Chapter 10

Inference from Small Sample

Small-Sample hypothesis Test for μ

Assumption : The sample is randomly selected from a normally distributed population.

1. Null hypothesis : $H_0 : \mu = \mu_0$

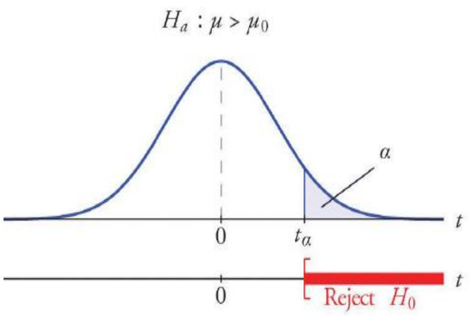
2. Alternative hypothesis :

One-Tailed Test	Two-Tailed Test
$H_a : \mu > \mu_0$	$H_a : \mu \neq \mu_0$
$H_a : \mu < \mu_0$	

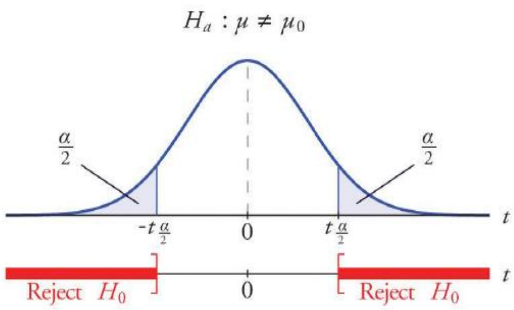
3. Test statistic : $t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ with (n-1) degrees of freedom

4. Rejection region : Reject H_0 when

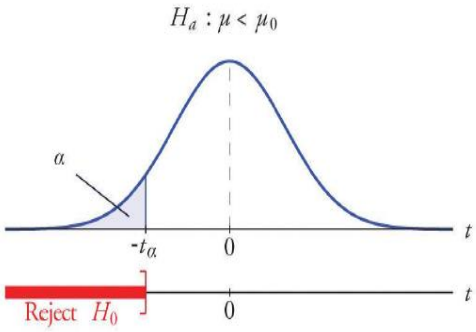
$t > t_\alpha$



$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$



$t < -t_\alpha$



The critical values t_α and $t_{\alpha/2}$ are based on (n-1) degrees of freedom.

Small-Sample Statistical Test for $\mu_1 - \mu_2$

Assumptions : 1. The samples are randomly and *independently* selected from *normally* distributed populations.

2. The variances of the populations σ_1^2 and σ_2^2 are equal.

1. Null hypothesis : $H_0 : \mu_1 - \mu_2 = D_0$,
   where D_0 is some specified difference that you wish to test. For many tests, you will hypothesize that there is no difference between μ_1 and μ_2.

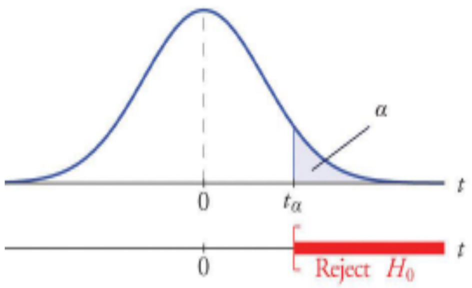
2. Alternative hypothesis :

One-Tailed Test	Two-Tailed Test
$H_a : \mu_1 - \mu_2 > D_0$	$H_a : \mu_1 - \mu_2 \neq D_0$
$H_a : \mu_1 - \mu_2 < D_0$	

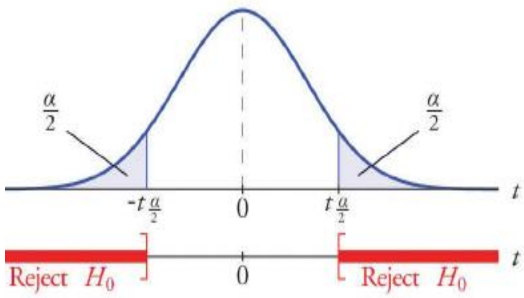
3. Test statistic : $$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

4. Rejection region : Reject H_0 when

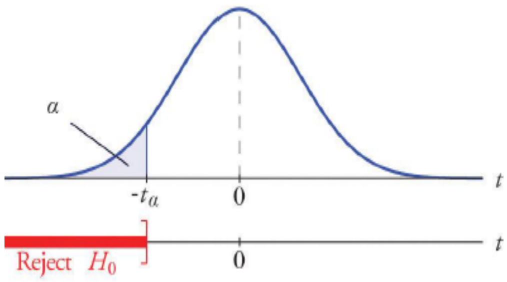
$t > t_\alpha$



$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$



$t < -t_\alpha$



The critical values t_α and $t_{\alpha/2}$ are based on $(n_1 + n_2 - 2)$ degrees of freedom.

Small-Sample Statistical Test for $\mu_1 - \mu_2 = \mu_d$: Dependent Samples

Assumptions : The experiment is designed as a paired-difference test so that the n differences represent a random sample from a normal population.

1. Null hypothesis : $H_0 : \mu_d = 0$

2. Alternative hypothesis :

One-Tailed Test	Two-Tailed Test
$H_a : \mu_d > 0$	$H_a : \mu_d \neq 0$
$H_a : \mu_d < 0$	

3. Test statistic : $t = \dfrac{\bar{d} - 0}{s_d / \sqrt{n}} = \dfrac{\bar{d}}{s_d / \sqrt{n}}$

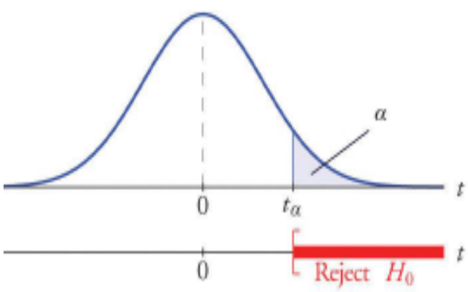
where n = Number of paired differences

$\bar{d}$ = Mean of the sample differences

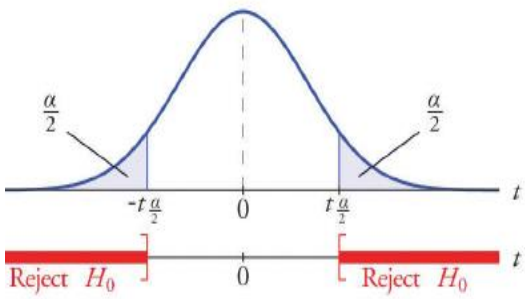
s_d = Standard deviation of the sample differences

4. Rejection region : Reject H_0 when

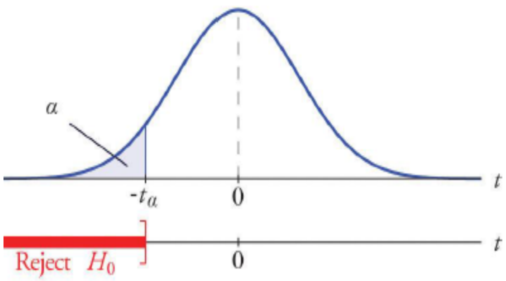
$t > t_\alpha$



$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$



$t < -t_\alpha$



The critical values t_α and $t_{\alpha/2}$ are based on $(n-1)$ degrees of freedom.