

Assignment 5

Stochastic Discount Factor

Suppose that consumption growth has lognormal distribution with the possibility of rare disasters:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\varepsilon} + \tilde{\nu}$$

Here ε is a standard normal random variable, while ν is an independent random variable that has value of either zero (with probability of 98.3%) or $\ln(0.65)$ (with probability of 1.7%).

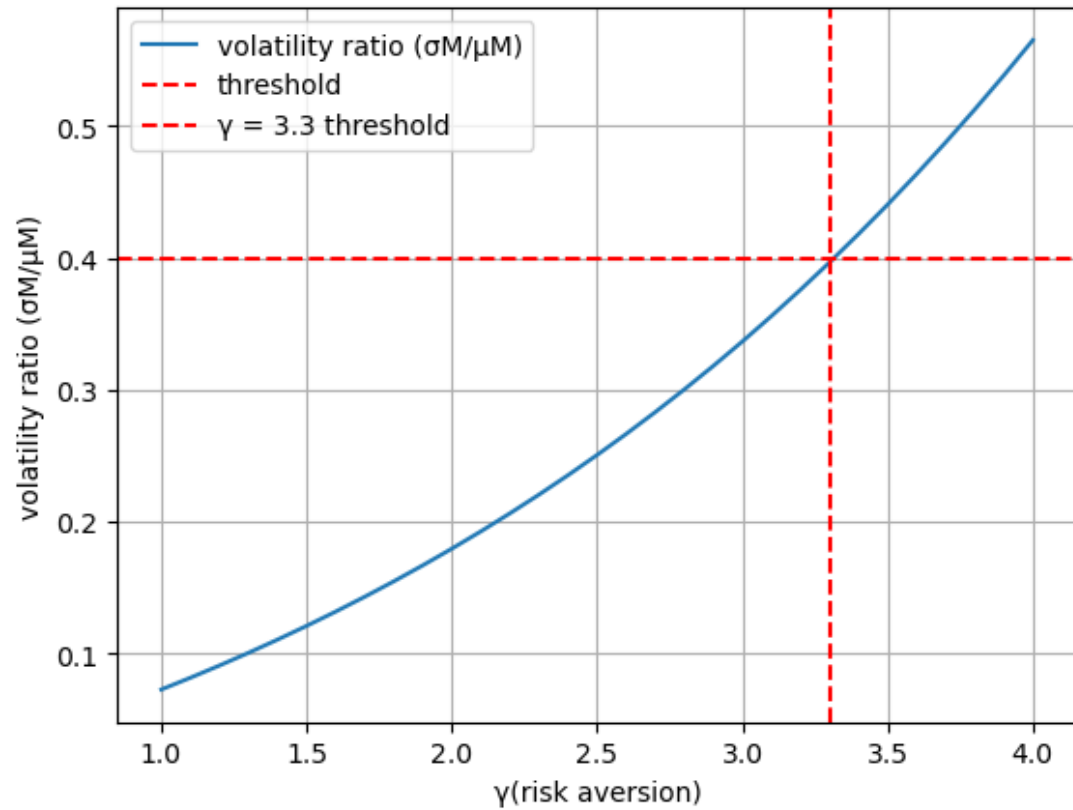
Simulate ε with (at least) 10^4 random draws from standard normal distribution, and simulate ν with (at least) 10^4 random draws from standard uniform distribution.

Use the simulated distribution of consumption growth to find the simulated distribution of the pricing kernel for power utility:

$$\tilde{M} = 0.99\tilde{g}^{-\gamma}$$

Repeat this process for values of γ in the range from 1 to 4, in increments of 0.1 (or less). (You can reuse the same simulated distribution of consumption growth for all values of γ)

- Calculate the mean (μ_M) and standard deviation (σ_M) of pricing kernel for each value of γ , and plot the volatility ratio (σ_M/μ_M) on the vertical axis vs γ on the horizontal axis.



- Find the smallest value of γ (in your data) for which $\sigma_M/\mu_M > 0.4$.

Smallest value of γ , risk aversion, where the volatility rate σ_M/μ_M , is more than 0.4 is 3.3.

- Explain (in words, without using mathematical equations or formulas) the economic significance of this result.

The economic significance of the volatility ratio σ_M/μ_M being greater than 0.4 relates to the behavior of the stochastic discount factor (SDF). The SDF represents the value of future consumption in terms of today's consumption. When the ratio exceeds 0.4, it indicates that the volatility of the SDF is relatively high compared to its mean, suggesting greater uncertainty or risk in the pricing kernel.

For asset pricing, this implies:

Risk Aversion:

As γ , which reflects the degree of risk aversion, increases, the volatility ratio rises. This suggests that as individuals become more risk-averse, the variability in how they value future consumption increases significantly relative to its expected value.

Asset Returns and Risk Premium:

A high volatility ratio means that the pricing of risky assets (such as stocks) requires a higher risk premium. Investors demand more compensation for bearing uncertainty, which is reflected in the higher risk premium.

Impact of Disasters:

In the presence of rare disasters (like economic crises), the volatility of the SDF increases because such events significantly impact future consumption. Even if these disasters are unlikely, their potential to drastically reduce consumption growth amplifies uncertainty and the demand for compensation when holding risky assets.

In essence, when the volatility ratio exceeds 0.4, it highlights an environment where risk aversion and the impact of uncertainty, such as disasters, play a crucial role in determining asset prices. Investors are more sensitive to fluctuations in consumption, leading to higher premiums

for taking on risk. However, by including rare disasters, it is demonstrated that a moderate level of risk aversion can explain the higher than expected returns on equities where gamma is more reasonable at around 3.3.