Technische Universität Braunschweig
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Assignment 4

Upload your solution until
Mon, 06. May 2024, 3:00 pm.

Assignment 4.1 *Summed trapezoidal rule* **(10 points)**

Create a file `summed_trapezoidal.py` in which you implement the summed trapezoidal rule. Hence, implement a function `summed_trapezoidal(f, a, b, J)`, which subdivides an interval $[a, b]$ into J equidistant subintervals and applies the trapezoidal rule on every subinterval in order to approximate $I(f) = \int_a^b f(x)\,\mathrm{d}x$. Make your `summed_trapezoidal`-function show a plot of f on the interval $[a, b]$, which also depicts the area which represents $I(f)$ and the area which represents $Q(f)$ (as in Figure 1). Try your implementation for $f(x) = \sin(\pi x)$ on the interval $[0, 2]$ for values $J \in [3, 5, 10, 20, 30]$. What do you observe? Make a comment in the script file.

Assignment 4.2 *Romberg extrapolation* **(10 points)**

Create a function `romberg_extrapolation(f, a, b, n)` within a file `romberg_extrapolation.py` which takes a function f, boundary values $a < b$ and an integer number n as input arguments and calculates the result of $n - 1$ steps of the Romberg extrapolation starting with the step size $h = a - b$.

Use this function file in order to compute the approximate value of

$$I(f) = \int_0^1 \mathrm{e}^x \cos x \,\mathrm{d}x$$

executing 2 steps of the Romberg extrapolation. Compare the resulting approximation with the exact value $I(f)$. (You may consider the approximations which you can achieve with `scipy.integrate.quad(...)` as exact). Now, use your implementation of Romberg extrapolation to find the required number of extrapolation steps to make the absolute error of the Romberg scheme smaller than 10^{-6}.

Hint: Look at the Romberg extrapolation schemes from the lecture. The entries in the column on the very left are created with the summed trapezoid rule for $1, 2, 4, 8, 16, \ldots$ intervals. You do not need to apply the summed quadrature rules outside of this column. The other columns may be computed with the stored values from the columns to their left. Think about how to extend the given schemes to an arbitrary number of extrapolation steps.

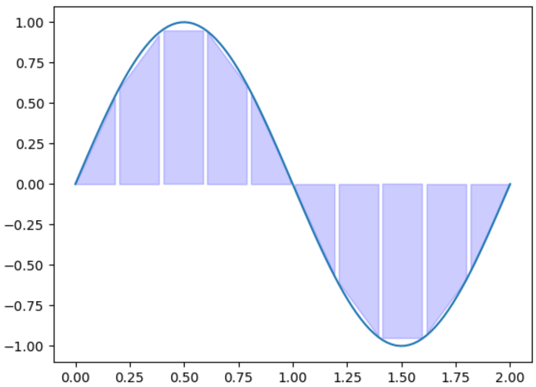

Figure 1: Summed trapezoidal quadrature for $f(x) = \sin(\pi x)$ as in Assignment 4.1 for $J = 10$.