Introduction to Probability and Statistics Eleventh Edition

Chapter 4

Probability and Probability Distributions

What is Probability?

- In Chapters 2 and 3, we used graphs and numerical measures to describe data sets which were usually **samples.**
- We measured "how often" using

Relative frequency = f/n

- As the sample size, n gets large (or $n \to \infty$),

Sample ⟶ **Population**
And "How often"
= Relative frequency ⟶ **Probability**

Basic Concepts

- An **experiment** is the process by which an observation (measurement or outcome) is obtained.

Example: Tossing a coin. There are 2 possible outcomes, head or tail.

Example: The opinions (agree/disagree) of voters concerning a new VAT is an observation of an experiment.

Basic Concepts

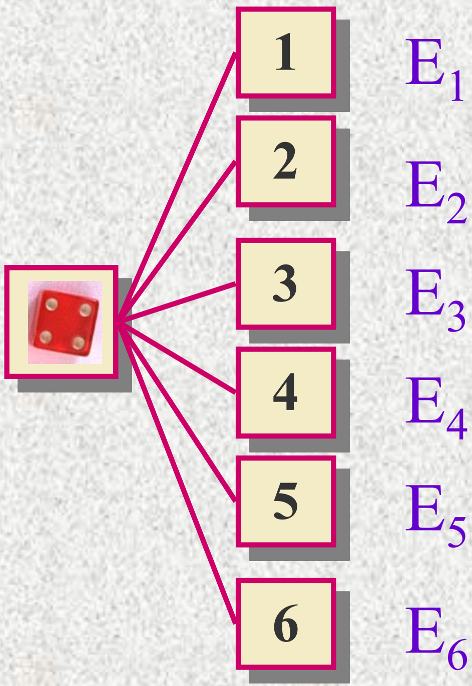
- A **simple event** is the outcome that is observed on a single repetition of the experiment.

  If there are *n* simple events, each will be denoted by E_i, $i = 1, 2, 3, \ldots, n$

- The set of all simple events is called the **sample space, *S***.

Example 1

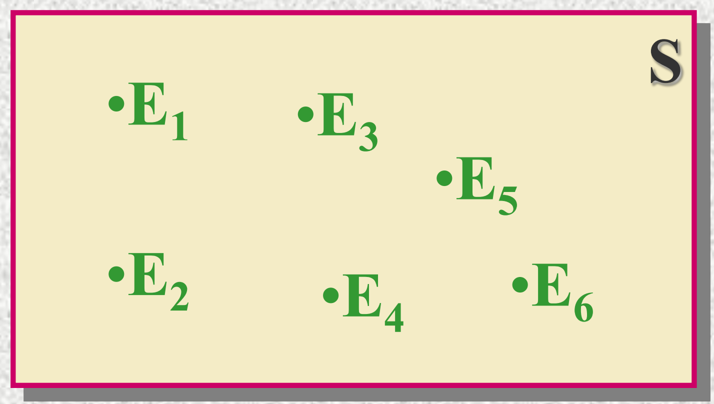
- **Experiment: Toss a die**
- Simple events:



Sample space:

$\boldsymbol{S} = \{E_1, E_2, E_3, E_4, E_5, E_6\}$

or $\boldsymbol{S} = \{1, 2, 3, 4, 5, 6\}$



Basic Concepts

- An **event** is a collection of simple events.

- **Back to Example 1: Toss a die**
  - Define the events *A* and *B* as

    A: Observe an odd number

    B: Observe a number greater than 2

Find *A* and *B*.

Example 1

- **Toss a die:**

  A: Observe an odd number

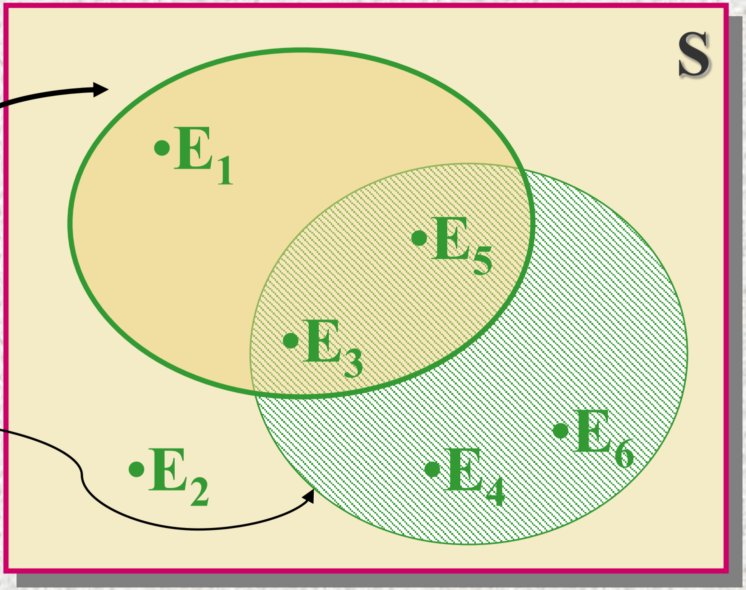
  B: Observe a number greater than 2

$A = \{E_1, E_3, E_5\}$

or $A = \{1, 3, 5\}$

$B = \{E_3, E_4, E_5, E_6\}$

or $B = \{3, 4, 5, 6\}$



Example 2

Experiment: Toss a coin.

The sample space ***S***, of possible outcomes when a coin is tossed, is

$$S = \{h, t\}$$

where **h** and **t** correspond to "head" and "tail", respectively, or

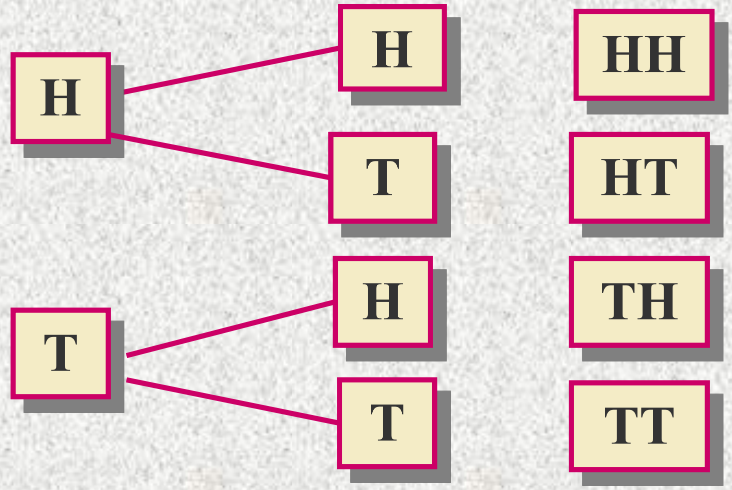
$$S = \{E_1, E_2\}$$

where $E_1 = h$ and $E_2 = t$.

Example 3 (a tree diagram)

- Toss two coins and record the outcome. Find the sample space and *A*, if *A* is the event that one head is observed.

1st Coin 2nd Coin E_i



$S = \{HH, HT, TH, TT\}$

or $S = \{E_1, E_2, E_3, E_4\}$

$A = \{HT, TH\}$

or $A = \{E_2, E_3\}$

Basic Concepts

- Two events are **mutually exclusive,** or **disjoint,** if when one event occurs, the other cannot, and vice versa.

- **Experiment: Tossing a die**
  - A: observe an odd number
  - B: observe a number greater than 2
  - C: observe a 6
  - D: observe a 3

Not Mutually Exclusive

Mutually Exclusive

B and C?
B and D?

The Probability of an Event

- Probability Rules

1. If A is an event, the $0 \leq P(A) \leq 1$.

   If A can never occur, $P(A)=0$.

   If A always occur, $P(A)=1$.

2. The sum of the probabilities for all simple events, in S equals 1.

$$\boxed{\sum_{i=1}^{n} P(E_i) = 1}$$

The Probability of an Event

- **Definition**
  - The **probability of an event** A is equal to the sum of the probabilities of the simple events contained in A.

If A contains E_1, E_2, …, and E_k , then

$$\boxed{P(A) = P(E_1) + P(E_2) + \cdots + P(E_k)}$$

Finding Probabilities

- Probabilities can be found using
  - Estimates from empirical studies
  - Common sense estimates based on equally likely events.

- **Examples:**
  - Toss a fair coin. P(Head) = 1/2
  - 10% of the U.S. population has red hair. Select a person at random. P(Red hair) = .10

Finding Probabilities

Suppose that an experiment involves N simple events and all the simple events are equally likely.

Then each simple event has the probability $1/N$, and **the probability of an event A** can be calculated as

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in } A}{\text{total number of simple events}}$$

Example 4

- Toss a fair coin twice. What is the probability of observing at least one head?

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

P(at least 1 head)

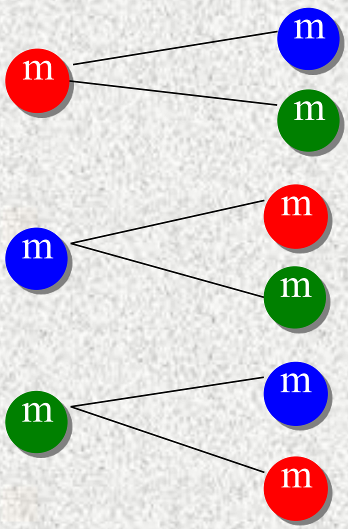
$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

Example 5

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms one at a time at random. What is the probability that at least one is red?

1st M&M 2nd M&M



E_i	$P(E_i)$
RB	1/6
RG	1/6
BR	1/6
BG	1/6
GB	1/6
GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Class Activity

1. A sample space S consists of five simple events, $S = \{E_1, E_2, E_3, E_4, E_5\}$, with these probabilities:

   $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$, $P(E_4) = 2\ P(E_5)$

   a) Find the probabilities for E_4 and E_5.

   b) Find the probabilities for these two events:

   $$A = \{E_1, E_3, E_4\}, \text{ and } B = \{E_2, E_3\}$$

   c) List the simple events that are in A, B or both.

   d) List the simple events that are in both A and B.

Class Activity

2. **Three coins.** A jar contains three coins: 1-baht, 5-baht, and 10-baht. Two coins are randomly selected one at a time without replacement.
   a) List the simple events in *S*.
   b) What is the probability that the selection will contain the 1-baht coin?
   c) What is the probability that the total amount drawn will equal 12 baht or more?

Counting Rules

- If the simple events in an experiment are **equally likely**, you can calculate

$$P(A)=\frac{n_A}{N}=\frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

- You can use **counting rules** to find n_A and N.

The *mn* Rule

- If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment.
- This rule is easily extended to ***k*** stages, with the number of ways equal to

$$n_1 n_2 n_3 \ldots n_k$$

Example 6: Toss two coins. The total number of simple events is:

$$\boxed{2 \times 2 = 4}$$

Example 7: Toss three coins. The total number of simple events is

$$2 \times 2 \times 2 = 8$$

Example 8: Toss two dice. The total number of simple events is

$$6 \times 6 = 36$$

Example 9: Three M&Ms are drawn one at a time without replacement from a dish containing ten candies. All candies have different colors. The total number of simple events is

$$10 \times 9 \times 8 = 720$$

Permutations (or Orderings)

- The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)...(2)(1)$ and $0! \equiv 1$.

Example 10: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4, assuming that the numbers cannot be repeated?

The order of the choice is important!

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Example 11

A lock consists of five parts and can be assembled in any order. All parts are different. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

Combinations

- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is

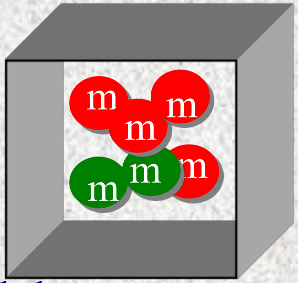
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example 12: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

The order of the choice is not important!

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example 13



- A box contains six M&Ms®, four red and two green, with different sizes. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

$4 \times 2 = 8$ ways to choose 1 red and 1 green M&M.

P(exactly one red) = 8/15

Class Activity

3. **100-Meter Run.** Three equally qualified runners, John, Peter, and Dave, run a 100-meter sprint, and the order of finish is recorded.
   a) How many simple events are in the sample space? $3P3 = 6$
   b) If the runners are equally qualified, what probability should be assigned to each simple event? $1/6$
   c) What is the probability that Dave wins the race?
   d) What is the probability that Dave finishes last?

$2/6$

$2/6$

Class Activity

4. **What to wear?** You own 4 pairs of jeans, 12 clean T-shirts, and 4 wearable pairs of sneakers. How many outfits (jean, T-shirt, and sneakers) can you create? $4 \times 12 \times 4 = 192$
5. **Traffic Problems.** Three city council members are to be selected from a total of seven to form a subcommittee to study the city's traffic problems.
   a) How many different subcommittees are possible? $_7C_3 = 35$
   b) If all possible council members have an equal chance of being selected, what is the probability that members Smith and Jones are both selected? $_5C_1$

leave out S&J

pick 3rd member

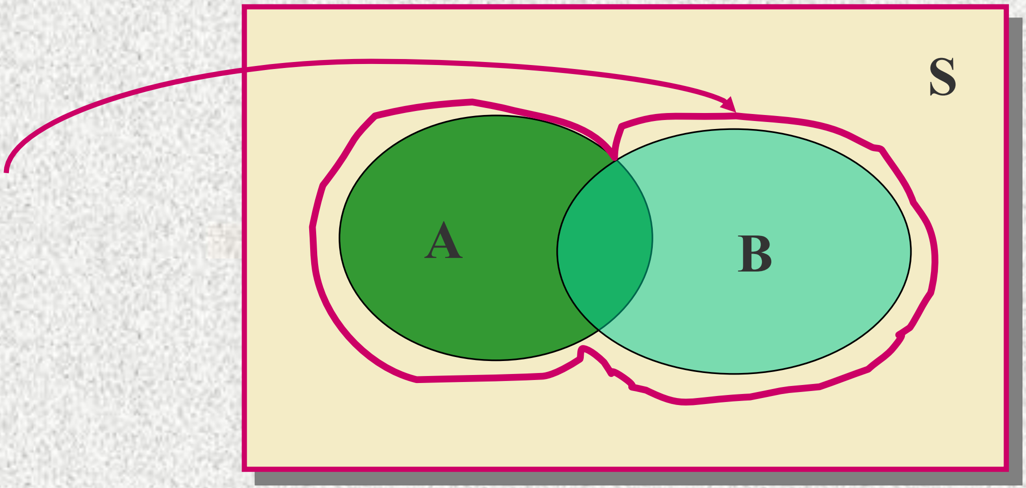
Class Activity

6. **Dice.** Three fair dice are tossed.
   a) How many simple events are in the sample space? $6^3 = 216$
   b) What is the probability that the third die shows a three? $36/216 = 1/6$
   c) What is the probability that the first and the third dice are different? $6 \times 6 \times 5$
   d) What is the probability that all 3 faces show different numbers. $6 \times 5 \times 4$

Event Relations

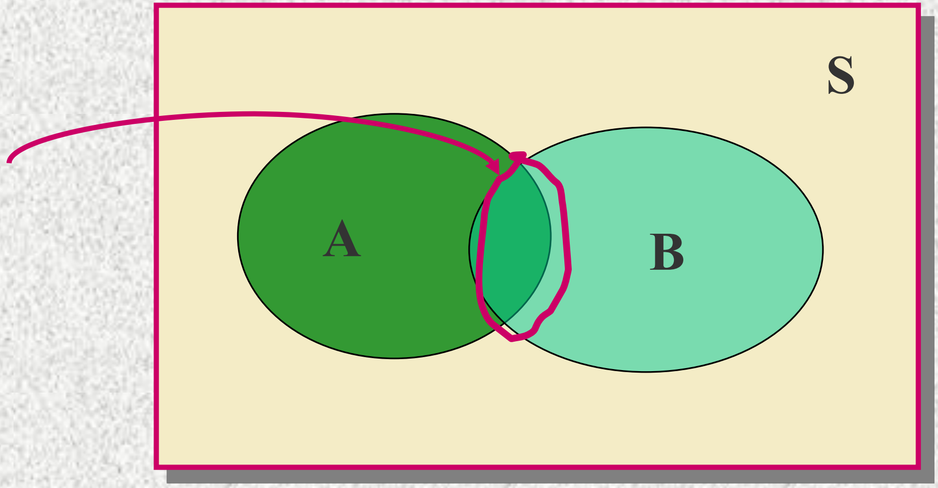
- The **union** of two events, A and B, is the event that either A **or** B **or both** occur when the experiment is performed. We write

$$A \cup B$$



Event Relations

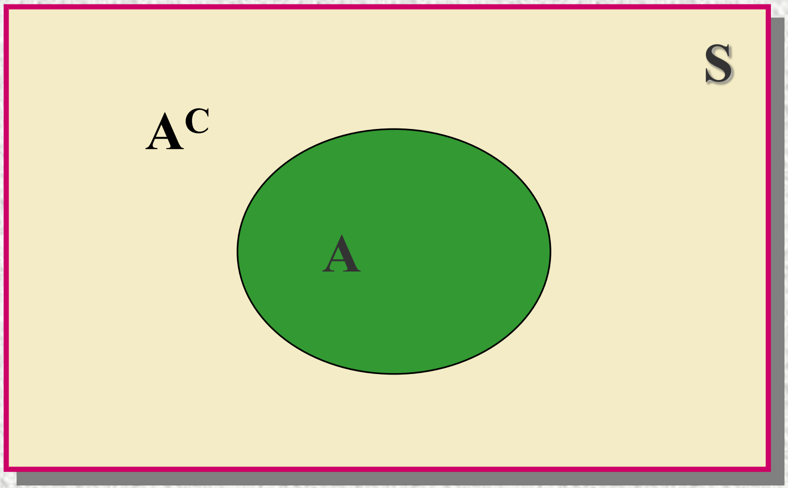
- The **intersection** of *two* events, **A** and **B**, is the event that both A **and** B occur when the experiment is performed. We write **A $\cap$ B.**



- If two events A and B are **mutually exclusive**, then $P(A \cap B) = 0$.

Event Relations

- The **complement** of an event **A** consists of all outcomes of the experiment that do not result in event A. We write $\mathbf{A^C}$.



Example 14

Select a student at random from a classroom and record his/her **hair color** and **gender**.

– **B:** student has brown hair
– **F:** student is female
– **M:** student is male

Mutually exclusive events; $F = M^C$

- What are these events, $\mathbf{B^C}$, $\mathbf{F \cap M}$ and $\mathbf{F \cup M}$?
- $\mathbf{B^C}$ **Student does not have brown hair**
- $\mathbf{F \cap M}$ **Student is both female and male =** $\varnothing$
- $\mathbf{F \cup M}$ **Student is either female or male = all students = S**

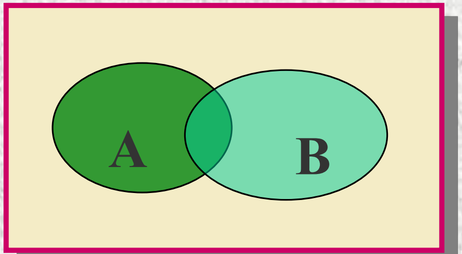
Calculating Probabilities for Unions and Complements

There are special rules that will allow you to calculate probabilities for composite events.

The Additive Rule for Unions:

For any two events, ***A*** and ***B***, the probability of their union, **P(*A*∪*B*)**, is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Example 15 : Additive Rule

Suppose that there are 110 students in a classroom, and that they could be classified as follows:

B: brown hair

$P(B) = 40/110$

F: female

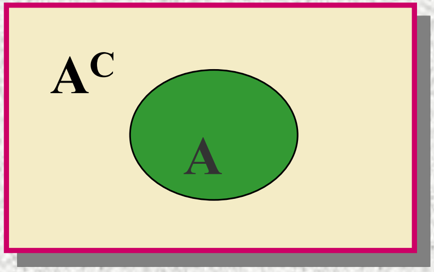
$P(F) = 60/110$

	Brown	Not Brown	Total
Male	10	40	50
Female	30	30	60

A student is selected a random, what is the probability that it is a female or has brown hair?

$$P(B \cup F) = P(B) + P(F) - P(B \cap F)$$
$$= 40/110 + 60/110 - 30/110$$
$$= 70/110$$
$$= 7/11$$

Calculating Probabilities for Complements



We know that for any event **A**:

– $P(A \cap A^C) = 0$

Since either **A** or **A^C** must occur,

$P(A \cup A^C) = 1$

so that $P(A \cup A^C) = P(A) + P(A^C) = 1$

$$\boxed{P(A^C) = 1 - P(A)}$$

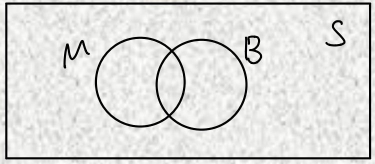
Class Activity

7. A survey of shoppers at a convenience grocery store showed that 70% purchase milk and 40% bread. If 25% of all the shoppers purchase both milk and bread, what is the probability that a randomly selected shopper
   a) will not purchase milk? 0.3
   b) will purchase at least one of these commodities?
   c) will purchase neither commodity? $P[(M \cup B)^c] = 0.15$

M = Milk
B = Bread

$P(M \cap B) = 0.25$

$P(M) = 0.7$
$P(B) = 0.4$



$P(M \cup B) = P(M) + P(B) - P(M \cap B)$
$= 0.85$

Class Activity

8. In an experiment to study the dependence of cancer and smoking habits, the following data were collected on 100 individuals:

	Cancer	No cancer	Total
Smoker	17	13	30
Nonsmoker	3	67	70
Total	20	80	100

If one of these individuals is selected at random, find the probability that the person

a) is a smoker
b) has cancer
c) is a nonsmoker and has no cancer
d) is a nonsmoker or has no cancer

Conditional Probabilities

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0$$

"given"

Example 15 (cont.)

Select a student at random from the classroom.

Define events:

M: male

B: brown hair

	Brown	Not Brown	Total
Male	10	40	50
Female	30	30	60
Total	40	70	110

If a male student is selected at random, then what is the probability that he has brown hair?

$$P(B/M) = 10/50 = 1/5$$

If a student with brown hair is selected at random, then what is the probability that it is a male?

$$P(M/B) = 10/40 = 1/4$$

Defining Independence

Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

$$P(A|B) = P(A|B^c) = P(A)$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$P(A \cap B) = P(A)\,P(B \text{ given that } A \text{ occurred}) = P(A)P(B|A)$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$P(A \cap B) = P(A)\,P(B)$$

Example 16

- Toss a fair coin twice. Define events
  - S: head on second toss
  - F: head on first toss

HH	1/4
HT	1/4
TH	1/4
TT	1/4

$P(S|F) = 1/2$

$P(S) = 2/4 = 1/2$

$P(S|F) = P(S)$ → Events S and F are independent!

P(S) does not change, whether F happens or not…

Example 19

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Two people are randomly selected from this population. What is the probability that exactly one of the two is high risk? Assume that being high risk of any people does not depend on the others.

Define H_i: i^{th} person has high risk.

N_i: i^{th} person does not have high risk.

$$\begin{aligned}
P(\text{exactly one high risk}) &= P(H_1 \cap N_2) + P(N_1 \cap H_2) \\
&= P(H_1)P(N_2) + P(N_1)P(H_2) \\
&= (0.1)(0.9) + (0.9)(0.1) \\
&= 0.18
\end{aligned}$$

Example 19 (cont.)

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a female and has a high risk?

Define H: high risk F: female

From the example, $P(F) = 0.49$ and $P(H|F) = 0.08$.

Use the Multiplicative Rule:

$$\begin{aligned} P(\text{high risk female}) &= P(H \cap F) \\ &= P(F)P(H|F) \\ &= 0.49(0.08) = 0.0392 \end{aligned}$$

Class Activity

9. Suppose that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$
   a) Find $P(A|B)$ and $P(B|A)$.
   b) Are events A and B mutually exclusive?
   c) Are events A and B independent?

Class Activity

10. In an experiment to study the dependence of cancer and smoking habits, the following data were collected on 100 individuals:

	Cancer	No cancer	Total
Smoker	17	13	30
Nonsmoker	3	67	70
Total	20	80	100

An individual is selected at random.

a) Find the probability that he is a smoker, given that he has cancer.
b) Find the probability that a nonsmoker is experiencing cancer
c) Are "smoking" and "having cancer" independent? Mutually exclusive?

Class Activity

11) Cast a die twice. Let A ={1 or 2 on the first roll}, and B ={2, 3, or 4 on the second roll}.

a) Find $P(A \cap B)$ and $P(A|B)$.

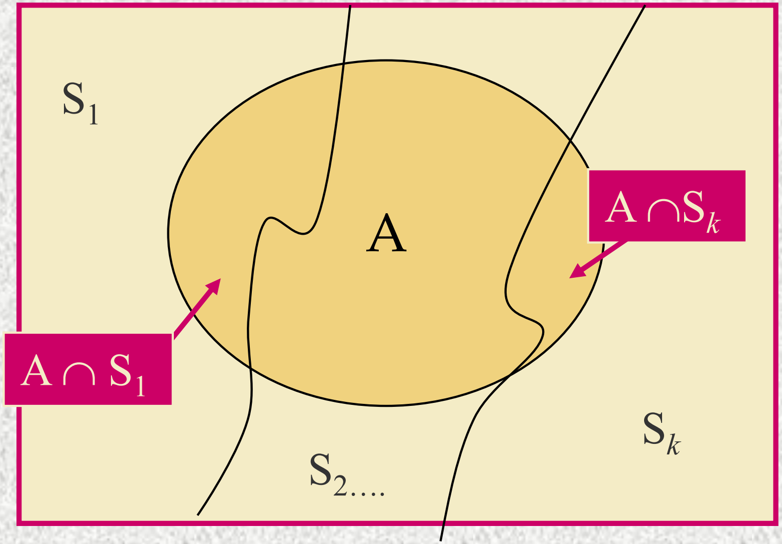
b) Are A and B independent events?

The Law of Total Probability

- Let S_1 , S_2 , S_3 ,..., S_k be mutually exclusive and exhaustive events (that is, one and only one must happen). Then the probability of another event A can be written as

$$P(A) = P(A \cap S_1) + P(A \cap S_2) + \ldots + P(A \cap S_k)$$
$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)$$

The Law of Total Probability



$$P(A) = P(A \cap S_1) + P(A \cap S_2) + \ldots + P(A \cap S_k)$$

$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)$$

Bayes' Rule

- Let S_1 , S_2 , S_3 ,...., S_k be mutually exclusive and exhaustive events with prior probabilities $P(S_1)$, $P(S_2)$,….,$P(S_k)$. If an event A occurs, the posterior probability of S_i, given that A occurred is

$$P(S_i \mid A) = \frac{P(S_i)P(A \mid S_i)}{\sum P(S_i)P(A \mid S_i)} \text{ for } i = 1, 2, \ldots k$$

Example 20

From a previous example, we know that 49% of the population are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. If a single person is selected at random and found to be high risk, what is the probability that it is a male?

Define **H: high risk** **F: female** **M: male**

We know:

P(F) =	.49
P(M) =	.51
P(H|F) =	.08
P(H|M) =	.12

$$P(M \mid H) = \frac{P(M)P(H \mid M)}{P(M)P(H \mid M) + P(F)P(H \mid F)}$$

$$= \frac{0.51(0.12)}{0.51(0.12) + 0.49(0.08)}$$

$$= 0.61$$

Class Activity/ Homework 5

12. **Worker Error.** A worker-operated machine produces a defective item with probability 0.01 if the worker follows the machine's operating instructions exactly, and with probability 0.03 if he does not. The worker follows the instructions 90% of the time. $P(D|I)=0.01, P(D|I^c)=0.03, P(I)=0.9$

   a) If a defective item is selected, what is the probability that it is produced when the worker did not follow the instructions. $P(I^c|D)=\frac{P(I^c\cap D)}{P(D)}=\frac{P(I^c)P(D|I^c)}{P(D\cap I)+P(D\cap I^c)}$

   b) what is the proportion of all items produced by the machine will be defective?

$$=\frac{P(I^c)P(D|I^c)}{P(I)P(D|I)+P(I^c)P(D|I^c)}$$

$= b)$

a) $=\frac{0.003}{0.012}=\frac{1}{4}$

Answers: a) 0.25, b) 0.012

Class Activity/ Homework 5

14. Golfing. Player A has entered a golf tournament but it is not certain whether player B will enter. Player A has probability 1/6 of winning the tournament if player B enters and probability 3/4 of winning if player B does not enter the tournament. Let the probability that player B enters the tournament be 1/3. $P(A|B) = 1/6$, $P(A|B^c) = 3/4$, $P(B) = 1/3 \rightarrow P(B^c) = 2/3$

a) Find the probability that player A wins the tournament.

$B = 1/3 - A|B = 1/6 \quad 1/6(1/3) = \frac{1}{18}$

$B^c = 2/3 - A|B^c = 3/4 \quad 2/3(3/4) = \frac{6}{12}$

$\frac{1}{18} + \frac{6}{12} = \frac{1}{18} + \frac{1}{2} = \frac{1}{18} + \frac{9}{18} = \frac{10}{18} = 5/9$

b) Given that player A wins, what is the probability that player B enters the tournament?

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)P(A|B)}{P(A)} = \frac{1/3(1/6)}{5/9} = \frac{1/18}{10/18} = \frac{1}{10} = 0.1$$

Answers: a) 5/9, b) 0.12

Random Variables

- A quantitative variable x is a **random variable** if the value that it assumes, corresponding to the outcome of an experiment, is a chance or random event.
- Random variables can be **discrete** or **continuous.**

Examples:

✓ x = SAT score for a randomly selected student

✓ x = number of people in a room at a randomly selected time of day

✓ x = number on the upper face of a randomly tossed die

Probability Distributions for Discrete Random Variables

- The **probability distribution for a discrete random variable *x*** resembles the relative frequency distributions we constructed in Chapter 1. It is a graph, table or formula that gives the possible values of x and the probability $p(x)$ associated with each value.

We must have

$$0 \le p(x) \le 1 \text{ and } \sum p(x) = 1$$

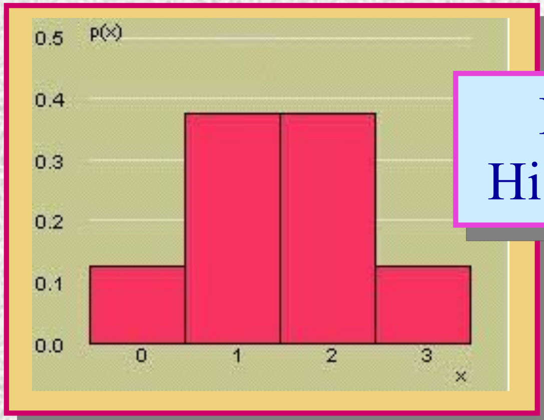
Example 21

Toss a fair coin 3 times and define x = **number of heads.**

		x
HHH	1/8	3
HHT	1/8	2
HTH	1/8	2
THH	1/8	2
HTT	1/8	1
THT	1/8	1
TTH	1/8	1
TTT	1/8	0

$P(x = 0) = 1/8$
$P(x = 1) = 3/8$
$P(x = 2) = 3/8$
$P(x = 3) = 1/8$

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8



Probability Histogram for x

Probability Distributions

- Probability distributions can be used to describe the population, just as we described samples in Chapter 1.
  - **Shape:** Symmetric, skewed, mound-shaped…
  - **Outliers:** unusual or unlikely measurements
  - **Center and spread:** mean and standard deviation. A population mean is called μ and a population standard deviation is called σ.

The Mean of x

- Let x be a discrete random variable with probability distribution $p(x)$. Then the **mean of x** or the **expected value of x**, is denoted by μ or $E(x)$, and is given as

$$\boxed{\mu = E(x) = \sum_x xp(x)}$$

- The mean is a value that you would expect to observe on average if the experiment is repeated over and over.

The Variance and Standard Deviation

- Let x be a discrete random variable with probability distribution $p(x)$. Then the variance of x is given as

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum_x (x-\mu)^2 p(x)$$

- The standard deviation of x is

$$\sigma = \sqrt{\sigma^2}$$

Example 21

- Toss a fair coin 3 times and record x the number of heads.

x	$p(x)$	$xp(x)$	$(x-\mu)^2p(x)$
0	1/8 TTT	0	$(-1.5)^2(1/8)$
1	3/8 HTT THT TTH	3/8	$(-0.5)^2(3/8)$
2	3/8 HHT HTH THH	6/8	$(0.5)^2(3/8)$
3	1/8 HHH	3/8	$(1.5)^2(1/8)$

$$\mu = \sum xp(x) = \frac{12}{8} = 1.5$$

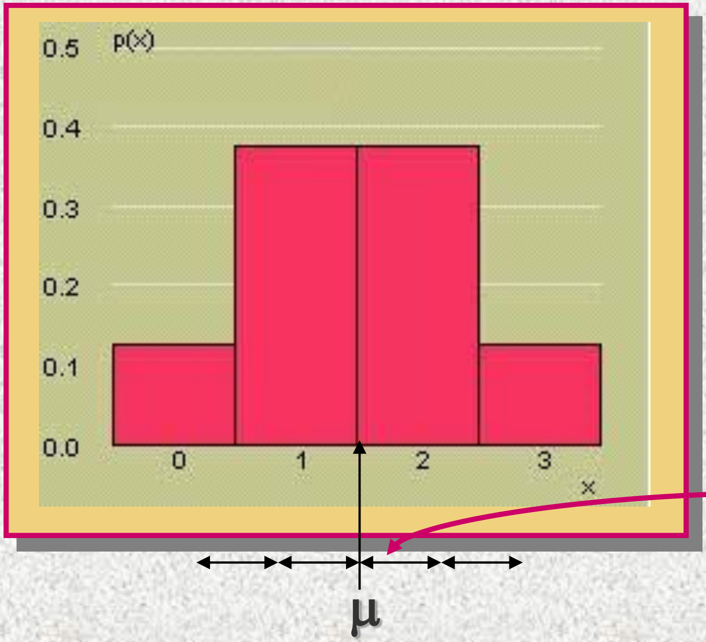
$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$\sigma^2 = .28125 + .09375 + .09375 + .28125 = .75$$

$$\sigma = \sqrt{.75} = .866$$

Example 21

The probability distribution for *x* the number of heads in tossing 3 fair coins.



- Shape? — Symmetric; mound-shaped
- Outliers? — None
- Center? — $\mu = 1.5$
- Spread? — $\sigma = .688$

Class Activity

15. A survey at a school asked students if they were ill with a cold or the flu last year. Let x be the number of times that they were ill with the following probability distribution:

x	0	1	2	5
Probability, $P(x)$	0.6	0.1	0.2	0.1

a) Find the probability that a student had cold/flu 3 times. 0

b) Find the probability that a student had cold/flu less than 4 times. 0.9

c) Find the mean of x. $0.1+0.4+0.5=1$

d) Find the standard deviation of x. $\Sigma(x-\mu)^2P(x)$

$= 0.6+0.2+0.8$

Answers: c) 1 time, d) 1.55 times

Class Activity

16. Gender Bias? A company has five applicants for two positions: two women and three men. Suppose that the five applicants are equally qualified and that no preference is given for choosing either gender.

Let x equal the number of women chosen to fill the two positions.

a) Find $p(x)$.

b) Find the mean and standard deviation of x.

0.8

a) $\dfrac{C^2_k C^3_{2-k}}{C^5_2}$ for $k=0, 1,$ and 2

b) $\mu = \left(\frac{2}{5}\right)2 = \frac{4}{5} = 0.8$