Anexo D

Tabla de Integrales

(PUEDE SUMARSE UNA CONSTANTE ARBITRARIA A CADA INTEGRAL)

1.
$$\int x^n dx = \frac{1}{n+1} x^{n+1} \quad (n \neq -1)$$

$$2. \quad \int \frac{1}{x} \, dx = \log |x|$$

$$3. \quad \int e^x \ dx = e^x$$

$$4. \quad \int a^x \ dx = \frac{a^x}{\log a}$$

$$5. \quad \int \sin x \ dx = -\cos x$$

$$6. \quad \int \cos x \ dx = \sin x$$

$$7. \quad \int \tan x \, dx = -\log|\cos x|$$

8.
$$\int \cot x \, dx = \log|\sin x|$$

9.
$$\int \sec x \ dx = \log|\sec x + \tan x| = \log\left|\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right|$$

10.
$$\int \csc x \ dx = \log|\csc x - \cot x| = \log\left|\tan\frac{1}{2} x\right|$$

11.
$$\int \arcsin \frac{x}{a} dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2} \quad (a > 0)$$

12.
$$\int \arccos \frac{x}{a} dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} \quad (a > 0)$$

13.
$$\int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \log \left(a^2 + x^2\right) \quad (a > 0)$$

14.
$$\int \sin^2 mx \ dx = \frac{1}{2m} \left(mx - \sin mx \cos mx \right)$$

15.
$$\int \cos^2 mx \ dx = \frac{1}{2m} \left(mx + \sin mx \cos mx \right)$$

16.
$$\int \sec^2 x \ dx = \tan x$$

17.
$$\int \csc^2 x \ dx = -\cot x$$

18.
$$\int \operatorname{sen}^{n} x \, dx = -\frac{\operatorname{sen}^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \operatorname{sen}^{n-2} x \, dx$$

19.
$$\int \cos^n x \ dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \ dx$$

20.
$$\int \tan^n x \ dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \ dx \quad (n \neq 1)$$

22.
$$\int \sec^n x \ dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \ dx \quad (n \neq 1)$$

23.
$$\int \csc^n x \ dx = \frac{\cot x \csc^{n-1} x}{n-2} + \frac{n-2}{n-1} \int \csc^{n-2} x \ dx \quad (n \neq 1)$$

24.
$$\int \operatorname{senh} x \, dx = \cosh x$$

$$25. \quad \int \cosh x \, dx = \operatorname{senh} x$$

26.
$$\int \tanh x \, dx = \log|\cosh x|$$

$$27. \quad \int \coth x \, dx = \log|\sin hx|$$

28.
$$\int \operatorname{sech} x \, dx = \arctan \, (\operatorname{senh} x)$$

29.
$$\int \operatorname{csch} x \, dx = \log \left| \tanh \frac{x}{2} \right| = -\frac{1}{2} \log \frac{\cosh x + 1}{\cosh x - 1}$$

30.
$$\int \operatorname{senh}^2 x \, dx = \frac{1}{4} \operatorname{sen} h \, 2x - \frac{1}{2} \, x$$

31.
$$\int \cosh^2 x \ dx = \frac{1}{4} \operatorname{sen} h \ 2x + \frac{1}{2} \ x$$

32.
$$\int \operatorname{sech}^2 x \ dx = \tanh x$$

33.
$$\int \operatorname{sen} h^{-1} \frac{x}{a} \, dx = x \operatorname{sen} h^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} \quad (a > 0)$$

34.
$$\int \cosh^{-1}\frac{x}{a} dx = \begin{cases} x\cosh^{-1}\frac{x}{a} - \sqrt{x^2 - a^2} & \left[\cosh^{-1}\left(\frac{x}{a}\right) > 0, a > 0\right] \\ x\cosh^{-1}\frac{x}{a} + \sqrt{x^2 - a^2} & \left[\cosh^{-1}\left(\frac{x}{a}\right) < 0, a > 0\right] \end{cases}$$

35.
$$\int \tanh^{-1} \frac{x}{a} \, dx = x \tanh^{-1} \frac{x}{a} + \frac{a}{2} \log |a^2 - x^2|$$

36.
$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left(x + \sqrt{a^2 + x^2}\right) = \sin h^{-1} \frac{x}{a} \quad (a > 0)$$

37.
$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} \quad (a > 0)$$

38.
$$\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} \quad (a > 0)$$

39.
$$\int (a^2 - x^2)^{\frac{3}{2}} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \arcsin \frac{x}{a} \quad (a > 0)$$

40.
$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} \quad (a > 0)$$

41.
$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a + x}{a - x} \right|$$

42.
$$\int \frac{1}{(a^2 - x^2)^{\frac{3}{2}}} dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$$

43.
$$\int \sqrt{x^2 \pm a^2} \ dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log \left| x + \sqrt{x^2 \pm a^2} \right|$$

44.
$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| = \cosh^{-1} \frac{x}{a} \quad (a > 0)$$

45.
$$\int \frac{1}{x(a+bx)} dx = \frac{1}{a} \log \left| \frac{x}{a+bx} \right|$$

46.
$$\int x\sqrt{a+bx} \ dx = \frac{2(3bx-2a)(a+bx)^{\frac{3}{2}}}{15b^2}$$

47.
$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} dx$$

48.
$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

49.
$$\int \frac{1}{x\sqrt{a+bx}} dx = \begin{cases} \frac{1}{\sqrt{a}} \log \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| & (a > 0) \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a+bx}{-a}} & (a > 0) \end{cases}$$

50.
$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \log \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

51.
$$\int x\sqrt{a^2 - x^2} \ dx = -\frac{1}{3} \left(a^2 - x^2\right)^{\frac{3}{2}}$$

52.
$$\int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} \left(2x^2 - a^2 \right) \sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} \quad (a > 0)$$

53.
$$\int \frac{1}{x\sqrt{a^2 - x^2}} \, dx = -\frac{1}{a} \log \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

54.
$$\int \frac{x}{\sqrt{a^2 - x^2}} \, dx = -\sqrt{a^2 - x^2}$$

55.
$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} \quad (a > 0)$$

56.
$$\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \log \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$$

57.
$$\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \arccos \frac{a}{|x|} = \sqrt{x^2 - a^2} - \operatorname{arcsec}\left(\frac{x}{a}\right) \quad (a > 0)$$

58.
$$\int x\sqrt{x^2 \pm a^2} \ dx = \frac{1}{3} \left(x^2 \pm a^2\right)^{\frac{3}{2}}$$

59.
$$\int \frac{1}{x\sqrt{x^2 + a^2}} \, dx = \frac{1}{a} \log \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right|$$

60.
$$\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \arccos \frac{a}{|x|} \quad (a > 0)$$

61.
$$\int \frac{1}{x^2 \sqrt{x^2 + a^2}} dx = \pm \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

62.
$$\int \frac{x}{\sqrt{x^2 \pm a^2}} \ dx = \sqrt{x^2 \pm a^2}$$

63.
$$\int \frac{1}{ax^2 + bx + c} dx = \begin{cases} \frac{1}{\sqrt{b^2 - 4ac}} \log \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| & (b^2 > 4ac) \\ \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & (b^2 < 4ac) \end{cases}$$

64.
$$\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \log |ax^2 + bx + c| - \frac{b}{2a} \int \frac{1}{ax^2 + bx + c} dx$$

65.
$$\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \begin{cases} \frac{1}{\sqrt{a}} \log|2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}| & (a > 0) \\ \frac{1}{\sqrt{-a}} \arcsin\frac{-2ax - b}{\sqrt{b^2 - 4ac}} & (a < 0) \end{cases}$$

66.
$$\int \sqrt{ax^2 + bx + c} \ dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{1}{\sqrt{ax^2 + b + c}} \ dx$$

68.
$$\int \frac{1}{x\sqrt{ax^2 + bx + c}} dx = \begin{cases} \frac{-1}{\sqrt{c}} \log \left| \frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x} \right| & (c > 0) \\ \frac{1}{\sqrt{-c}} \arcsin \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} & (c < 0) \end{cases}$$

69.
$$\int x^3 \sqrt{x^2 + a^2} \, dx = \left(\frac{1}{5}x^2 - \frac{2}{15}a^2\right) \sqrt{(a^2 + x^2)^3}$$

70.
$$\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \frac{\pm \sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}$$

71.
$$\int \operatorname{sen} ax \operatorname{sen} bx \ dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)} \quad \left(a^2 \neq b^2\right)$$

72.
$$\int \operatorname{sen} ax \cos bx \, dx = \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

73.
$$\int \cos ax \cos bx \ dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

74.
$$\int \sec x \, \tan x \, dx = \sec x$$

75.
$$\int \csc x \cot x \, dx = -\csc x$$

76.
$$\int \cos^m x \sin^n x \, dx = \frac{\cos^{m-1} x \sin^{n-1} + x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x \, dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x \, dx$$

77.
$$\int x^n \sin ax \ dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \ dx$$

78.
$$\int x^n \cos ax \ dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \ dx$$

79.
$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

80.
$$\int x^n \log(ax) \ dx = x^{n+1} \left[\frac{\log ax}{n+1} - \frac{1}{(n+1)^2} \right]$$

81.
$$\int x^n (\log ax)^m \ dx = \frac{x^{n+1}}{n+1} (\log ax)^m - \frac{m}{n+1} \int x^n (\log ax)^{m-1} \ dx$$

82.
$$\int e^{ax} \operatorname{sen} bx \ dx = \frac{e^{ax} \left(a \operatorname{sen} bx - b \cos bx \right)}{a^2 + b^2}$$

83.
$$\int e^{ax} \cos bx \, dx = \frac{e^{ax} (b \sin bx + a \cos bx)}{a^2 + b^2}$$

84.
$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$$

85.
$$\int \operatorname{csch} x \operatorname{coth} x \, dx = -\operatorname{csch} x$$