Definitions

$$E[X] = \int_\Omega x f_X(x)\,\mathrm{d}x$$

$$\mathrm{Var}(X) = E\left[(X-\mu)^2\right] = E\left[X^2\right] - E[X]^2$$

$$\mathrm{Cov}(X,Y) = E\left[\left(X-\mu_X\right)\left(Y-\mu_Y\right)\right] = E[XY] - E[X]E[Y]$$

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$$

$$M_X(t) = E[e^{tX}] = \int_\Omega e^{tx} f_X(x)\,\mathrm{d}x$$

Useful results

Independence

- X, Y independent iff
  - $f_{X,Y}(x,y) = f_X(x)f_Y(y)$
  - $f_{X|Y}(x|y) = f_X(x)$ or vice versa
  - $F_{X,Y}(x,y) = F_X(x)F_Y(y)$
  - $M_{X,Y}(s,t) = M_X(s)M_Y(t)$
- Independent $\Rightarrow \mathrm{Cov}(X,Y) = 0$.

Sum of random variables

$$F_{X+Y}(x) = \int_{-\infty}^{\infty} F_X(x-y)f_Y(y)\,\mathrm{d}y$$

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- Expectation and covariance are linear
  - $\Rightarrow$ Variance is linear **when independent**
- Sum of **independent**
  - Normals is normal
  - Poissons is Poisson
  - Gammas with same λ is gamma
  - Binoms with same p is binom

Density stretching theorem

Suppose $g : \mathbb{R}^n \supset \Omega \to \mathbb{R}^n$ is differentiable with global differentiable inverse, thought of as $Y_i = g_i(X_1, \ldots, X_n)$. Then

$$f_{Y_1,\ldots,Y_n}\left(y_1, \ldots, y_n\right) = |J(g)|^{-1} f_{X_1,\ldots,X_n}(x_1, \ldots, x_n).$$

Equations, inequalities, formulas

- Total expectation:

$$E[X] = E[E[X|Y]]$$

- Total variance:

$$\mathrm{Var}(X) = E[\mathrm{Var}(X|Y)] + \mathrm{Var}(E[X|Y])$$

- Markov's inequality (nonnegative X):

$$P(X \geq \alpha) \leq \frac{E[X]}{\alpha}$$

- Chebyshev's inequalities:

$$P(|X-\mu| \geq \alpha) \leq \frac{\mathrm{Var}(X)}{a^2}$$

$$P(X \geq \mu + \alpha) \leq \frac{\mathrm{Var}(X)}{\mathrm{Var}(X) + a^2}$$

- Weak Law of Large Number (X_n i.i.d.):

$$P\left(\left|\overline{X}_n - \mu\right| \geq \varepsilon\right) \to 0$$

- Strong Law of Large Number (X_n i.i.d.):

$$\lim_{n\to\infty} \overline{X}_n = \delta(\mu)$$

- Central Limit Rheorem (X_n i.i.d.):

$$\sqrt{n}(\overline{X}_n - \mu) \to N(0, \sigma^2)$$

Name	pdf	$E[X]$	$\mathrm{Var}(X)$	mgf
$\mathrm{Bin}(n,p)$	$\binom{n}{k}p^k q^{n-k}$	np	npq	$(q+pe^t)^n$
$\mathrm{Geo}(p)$	pq^{k-1}	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}$
$\mathrm{HG}(n,m,N)$	$\frac{\binom{N-m}{n-m}\binom{n}{m}}{\binom{N}{m}}$	np	$npq\frac{N-n}{N}$	$* \, p=\frac{m}{N}$
$\mathrm{Po}(\lambda)$	$e^{-\lambda}\frac{\lambda^k}{k!}$	λ	λ	$\exp(\lambda e^{t-1})$

Name	pdf	$E[X]$	$\mathrm{Var}(X)$	mgf
$U(\alpha,\beta)$	$\frac{1}{\beta-\alpha}$	$\frac{\alpha+\beta}{2}$	$\frac{1}{12}(\beta-\alpha)^2$	$\frac{e^{\beta t}-e^{\alpha t}}{(\beta-\alpha)t}$
$\mathrm{Exp}(\lambda)$	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$, $t<\lambda$
$N(\mu,\sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	μ	σ^2	$\exp\left(\mu t+\frac{\sigma^2 t^2}{2}\right)$
$\Gamma(\alpha,\lambda)$	$\frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}$ $*\,\Gamma(\alpha)=\int_0^\infty e^{-y}y^{\alpha-1}\,\mathrm{d}y$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(1-\frac{t}{\lambda}\right)^{-\alpha}$, $t<\lambda$