Monte Carlo Simulationm and the CLT

Simulating Buffon-Laplace Method

```python
def throwNeedles(numNeedles):
    inCircle = 0
    for Needles in range(1, numNeedles + 1, 1):
        x = random.random()
        y = random.random()
        if (x*x + y*y)**0.5 <= 1.0:
            inCircle += 1
    return 4*(inCircle/float(numNeedles))
```

```python
def getEst(numNeedles, numTrials):
    estimates = []
    for t in range(numTrials):
        piGuess = throwNeedles(numNeedles)
        estimates.append(piGuess)
    sDev = stdDev(estimates)
    curEst = sum(estimates)/len(estimates)
    print('Est. = ' + str(curEst) +\
          ', Std. dev. = ' + str(round(sDev, 6))\
          + ', Needles = ' + str(numNeedles))
    return (curEst, sDev)
```

```python
def estPi(precision, numTrials):
    numNeedles = 1000
    sDev = precision
    while sDev >= precision/2:
        curEst, sDev = getEst(numNeedles, numTrials)
        numNeedles *= 2
    return curEst
```

```
Est. = 3.14844000000012, Std. dev. = 0.047886, Needles = 1000
Est. = 3.1391799999999987, Std. dev. = 0.035495, Needles = 2000
Est. = 3.1410799999999997, Std. dev. = 0.02713, Needles = 4000
Est. = 3.141435, Std. dev. = 0.016805, Needles = 8000
Est. = 3.141355, Std. dev. = 0.0137, Needles = 16000
Est. = 3.1413137500000006, Std. dev. = 0.008476, Needles = 32000
Est. = 3.141171874999999, Std. dev. = 0.007028, Needles = 64000
Est. = 3.141589687499993, Std. dev. = 0.004035, Needles = 128000
Est. = 3.141741406249995, Std. dev. = 0.003536, Needles = 256000
Est. = 3.14155671875, Std. dev. = 0.002101, Needles = 512000
```

Being Right is Not Good Enough

- Not sufficient to produce a good answer

- Need to have reason to believe that it is close to right

- In this case, small standard deviation implies that we are close to the true value of π

Right?

Is it Correct to State

- 95% of the time we run this simulation, we will estimate that the value of pi is between 3.13743875875 and 3.14567467875?

- With a probability of 0.95 the actual value of π is between 3.13743875875 and 3.14567467875?

- Both are factually correct

- But only one of these statement can be inferred from our simulation

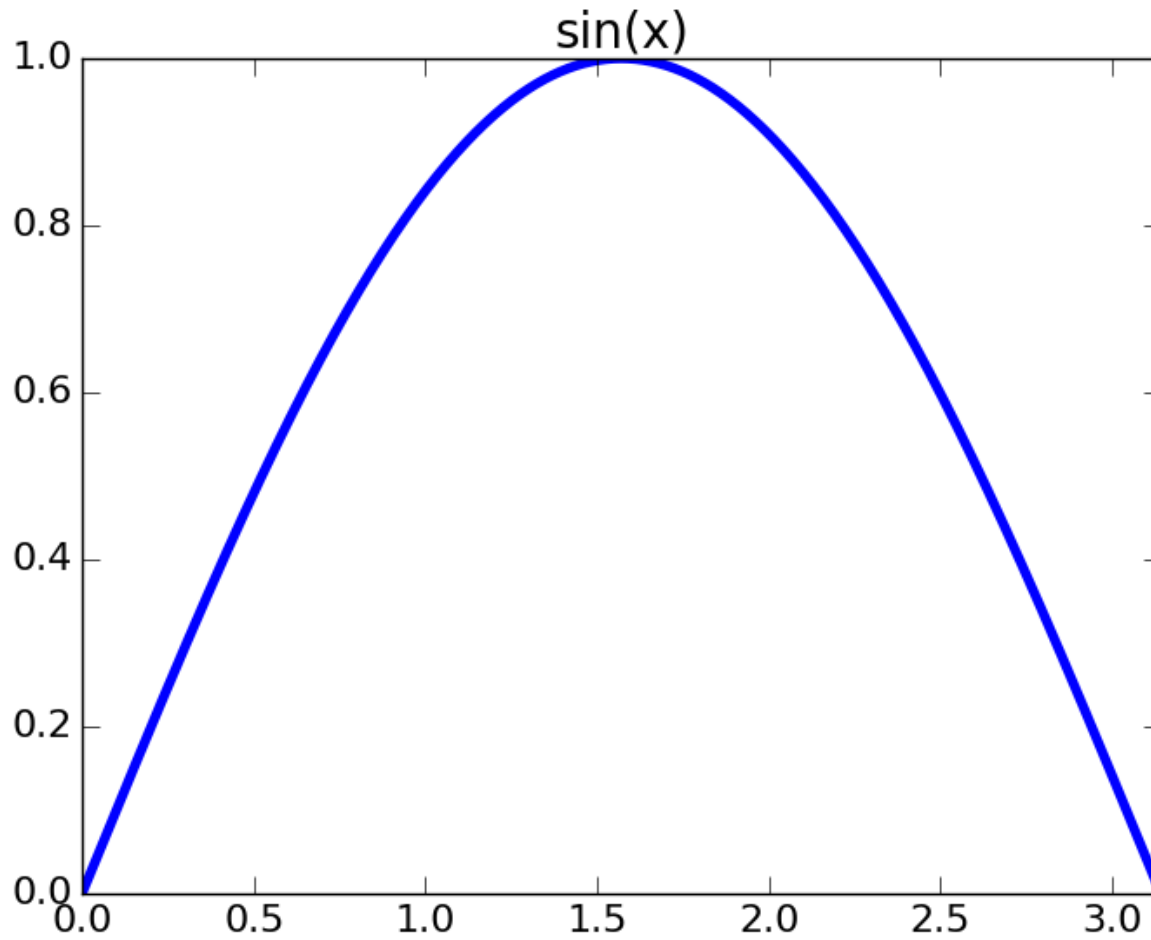- $statiscally\ valid\ \neq\ true$

```python
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    inCircle = 0
    for Needles in range(1, numNeedles + 1, 1):
        x = random.random()
        y = random.random()
        if (x*x + y*y)**0.5 <= 1.0:
            inCircle += 1
    return 2*(inCircle/float(numNeedles))
```

Generally Useful Technique

- To estimate the area of some region, R
  - Pick an enclosing region, E, such that the area of E is easy to calculate and R lies completely within E
  - Pick a set of random points that lie within E
  - Let F be the fraction of the points that fall within R
  - Multiply the area of E by F

- Way to estimate integrals

$$\int_0^\pi \sin(x)$$

Sin(x)

Random Points