Probe phase estimation

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Using the convention in Give'on, Kern and Shaklan 2011, the pairwise probing creates an approximate probe field Δp with a relationship to intensity images as:

$$I_0 = |C[E_0]|^2 + I_{inc} \tag{1}$$

$$I_{+,1} = \left|C[E_0 e^{i\Delta\psi_1}]\right|^2 + I_{inc} \approx |C[E_0] + iC[E_0\Delta\psi_1]|^2 + I_{inc} \equiv |C[E_0] + i\Delta p_1|^2 + I_{inc} \tag{2}$$

$$I_{-,1} = \left|C[E_0 e^{-i\Delta\psi_1}]\right|^2 + I_{inc} \approx |C[E_0] - iC[E_0\Delta\psi_1]|^2 + I_{inc} \equiv |C[E_0] - i\Delta p_1|^2 + I_{inc} \tag{3}$$

If we have a model for C, we don't have necessarily play approximation games, but what we do need to do is give the model what it wants. We have defined

$$\text{From } I_{+,1} : \Delta p_1 = \frac{C[E_0 e^{i\Delta\psi_1}] - C[E_0]}{i} \tag{4}$$

$$\text{From } I_{-,1} : \Delta p_1 = \frac{C[E_0] - C[E_0 e^{-i\Delta\psi_1}]}{i} \tag{5}$$

on the assumption the two are the same. We want to estimate probe phase (angle of Δp_1) while reconciling these two. Easiest way is to average the two:

$$\Delta p_{1,\text{effective}} = \frac{\left(C[E_0 e^{i\Delta\psi_1}] - C[E_0]\right) + \left(C[E_0] - C[E_0 e^{-i\Delta\psi_1}]\right)}{2i} \tag{6}$$

$$= \frac{\left(C[E_0 e^{i\Delta\psi_1}] - C[E_0 e^{-i\Delta\psi_1}]\right)}{2i} \tag{7}$$

This will be an identity operation if the positive and negative probe effect on the field is in fact the same, and will split the difference otherwise.

To get a model-based probe phase (and probe amplitude), use `np.abs()` and `np.angle()` on $\Delta p_{1,\text{effective}}$, which is complex-valued.