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1.1

Given their principal axes interesect at a point.

p1=[0 0 1]

p2=[0 0 1]

The fundamental matrix, equation should satisfy,

$p1^T Fp2=0$

or, $[0\ 0\ 1] \begin{matrix} F11 & F12 & F13 \\ F21 & F22 & F23 \\ F31 & F32 & F33 \end{matrix} \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} = 0$

or, $0\ 0\ 1 \begin{matrix} F13 \\ F23 \\ F33 \end{matrix} = 0$

or, F13*0 + F23*0 + F33*0 =0

or, F33=0

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1.2

Given translation is parallel to x-axis. It will be of form-

$$t = \begin{matrix} t \\ 0 \\ 0 \end{matrix}$$

writing in skew-symettric form, we have

$$t_x = \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -t \\ 0 & t & 0 \end{matrix}$$

Since there is no rotation, the rotation matrix will be identity

$$R = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

We have, Essential matrix, $E = t_x R$

$$E = \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -t \\ 0 & t & 0 \end{matrix}$$

For the points [x1 y1 1] and [x2 y2 1] in the two cameras,

Epipolar lines will satisfy

$$[x1\ y1\ 1]\ E \begin{matrix} x2 \\ y2 \\ 1 \end{matrix} = 0$$

$$\text{Or, } x1\ y1\ 1 \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -t \\ 0 & t & 0 \end{matrix} \begin{matrix} x2 \\ y2 \\ 1 \end{matrix} = 0$$

Or, $t(y1-y2)=0$

Or, $y1=y2$

Hence, epipolar lines are parallel to x axis.

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1.3

Extrinsic matrix for Camera 1, $U_1=[R_1\ t_1]$

Extrinsic matrix for Camera 2, $U_2=[R_2\ t_2]$

Relative extrinsics for camera 2 with respect to camera 1,

$U_{rel}= U_1^{-1}U_2$

$=[R_1\ t_1]^{-1}[R_2\ t_2]$

$$=\begin{matrix} R_1^{-1}R_2 & R_1^{-1}t_1-R_1^{-1}t_2 \\ 0 & 1 \end{matrix}$$

Therefore $R_{rel}=R_1^{-1}R_2$

And $t_{rel}=R_1^{-1}t_1-R_1^{-1}t_2$

Essential Matrix, $E=t_{rel}\times R_{rel}$

Fundamental Matrix, $F = KE= K\ t_{rel}\times R_{rel}$

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1.4

Let K, R and t be the intrinsic matrix, rotation matrix and translation vector respectively.

There is no rotation(the second image is reflection from the plane mirror)

The rotation matrix is therefore.

$$R=\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\text{and } t=\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}$$

The essential matrix, E= [t]R

$$E=\begin{matrix} 0 & -t_y & -t_z \\ t_y & 0 & -t_x \\ t_z & t_x & 0 \end{matrix}$$

$E^T=-E$. Hence, E is a skew symettric matrix.

The fundamental matrix, F=KE, which therefore is also a skew symmetric matrix.

The points should satisfy.

$P_1^T F P_2 = 0$

Hence, points are seprated by skew symmetric fundamental matrix

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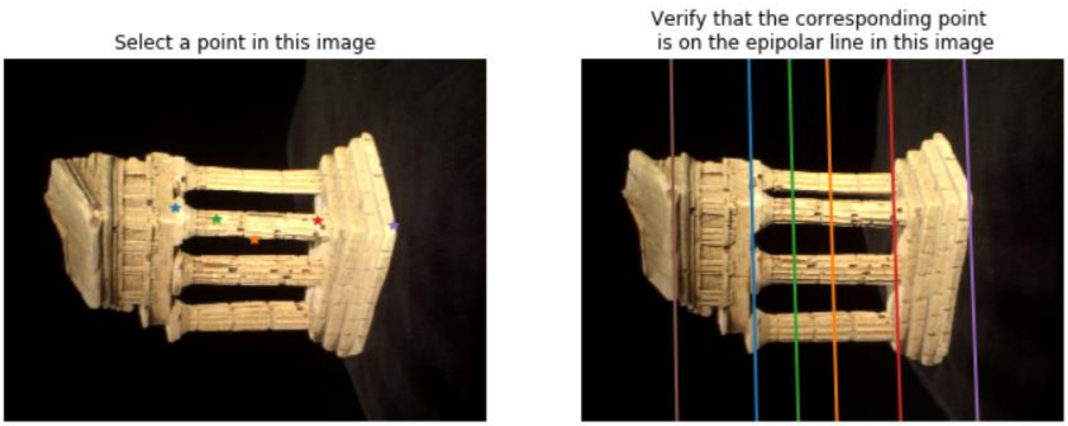
2.1

The fundamental matrix:

```
In [8]: f
Out[8]:
array([[ 9.80213863e-10, -1.32271663e-07,  1.12586847e-03],
       [-5.72416248e-08,  2.97011941e-09, -1.17899320e-05],
       [-1.08270296e-03,  3.05098538e-05, -4.46974798e-03]])
```



3.1

```
In [37]: e
Out[37]:
array([[ 2.26587821e-03, -3.06867395e-01,  1.66257398e+00],
       [-1.32799331e-01,  6.91553934e-03, -4.32775554e-02],
       [-1.66717617e+00, -1.33444257e-02, -6.72047195e-04]])
```

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3.2

For a set of points in the cameras, i=1,2,…N and j=1,2,…N –(u_i, v_i) and (u_j, v_j) and w_i be the corresponding 3D point.

Let M_1 and M_2 be the camera projection matrices for the two cameras.

Let M_1^k andM_2^k represent the k^{th} of the two matrices.

We have,

$u_i=M_1^1w_i/M_1^3$

and, $v_i=M_1^2w_i/M_1^3$

Similarly,

$u_j=M_2^1w_i/M_2^3$

and, $v_j=M_2^2w_i/M_2^3$

Arranging the above equations, we get

$(M_1^3u_i-M_1^1)w_i=0$,

$(M_1^3v_i-M_1^2)w_i=0$,

$(M_2^3u_j-M_2^1)w_i=0$,

and, $(M_2^3v_j-M_2^2)w_i=0$

These can be represented in the following form:

$A_iw_i=0$

Where, $A_i=\begin{matrix} {M^3}_1u_i-{M^1}_1 \\ {M^3}_1v_i-{M^2}_1 \\ {M^3}_2u_j-{M^1}_2 \\ {M^3}_2v_j-{M^2}_2 \end{matrix}$

Each row of A_i is a 1×4 vector.

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3.3

Correct M

```
In [43]: a['M']
Out[43]:
array([[ 0.99942697,  0.03331533,  0.00598477, -0.02599827],
       [-0.03372859,  0.96531605,  0.25889634, -1.        ],
       [ 0.00284802, -0.25894984,  0.96588657,  0.07961991]])
```

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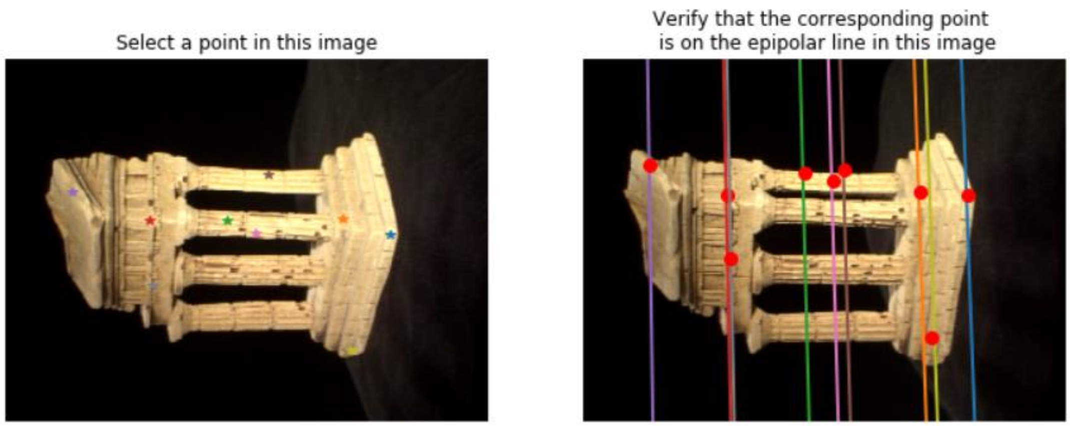
4.1



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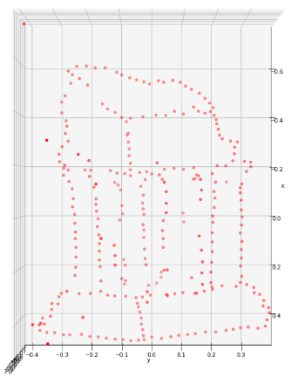
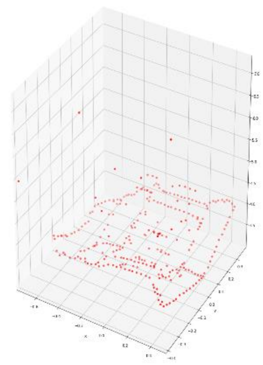
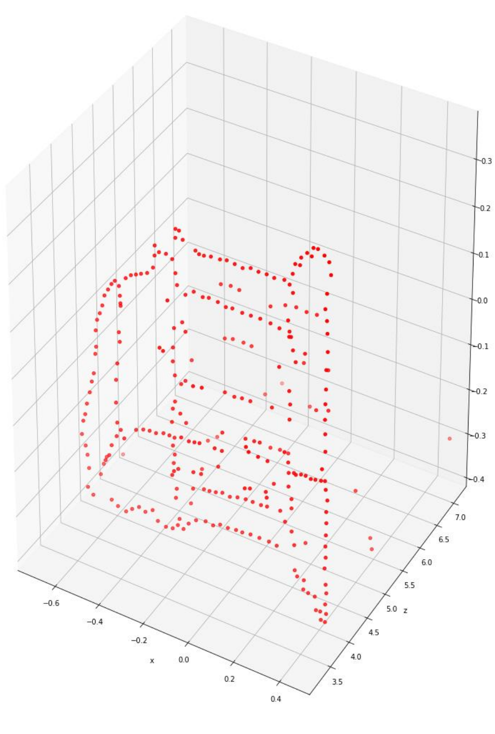
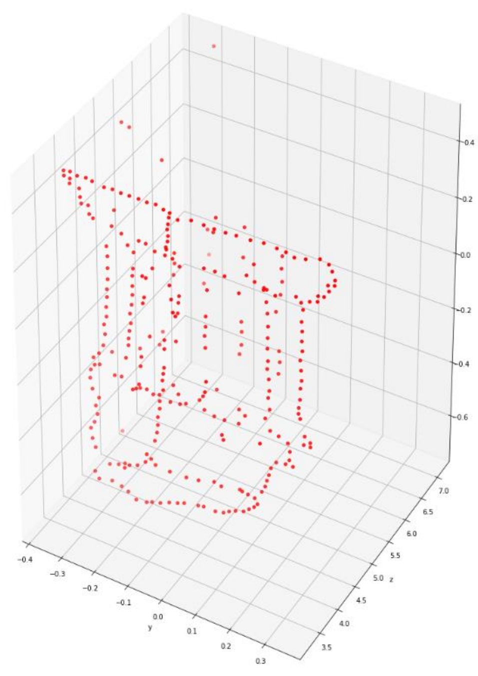
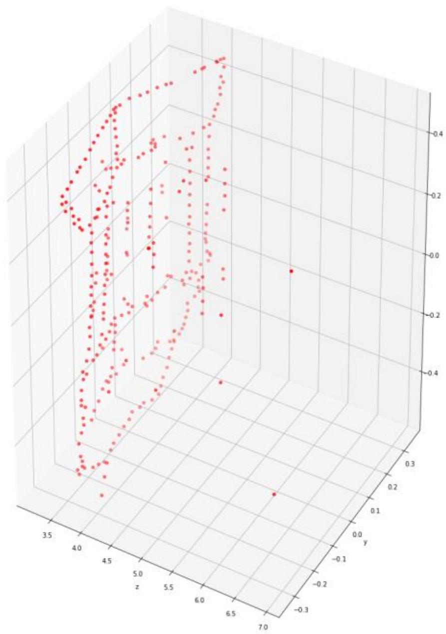
4.2










3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
z
0.3
0.2
0.1
0.0
y
−0.1
−0.2
−0.3
0.4
0.2
0.0
−0.2
−0.4

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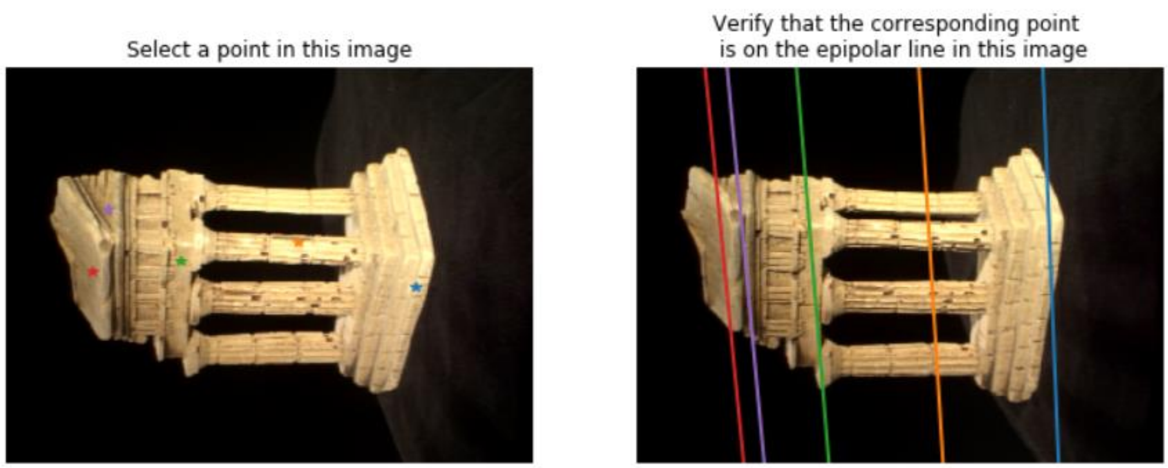
5.1

I calculated tolerance as the distance between the correspondence on right image and the epipolar line computed by the fundamental matrix. The points that were closer than the tolerance value were saved as inliers. The fundamental matrix with the maximum number of inliers was chosen to be the final fundamental matrix. The hyperparameters are the number of iterations and the tolerance value. I got the most optimal result for tol=0.8 and number of iteration=200

The performance improves by increasing the number of iteration but converges after 200, after which no improvement is observed. By increasing the tolerance value, some noise is also calculated as inliers and the result isn't good. Performance declines as well on decreasing the tolerance as the algorithm misses out on some of the inliers.

Compared to the result in Q2, the epipolar lines are not exactly vertical. This can be attributed to the noisy set of correspondences.



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5.2

Implemented in Code