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ICMA151 Final Exam Review

Review Problems

1. Give short answers.

1.1 If many samples are taken and a 90% confidence interval for p is constructed for each sample, then what percents of the intervals one can expect not to cover the unknown p?

1.2 A 90% confidence interval estimate for a population mean μ is determined to be 62.8 to 73.4. If the confidence level is reduced to 80%, would the confidence interval for μ become narrower or wider?

1.3 As the significance level α increases, would the probability of a Type I error increase? Would the size of the rejection region increase as well?

1.4 If a null hypothesis is rejected at the 0.05 level of significance, must it be rejected at the 0.01 level?

1.5 An instructor believes that on the average female students spend more time to study than the male students. To test her claim, the hypotheses are

H_0: ____________________ vs. H_a : ________________________

1.6 A candidate running for president of student committee believes that 58% of the voters are supporting him. His supporters believe this estimate maybe higher. The appropriate hypotheses to be tested by the supporters are

H_0: ____________________ vs. H_a : ________________________

1.7 Consider testing the hypotheses: $H_0: \mu = 10$ versus $H_a: \mu \neq 10$. If the value of the test statistic is $z = 1.24$, then what is the p-value of this test?

1.8 Consider testing the hypotheses: $H_0: p = 0.4$ versus $H_a: p < 0.4$. If a sample of n =100 shows that there are x = 30 successes, then what is the p-value of this test?

1.9 To test $H_0: \mu = 10$ versus $H_a: \mu \neq 10$ at 0.1 significance level. If a sample of size 10 is taken and the normal distribution can be assumed, then what is (are) the critical value(s) of the test?

2)

The diameters of Douglas firs grown at a Christmas tree farm are normally distributed with a mean of 4 inches and a standard deviation of 1.5 inches.

a) What proportion of the trees will have diameters between 3 and 5 inches?

b) What proportion of the trees will have diameters less than 3 inches?

(*An extra question*) What is the 70th percentile of the growth?

***NOTE to students*:** *For any problem involving hypothesis test, be sure that you state clearly what H_0 and H_a* (unless stated already in the problem), *test statistic, rule of decision and conclusion*.

3) Historically, the average time it takes Amy to swim the 200 meter butterfly is 148 seconds. Amy would like to know if her average time has changed. She records

her time on 50 randomly selected occasions and computes the mean to be 147.2 seconds with a standard deviation of 2 seconds.

3.1) Test $H_0 : \mu = 148$ vs. $H_a : \mu \neq 148$ at 0.05 level of significance.

3.2) Find a point estimate of μ.

3.3) Find a 99% confidence interval for μ.

3.4) A few days later Amy has a new hypothesis, she wants to convince her mom that her average time is better than her old record? What are H_0 and H_a?

4) The proportions of defective parts produced by machines 1 and 2, denoted by p_1 and p_2, were compared. The following data were collected.

Machine 1: $n = 100$, number of defective parts = 10
Machine 2: $n = 120$, number of defective parts = 6

a) Find a 90% confidence interval for p_1 .

b) From the result in part (a), is it reasonable to conclude that percentage of defective parts produced by machine 1 is 12%? Why or why not?

c) Test $H_0 : p_1 - p_2 = 0$ vs. $H_a : p_1 - p_2 \neq 0$ at a significance level of $\alpha = 0.03$. What is the conclusion?

5) A telephone company wants to advertise that more than 30% of all its customers have more than two telephones. To support this ad, the company selects a sample of 200 customers and finds that 70 have more than two telephones. Tests $H_0 : p = 0.3$ vs. $H_a : p > 0.3$ using 0.1 level of significance.

6) Suppose it is known that the mean blood glucose (mg/mL) in a group of diabetic rats treated with a drug is $\mu = 1.85$ mg/mL with $\sigma = 0.4$ mg/mL. Given a sample of 36 diabetic rats and are treated with drug. Approximate the probability that the sample mean will be less than 1.9 mg/mL.

7) The average total math scores were recorded for two groups of students: one group planning to major in Asian Studies and one group planning to major in African Studies.

Asian Studies (Group 1)	African Studies (Group 2)
$\bar{x}_1 = 80$ $s_1 = 12$ $n_1 = 7$	$\bar{x}_2 = 75$ $s_2 = 9$ $n_2 = 8$

Assume that the score of each group is normally distributed, the population variances are equal.

7.1) Do the data provide sufficient evidence to indicate the mean score of students planning to major in African Studies is less than the mean score of students planning to major in Asian Studies? Test using $\alpha = 0.10$.

7.2) Find a 95% confidence interval for the difference between the mean scores of these two groups of students. Based on the result, is there a difference **between the mean math scores among two groups of students.**

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8) A study of 392 healthy children living in the area of Tours, France, was designed to measure the serum levels of certain fat-soluble vitamins—namely, vitamin A, vitamin E, β-carotene (in micrograms per liter), and cholesterol (in rams per liter). Knowledge of the reference levels for children living in an industrial country, with normal food availability and feeding habits, would be important in establishing borderline levels for children living in other, less favorable conditions. Results of the study are shown in the table.

	Mean ±SD (*n* = 392)	**Boys (*n*= 207)**	**Girls (*n*= 185)**
Retinol (µg/dl)	42.5 ± 12.0	43.0 ± 13.0	41.8 ± 10.7
β-carotene (µg/l)	572 ± 381	588 ± 406	553 ± 350
Vitamin E (mg/l)	9.5 ± 2.5	9.6 ± 2.7	9.5 ± 2.2
Cholesterol (g/l)	1.84 ± .42	1.84 ± .47	1.83 ± .38
Vitamin E/cholesterol (mg/g)	5.26 ± 1.04	5.26 ± 1.11	5.26 ± .25

a. Find a 90% confidence interval for the difference in the mean serum cholesterol levels of boys versus girls.
b. Find a 95% confidence interval for the difference in the mean β-carotene levels of boys versus girls.
c. Do the intervals in parts a and b contain the value $(\mu_1 - \mu_2) = 0$? Why is this of interest to the researcher?
d. Based on part b, do the data indicate a difference between boys and girls in average β-carotene or serum cholesterol levels? Explain.
e. Do the data indicate a significant difference between the mean β-carotene levels for boys versus girls? Test the hypotheses at $\alpha = .01$.
f. Would the results of part e change if you had used $\alpha = .05$?

9. Six men compared two brands of razors. One side of the face was shaved by brand A, and the other was shaved by brand B. A "smoothness score" (from 1 to 10) was given by each person for each side. The side on which a given shaver was used was assigned by the flip of a coin. Using the following scores, test $H_0 : \mu_d = 0$ vs. $H_a : \mu_d \neq 0$ at 0.1 significance level. Assume that both scores A and B follow the normal distributions.

Man	*1*	*2*	*3*	*4*	*5*	*6*
A score	7	10	8	7	4	4
B score	5	6	8	4	6	5

OpenIntro Stat Exercises: 6.47, 6.50, 7.40, 8.24

Answers

1.1 10%
1.2 narrower
1.3 Yes, because the significance level, α, is also the probability of Type I error. Consequently, the rejection region increases as well.
1.4 No, not necessary.
1.5 $H_0 : \mu_f - \mu_m = 0$ vs. $H_a : \mu_f - \mu_m > 0$ or $H_0 : \mu_f = \mu_m$ vs. $H_a : \mu_f > \mu_m$
1.6 $H_0 : p = 0.58$ versus $H_a : p > 0.58$.
1.7 *p*-value = 0.215
1.8 *p*-value = 0.0207
1.9 The test is two-tailed, so with *n*-1 = 9 degrees of freedom, the critical values are ±1.833.

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2a) 0.4972 2b) 0.2514

Extra: 4.78 inches.

3.1 Test statistic: z = -2.83,
critical values are +1.96 and -1.96.
Thus, reject H_0. Therefore, Amy's performance has not changed.

3.2 147.2

3.3 147.2 ± 0.73

3.4 $H_0 : \mu = 148$ vs. $H_a : \mu > 148$

5 Test statistic: z = 1.54; critical value = 1.28
Thus, reject H_0. We conclude that the evidence shows that more than 30% of customers have more than two telephones.

6 $P(\bar{x} < 1.9) = P(z < 0.75) = 0.7734$

7.1 $H_0 : \mu_2 - \mu_1 = 0$ vs. $H_0 : \mu_2 - \mu_1 < 0$,

$$t = \frac{\bar{x}_2 - \bar{x}_1 - 0}{\sqrt{s^2\left(\frac{1}{n_2} + \frac{1}{n_1}\right)}}$$ ≈ -0.92, critical value = -1.35 , … ....

Do not reject H_0. Data do not indicate that the students planning to major in African Studies have a lesser mean score.

7.2 A 95% confidence interval for $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 80 - 75 \pm 2.160\sqrt{\cdots}$$

You can also construct the CI for $\mu_2 - \mu_1$

8 **a** The 90% confidence interval for $\mu_1 - \mu_2$ is approximately
0.01±0.071 or -0.061 < $\mu_1 - \mu_2$ <0.081

b The 95% confidence interval for $\mu_1 - \mu_2$ is approximately
35±74.852 or -39.852 < $\mu_1 - \mu_2$ < 109.852

c Both of the intervals contain the value $(\mu_1 - \mu_2) = 0$. If $(\mu_1 - \mu_2) = 0$ is contained in the confidence interval, the it is not unlikely that μ_1 could equal μ_2, implying no difference in the vitamin levels between boys and girls. This would be of interest to the experimenter.

d Since the value $(\mu_1 - \mu_2) = 0$ is in the confidence interval, it is possible that $\mu_1 = \mu_2$. You should not conclude that there is a difference in the average β- carotene or serum cholesterol levels for boys and girls.

e The hypothesis to be tested is

$$\mathrm{H_0} : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad \mathrm{H_a} : \mu_1 - \mu_2 \neq 0$$

and the test statistic is

$$z \approx \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{588 - 553}{\sqrt{\frac{406^2}{207} + \frac{350^2}{185}}} = .92$$

The rejection region, $\alpha = .01$, is two-tailed or $|z| > 2.58$ and the null hypothesis is not rejected.

f The *p*-values is 0.3576. Hence, the null hypothesis could not be rejected in either case for $\alpha = .05$.

6.47 Use a chi-squared goodness of fit test. The hypotheses are as follows:

H_0: Each option is equally likely.

H_A: Some options are preferred over others.

Before calculating the test statistic we should check that the conditions are satisfied.

1. Independence: We are checking to see if the decisions for rock, paper, scissors are independent or not, so we can't validate this condition.
2. Sample size: Total sample size is 99, and expected counts are $(1/3) \times 99 = 33$ for each option. These are all above 5, so conditions are satisfied.

The chi-squared statistic, the degrees of freedom associated with it, and the p-value can be calculated as follows:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(43-33)^2}{33} + \frac{(21-33)^2}{33} + \frac{(35-33)^2}{33} = 7.52$$

$$df = 3 - 1 = 2$$

$$p - value = P(\chi^2_2 > 7.52) \rightarrow p - value = 0.023$$

Since the p-value is less than 5%, we reject H_0. The data provide convincing evidence that some options are preferred over others.