Introduction to Probability and Statistics Eleventh Edition

Chapter 9

Large-Sample Tests of Hypotheses

Introduction

- A statistical test of hypothesis concerning these 4 population parameters.
  - The population mean, μ
  - The population proportion, p
  - The difference between 2 population means, $\mu_1 - \mu_2$
  - Th difference between 2 population proportions, $p_1 - p_2$

A Large-Sample Test about a Population Mean

- Let μ_0 be a hypothesized value of μ.
- A **one tailed** test:

  1) $H_0: \mu = \mu_0$ vs. $H_a: \mu > \mu_0$ (right-tailed test)

     To test whether or not μ is greater than μ_0.

  2) $H_0: \mu = \mu_0$ vs. $H_a: \mu < \mu_0$ (left-tailed test)

     To test whether or not μ is less than μ_0.

- A **two-tailed** test:

  3) $H_0: \mu = \mu_0$ vs. $H_a: \mu \neq \mu_0$

     To test whether or not μ is different from μ_0.

Example 1

a) The mayor of a small city claims that the average income per family in his city is more than 50,000 baht per month. Thus,

H_0: $\mu = 50{,}000$ baht vs. H_a: $\mu > 50{,}000$ baht

b) Should one believe that the mean time to finish an exam is different from 90 minutes? Thus,

H_0: $\mu = 90$ min. vs. H_a: $\mu \neq 90$ min.

Class Activity

1. State the null hypothesis H_0, and the alternative hypothesis H_a that would be used for a hypothesis test related to each of the following statements.
   a) You wish to show that the mean age of the students enrolled in evening classes at a certain college is greater than 25 years.
   b) You would like to test that the average starting salary of a new graduate is different from 18,000 baht per month.
   c) An instructor of mathematics claims that an average student spends less than 5 hours studying for the final exam. To test the claim, what are the hypotheses?

Example 2

- A homeowner randomly samples 64 homes similar to her own and finds that the average selling price is \$252,000 with a standard deviation of \$15,000. Is this sufficient evidence to conclude that the average selling price is greater than \$250,000? Use α = .01.

$$H_0 : \mu = 250{,}000$$
$$H_a : \mu > 250{,}000$$

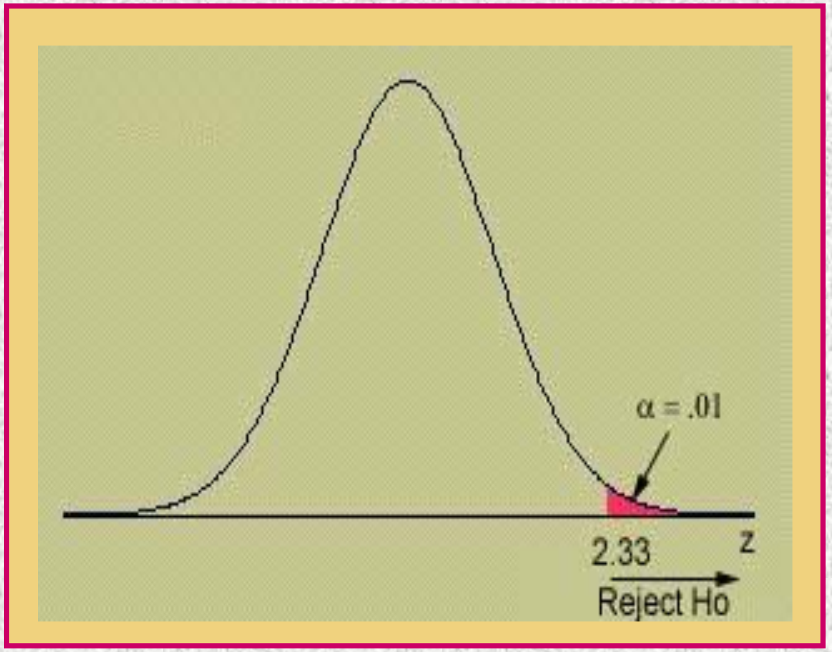
Test Statistic:

$$z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{252{,}000 - 250{,}000}{15{,}000/\sqrt{64}} = 1.07$$

Critical Value Approach

What is the critical value of z that cuts off exactly $\alpha = 0.01$ in the right-tail of the z distribution?

Rejection Region: Reject H_0 if $z > 2.33$.



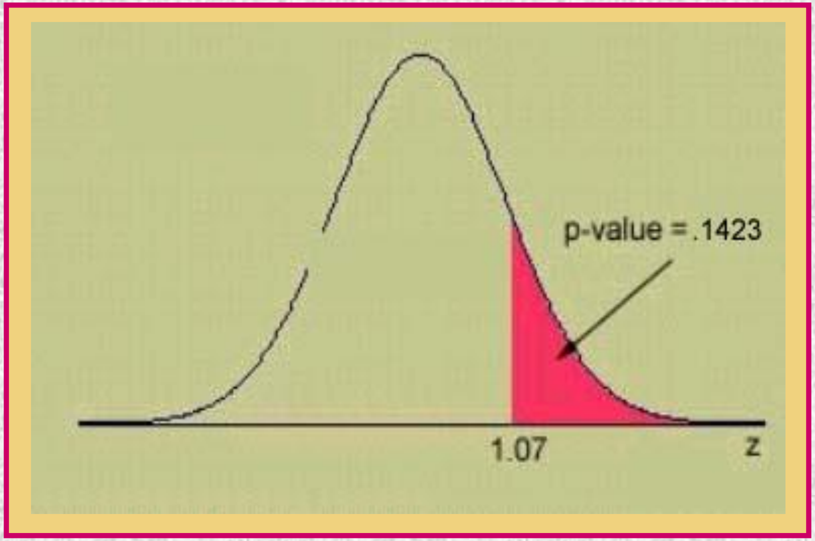
For our example, $z = 1.07$ does not fall in the rejection region and H_0 is not rejected.

There is not enough evidence to indicate that μ is greater than \$250,000.

p-Value Approach

- The probability that our sample results or something even more unlikely would have occurred *just by chance*, when $\mu = 250{,}000$.

$$p-\text{value} = P(z > 1.07) = 1 - 0.8577 = 0.1423$$



Since p-value $> \alpha = 0.01$, H_0 is not rejected.

There is insufficient evidence to indicate that μ is greater than \$250,000.

Example 3

The daily yield for a chemical plant has averaged 880 tons for several years. The quality control manager wants to know if this average has changed. She randomly selects 50 days and records an average yield of 871 tons with a standard deviation of 21 tons. At 1% significance level what would she conclude?

$$H_0 : \mu = 880$$
$$\text{vs. } H_a : \mu \neq 880$$

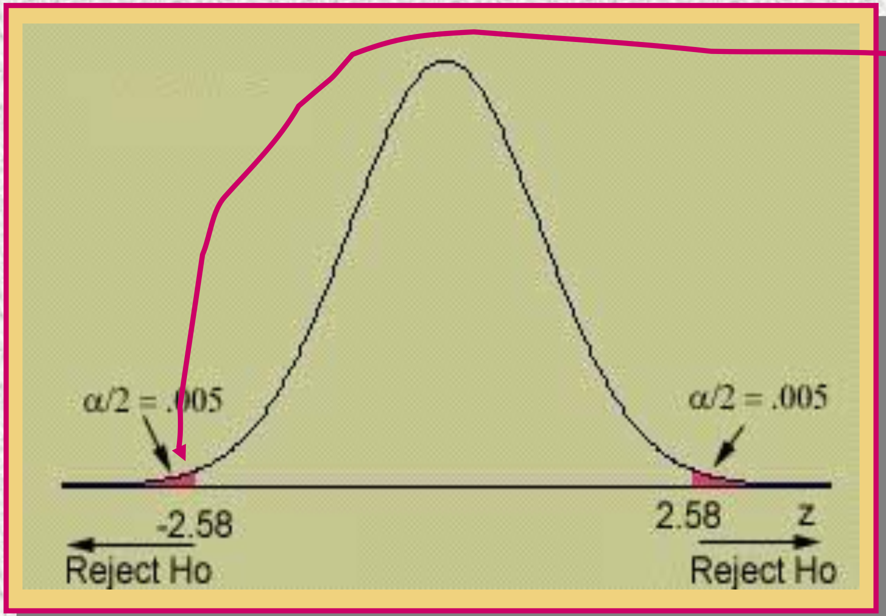
Test Statistic:

$$z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{871 - 880}{21/\sqrt{50}} = -3.03$$

Critical Value Approach

If $\alpha = 0.01$, what is the critical value of z that cuts off exactly $\alpha/2 = 0.01/2 = 0.005$ in the tail of the z distribution?

Rejection Region: Reject H_0 if $z > 2.58$ or $z < -2.58$.



For our example,

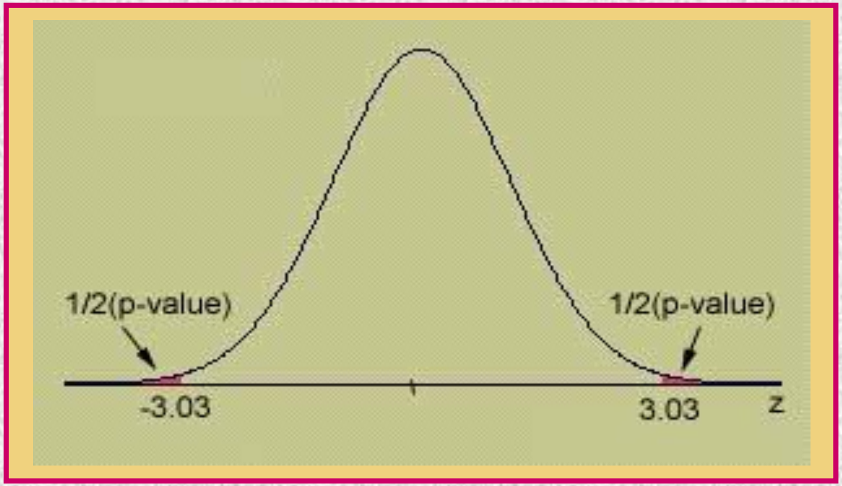
$z = -3.03$ falls in the rejection region and H_0 is rejected at the 1% significance level.

Thus, the average yield is different from 880 tons.

p –Value Approach

What is the probability that this test statistic or something even more extreme (far from what is expected if H_0 is true) could have happened *just by chance*? 2 tailed → P X 2

$$
\begin{aligned}
p-\text{value} &= P(z<-3.03)+P(z>3.03) \\
&= 2P(z<-3.03) = 2(0.0012) = 0.0024
\end{aligned}
$$



Since our *p*-value = 0.0024 is less than α=0.01, we reject H_0 and conclude that the average yield has changed.

Class Activity

2. An instructor of mathematics claims that his students do not like the course he teaches so that they spend less that 5 hours on average studying for the final exam. To test the claim, he takes a sample of 80 students. The mean and standard deviation of time spent are 4.7 and 1.2 hours, respectively. Test his claim at significance level $\alpha = 0.05$.

$H_0: \mu = 5 \quad H_1: \mu < 5$

$$z = \frac{4.7 - 5}{1.2/\sqrt{80}} = -2.236$$

-1.645

reject H_0,

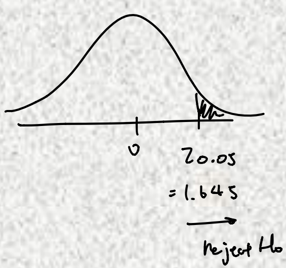
there is sufficient evidence

that.....

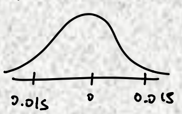
Class Activity

3. For each of the following find the appropriate rejection region(s) for the large-sample test statistic z.
   a) A right-tailed test with $\alpha = 0.05$
   b) A left-tailed test with $\alpha = 0.05$
   c) A two-tailed test with 3% significance level

a)



c) $\alpha = 0.03$



$z = \pm 2.17$

Class Activity

4. Test $H_0 : \mu = 30$ vs. $H_a : \mu > 30$ using the *p*-value approach. Given that a sample of $n = 36$ yields $\bar{x} = 32$, and $S = 5$.

Answer: $Z = 2.4$; *p*-value = 0.0082
Reject the null hypothesis.

Class Activity

5. For the *p*-value for the following large-sample z test:
   a) A left-tailed test with observed $z = -2.14$
   b) A two-tailed test with observed $z = -2.14$

Answer

5a) *p*-value = 0.0162

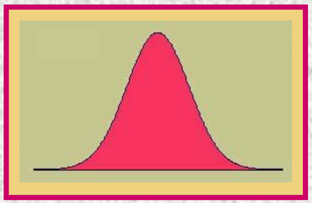
Statistical Significance

- The critical value approach and the *p*-value approach produce identical results.
- The *p*-value approach is often preferred because
  - Computer printouts usually calculate *p*-values
  - You can evaluate the test results at any significance level you choose.
- What should you do if you are the experimenter and no one gives you a significance level to use?

Statistical Significance

- If the *p*-value is less than **0.01**, reject H_0. The results are **highly significant.**
- If the *p*-value is between **0.01** and **0.05**, reject H_0. The results are **statistically significant.**
- If the *p*-value is between **0.05** and **0.10**, do not reject H_0. But, the results are **tending towards significance.**
- If the *p*-value is greater than **0.10**, do not reject H_0. The results are **not statistically significant.**

Other Large Sample Tests



- There were three other statistics in Chapter 8 that we used to estimate population parameters.
- These statistics had approximately normal distributions when the sample size(s) was large.
- These same statistics can be used to test hypotheses about those parameters, using the general test statistic:

$$z = \frac{\text{statistic} - \text{hypothesized value}}{\text{standard error of statistic}}$$

A Large-Sample Test for the Difference between Two Population Means

- Let D_0 be a hypothesized value of $\mu_1 - \mu_2$.
- A **one tailed** test:
  1) $H_0: \mu_1 - \mu_2 = D_0$ vs. $H_a: \mu_1 - \mu_2 > D_0$
     (right-tailed test)
  2) $H_0: \mu_1 - \mu_2 = D_0$ vs. $H_a: \mu_1 - \mu_2 < D_0$
     (left-tailed test)
- A **two-tailed** test:
  3) $H_0: \mu_1 - \mu_2 = D_0$ vs. $H_a: \mu_1 - \mu_2 \neq D_0$

A Large-Sample Test for the Difference between Two Population Means

- If $D_0 = 0$, then

1) $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 > 0$

   To test whether or not μ_1 is greater than μ_2.

2) $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 < 0$

   To test whether or not μ_1 is less than μ_2.

3) $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 \neq 0$

   To test whether or not there exists a difference between μ_1 and μ_2.

Example 4

Average Daily Intakes	Men	Women
Sample size	50	50
Sample mean	756	762
Sample Std Dev	35	30

- Is there a difference in the average daily intakes of dairy products for men versus women? Use $\alpha = .05$.

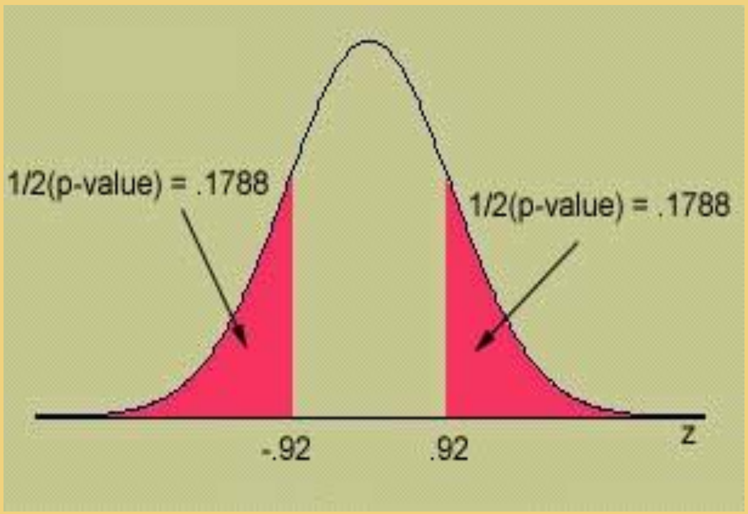
H_0: $\mu_m - \mu_w = 0$ (same) vs. H_a: $\mu_m - \mu_w \neq 0$ (different)

Test Statistic:

$$z \approx \frac{\bar{x}_m - \bar{x}_w - 0}{\sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}} = \frac{756 - 762}{\sqrt{\frac{35^2}{50} + \frac{30^2}{50}}} = -0.92$$

p-Value Approach

$$
\begin{aligned}
p-\text{value} &= P(z<-0.92)+P(z>0.92) \\
&= 2(0.1788) \quad = \ 0.3576
\end{aligned}
$$



Since p-value > $\alpha = 0.05$, H_0 is not rejected.

There is insufficient evidence to indicate that men and women have different average daily intakes of dairy products.

Class Activity

6. A study was conducted to compare the mean numbers of police emergency calls per 8-hour shift in two districts of a large city. Samples of 100 8-hour shifts were randomly selected from the police records for each of the two regions, and the number of emergency calls was recorded for each shift. The sample statistics are listed here:

Region	1	2
Sample Size	100	100
Sample Mean	2.4	3.1
Sample Variance	1.44	2.64

Test whether Region 2 gets more calls on the average than Region 1 . Test at 0.04 significance level.

$H_0: \mu_2 = \mu_1$ $\alpha = 0.04$ $z = \frac{2.4 - 3.1}{\sqrt{\frac{1.44}{100} + \frac{2.64}{100}}} = -2.33$, p-value = 0.0003

$H_a: \mu_2 > \mu_1$

A Large-Sample Test for a Binomial Proportion

- Let p_0 be a specified value, and $0 < p_0 < 1$

1) $H_0: p = p_0$ vs. $H_a: p > p_0$

To test whether or not p is greater than p_0.

2) $H_0: p = p_0$ vs. $H_a: p < p_0$

To test whether or not p is less than p_0.

3) $H_0: p = p_0$ vs. $H_a: p \neq p_0$

To test whether or not p is different from p_0

Example 5: A Test for *p*

- Regardless of age, about 20% of American adults participate in fitness activities at least twice a week. Is there evidence of a decline in participation after age 40? A random sample of 100 adults over 40 years old found 15 who exercised at least twice a week. Perform a statistical test at $\alpha = 0.05$.

$$H_0 : p = 0.2$$
$$H_a : p < 0.2$$

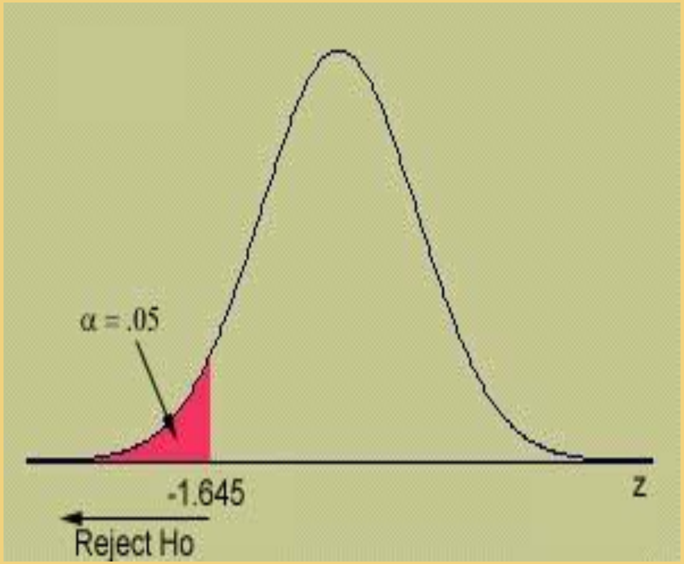
Test Statistic:

$$z \approx \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.15 - 0.2}{\sqrt{\frac{0.2(0.8)}{100}}} = -1.25$$

Critical Value Approach

What is the critical value of z that cuts off exactly α= .05 in the left-tail of the z distribution?

Rejection Region: Reject H_0 if $z < -1.645$.



For our example, $z = -1.25$ does not fall in the rejection region and H_0 is not rejected.

There is not enough evidence to indicate that p is less than 0.2 for people over 40 years old.

Class Activity

7. It is believed that a new insect spray, type A, is 55% effective. That is it can kill at least 55% of insects for each 30-second spray. Two hundred insects are released into a room, and after 1 hour there are 120 dead insects.
   a) Test for effectiveness of spray using the *p-value* approach.
   b) Is there any assumption of the test statistic you chose in Part (a)?

A Large-Sample Test for $p_1 - p_2$

1) $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 > 0$

Or $H_0: p_1 = p_2$ vs. $H_a: p_1 > p_2$

To test whether or not p_1 is greater than p_2.

2) $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 < 0$

To test whether or not p_1 is less than p_2.

3) $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 \neq 0$

To test whether or not there exists a difference between p_1 and p_2.

Example 6: A Test for $p_1 - p_2$

Obesity	Male (1)	Female (2)
Sample size	80	70
# of obese students	65	39

- Do male students have a higher percentage of obesity than female students? Eighty male students and 70 females students are randomly selected. The results are shown in the table. To answer the question test hypothesis at 0.03 significance level

$\hat{p}_2 = 39/70 = .56$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{104}{150} = .69$$

Example 6: A Test for $p_1 - p_2$

Obesity	Male (1)	Female (2)
Sample size	80	70
# of obese students	65	39

$H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 > 0$

Calculate

$$\hat{p}_1 = \frac{65}{80} = 0.81 \qquad \hat{p}_2 = \frac{39}{70} = 0.56$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{104}{150} = 0.69$$

Example 6: A Test for $p_1 - p_2$

Obesity	Male (1)	Female (2)
Sample size	80	70
# of obese students	65	39

Test Statistic :

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.81 - 0.56}{\sqrt{0.69(0.31)\left(\frac{1}{80} + \frac{1}{70}\right)}} = 3.30$$

$$p\text{-value} = P(z > 3.30) = 0.0005$$

Since the p-value is less than $\alpha = .02$, H_0 is rejected. The results are highly significant. There is evidence to indicate that the percentage of obesity in male students is higher than that of female students.

Class Activity

8. A new insect spray, type A, is to be compared with a spray, type B, that is currently in use. Two rooms of equal size are sprayed with the same amount of spray, one room with A, the other with B. Two hundred insects are released into each room, and after 1 hour the numbers of dead insects are counted. There are 120 dead insects in the room sprayed with A and 90 in the room sprayed with B.

   Do the data provide enough evidence to indicate that spray A is more effective than spray B? Write the null and alternative hypotheses and test at significance level $\alpha = 0.05$.