Technische Universität Braunschweig

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Assignment 8

Upload your solution until
Mon, 10. June 2024, 3:00 pm.

Assignment 8.1 *Numerical methods for ODEs* **(10 points)**

Consider an initial value problem

$$y' = f(t, y), \quad y(0) = y_0$$

with $y : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $y_0 \in \mathbb{R}^n$. Create a file `euler.py` in which you implement the explicit Euler method, the implicit Euler method and the Crank–Nicolson method. The inputs of the functions are supposed to be the right hand side function $f(t, y)$, the end time t_end and the constant stepsize $h > 0$. should not only contain the solution of the problem but should also include intermediate values for each step (path to the solution at t_{end} from the initial time $t_0 = 0$).

Remarks:

- *Your implementation has to work for vector valued solutions $y(t)$ in $\mathbb{R}^n$.*
- *You have to solve a non-linear equation during the implicit method. Use your implementation of Newton's method from Assignment 6 to do so.*

Assignment 8.2 *Test of the methods in 1D* **(1 + 4 = 5 points)**

Consider the initial value problem

$$y' = 2t(1 + y), \quad y(0) = 0.$$

(a) Find the analytical solution of this ODE.

(b) Create a file `euler_1d.py` for the implementation of this assignment. Use Assignment 8.1 to compute numerical solutions of the initial value problem at $t_{\text{end}} = 2$, as well as the paths which lead to the solution. Use the explicit Euler method, the implicit Euler method and the Crank–Nicolson method. Therefore, consider step sizes $h \in \{1, 0.5, 0.1, 0.01\}$. Plot for each choice of h the paths together with the analytic solution in a common plot over the interval $[0, 2]$.

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Assignment 8.3 *Test of the methods in 2D* **(5 points)**

Create a file euler_2d.py for the implementation of this assignment. Use Assignment 8.1 in order to compute a numerical solution of the initial value problem

$$y'(t) = \begin{pmatrix} -4 & 6 \\ 31 & -189 \end{pmatrix} y(t), \qquad y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}, \qquad y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

with end time $t_{end} = 2$ and step sizes $h \in \{0.01, 0.02\}$.

For each step size, plot the path of $y_1(t)$ and $y_2(t)$ separately. Each plot should include the paths from both the methods together with the analytical solution in the interval [0, 2]. Observe how the paths vary. Of the given step sizes, for which step sizes does which method work as expected?