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DAMPING CHARACTERISTIC IDENTIFICATION FOR A NONLINEAR SEISMIC ISOLATION SYSTEM

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The steady-state response of structures to harmonic excitation is of both direct and indirect importance. Such a response is of obvious direct importance in problems which involve excitation from rotating machinery or other sources of steady harmonic excitation. It is also of indirect importance in problems involving transient excitation where knowledge of the harmonic response may be used in estimating and interpreting the transient structural response. For example, the "effective" natural frequencies, damping ratios and mode shapes identified from full-scale harmonic tests of structures are often used to interpret the transient non-linear response of these structures to earthquakes. The non-linearity is confined to the connection between the structure and the moving base. This system might be a highly idealized model for a reactor structure including non-linear seismic isolation effects.

In this paper a phase resonance method is given for damping characteristic identification of the nonlinear device seismic isolation in the harmonic excitation case. The method allows the shape of the symmetric function $F_d(\dot{x})$ of the system to be determined, when $F_e(x)$ is not known. If $F_d(\dot{x})$ is asymmetric then the elasticity characteristic should be known.

The identification algorithm has been derived assuming that $F_d(\dot{x})$ is given in an analytic form. The validity of the method was checked for some systems with strongly nonlinear damping characteristics.

1. Introduction

The dynamic response of nonlinear models differs qualitatively in several aspects from that of linear models. It is possible that the behaviour of the structure is linear elastically but its connection or soil-interaction is nonlinear. Examples also include buildings with a flexible first storey, adjacent structures with a flexible seismic connection and equipment mounted on flexible supports. Because the system is complex certain parameters or characteristic functions are not determined.

In this case the identification methods are utilised with efficiency. The identification problem as a part of the structural problem can be divided into the parametric and black box identification. The parametric identification is characterized by a given mathematical model, it therefore consists of a parameter determination. Most of the ground and vibration test methods use a parametric test model. If the parameter determination is done with an open loop process, the method is a direct one. The classification methods in identification problems is given by Natke [1]. The iterative method attempts to approximate the individual model and/or the model parameters of the real elastomechanical structure as accurately as possible by means of given error criteria and subsequent adjustment strategy.

The damping determination for the steady state and transient case is given for time invariance of the structural parameters and linearity in the sense of the classical theory of elasticity by Natke [2]. The classical phase resonance test presented by Natke [3,4], and de Vries and Beatrix [5] belongs to the methods in the frequency domain for the linear systems.

Natke [6] presented his phase separation method with the assumption of a diagonal generalized damping matrix, related to the eigenmodes of the associated conservative system and Breitbach [7] gave some methods concerning the identification of nonlinear systems using modal synthesis concepts for the aeroelastic problems. All the identification methods are given for mechanical systems.

In this paper is given a phase resonance method for the identification of the damping characteristic of a nonlinear device seismic isolation or structure–soil interaction with hysteretic behaviour in the harmonic excitation case.

The method enables the identification of damping characteristics $F_d(\dot{x})$ of the system defined by the model

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to be identified for an arbitrary elasticity function $F_e(x)$, the shape of which need not be known.

Because the system is nonlinear there is a primary resonance and a wide variety of secondary nonlinear resonances in which the excitation frequency differs from the natural frequency of the response. These are called superharmonic or ultraharmonic resonances when the excitation frequency is less than the natural frequency and subharmonic resonances when the excitation frequency is greater than the natural frequency. In this case the resonance is generally rich in harmonic content with the major component having natural frequency of the system and only a minor component having the frequency of excitation. In this paper, the higher harmonics are considered.

2. Description of the general model

Many engineering structures may be adequately modelled by a one-dimensional linear system. Consider such a system attached at a point P to an external non-linear compliant constraint, as indicated in fig. 1. The constraint force is assumed to consist of separate contributions from a linear viscous element (with damping coefficient γ) and a non-linear elastic or hysteretic element. The linear system is assumed to be undamped. An extension to include lightly damped linear systems is possible, but only at the expense of a substantial increase in numerical difficulty. If the unidirectional displacement of point P is denoted by $x(t)$, then the non-linear restoring force may be expressed as $F(x, \dot{x})$. Let $\mathscr{F}$ be the force transmitted to the external constraint force $f(x, \dot{x}) + \gamma\dot{x}$. The force $\mathscr{F}$ will be a function of the total response of the linear system including the motion of point P. However, since the system is linear, $\mathscr{F}$ may be expressed as the sum of two separate forces $h(x)$ and $g(t)$. In this decomposition $h(x)$ is the force which would develop at P in response to an imposed displacement time history $x(t)$ at this point with no other external forces acting. $g(t)$ is the force which would develop at P in response to any prescribed external forces applied to the linear system with P fixed. Thus, the motion of point P is governed by the equation

$$m\ddot{x} + F(x, \dot{x}) + \gamma\dot{x} - h(x) = g(t). \tag{1}$$

Let all external forces applied to the linear system have the same harmonic time dependence represented by

$$z(t) = z_0 \cos \omega t. \tag{2}$$

Then, for steady-state response, there exists a real-valued transfer function $\mathscr{G}(\omega)$ such that

$$g(t) = -\mathscr{G}(\omega)z(t) \text{ and } h(x) = -\mathscr{H}(\omega)x(t). \tag{3}$$

Eq. (1) may therefore be written as

$$m\ddot{x} + F(x, \dot{x}) + \gamma\dot{x} - h(x) = -z_0\mathscr{G}(\omega)\cos \omega t. \tag{4}$$

In most cases of practical interest, the non-linear function $F(x, \dot{x})$ is sufficiently complex that no exact analytical solution of eq. (4) is possible. Therefore approximate techniques of analysis must be used.

To illustrate the general mode the particular case in fig. 2 is presented. The non-linearity is confined to the connection between the structure and the moving base. This system might be a highly idealized model for a reactor structure including non-linear seismic isolation effects.

Let the uniform elastic shear beam have length l, shear modulus μ, cross-sectional area A and shear wave velocity β. Then the transfer function $\mathscr{H}(\omega)$ will have the form

$$\mathscr{H}(\omega) = -\frac{\mu A}{l}\frac{\omega l}{\beta}\tan\frac{\omega l}{\beta}. \tag{5}$$

As indicated in fig. 2, let the system be excited by a

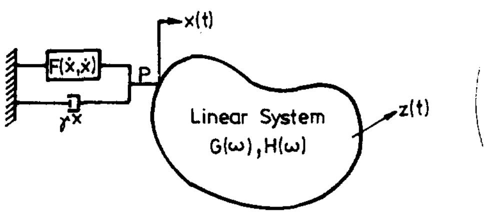

Fig. 1. General linear system attached to the non-linear constraint.

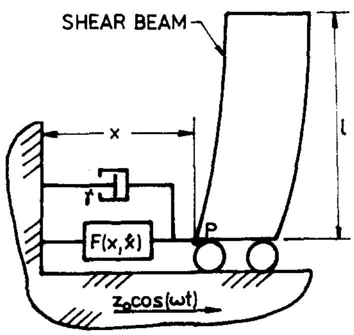

Fig. 2. Base excitation of a shear beam with the constraint $F(x, \dot{x})$.

base motion which is transmitted through the non-linear element $F(x, \dot{x})$. In this case, x measures the displacement of point P relative to the moving base and it can be shown that

$$\mathscr{G}(\omega)=\mathscr{H}(\omega). \tag{6}$$

If the system is excited by a point loading rather than by base excitation (the same value of the force as earlier at the distance ξl), the transfer function $\mathscr{G}(\omega)$ will have the form

$$\mathscr{G}(\omega)=-\left(\cos\frac{\xi\omega l}{\beta}+\sin\frac{\xi\omega l}{\beta}\tan\frac{\omega l}{\beta}\right). \tag{7}$$

3. Method of phase resonance for the non-linear dynamic system

If we consider that the eq. (1) can be written in the form:

$$m\ddot{x}+F_{\mathrm{d}}(\dot{x})+F_{\mathrm{e}}(x)=p(t)$$

or

$$m\ddot{x}+\sum_{\nu'=1}^{n'}K_{\nu'}\dot{x}^{\nu'}+\sum_{\nu''=2}^{n''}K_{\nu''}\dot{x}^{\nu''}+F_{\mathrm{e}}(x)=p(t), \tag{8}$$

where $\nu'=1,3,5,\ldots,n'$; $\nu''=2,4,6,\ldots,n''$ (n' and n'' are arbitrarily large odd and even numbers, respectively). Let us assume furthermore that the function

$$x(t)=x_0+X\mathrm{e}^{\mathrm{j}\omega t}+X\mathrm{e}^{-\mathrm{j}\tau t}=x_0+2X\cos(\omega t+\varphi), \tag{9}$$

($X=X\mathrm{e}^{\mathrm{j}\varphi}$) is a sufficiently accurate approximation of a steady state of the system considered if the exciting force $p(t)$ with the period $T=2\pi/\omega$ is of the type

$$p(t)=P\mathrm{e}^{\mathrm{j}\omega t}+P\mathrm{e}^{-\mathrm{j}\omega t}+\Delta p(t)=2P\cos\omega t+\Delta p. \tag{10}$$

The term $\Delta p(t)$ in the above expression contains superharmonic components in case there are any.

Introducing a new variable

$$\psi=\omega t+\varphi \tag{11}$$

we can rewrite function (9) in the form

$$x=x_0+X\mathrm{e}^{\mathrm{j}\psi}+X\mathrm{e}^{-\mathrm{j}\psi} \tag{12}$$

hence

$$\dot{x}=\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{j}\omega X\mathrm{e}^{\mathrm{j}\psi}-\mathrm{j}\omega X\mathrm{e}^{\mathrm{j}\psi};$$

$$\ddot{x}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\omega^2X\mathrm{e}^{\mathrm{j}\psi}-\omega^2X\mathrm{e}^{-\mathrm{j}\psi}. \tag{13}$$

In order to insert functions (12), (13) into eq. (8) the form of arbitrary powers $\dot{x}^{\nu'}$ and $\dot{x}^{\nu''}$ should be determined. Thus the power $\dot{x}^{\nu'}$ has the form

$$\dot{x}^{\nu'}=\mathrm{j}(\omega X)^{\nu'}\left\{\left[\alpha_{\nu'}\mathrm{e}^{\nu'\mathrm{j}\psi}+\alpha_{\nu'-2}\mathrm{e}^{(\nu'-2)\mathrm{j}\psi}+\ldots+\alpha_3\mathrm{e}^{3\mathrm{j}\psi}\right]+\alpha_1\mathrm{e}^{\mathrm{j}\psi}-[\ldots]^*\right\}, \tag{14}$$

where the coefficients α are expressed by the recurrent relation

$$\alpha_{l'}=-\alpha_{l'+2}+2\alpha_{l'}-\alpha_{l'-2}\quad(l'=3,5,\ldots,\nu'),$$
$$\alpha_1=-\alpha_3+3\alpha_1 \tag{15}$$

with the additional requirement that for $l'=\nu'$, $\nu'+2,\ldots,$
$\tilde{\alpha}_{\nu'}=\tilde{\alpha}_{\nu'+2}=\tilde{\alpha}_{\nu'+4}=\ldots=0.$

Formulae (15) enable the "triangle" of superharmonic amplitude to be built quickly for odd powers of the velocity $\dot{x}$ in eq. (8). Table 1 shows some first numbers of this "triangle" which could be developed further to infinity. Proceeding in a similar manner in the case of even powers we can arrive at the following formula for an arbitrary power

$$\dot{x}^{\nu''}=(\omega X)^{\nu''}\left\{\left[\alpha_{\nu''}\mathrm{e}^{\nu''\mathrm{j}\psi}+\alpha_{\nu''-2}\mathrm{e}^{(\nu''-2)\mathrm{j}\psi}+\ldots+\alpha_2\mathrm{e}^{2\mathrm{j}\psi}\right]+\alpha_0+[\ldots]^*\right\}, \tag{16}$$

where the coefficients α are defined by the corresponding coefficients $\tilde{\alpha}$ of the preceding power

$$\alpha_{l''}=(-\tilde{\alpha}_{l''+2}+2\tilde{\alpha}_{l''}-\tilde{\alpha}_{l''-2})$$
$$\text{for}\quad l''=2,4,\ldots,\nu'';\ \alpha_0=2(\tilde{\alpha}_0-\tilde{\alpha}_2), \tag{17}$$

where $\tilde{\alpha}_{\nu''}=\tilde{\alpha}_{\nu''+2}=\ldots=0.$

Formulae (17) define the "triangle" of superharmonic amplitude values for even powers of the velocity $\dot{x}^{\nu''}$ in the system (8). A few first numbers of this "triangle" are given in table 2. Substitution of eqs. (10), (12), (14) and (16) in eq. (8) gives

$$\Bigg[-m\omega^2X\,\mathrm{e}^{\mathrm{j}\psi}+\mathrm{j}\sum_{\nu'=1}^{n'}\left\{K_{\nu'}(\omega X)^{\nu'}\left[\alpha_{\nu'}\mathrm{e}^{\nu'\mathrm{j}\psi}+\alpha_{\nu'-2}\,\mathrm{e}^{(\nu'-2)\mathrm{j}\psi}+\ldots+\alpha_3\,\mathrm{e}^{3\mathrm{j}\psi}+\alpha_1\,\mathrm{e}^{\mathrm{j}\psi}\right\}\Bigg]\Bigg]$$
$$+\sum_{\nu''=2}^{n''}\left\{K_{\nu''}(\omega X)^{\nu''}\left[\alpha_{\nu''}\,\mathrm{e}^{\nu''\mathrm{j}\psi}+\alpha_{\nu''-2}\,\mathrm{e}^{(\nu''-2)\mathrm{j}\psi}+\ldots+\alpha_2\,\mathrm{e}^{2\mathrm{j}\psi}+\alpha_1\right]\right\}+[\ldots]^*F_{\mathrm{e}}(x_0+X\,\mathrm{e}^{\mathrm{j}\psi}+X\,\mathrm{e}^{-\mathrm{j}\psi})$$
$$=(P_1\,\mathrm{e}^{-\mathrm{j}\varphi})\mathrm{e}^{\mathrm{d}\psi}+(P_1\,\mathrm{e}^{\mathrm{j}\varphi})\mathrm{e}^{-\mathrm{j}\psi}+\Delta p,$$

Table 1
Initial harmonic component amplitude values for odd powers

Powers	Harmonic component amplitudes										
	α_1	α_3	α_5	α_7	α_9	α_{11}	α_{13}	α_{15}	α_{17}	α_{19}	α_{21}
$\dot{x}^1$	1	0	0	0	0	0	0	0	0	0	0
$\dot{x}^3$	3	−1	0	0	0	0	0	0	0	0	0
$\dot{x}^5$	10	−5	1	0	0	0	0	0	0	0	0
$\dot{x}^7$	35	−21	7	−1	0	0	0	0	0	0	0
$\dot{x}^9$	126	−84	36	−9	1	0	0	0	0	0	0
$\dot{x}^{11}$	462	−330	165	−55	11	−1	0	0	0	0	0
$\dot{x}^{13}$	1716	−1287	715	−286	78	−13	1	0	0	0	0
$\dot{x}^{15}$	6435	−5005	3003	−1365	466	−105	15	−1	0	0	0
$\dot{x}^{17}$	24310	−19448	12376	−6188	2380	−680	136	17	1	0	0
$\dot{x}^{19}$	92378	−75582								−1	0
$\dot{x}^{21}$	3×92378+ +75582							2×680+ 136+2380			

where the bracket [...]* contains appropriate coupled terms. Since argument x of the function $F_e(x)$ is a periodic and even function with respect to ψ, then the function $F_e(\psi)$ is also periodic and even and can be expended into a Fourier series

$$F_e(\psi)=f_0(x_0, X)$$
$$=f_1(x_0, X)e^{j\psi}+f_1(x_0, X)e^{-j\psi}+\Delta f, \tag{19}$$

where functions $f_0(x_0, X)$, $f_1(x_0, X)$ are the initial "coefficients" of this series, while the term Δf contains higher harmonics.

Putting expansion (19) in eq. (18) and equating the terms accompanying the corresponding higher harmonics we obtain

$$\text{for } e^{j0}\text{:}\quad \sum_{\nu''=2}^{n''} K_{\nu''}(\omega X)^{\nu''}\alpha_{0\nu''}+f_0(x_0, X)=0, \tag{20}$$

$$\text{for } e^{j\psi}\text{:}\quad \left[-m\omega^2 X+f_1(x_0, X)\right]$$
$$+j\left[\sum_{\nu'=1}^{n'} K_{\nu'}(\omega X)^{\nu'}\alpha_{1\nu'}\right]=P_1\,e^{-j\varphi}, \tag{21}$$

where the terms $\alpha_{0\nu''}$ and $\alpha_{1\nu'}$ denote coefficients α_0 and α_1 for the power $\dot{x}^{\nu''}$ and $\dot{x}^{\nu'}$, respectively.

For real numbers relation (21) yields

$$\left[-m\omega^2 X+f_1(x_0, X)\right]^2+\left[\sum_{\nu'=1}^{n'}\alpha_{1\nu'}K_{\nu'}(\omega X)^{\nu'}\right]^2=P_1^2 \tag{22}$$

Table 2
Initial harmonic component amplitude values for even powers

Powers	Harmonic component amplitudes								
	α_0	α_2	α_4	α_6	α_8	α_{10}	α_{12}	α_{14}	α_{16}
$\dot{x}^2$	2	−1	0	0	0	0	0	0	0
$\dot{x}^4$	6	−4	1	0	0	0	0	0	0
$\dot{x}^6$	20	−15	6	−1	0	0	0	0	0
$\dot{x}^8$	70	−56	28	−8	1	0	0	0	0
$\dot{x}^{10}$	252	−210	120	−45	10	−1	0	0	0
$\dot{x}^{12}$	924	−792	495	−220	66	−12	1	0	0
$\dot{x}^{14}$	3432	−3003	2002	−1001	364	−91	14	−1	0
$\dot{x}^{16}$	3432+ +3003×2						2×91+14+ +364		

and

$$\tan(-\varphi) = \frac{\sum_{\nu'=1}^{n'} \alpha_{1\nu'} K_{\nu'} (\omega X)^{\nu'}}{-m\omega^2 X + f_1(x_0, X)}. \tag{23}$$

Let ω_r stand for the phase resonance pulsation, such that

$$-m\omega_r^2 X_r + f_1(x_{0r}, X_r) = 0. \tag{24}$$

This requirement is satisfied when $\varphi = \varphi_r = -\pi/2$. For $\omega = \omega_r$ expression (20) is reduced to the form

$$\sum_{\nu'=1}^{n'} \alpha_{1\nu'} K_{\nu'} (\omega_r X_r)^{\nu'} = P_1. \tag{25}$$

Thus at phase resonance the amplitude of excitation P is an odd function of the argument $V_r = \omega_r X_r$

$$P_1 = \sum_{\nu'=1}^{n'} (\alpha_{1\nu'} K_{\nu'}) V_r^{\nu'}. \tag{26}$$

On the other hand, the expression (20) yields an even function of the argument $V = X\omega$ (denoting amplitudes of the system velocity response)

$$-f_0(x_0, X) = \sum_{\nu''=2}^{n''} (\alpha_{0\nu''} K_{\nu''}) V^{\nu'}. \tag{27}$$

Formulae (26) and (27) enable the damping characteristic $F_d(\dot{x})$ of any one-degree-of-freedom systems defined by eq. (8) to be fully identified.

The procedure of identification consists in determining the empirical relation $\hat{P}_1(\hat{V}_r)$ for the given object and next approximating it by function (26) which yields the coefficients $(\alpha_{11}K_1), (\alpha_{13}K_2), \ldots, (\alpha_{1n'}K_{n'})$. These in turn, the numbers α_1 being known (see the first column of table 1), will give the odd coefficients $K_1, K_3, \ldots, K_{n'}$. As can be seen when treating systems with the symmetric odd damping characteristic, the elasticity characteristic need not be known. This is not the case for systems showing nonsymmetric damping, since in such instances the empirical dependence $\hat{f}_0(\hat{V})$ must be additionally approximated by function (26) which requires the knowledge of elasticity $F_e(x)$ (for example for a linear spring $F_e(x) = c_1 x$ we have $f_0(x_0, X) = c_1 x_0$, hence the knowledge of the stiffness coefficient c_1 and the measurement of the variable x_0 are indispensable). It seems however, that for the majority of dynamic systems one may assume that their damping characteristics are symmetric (e.g. constructional materials) [this assumption may be easily verified experimentally by determining the empirical relationship $\dot{x}_0(X)$ for a few different frequencies ω].

4. Numerical example

The best way of checking the validity of a method and evaluating its practical usability is its application to real systems with various precisely known characteristics.

The system was set up so that its behaviour within the range of velocities $x \in (-0.8; +0.8)$ and displacements $x \in (-1.5; +1.5)$ obeyed exactly the equation

$$\ddot{x} + 0.50\dot{x} - 1.45\dot{x}^3 + 2.12\dot{x}^5 + 0.562x - 1.14x^2 + 1.01x^3 = p(t). \tag{28}$$

Here, similarly to the system (1): $\gamma = 0.5$; $F(x, \dot{x}) = -1.45\dot{x}^3 + 2.12\dot{x}^5 - 1.14x^2 + 1.01x^3$; $h(x) = 0.562x$.

Damping $F_d(\dot{x})$ and elastic $F_e(x)$ characteristics of the system are shown in figs. 3 and 4, respectively. As can be seen both functions are monotonic, although the assumed nonlinearity was quite large. The system under investigation was put to phase resonance (using the pulsating harmonic excitation) and two shapes $\hat{P}(\hat{V}_r)$ of the function $P(V_r)$ were obtained experimentally (fig. 5), by measuring the variable $\hat{V}_r$ in two ways. In the first one the peak values of the steady-state velocities $\hat{\dot{x}}_r(t)$

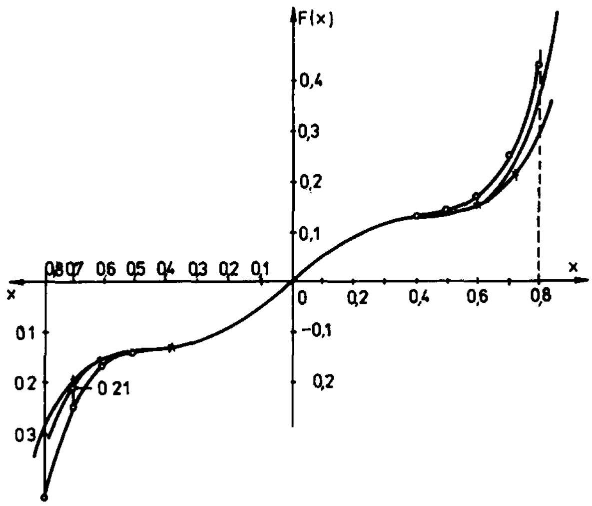

Fig. 3. Evaluation of the method as tested for the system with a symmetric elasticity characteristic: (a) plot of given function $F_d(\dot{x})$; (b) x–x–x, plot of the experimentally measured values of $F_d(\dot{x})$ (based on displacements); (c) 0–0–0–, same as (b) but based on velocity amplitudes.

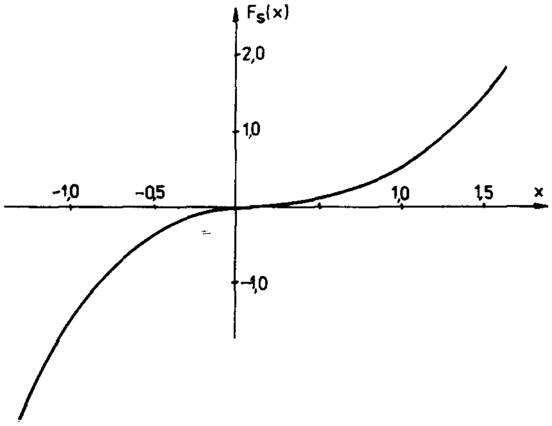

Fig. 4. An elasticity plot for the system investigated.

were measured and $\hat{V}_r$ was determined by the formula

$$\hat{V}_r = \tfrac{1}{4}(\hat{\dot{x}}_{r\,max} - \hat{\dot{x}}_{r\,min}), \tag{29}$$

while in the second approach the peak values of the displacement function $\hat{x}_r(t)$ were measured, and $\hat{V}_r$ was calculated according to

$$\hat{V}_r = \tfrac{1}{4}\omega_r(\hat{x}_{r\,max} - \hat{x}_{r\,min}). \tag{30}$$

It can be easily seen that if function (26) described the system vibrations exactly, then both the methods (28) and (29) would yield the same results. If however only the harmonic excitation ($\Delta p = 0$!) is applied to the nonlinear system then these two ways of computation lead to different values of $\hat{V}_r$, and hence to different functions $\hat{P}(\hat{V}_r)$. The differences will grow with the increasing nonlinearity of the system. From the calculations performed it follows that (fig. 5) the differences

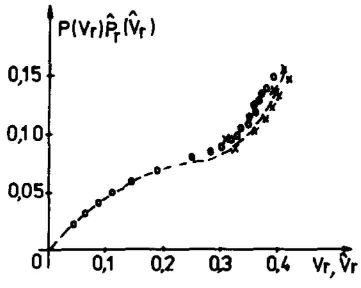

Fig. 5. Experimentally measured values of $\hat{P}(\hat{V}_r)$ for the system with nonlinear damping (without a "saddle"). A dashed line denotes the ideal function $P(V_T)$; (a) 0–0–0, $\hat{P}(\hat{V}_r)$ for the directly measured velocity amplitudes; (b) xxx, $\hat{P}(\hat{V}_r)$ for the computed velocity amplitudes ($\hat{V}_r = \omega_r \hat{X}_r$).

for the system (28) are negligible and vanish as the vibration amplitude decreases. Approximation (by the regression analysis method) of the empirical relations $\hat{P}(\hat{V}_r)$ by function (26) gave the following results
(a) for the method (29)

$$\hat{F}_d(\dot{x}) = 0.50\dot{x} - 1.44\dot{x}^3 + 2.34\dot{x}^5, \tag{31}$$

(b) for the method (30)

$$\hat{F}_d(\dot{x}) = 0.48\dot{x} - 1.22\dot{x}^3 + 1.66\dot{x}^5. \tag{32}$$

For comparative reasons the functions (31) and (32) are given in fig. 3.

In general, they differ only a little from the given characteristic $F_d(\dot{x})$ and the two curves coincide almost ideally up to a half range of velocity changes.

5. Conclusion

These results may be of significance in dynamic investigations of constructional materials where the effect of friction and viscous damping on vibration attenuation is not known or in the concept of non-linear systems of seismic isolation.

Summing up, it should be stated that the experiments fully confirmed the validity of the method. The fact that the accuracy of the results is not affected even if nonlinearities are large and excitations applied are merely harmonic is very optimistic. This permits the application of commonly available harmonic excitation generators and makes experiments quite simple. All that should be done is to determine the function $\hat{P}(\hat{V}_r)$ and–in the case of an asymmetry–also the function $\hat{f}_0(\hat{V})$.

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