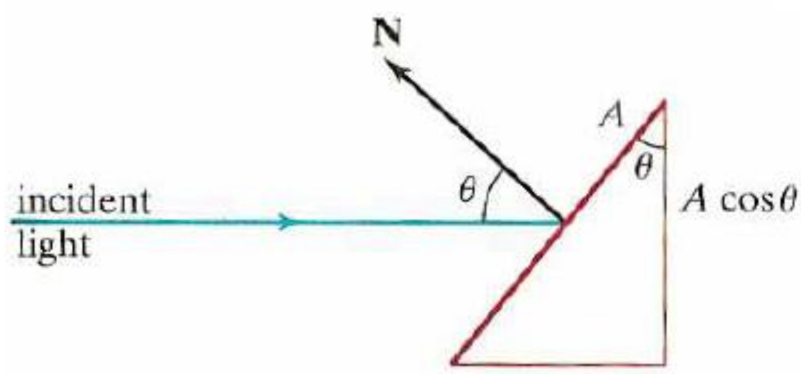
Kartik Narula
HW6

Question 1

a. From Lambert's cosine law:



For Intensity of light, l, falling at an angle θ at an area A, the irradiance is given by,

I_o=LAcosθ

= A|L||n|cosθ, where |l| is the magnitude of the vector l and n is the unit vector in the normal direction

=A n.L

=or A_o|L|, where A_o=Acosθ, the projected area.

The irradiance observed the viewer is same from every angle because the solid angle suspended by the visible surface of the viewer is changed by the same amount as the cosine of emission angle.

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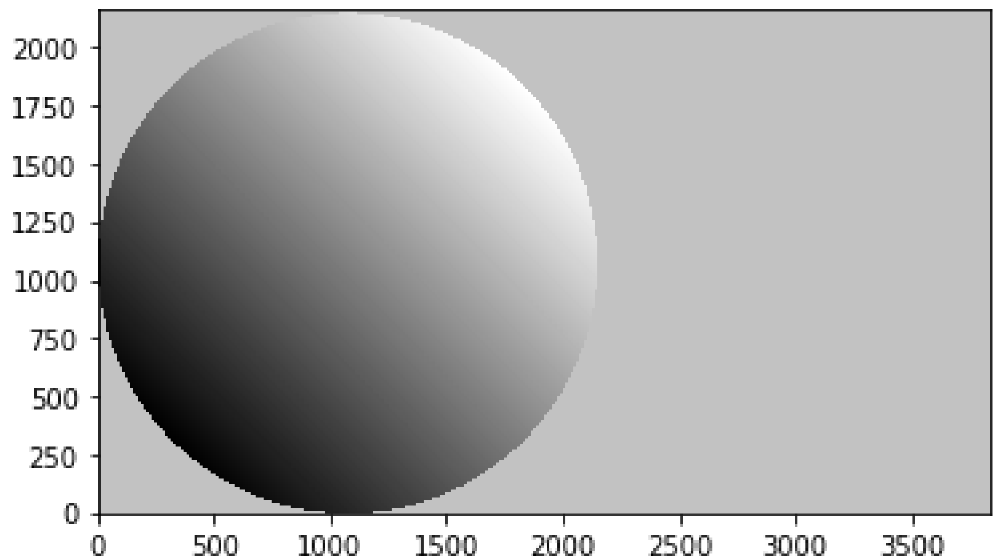
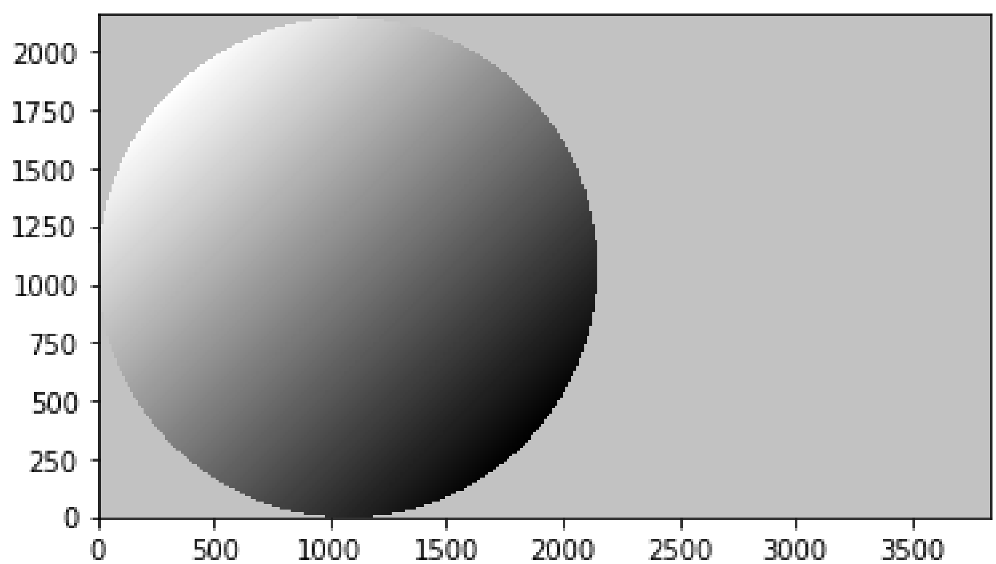
b.



Light direction=$(1,1,1)/(3)^{1/2}$



Light direction=$(1,-1,1)/(3)^{1/2}$

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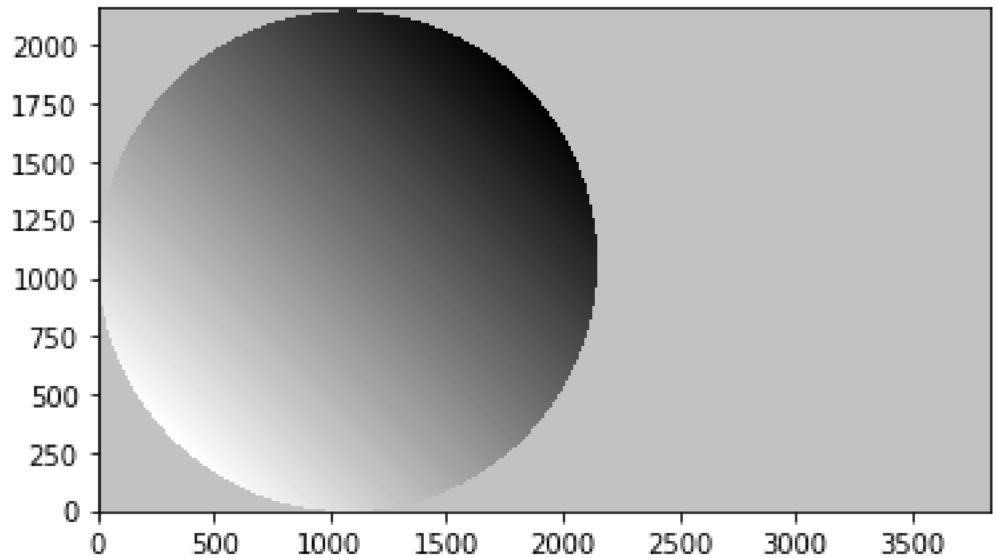

Light direction=(-1,-1,1)/$(3)^{1/2}$

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c.

```python
def loadData(path = "../data/"):


    b=[]
    for i in range(7):
        a=plt.imread(path+'input_'+str(i+1)+'.tif')
        a=a.astype('uint16')
        xy= rgb2xyz(a)
        l=xy[:,:,1]
        l=l.flatten()
        b.append(l)

    I=np.array(b)
    g= np.load(path+'sources.npy')
    L= np.transpose(g)

   # I = None
   # L = None
    s = a.shape

    return I, L, s
```

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d.

We have,

$I=L^TB$

B is of dimension 3×P and has a rank 3.

L is of dimension 3×N, where N is the number of light sources.

The normal vector, n has three degrees of freedom and hence we need a minimum of 3 light sources to estimate n(or B, which is ρ.n). Hence L has the minimum rank 3.

Since I is the product of B which has rank 3 and L which has minimum rank 3.

I has rank 3.

```
In [13]: scipy.linalg.svdvals(I)
Out[13]:
array([0.07576378, 0.00906763, 0.00635114, 0.00194115, 0.00146786,
       0.00115865, 0.00094721])
```

Figure 1 The reported singular values of I

No, the singular values do not meet the rank 3 requirements. However, the last eigen values are very close to zero. This can be attributed to the noise in the data and so the singular values are not exactly zero.

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e.

We have,

$I_1=L_1^TB$

$I_2= L_2^TB$

.

.

$I_7= L_7^TB$

Or, $L^T B=I$

Which is of form, $AX=y$

$A=L^T$, $X=B$ and $y=I$

Where X and y are flattened matrices(Vectors)

B can be estimated as

$B=(LL^T)^{-1}L\ I$

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f

Figure 2 Albedo Image

The neck, nostrils and ears have an higher albedo values the lips and eyelids have lower albedo values. The lips and eyelids are darker than other parts of the face and hence have lower albedo values. The ears and nostrils are hollow and reflect more light(probably) and hence have higher albedo values.

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Figure 3 Normals as an Image

This is expected as I see the normals on the left and the right side of the face having similar values(the red and green regions)

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g.

Let, the surface be represented by, $F(x,y,z)=0$

$F(x,y,z)=z-f(x,y)=0$

Let normal at a point(X,Y,Z) be given by

$N= (n1,n2,n3)$ ----(i), where N is a unit vector

N is normal (or perpendicular) to the surface at $F(X,Y,Z)$

$\delta F(X,Y,z)/ \delta X=0$, $\delta F(X,Y,Z)/ \delta y=0$, and $\delta F(X,Y,Z)/ \delta Z=0$

Solving which gives us the x,y and z coordinates respectively

The normal can be represented by

$N=(-\delta f(X,Y)/ \delta X, -\delta f(X,Y)/ \delta y, 1)$, where $z=f(x,y)$

From (i)

$N= (n1/n3, n2/n3, 1)$

Comparing we have,

$\delta f(X,Y)/ \delta X =-n1/n3$

and $\delta f(X,Y)/ \delta Y =-n2/n3$

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h.

Yes, constructing g by these two ways give the same result.

We have, $g_x = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$

$g_y = \begin{matrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{matrix}$

Modifying $g_x = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$

We get, using the first method

$G = \begin{matrix} 1 & 2 & 4 & 7 \\ 5 & 6 & 8 & 8 \\ 9 & 10 & 12 & 12 \\ 13 & 14 & 16 & 16 \end{matrix}$

Using the second method

$G = \begin{matrix} 1 & 2 & 4 & 7 \\ 5 & 9 & 14 & 20 \\ 9 & 16 & 24 & 33 \\ 13 & 14 & 15 & 16 \end{matrix}$

Which are not the same.

The gradients calculated in g) may not be integrable if the surfaces in x and y direction are not smooth and are discontinuities.

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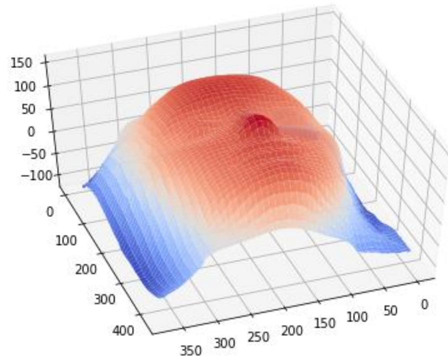
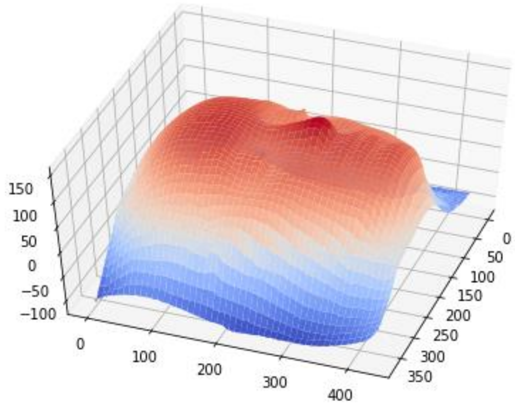
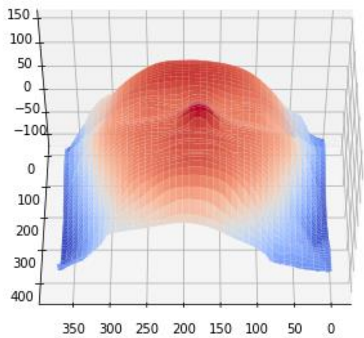
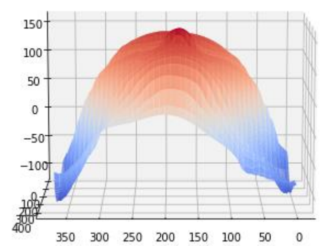
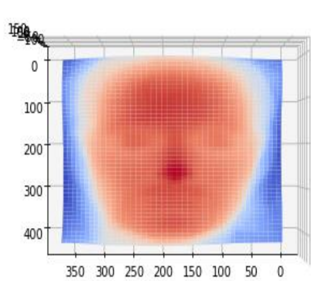
i.

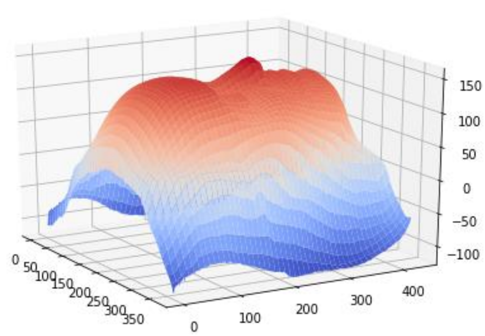












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Question 2

a.

We have,

$I=L^TB$

Applying Singluar value decomposition on I

$I = U\Sigma V^T$

Making I, rank 3 by making all the singular values except the top 3 zero.

Let s be the diagonal matrix with top 3, singular values

And u and v be the matrices with the first three columns of U and V respectively.

We decompose I as follows

$L^T=us^{1/2}$

And, $B=s^{1/2}v^T$

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b

Albedo Image

Normal Image

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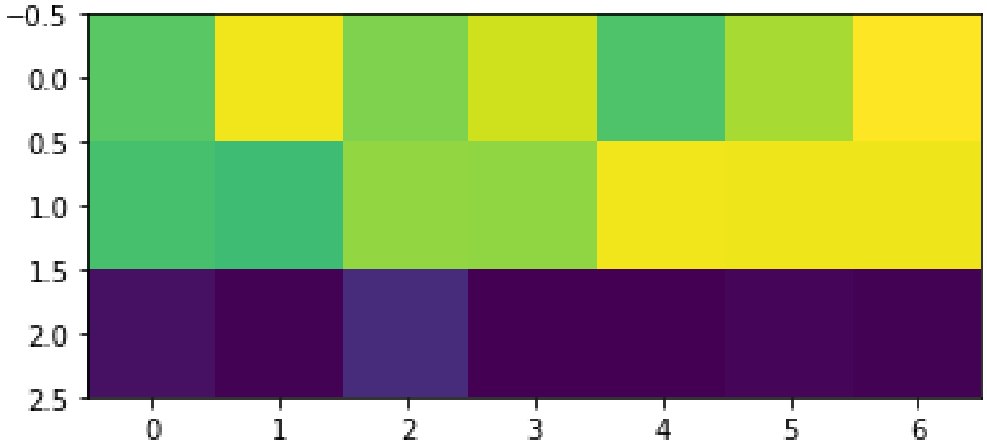
c.

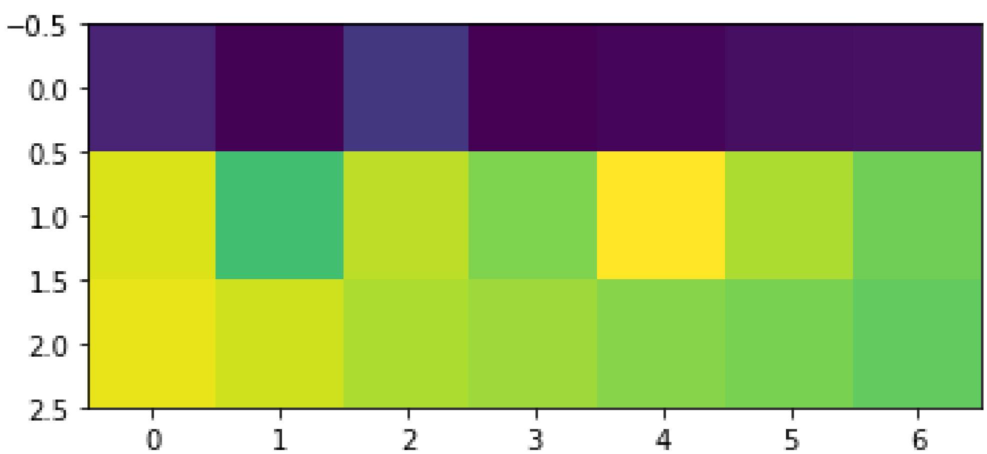
Ground truth lighting directions

```
In [239]: L
Out[239]:
array([[-0.1418,  0.1215, -0.069 ,  0.067 , -0.1627,  0.    ,  0.1478],
       [-0.1804, -0.2026, -0.0345, -0.0402,  0.122 ,  0.1194,  0.1209],
       [-0.9267, -0.9717, -0.838 , -0.9772, -0.979 , -0.9648, -0.9713]])
```



Estimated lighting directions

```
In [245]: Lp.T
Out[245]:
array([[-0.0269, -0.0302, -0.0246, -0.0304, -0.0298, -0.0291, -0.0288],
       [ 0.0032, -0.0057,  0.0016, -0.0018,  0.0051,  0.0008, -0.0026],
       [ 0.004 ,  0.0025,  0.0007, -0.0001, -0.0013, -0.0021, -0.0034]])
```



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No, they are not same

A possible change in factorization is follows:

$L^T = U\Sigma$

And, $B = V$

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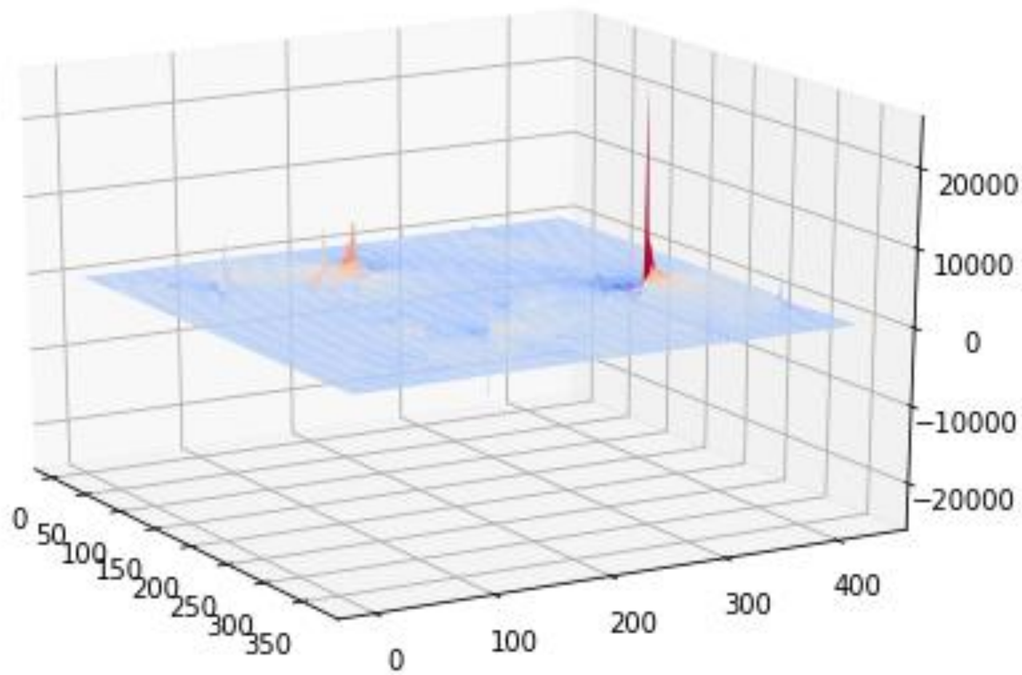
d.



Depth Map without enforcing Integrability.

No, this does not look like a face.

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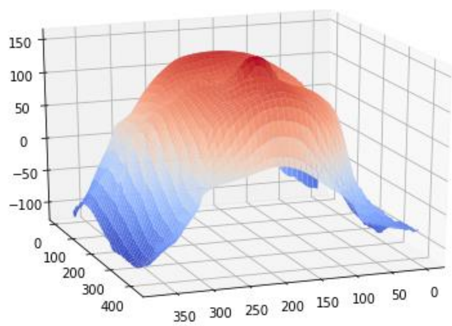
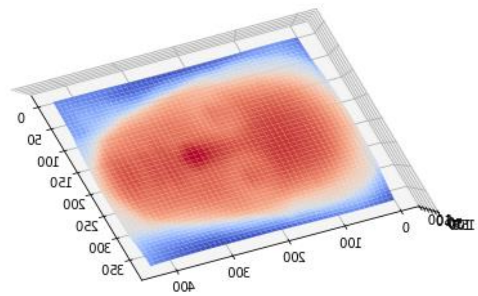
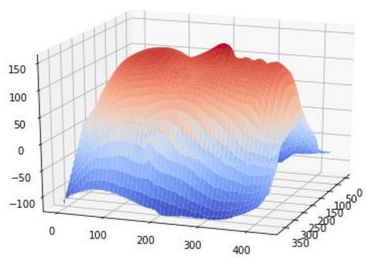
e.







Yes, this looks like a face output by a photometric stereo.

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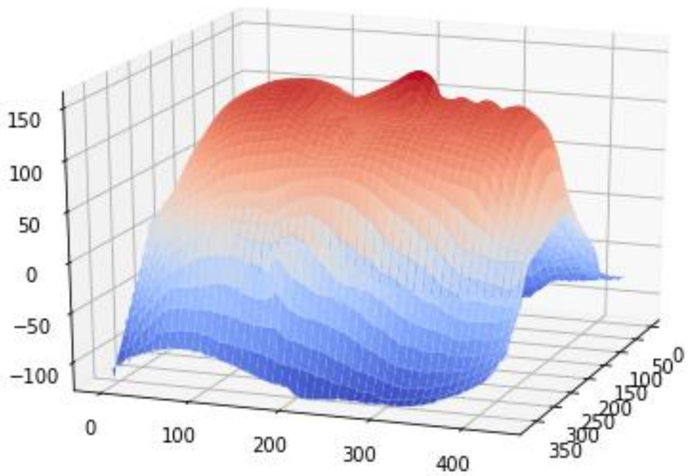
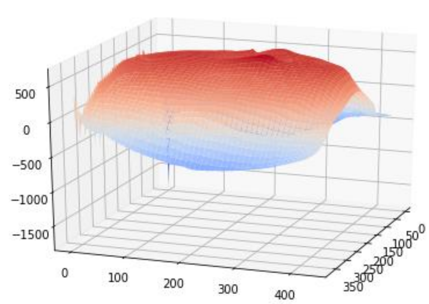
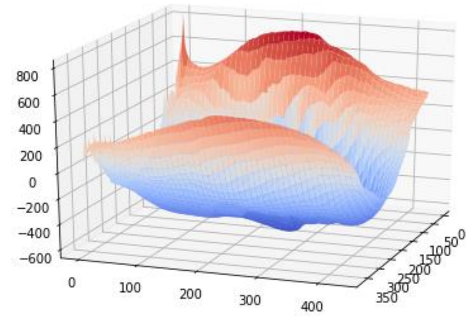
f.

With G as the identity matrix(μ=0, ν=0, λ=1), we have



μ=0, λ=1





, ν=-0.6 , ν=-0.4

ν=-0.6, λ=1

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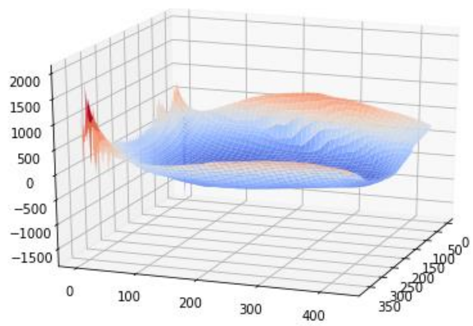
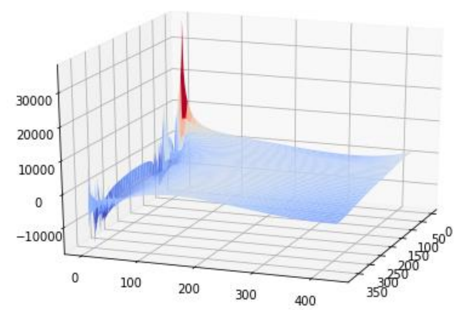
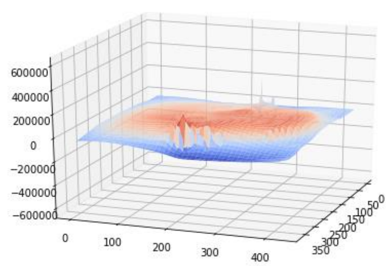


μ=0.4

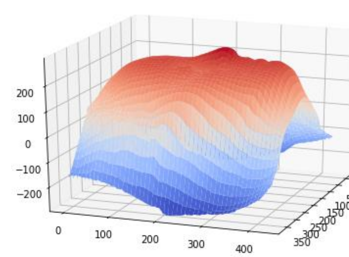


μ=0.2

μ=0, ν=-0



Λ=0.002



Λ=1.8

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The parameters μ and , ν add additive planes in horizontal and vertical direction while λ has an effect of elongating/flattening the surface.

High relief sculptures are the ones which are distinguishable from their background, while low relief sculptures are close to their background. , ¯

Bas relief ambiguity is defined as:

$f(x, y) = \lambda f(x, y) + \mu x + \nu y$

The resulting transformation is a function of x and y and is attached to the background and is therefore, low relief.

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g. Making the λ value as low as possible(0.001 to 0.002) gives flattest surfaces.

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h.

No, acquiring more pictures will not help resolve the ambiguity. Because there will still exist an ambiguity matrix A that satisfies.

$I= (L^T A^{-1}) (AB)$