SECI1013 - Discrete Structure

Assignment 2

Group 9

Lecturer name : Dr. Noorfa Haszlinna binti Mustaffa
Section : 02
Group 9 members : Muhammad Hafiz bin Suhaili - A25CS0106
Nasaaie bin Noriskamar - A25CS0118
Ng Yaw Kuan - A25CS0292

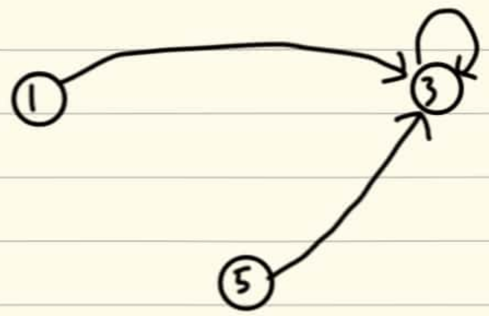
CS CamScanner

Question 1

(i) $R = \{(1,3), (3,3), (5,3)\}$

(ii) Domain $= \{1, 3, 5\}$
Range $= \{3\}$

(iii) Digraph



(iv) R is not asymmetric. It is because this relation is not reflexive and antisymmetric.
It is not reflexive since $(1,1) \notin R$ and $(5,5) \notin R$ but $(3,3) \in R$
It is antisymmetric since $(1,3) \in R$ but $(3,1) \notin R$

Question 2

- $R = \{(x,x),(y,y),(z,z),(x,z),(y,x),(x,y),(y,z),(z,x),(z,y)\}$
- R is an equivalence relation because it is reflexive, transitive and symmetric.

- For Reflexive, $(x,x) \cap (y,y) \cap (z,z) \in R$

- For Symmetric, $\forall a,b \in A, (a,b) \in R \rightarrow (b,a) \in R$

Hence, $(x,y),(y,x) \in R$

$(y,z),(z,y) \in R$

$(x,z),(z,x) \in R$

- For Transitive, $(a,b) \cap (b,c) \in R$, then $(a,c) \in R$

Hence, $(x,y),(y,z) \in R$, $(x,z) \in R$

, $(y,x),(x,z) \in R$, $(y,z) \in R$

, $(z,x),(x,y) \in R$, $(z,y) \in R$

, $(x,z),(z,y) \in R$, $(x,y) \in R$

, $(y,z),(z,x) \in R$, $(y,x) \in R$

, $(z,y),(y,x) \in R$, $(z,x) \in R$

Proving by boolean matrics

$$M_R^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = M_R$$

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{M_R} \otimes \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{M_R} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Symmetric because $M_R^T = M_R$

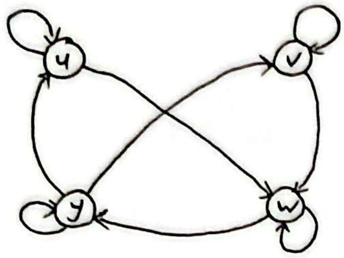
Reflexive because M_r has 1 on the main diagonal

Transitive because $M_R \otimes M_R = M_R$ at 1.

Question 3

i) $M_R:$
$$\begin{array}{c c} & \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{array}{c} u \\ v \\ w \\ y \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii)



	u	v	w	y
in-degrees	2	2	3	2
out-degrees	2	2	2	3

iii)

$$M_R = \begin{array}{c c} & \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{array}{c} u \\ v \\ w \\ y \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

R is reflexsive since for each element $x \in B$, $(x,x) \in R$ such that the main diagonal $M_R = 1$

$(u,w) \in R$, $(w,u) \notin R$ $\qquad$ $(u,u) \in R$, $a=b$

$(v,w) \in R$, $(w,v) \notin R$ $\qquad$ $(v,v) \in R$, $a=b$

$(w,y) \in R$, $(y,w) \notin R$ $\qquad$ AND $\quad$ $(w,w) \in R$, $a=b$

$(y,u) \in R$, $(u,y) \notin R$ $\qquad$ $(y,y) \in R$, $a=b$

$(y,v) \in R$, $(v,y) \notin R$

R is antisymmetric since for each $(a,b) \in R$, $(b,a) \notin R$ $\wedge$ or $\forall a,b \in B$, $(a,b),(b,a) \in R$, $a=b$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \otimes M_R \neq M_R$, thus the relation is not transitive

The relation is reflexive, antisymmetric but not transitive. Thus, it is not portial order

Question 4

let

$$f:[1,\infty) \to [0,\infty),\ f(x) = (x-1)^2$$

let $f(x_1) = f(x_2)$, ~~then~~ for $x_1, x_2 \in [1,\infty)$, then

$$(x_1-1)^2 = (x_2-1)^2$$

$$\sqrt{(x_1-1)^2} = \sqrt{(x_2-1)^2}$$

$$|x_1-1| = |x_2-1|$$

since $x_1-1, x_2-1 \geq 0$, then

$$x_1-1 = x_2-1$$

$$x_1 = x_2$$

therefore, f is one-to-one

let $y \in [0,\infty)$ be element of range

$$y = (x-1)^2$$

$$\sqrt{y} = |x-1|$$

$$\sqrt{y} = x-1$$

$$x = \sqrt{y}+1$$

~~since $y \in [0,\infty)$, $\sqrt{y} \geq 0$. thus $x = \sqrt{y}+1 \geq 1$ is in domain range $[1,\infty)$~~

~~therefore, f is onto.~~

~~since f is one-to-one and onto, it is bijective~~

Since $y \in [0,\infty)$ where $\sqrt{y} \geq 0$. Thus $x + \sqrt{y} + 1 \geq 1$ is in domain $[1,\infty)$. f is onto.

The function f is bijective, since it complies to the conditon one-to-one and onto.

Question 5

$$f(x) = 9x + 4 \quad , g(x) = \frac{3}{2}x - 1$$

a) let $y = \frac{3}{2}x - 1$

$$\frac{3}{2}x = y + 1$$

$$x = \frac{2}{3}(y+1)$$

$$g^{-1}(y) = \frac{2}{3}(y+1)$$

b) $(g \circ f)(x)$

$$g[f(x)] = \frac{3}{2}(9x+4) - 1$$

$$= \frac{27x + 12}{2} - 1$$

$$= \frac{27x}{2} + 6 - 1$$

$$\cancel{= \frac{27x}{2}} = \frac{27}{2}x + 5$$

c) $(f \circ g)(x)$

$$f[g(x)] = 9\left(\frac{3}{2}x - 1\right) + 4$$

$$= \frac{27}{2}x - 9 + 4$$

$$= \frac{27}{2}x - 5$$

d) $(f \circ g \circ g)(x)$

$$g[g(x)] = \frac{3}{2}\left(\frac{3}{2}x - 1\right) - 1$$

$$= \frac{9}{4}x - \frac{3}{2} - 1$$

$$= \frac{9}{4}x - \frac{5}{2}$$

$$(f \circ g \circ g)(x) = 9\left(\frac{9}{4}x - \frac{5}{2}\right) + 4$$

$$= \frac{81}{4}x - \frac{45}{2} + 4$$

$$= \frac{81}{4}x - \frac{37}{2}$$

Question 6

a) $P_t = P_{t-1} + \frac{1}{4}P_{t-2},\ t \geq 2,\ P_0 = 4.0,\ P_1 = 5.0$

b) $P_0 = 4.0°F$

$P_1 = 5.0°F$

$P_2 = P_{2-1} + \frac{1}{4}P_{2-2}$
$= P_1 + \frac{1}{4}P_0$
$= 5 + \frac{1}{4}(4)$
$= 6.0°F$

$P_3 = P_{3-1} + \frac{1}{4}P_{3-2}$
$= P_2 + \frac{1}{4}P_1$
$= 6 + \frac{1}{4}(5)$
$= 7.25°F$

$P_4 = P_{4-1} + \frac{1}{4}P_{4-2}$
$= P_3 + \frac{1}{4}P_2$
$= 7.25 + \frac{1}{4}(6)$
$= 8.75°F$

$P_5 = P_{5-1} + \frac{1}{4}P_{5-2}$
$= P_4 + \frac{1}{4}P_3$
$= 8.75 + \frac{1}{4}(7.25)$
$= 10.5625°F$

Question 7

a)
```
S(n)
{   if (n=1)
        return 2
    else
        return S(n-1)* S(n-1)-1
}
```

7 b)

S(4)

n=4
because $n \neq 1$
return $S(3) * S(3) - 1$

$S(4) = 63$
return $8 \times 8 - 1$

S(3)

n=3
becouse $n \neq 1$
return $S(2) * S(2) - 1$

$S(3) = 8$
return $3 \times 3 - 1$

S(2)

n=2
becouse $n \neq 1$
return $S(1) * S(1) - 1$

$S(2) = 3$
return $2 \times 2 - 1$

S(1)

n=1
becouse $n=1$
return 2

$S(1) = 2$
return 2

~~$\therefore S_4 = 63$~~ $\therefore S_4 = 63$