frac{0.006}{0.006 + 0.035} = \frac{6}{41} \cong 0.146$$

$$P(D) = 0.041$$

15) a) discrete b) discrete c) continuous d) continuous e) continuous

16a-c) Yes, Yes, No

Students must be able to justify all of the above answers.

17a) 0.3

17b) 0.1; 0

17c) 0.6

17d) 1.5

17e) 3.45

18a-b) 3 customers, $\sqrt{3.2}$

19) 98,100 baht