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Assignment 3

Upload your solution until
Mon, 29. April 2024, 3:00 pm.

Assignment 3.1 *Quadrature rules* **(2 + 7 + 10 + 1 = 20 points)**

Quadrature rules can be used to approximate the value of an integral. In this assignment, we aim to approximate

$$I(f) = \int_a^b f(x)\,\mathrm{d}x.$$

First, we start with finding the exact value of the integral (or actually just another approximation with a quadrature, but a really good one).

(a) Create a file `quadratures.py` (in which you can put the code of this part of the exercise and the code of the following parts of the exercise as well). Use the `scipy.integrate` library to compute $I(f)$ for $f(x) = e^x$ and $a = 0, b = 2\pi$. Print the value in the console.

The idea behind a quadrature $Q(f)$ is to replace the function $f(x)$ by a Lagrange interpolation polynomial $p(x)$ of degree N, such that $I(f) \approx Q(f)$. Depending on N and how we choose the nodes $x_0, \ldots, x_N$, we end up with different quadrature rules. In this exercise we consider the

- Left rectangle rule,
- Right rectangle rule,
- Midpoint rule,
- Trapezoid rule,
- Simpson's/ Kepler's barrel rule,
- A rule for which it holds that $N = 3$ and $x_0 = a$, $x_1 = \frac{2a+b}{3}$, $x_2 = \frac{a+2b}{3}$, $x_3 = b$ (the corresponding p and $Q(f)$ has to be derived by you for this exercise).

Let us use each of these quadratures to approximate the value of the integral over an arbitrary function and visualise how a quadrature works. Follow the next steps to do so:

(b) Create a functions `left_rectangle`, `right_rectangle`, `midpoint`, `trapezoid`, `kepler`, `own` which take the integral boundaries `a`, `b` and an arbitrary function `f` as inputs. Compute the approximations with the corresponding quadrature rules in each of the functions and print the values in the console.

(c) Extend the functions from the previous exercise, such that they create plots wich include the function $f(x)$, the area which we want to approximate $I(f)$, the polynomial $p(x)$ with which $f(x)$ is replaced during the derivation of the quadrature and the area which is relevant for the computation of $Q(f)$.

(d) Call all functions for $f(c) = e^x$ and $a = 0, b = 2\pi$. You may find reference plots in Figure 1.

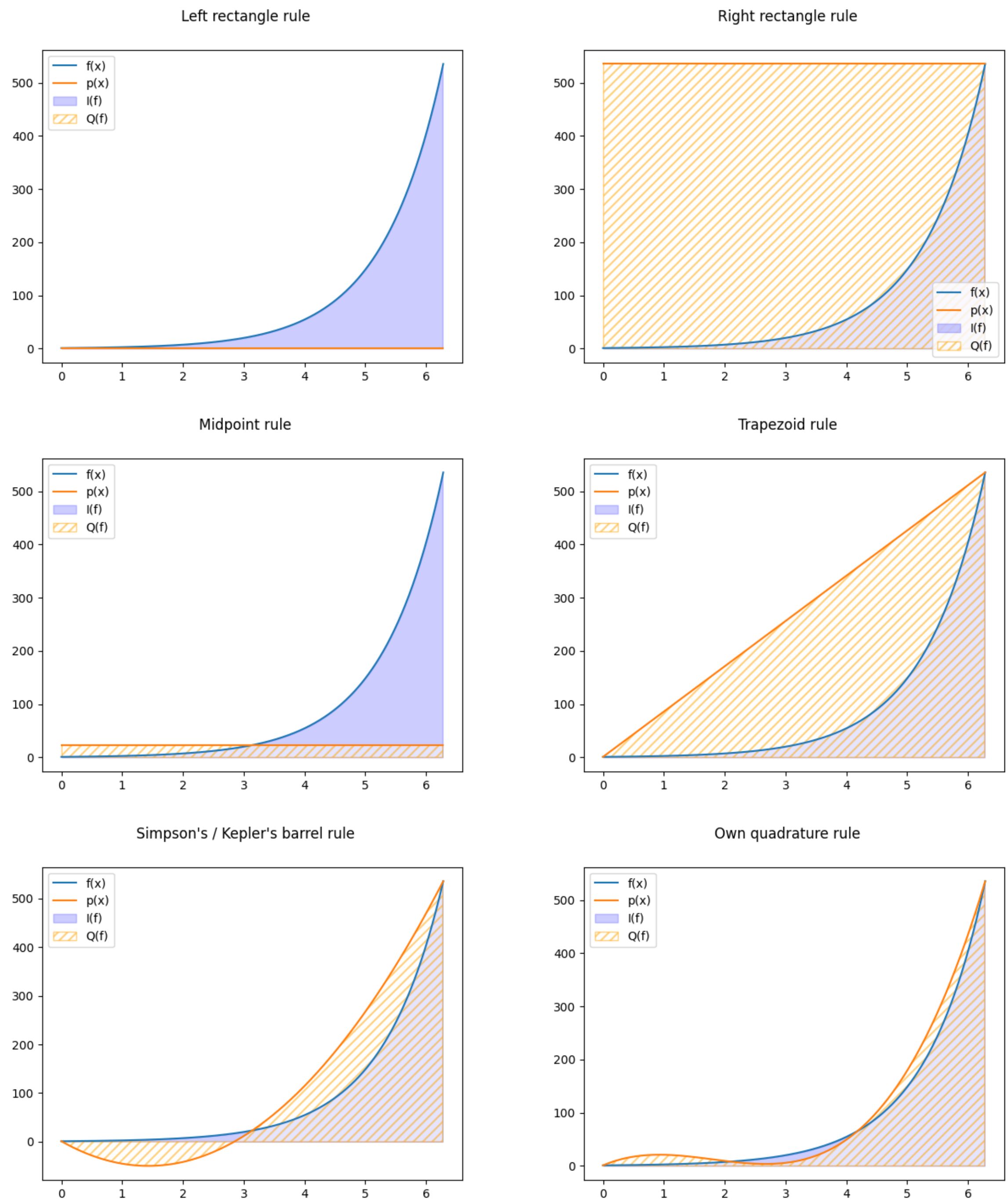

Figure 1: Exemplary plots for Assignment 3.1