ICMA151 Midterm Exam Solution

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Problem 22 A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a) Find P(A).

Answer P(A)= 300/1000 = 0.3

b) Find P(B|A).

Answer $P(B|A) = \frac{P(B \cap A)}{A}$.

P(A intersect B)=(300/1000) $*$ (299/999) = (3/10)(299/999) = 299/3330 =

```
300/1000*299/999

## [1] 0.08978979
```

Given that A occurs, there are 999 components remaining, of which 299 are defective. Therefore, P(B|A)=299/999.

c) Find $P(A \cap B)$.

Answer $P(A \cap B) = P(A)P(B|A) = (300/1000)(299/999) = 0.0897898$

d) Find P(A^c ∩B).

Answer $P(A^c \cap B) = (7/10)(300/999) = 70/333 = 0.2102102$

e) Find B.

Answer P(B)=P($A \cap B$)+P($A^c \cap B$)=299/3330+70/333=3/10=0.3

f) Find P($A|B$).

Answer P(A|B)=(299/3330)/(3/10)=299/999=0.2992993

g) Are A and B independent? Is it reasonable to treat A and B as though they were independent? Explain.

Answer A and B are not independent, but they are very nearly independent. To see this note that $P(A)$ = 0.3, while $P(A|B)$ = 0.2992993. So $P(A)$ is very nearly equal to $P(A|B)$, but not exactly equal. Therefore in most situations it would be reasonable to treat A and B as though they were independent.

Problem 23 In 2020, the USA has a population of 331,002,651 people. As of 13th August 2020, it has found 5,176,018 cases of COVID-19.

A) Assume that the US government randomly samples 100,000 people on the same day. What is the probability that there are at most 1,500 cases of COVID-19 (including the people who have recovered from COVID-19)?

```
p<-5176018/331002651
p

## [1] 0.01563739
```

Let X=number of COVID-19 cases out of 100,000 samples

X~B(n,p), n=100000, p=0.01563739

P(X<= 1500)=

```
pbinom(1500,100000,p)

## [1] 0.05276799
```

B) Assume that the US government tests people one by one. What is the probability that the US government will test at most 100,000 people to find 1,500 cases of COVID-19.

Let Y=number people to test for 1,500 cases of COVID-19. Y~Negative binomial

$Y \sim Negbin(r = 1500, p = 0.01563739), p = P(success)$ r =number of successes

We want to find 1500 cases by testing at 100,000 people. Therefore, we want to commpute $P(Y \leq 100{,}000) =$

```
pnbinom(q=100000-1500,size = 1500,p=0.01563739)

## [1] 0.9499555
```

Note that size=r and q= the number of failures before r successes.

Problem 24 Weights of female cats of a certain breed are normally distributed with mean 4.1 kg and standard deviation 0.6 kg.

a) What proportion of female cats have weights between 3.7 and 4.4 kg?

Answer Let X =weights of female cats in the population. $X \sim N(4.1{,}0.6)$. $\mu = 4.1$ and $\sigma = 0.6$. The proportion is

```
pnorm(4.4,mean = 4.1,sd = 0.6)-pnorm(3.7,mean = 4.1,sd = 0.6)

## [1] 0.4389699

mu1<-4.1
sd1<-0.6
```

b) A certain female cat has a weight that is 0.5 standard deviations above the mean. What proportion of female cats are heavier than this one?

Answer We want to find $P(X > \mu + 0.5 * \sigma)$ = P(X>4.4)= 1-P(X<=4.4)=

```
1-pnorm(mu1+0.5*sd1,mean = mu1,sd = sd1)

## [1] 0.3085375
```

c) How heavy is a female cat whose weight is on the 80th percentile?

Answer We want to find q such that $P(X < p) = 0.8$. Therefore, we can use the command 'qnorm()'.

```
qnorm(0.8,mean = mu1,sd=sd1)

## [1] 4.604973
```

Therefore, the 80th percentile is 4.604973 kg.

d) A female cat is chosen at random. What is the probability that she weighs more than 4.5 kg?

Answer The probability is $P(X > 4.5) = 1 - P(X \leq 4.5) =$

```
1-pnorm(4.5,mean = mu1,sd=sd1)

## [1] 0.2524925
```

e) Six female cats are chosen at random. What is the probability that exactly one of them weighs more than 4.5 kg

Answer Let Y = number of cats weight more than 4.5. Assume that choosing each cat (six cats total) is independent and the probability of choosing a cat with weight more than 4.5 kg. is the same for each trial. Thus, $Y \sim Bin(n = 6, p)$ where p =P(choosing a cat with weight more than 4.5 kg.)=P(X>4.5)=1-P(X<=4.5)=0.2524925

Thus, $P(Y = 1) =$

```
dbinom(x=1,size = 6, prob = 0.2524925)

## [1] 0.3535717
```

Problem 25 The number of hits on a certain website follows a Poisson distribution with a mean rate of 4 hits per minute.

a) What is the probability that 5 hits are received in a given minute?

Answer Let X_1 = number of hits in a minute. Thus, $\lambda_1 = 4$ hits/minute.

Thus, $P(X_1 = 5) =$

```
dpois(5,4)
```

```
## [1] 0.1562935
```

b) What is the probability that 9 hits are received in 1.5 minutes?

Answer Let X_2 = number of hits in 1.5 minutes. $\lambda_2 = 1.5 \times 4 = 6$ hits/1.5 minutes.

$P(X_2 = 9) =$

```r
dpois(9,6)

## [1] 0.06883849
```

c) What is the probability that fewer than 3 hits are received in a period of 30 seconds?

Answer Let X_3 =number of hits in 30 seconds =1/2 minute. $\lambda_3 = 4 \times \frac{1}{2} = 2$.

Thus, $P(X_3 < 3) = P(X_3 \leq 2) =$

```r
ppois(2,2)

## [1] 0.6766764
```