Graph-theoretic Models, Lecture 3, Segment 1

John Guttag

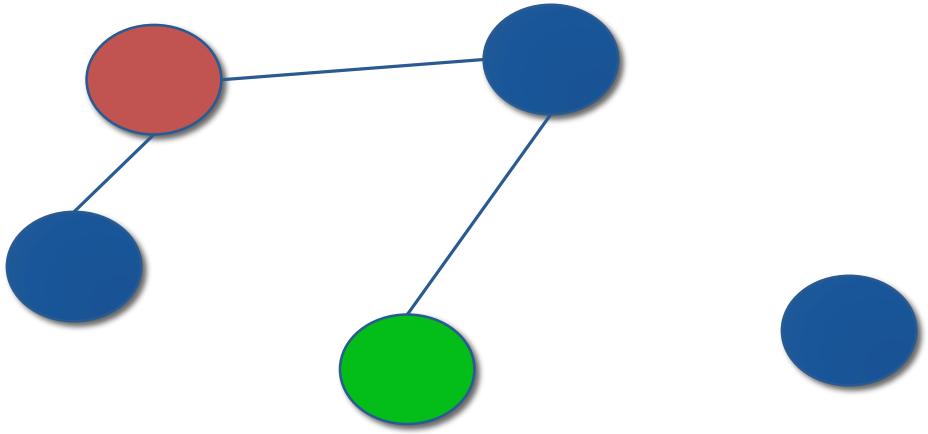
MIT Department of Electrical Engineering and Computer Science

Computational Models

- Programs that help us understand the world and solve practical problems
- Saw how we could map the informal problem of choosing what to eat into an optimization problem, and how we could design a program to solve it
- Now want to look at class of models called graphs

What's a Graph?

- Set of nodes (vertices)
  - Might have properties associated with them
- Set of edges (arcs) each consisting of a pair of nodes
  - Undirected (graph)
  - Directed (digraph)
    - Source (parent) and destination (child) nodes
  - Unweighted or weighted



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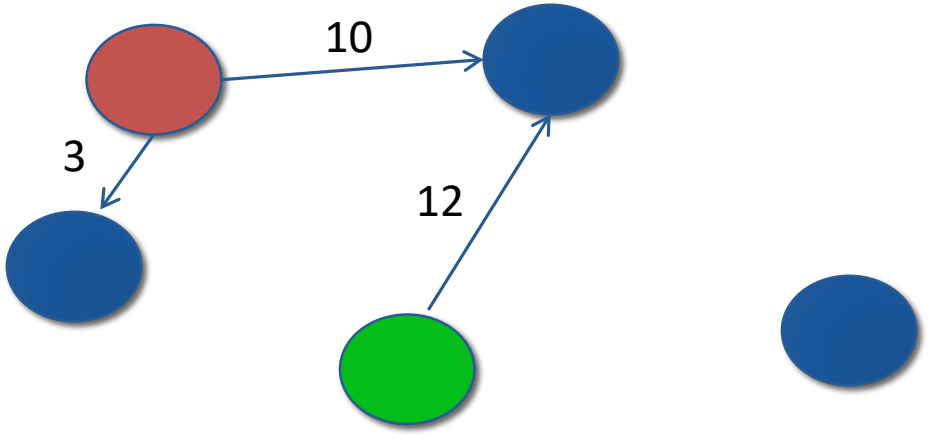
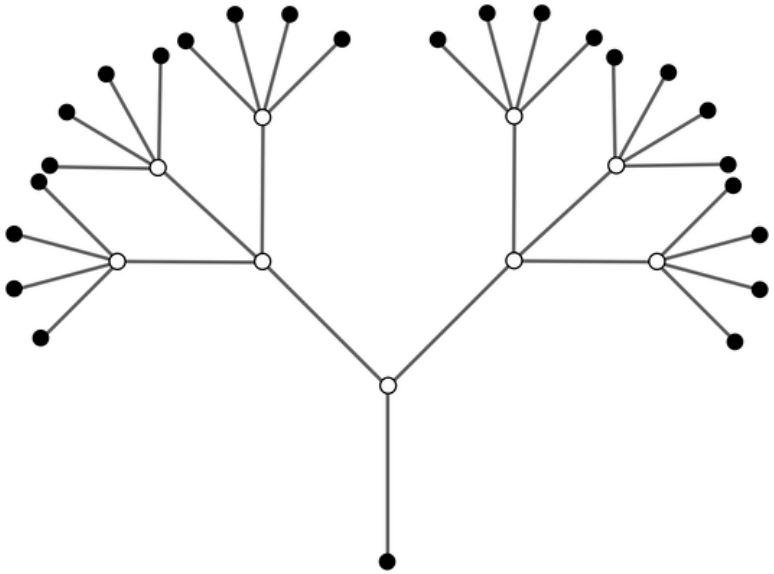


Why Graphs?

- To capture useful relationships among entities
  - Rail links between Paris and London
  - How the atoms in a molecule related to one another
  - Ancestral relationships

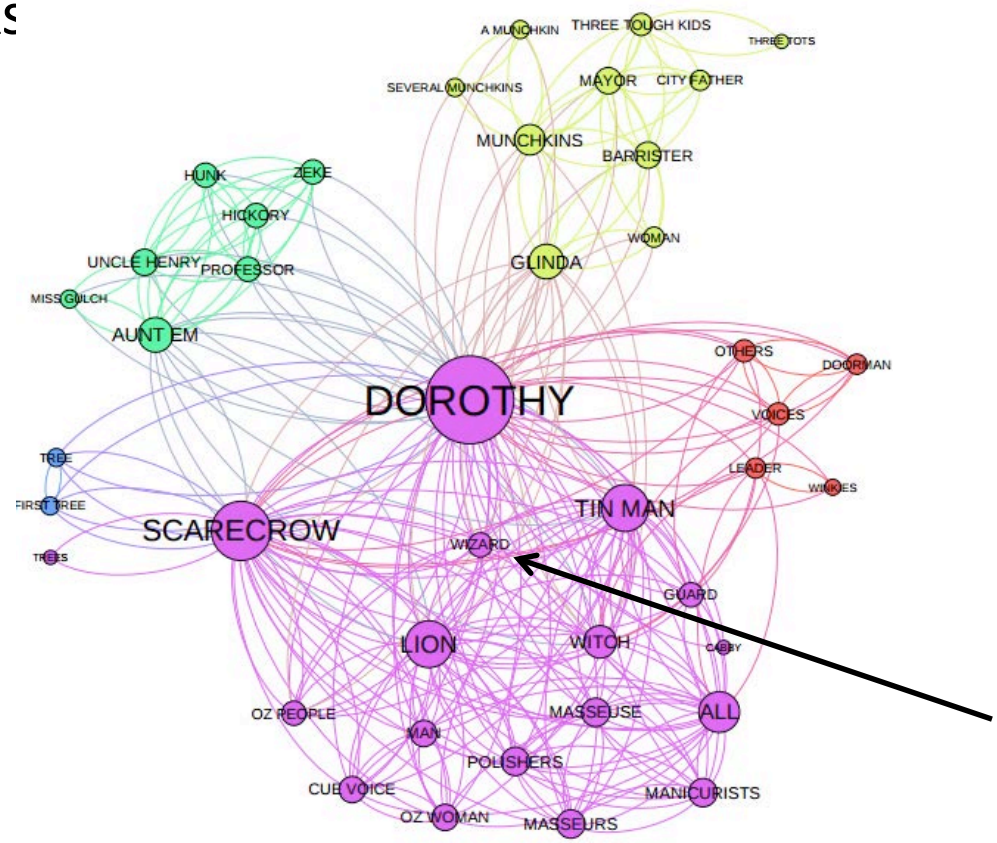
Trees: An Important Special Case

- A directed graph in which each pair of nodes is connected by a single path
  - Recall the search trees we used to solve knapsack problem



Why Graphs Are So Useful

- World is full of networks based on relationships
  - Computer networks
  - Transportation networks
  - Financial networks
  - Sewer networks
  - Political networks
  - Criminal networks
  - Social networks
  - Etc.



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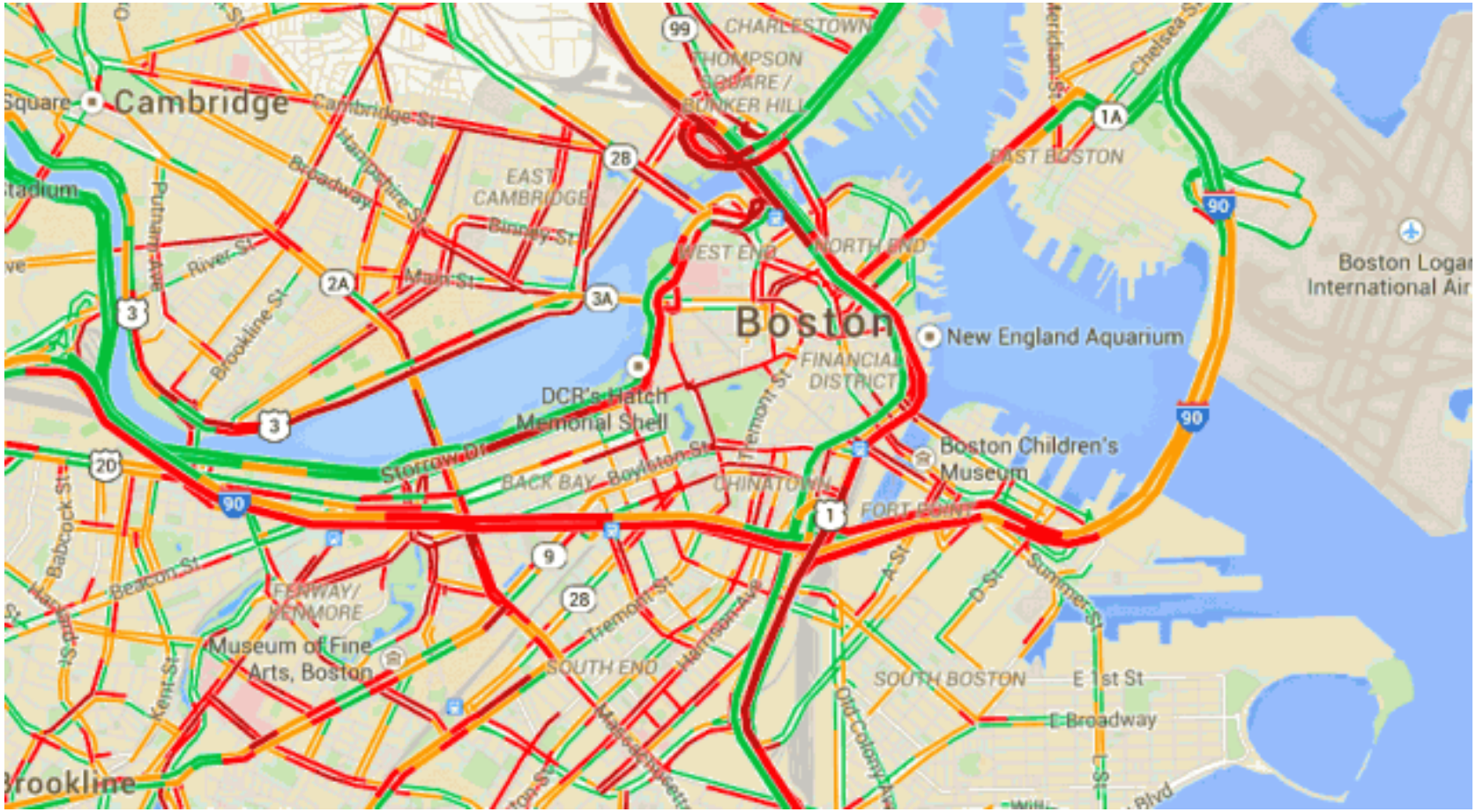
Graph Theory Saves Me Time Every Day



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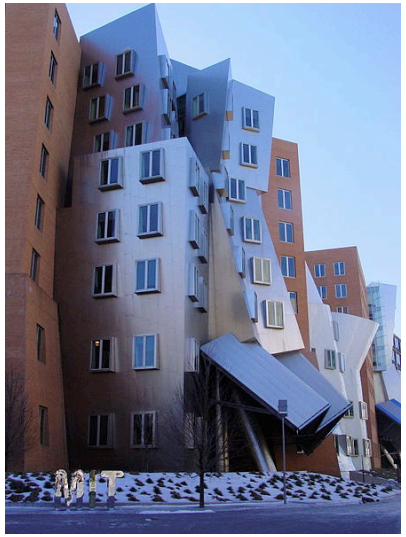
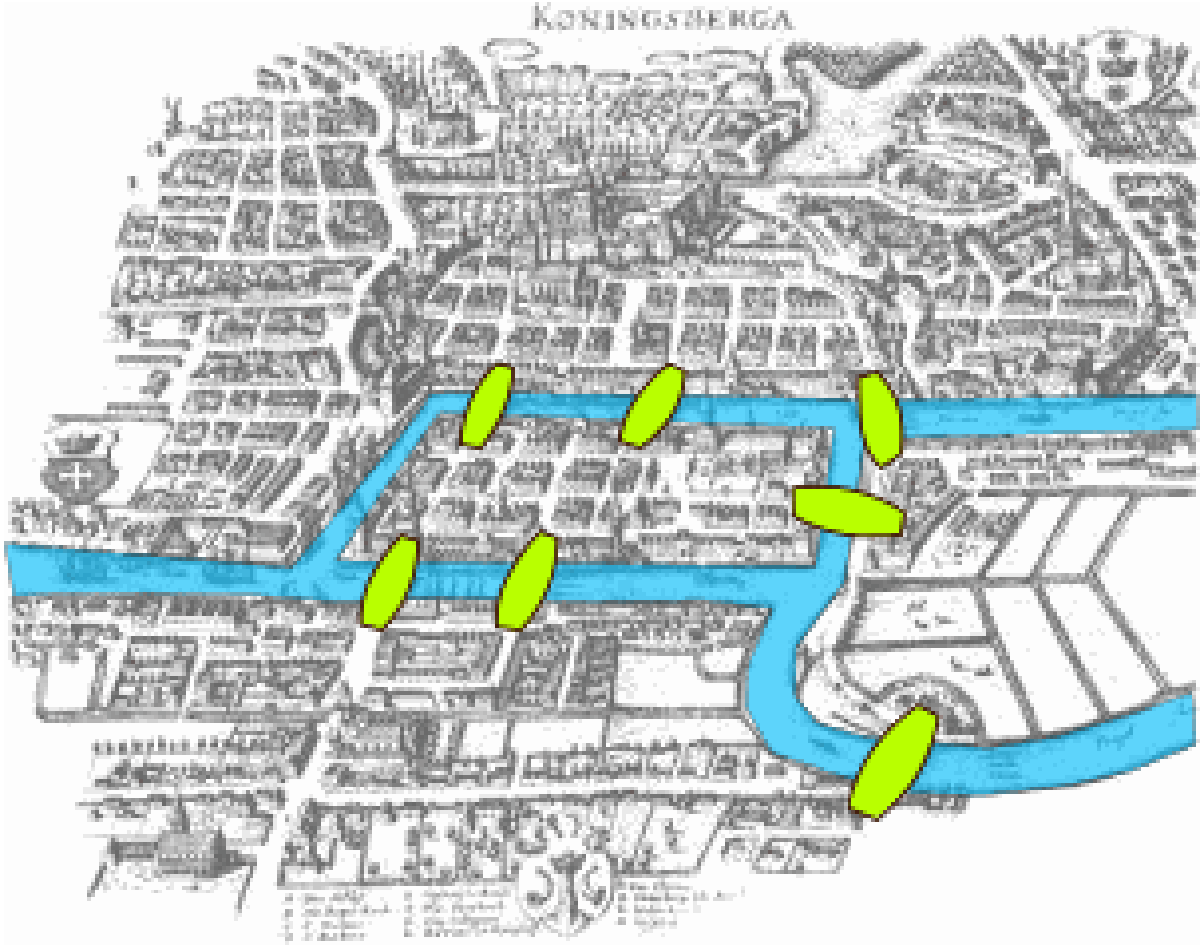
Getting John to the Office

- Model road system using a digraph
  - Nodes: points where roads end or meet
  - Edges: connections between points
    - Each edge has a weight indicating time it will take to get from source node to destination node for that edge
- Solve a graph optimization problem
  - Shortest weighted path between my house and my office



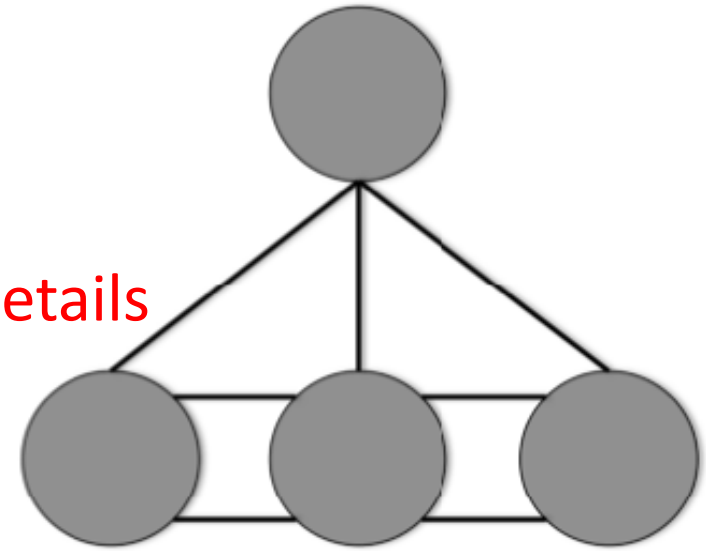
First Reported Use of Graph Theory

- Bridges of Königsberg (1735)
- Possible to take a walk that traverses each of the 7 bridges exactly once?



Leonhard Euler's Model

- Each island a node
- Each bridge an undirected edge
- Model abstracts away irrelevant details
  - Size of islands
  - Length of bridges



- Is there a path that contains each edge exactly once?

Next Segment

- Implementing graphs
- Some classic graph optimization problems