Approximate seismic analysis of inelastic asymmetric structures

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This paper examines the approximate method of the single element model for analysing inelastic asymmetric structures. It simplifies the analysis by treating only a single resisting element having dynamic as well as yielding properties equivalent to the original structure with many resisting elements. The range of validity of the single element model for earthquake response calculations is defined in the paper. The study considers inelastic asymmetric one-storey structural models whose properties are varied to simulate a wide range of typical structural buildings. These properties include lateral period, torsional-to-lateral frequencies ratio, magnitude of structural eccentricity, number of resisting elements in the original structure, and yield strength of the structural model. The prime concern of the paper is to assess the accuracy of the displacement ductility demand estimated using the approximate method. For long period structures the single element model generally gives conservative estimates of displacement ductility demand. For such a range a better estimate using a modified single element model is proposed in which the single element is located at the plastic centre of strength instead of the centre of stiffness. The accuracy of the approximate method is found to considerably improve as the number of resisting elements in the original structure increases. For highly inelastic stiff structures, no approximate method can reasonably estimate their ductility demand values.

Keywords: earthquake engineering, inelastic behaviour, asymmetric structures

An elastic asymmetric structure with many resisting elements under earthquake excitations can be exactly modelled by an equivalent model with a single element located at the centre of stiffness and having the same overall dynamic properties as the original structure These properties are lateral period T_x, torsional-to-lateral frequencies ratio Ω, and fractional damping ξ. For approximate earthquake analysis of inelastic asymmetric structures, a conceptually similar treatment has been proposed by Kan and Chopra[1]. The approximate method simplifies the analysis through replacing the original structure with many resisting elements by an equivalent single element model whose yielding properties are synthesized to approximate the original properties. The use of such an equivalent single element model in the time history analysis considerably reduces the computation needed. Nevertheless, its application has been held back because the model was insufficiently tested by Kan and Chopra[1] especially against stiff asymmetric structures which have been reported[2] to be vulnerable to the combined effect of asymmetry and inelasticity. The present paper aims to identify the range of validity of this approximate method and to further substantiate the approximation of the analysis by proposing a modified single element model.

The earthquake inelastic response of asymmetric one-storey structural models whose dynamic properties are varied to simulate a wide range of typical structural

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buildings are considered These properties include lateral period, torsional-to-lateral frequencies ratio, magnitude of structural eccentricity, number of resisting elements in the original structure, and yield strength of the structural model. The prime concern of the paper is to assess the accuracy of the displacement ductility demand estimated using the approximate method To avoid results being dependent on a specific ground motion, two different earthquake records are used.

Original structural model

Let the original structure be taken as a one-storey structure consisting of a rigid deck of mass m, square in plan with side length D as shown in *Figure 1* The rigid deck is supported on four massless inextensible columns on rigid foundations at the extremities of a square of dimension a. A one-way eccentricity e is introduced by shifting the centre of stiffness CS away from the centre of mass CM of the rigid deck along the y-axis. The shift is achieved by assigning larger stiffness values to columns 1 and 2 than those assigned to columns 3 and 4. The earthquake ground motion $\ddot{u}_{gx}(t)$ is assumed to act horizontally along the x-axis and thus induces coupled lateral and torsional motions, q_x and q_θ respectively

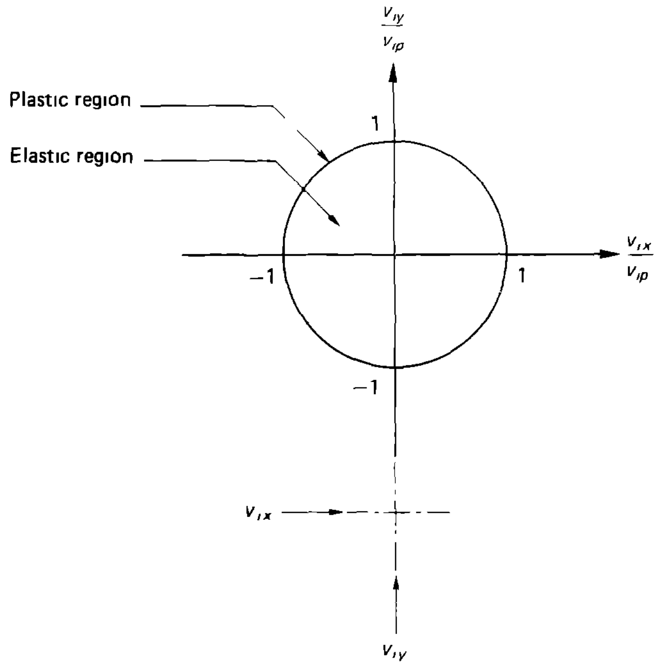


Figure 2 Properties of resisting elements

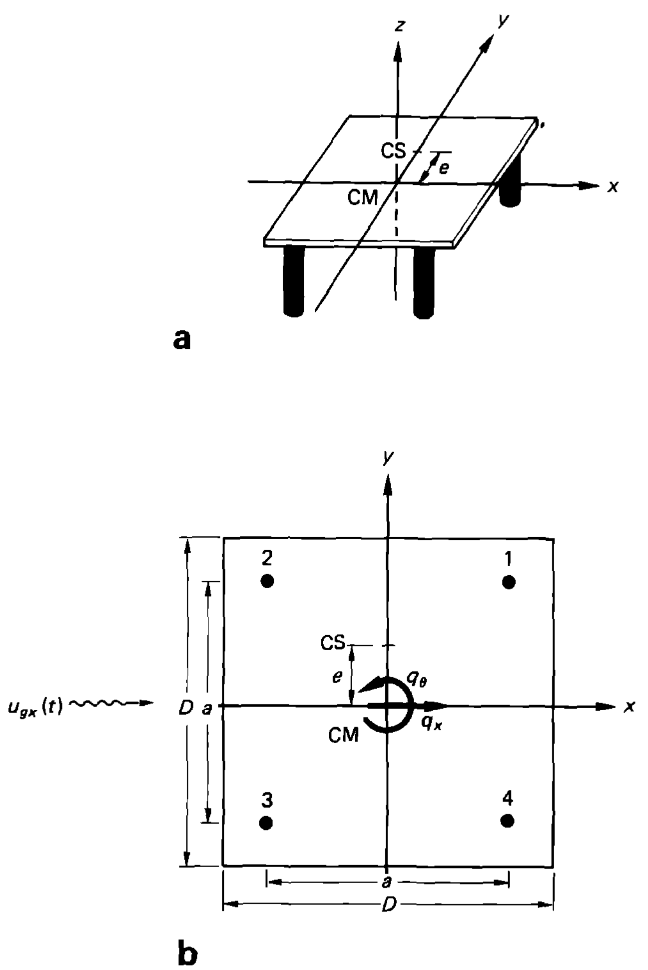


Figure 1 Original structure. (a) isometric; (b) plan

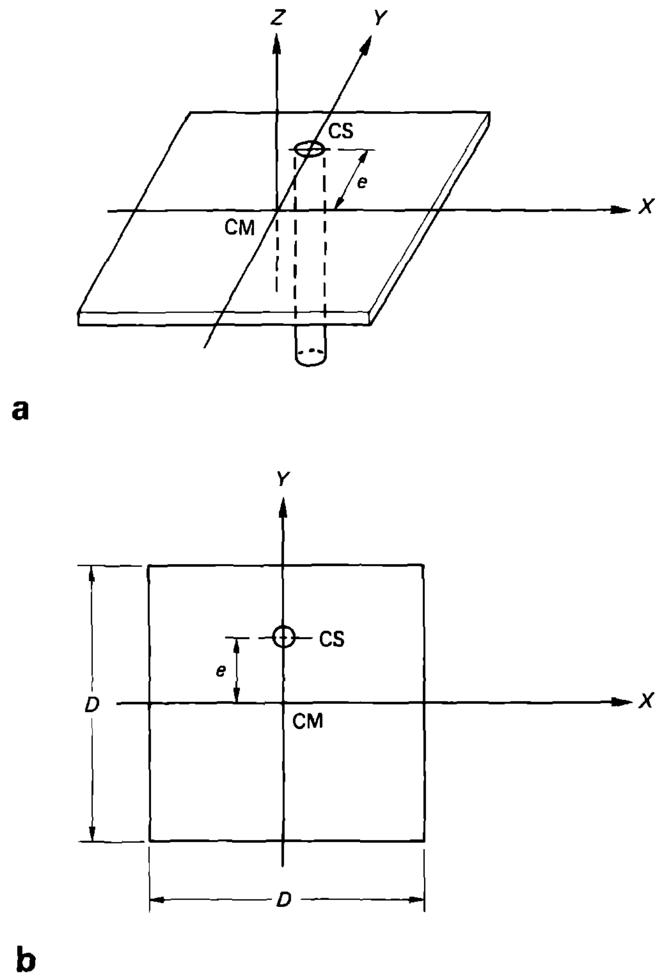


Figure 3 Equivalent single-element model: (a) isometric, (b) plan

Equations of motion

The incremental form of equations of motion of the above model is given by

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \Delta\ddot{q}_x \\ r\Delta\ddot{q}_\theta \end{Bmatrix} + 2\zeta m\begin{bmatrix} \omega_x & 0 \\ 0 & \omega_\theta \end{bmatrix}\begin{Bmatrix} \Delta\dot{q}_x \\ r\Delta\dot{q}_\theta \end{Bmatrix} + [K_t]\begin{Bmatrix} \Delta q_x \\ r\Delta q_\theta \end{Bmatrix} = -m\begin{Bmatrix} \Delta\ddot{u}_{gx} \\ 0 \end{Bmatrix} \tag{1}$$

where r is the polar radius of gyration of the deck with reference to CM, ω_x and ω_θ are the uncoupled lateral and torsional frequencies of the structure, respectively, ζ is the fraction of critical damping, and dots in the equations represent differentiation with respect to the time variable t. The overall stiffness matrix $[K_t]$ is assembled from the updated columns stiffness matrices $[S]_i$ as follows

$$[K_t] = \sum_i [d]_i^T[S]_i[d]_i \tag{2}$$

For the ith column with position coordinates (x_i, y_i) with respect to CM, the position matrix $[d]_i$ is given by

$$[d]_i = \begin{bmatrix} 1 & -y_i/r \\ 0 & x_i/r \end{bmatrix} \tag{3}$$

Yield and stiffness properties of columns

Each column is assumed to be elasto-plastic and the yield criterion under the combined action of biaxial shear v_{ix} and v_{iy} is described by a circular yield curve (*Figure 2*) and is given by

$$\phi = (v_{ix}/v_{ip})^2 + (v_{iy}/v_{ip})^2 = 1 \tag{4}$$

where v_{ip} is the plastic shear capacity of the column along each of the x and y directions. The column is said to be elastic when the tip of the force vector lies inside the yield curve and the matrix $[S]_i$ is given by

$$[S]_i = [S]_i^e = \begin{bmatrix} k_i & 0 \\ 0 & k_i \end{bmatrix} \tag{5}$$

where k_i is the elastic column stiffness. The column is said to be plastic when the tip of the force vector hits the yield curve and the column stiffness matrix is given by

$$[S]_i = [S]_i^e - [S]_i^P \tag{6}$$

where

$$[S]^P = k_1/(\partial\phi/\partial v_{1x})^2 + (\partial\phi/\partial v_{1y})^2) \times \begin{bmatrix} (\partial\phi/\partial v_{1x})^2 & (\partial\phi/\partial v_{1x})(\partial\phi/\partial v_{1y}) \\ (\partial\phi/\partial v_{1x})(\partial\phi/\partial v_{1y}) & (\partial\phi/\partial v_{1x})^2 \end{bmatrix} \tag{7}$$

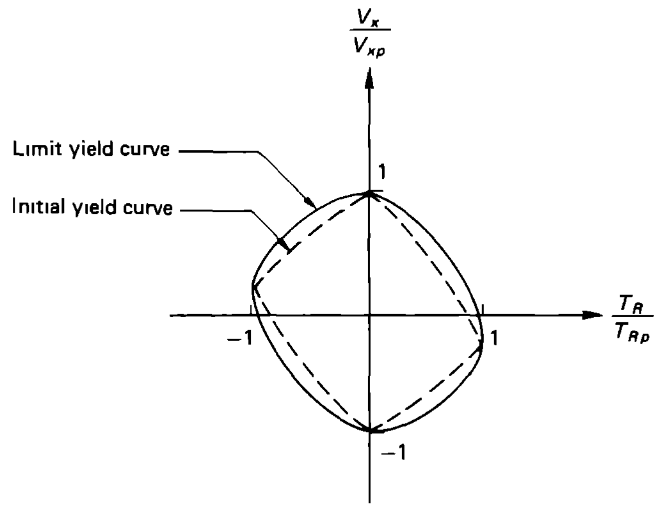


Figure 4 Initial and limit yield curves for 4-element model (e/r = 0 4)

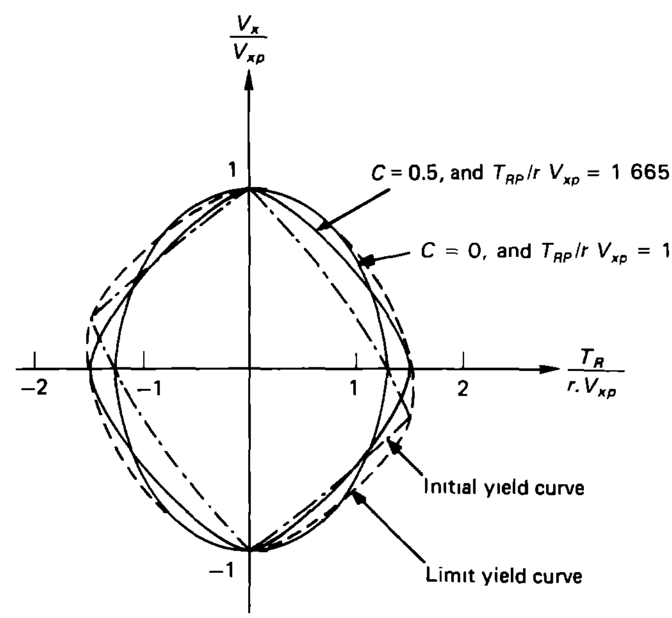


Figure 5 Yield curves for 4-element and single element models (e/r = 0 4)

Normalized equations of motion

Equation (1) can be put into the following nondimensional form

$$\begin{Bmatrix} \Delta\ddot{u}_x \\ \Delta\ddot{u}_\theta \end{Bmatrix} + 2\zeta\omega_x\begin{bmatrix} 1 & 0 \\ 0 & \Omega \end{bmatrix}\begin{Bmatrix} \Delta\dot{u}_x \\ \Delta\dot{u}_\theta \end{Bmatrix} + \omega_x^2[\bar{K}_t]\begin{Bmatrix} \Delta u_x \\ \Delta u_\theta \end{Bmatrix} = -R\omega_x^2\begin{Bmatrix} \Delta\ddot{\bar{u}}_{gx} \\ 0 \end{Bmatrix} \tag{8}$$

where u_x and u_θ are the lateral and torsional deformations normalized by the respective yield deformation, Ω is the torsional to lateral frequency ratio, and $[\bar{K}_t] = [K_t]/K_x$. R is the strength reduction factor defined[2] as the ratio of the maximum induced earthquake force mS^*_a to the system yield strength V_p.

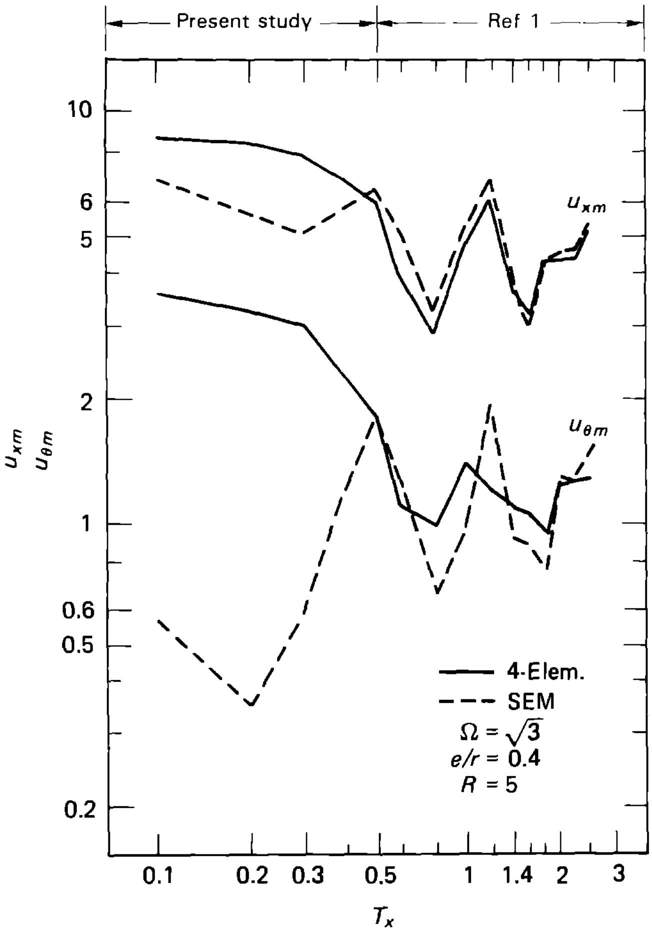


Figure 6 Comparison of maximum response for single- and 4-element models

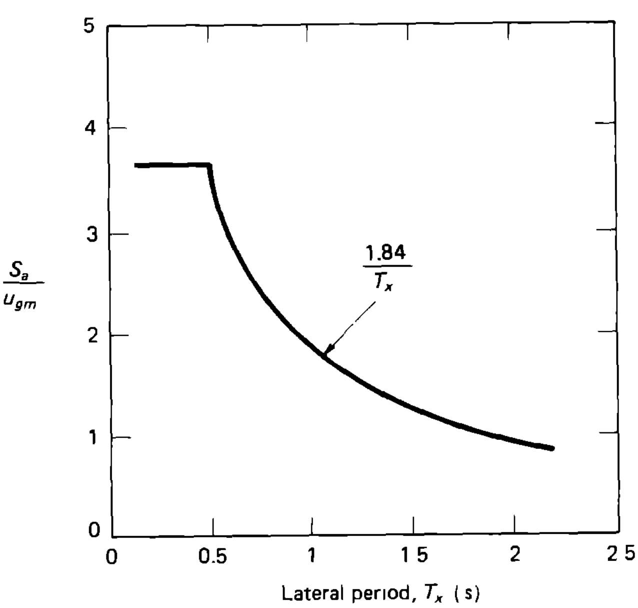


Figure 7 Normalized elastic smooth acceleration spectrum[4]

Equivalent single element model

In the equivalent model the single element is located at the elastic centre of stiffness CS as shown in *Figure 3*. In the elastic range, equivalence is satisfied by assuming dynamic properties, i.e. T_x, Ω, and ξ, to be the same as those of the original structure. With regard to the equivalence in the post-elastic range, it is assumed[1] that the total shear force V_x-torque T_R yield curve of the single element should lie between the initial and limit yield curves of the original structure. The initial yield curve is defined by combinations of shear force V_x and torque T_R at the centre of stiffness for which yielding of any resisting element in the system is initiated. The limit yield curve is defined by force combinations at which all resisting elements are fully plastic. The initial and limit yield curves of the four-element model in *Figure 1* are shown in *Figure 4* for the eccentricity ratio $e/r = 0.4$. Yield curves are plotted in the normalized force coordinate system V_x/V_{xp} and T_R/T_{RP} where V_{xp} is the total yield shear of the system and T_{RP} is the pure torque at CS causing yield. The yield curve ϕ_s of the single element is then approximated by the following expression[1]

$$\phi_s = (V_x/V_{xp})^2 + c(V_x/V_{xp})(T_R/T_{RP}) + (T_R/T_{RP})^2 = 1 \tag{9}$$

The yield curve ϕ_s is mapped onto the alternative force space $V_x/V_{xp} - T_R/rV_{xp}$ as shown in *Figure 5*. This is to reflect the relative strength of the original structure in both shear and torque. Selecting an appropriate value for c will control the curvature of the yield curve and the value of the ratio T_{RP}/rV_{xp} controls the point of intercept of ϕ_s with the torque axis. Two equivalent yield curves are plotted in *Figure 5*. The first is for torque to shear ratio of 1.665 which is the same as that of the original structure together with $c = 0.5$. The second curve is for torque to shear ratio of 1.3 and $c = 0.0$. The reduction of the torque capacity in the second curve is necessary to bring the equivalent yield curve between the initial and limit yield curves.

Stiffness properties

The tangential stiffness matrix $[K_t]$ of the single element model is given by

$$[K_t] = [K]^e - [K]^p \tag{10}$$

where $[K]^e$ is the elastic stiffness matrix given by

$$[K]^e = \omega_x^2 \begin{bmatrix} 1 & -e/r \\ -e/r & \Omega^2 \end{bmatrix} \tag{11}$$

and $[K]^p$ is the plastic stiffness matrix whose value depends on the state of the single element as follows

$$\text{Elastic} \quad [K]^p = [0] \tag{12}$$

Plastic

$$[K]^p = (1/K_x(\partial\phi_s/\partial V_x) + K_{\theta R}(\partial\phi_s/\partial T_R))$$

$$\begin{bmatrix} K_x^2(\partial\phi_s/\partial V_x)^2 & K_x K_{\theta R}(\partial\phi_s/\partial V_x)(\partial\phi_s/\partial T_R) \\ K_x K_{\theta R}(\partial\phi_s/\partial V_x)(\partial\phi_s/\partial T_R) & K_{\theta R}^2(\partial\phi_s/\partial T_R)^2 \end{bmatrix}$$

where $K_{\theta R}$ is the torsional stiffness of the structure evaluated at CS and is given by

$$K_{\theta R} = K_\theta - e^2 K_x \tag{13}$$

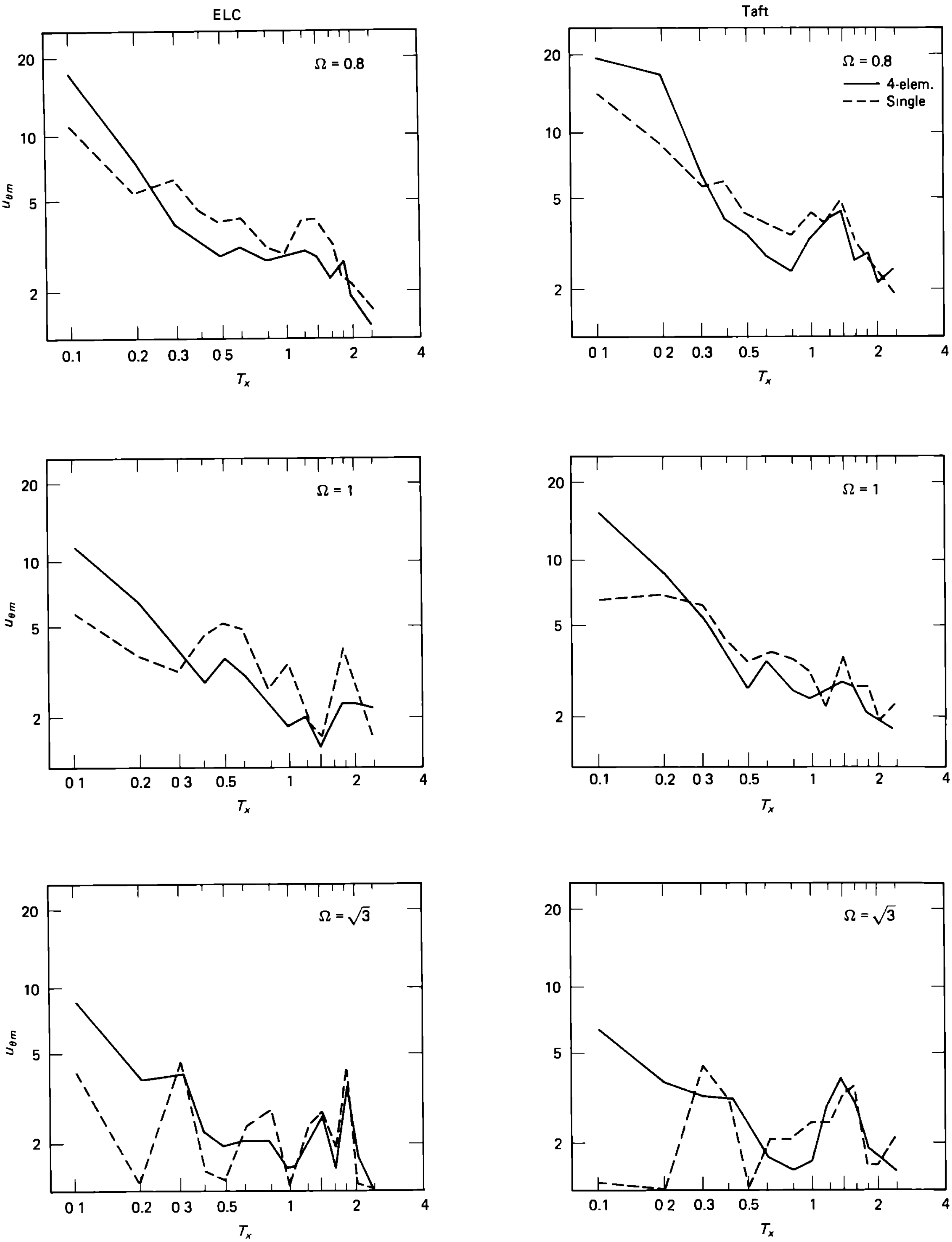


Figure 8 Comparison of rotational response of the 4-element and single-element models for different values of Ω (with e/D = 0.2, R = 5)

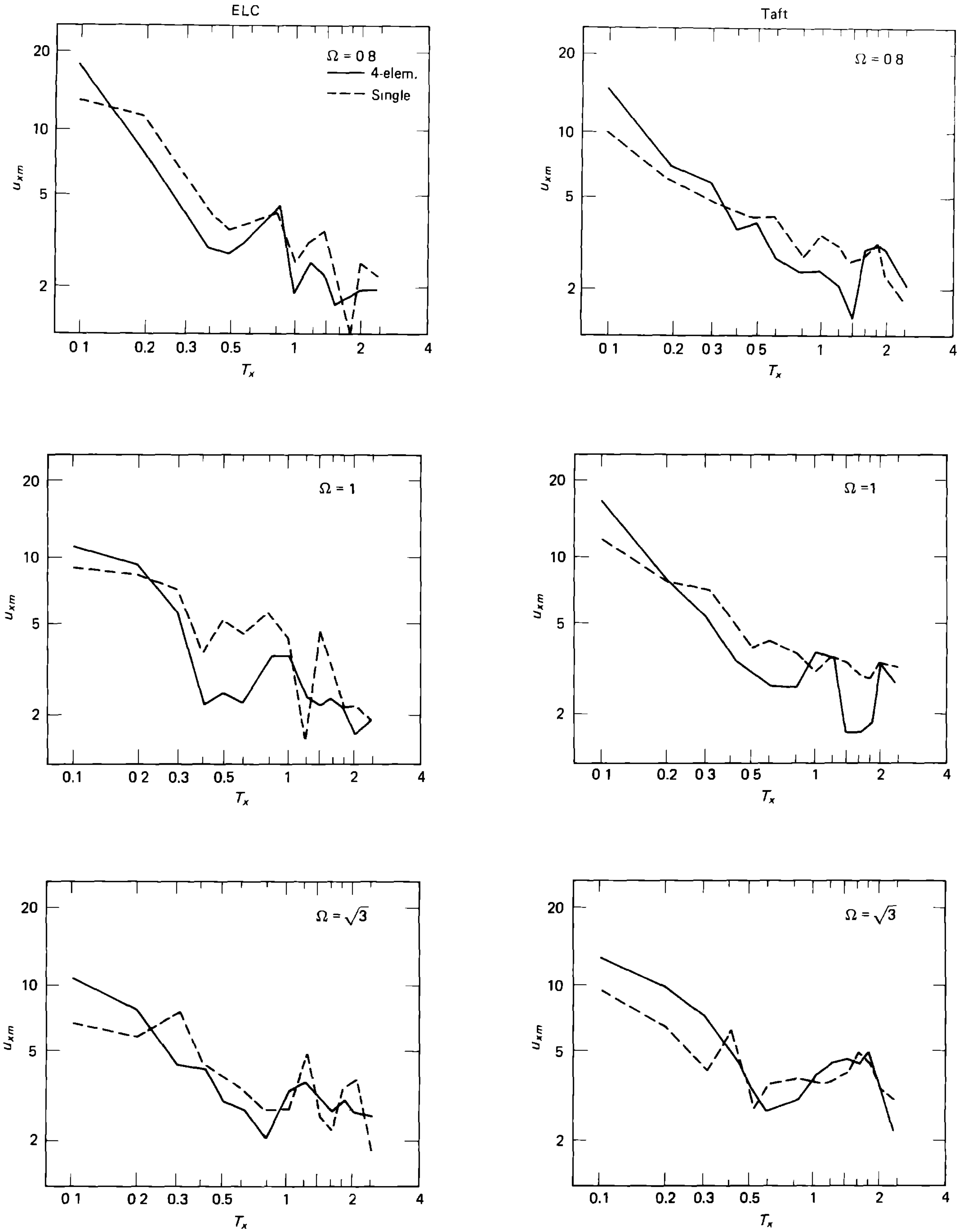


Figure 9 Comparison of translational response of the 4-element and single-element models for different values of Ω (e/D = 0.2, R = 5)

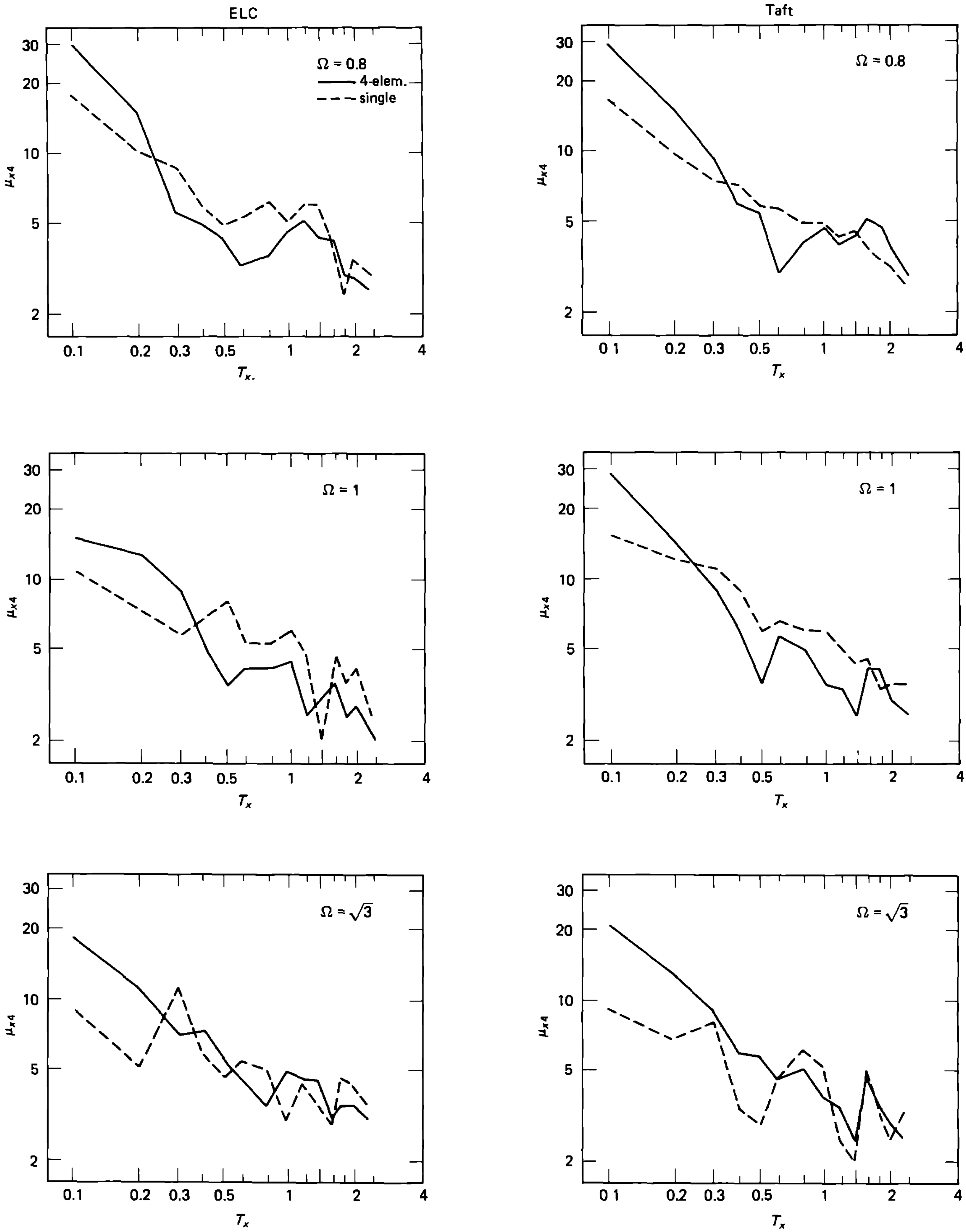


Figure 10 Comparison of maximum ductility demand of 4-element and single-element models for different values of Ω (e/D = 0.2, R = 5)

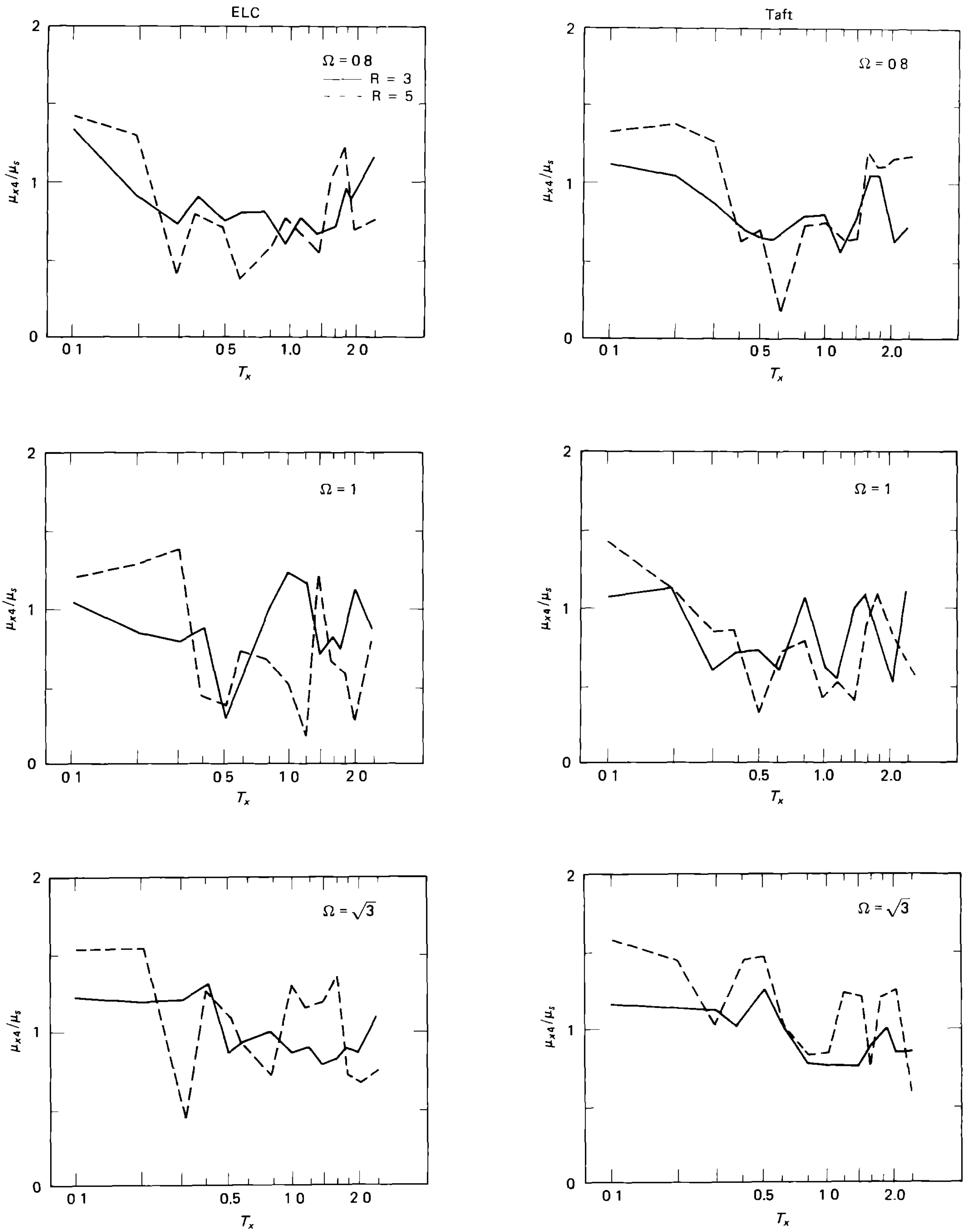


Figure 11 Ductility ratio μ_{x4}/μ_s for systems with R = 3 and 5 and e = 0 2 D

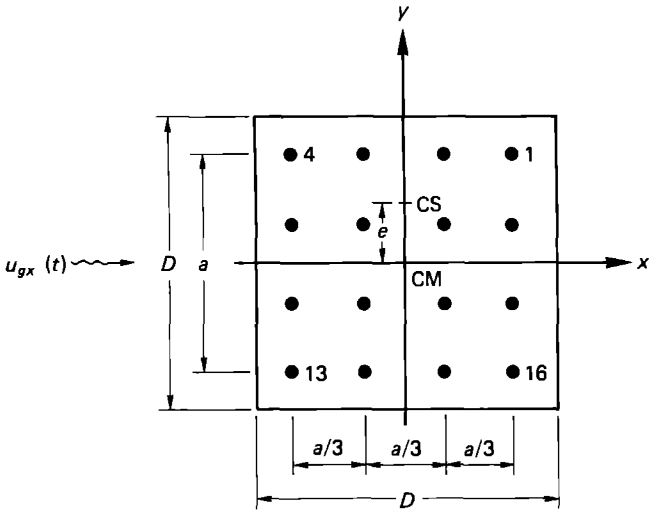


Figure 12 16-element model. System properties: $\Omega = 1.0$, $e/D = 0.2$, $R = 5$, $a/D = 0.7716$

Validity of the single-element model

As evidence of the success of the proposed single element model, Kan and Chopra[1] compared its estimates of maximum torsional and lateral deformations with those obtained from an accurate analysis using the original structure model. Structures with $\Omega = \sqrt{3}$ and whose lateral period ranged from 0.5 s to 2.5 s were used and the comparison is reproduced in *Figure 6* (other system parameters are $e/r = 0.4$, the N-S component of the 1940 El Centro earthquake record). For such a range of periods good agreement was reported. However, it should be noted that the choice of system parameters in Kan and Chopra[1] is rather limited and in favour of the proposed model. Not only did they select torsionally stiff structures, as is evident from the large value of Ω, but they also limited the comparison to structures of long or intermediate lateral periods. The more critical case of the torsionally-flexible laterally-stiff structures, which is reported[2,3] to be very susceptible to inelastic torsional deformations, was not examined. By extending the comparison in *Figure 6* down to $T_x = 0.1$ s, a sizable difference is evident between the approximate and accurate estimates of torsional as well as lateral deformations. This discussion indicates that although the single element modelling is a very useful approximate method it has not been properly examined and its range of validity in terms of system parameters has not been identified. A systematic parametric study is performed in the next section to establish the range of validity of the method.

Parametric study

The following parameters are considered in the present systematic comparison of the approximate and accurate methods

- eccentricity ratio $e/D = 0.2$; represents the case of large eccentricity
- uncoupled torsional to lateral frequencies ratio $\Omega = 0.8$, 1, and $\sqrt{3}$; represent the cases of torsionally flexible and stiff structures
- uncoupled lateral period $T_x = 0.1$ s to 2.2 s
- strength reduction factor $R = 3,5$; represent the cases of moderate and severe excitations respectively. In terms of structural behaviour, such values represent the cases of structures being moderately or severely excited into the inelastic range
- ground motion two earthquake records are used as the input ground motion, the N-S component of the 1940 El-Centro earthquake and the S69E component of the 1952 Taft earthquake. The smooth elastic spectrum proposed by Newmark and Hall[4] and shown in *Figure 7* is used for normalizing earthquake ground records.

Figures 8 to *10* show the variation of the maximum torsional deformation $u_{\theta m}$, maximum lateral deformation at CM, u_{xm}, and the ductility demand on the most stressed column μ_{x4} as functions of the lateral period for original and single element models. Values of $\Omega = 0.8$, 1, and $\sqrt{3}$ and $R = 5$ are compared. From the figures the following three observations can be made.

First from comparing the broken and solid lines in *Figure 8* it is clear that for short period structures ($T_x < 0.3$ s) the maximum rotational deformation responses $u_{\theta m}$ are considerably lower in the single element model than those accurately evaluated using the original structural model. This observation holds true for all the cases of Ω considered. For long period structures, the single element model tends to overestimate the rotational response particularly for torsionally flexible cases ($\Omega = 0.8$ and 1). Good agreement is only limited to the cases of torsionally stiff ($\Omega = \sqrt{3}$) laterally flexible structures ($T_x > 0.5$).

Secondly *Figure 9* showing u_{xm} values depicts a similar trend as mentioned above but to a lesser extent and in an unsystematic manner

Thirdly the comparison of the ductility demand curves in *Figure 10* strongly confirms the first observation. For very stiff structures ($T_x = 0.1$ s) the accurate value of ductility demand may be twice as much as the approximate value. Hence it can be said that for such a range accurate analysis is indispensible.

Effect of level of excitation, R

The above discussion is based on $R = 5$. It is useful, however, to examine the performance of the single element model in approximating moderately excited structures ($R = 3$). *Figure 11* compares the ductility ratio μ_{x4}/μ_s, defined as the ratio of the peak ductility demand accurately evaluated using the original model to that evaluated using the equivalent single element model, for $R = 3$ and 5. It can be seen that ratios associated with $R = 3$ (solid line) are generally closer to unity than the ratios of $R = 5$ (broken line). The improvement is pronounced in the short period range ($T < 0.5$). One can conclude that the accuracy of the approximate model improves for moderately excited inelastic structures.

Effect of number of resisting elements

Discussion so far has been limited to structures with only four columns. In actual buildings, however, the

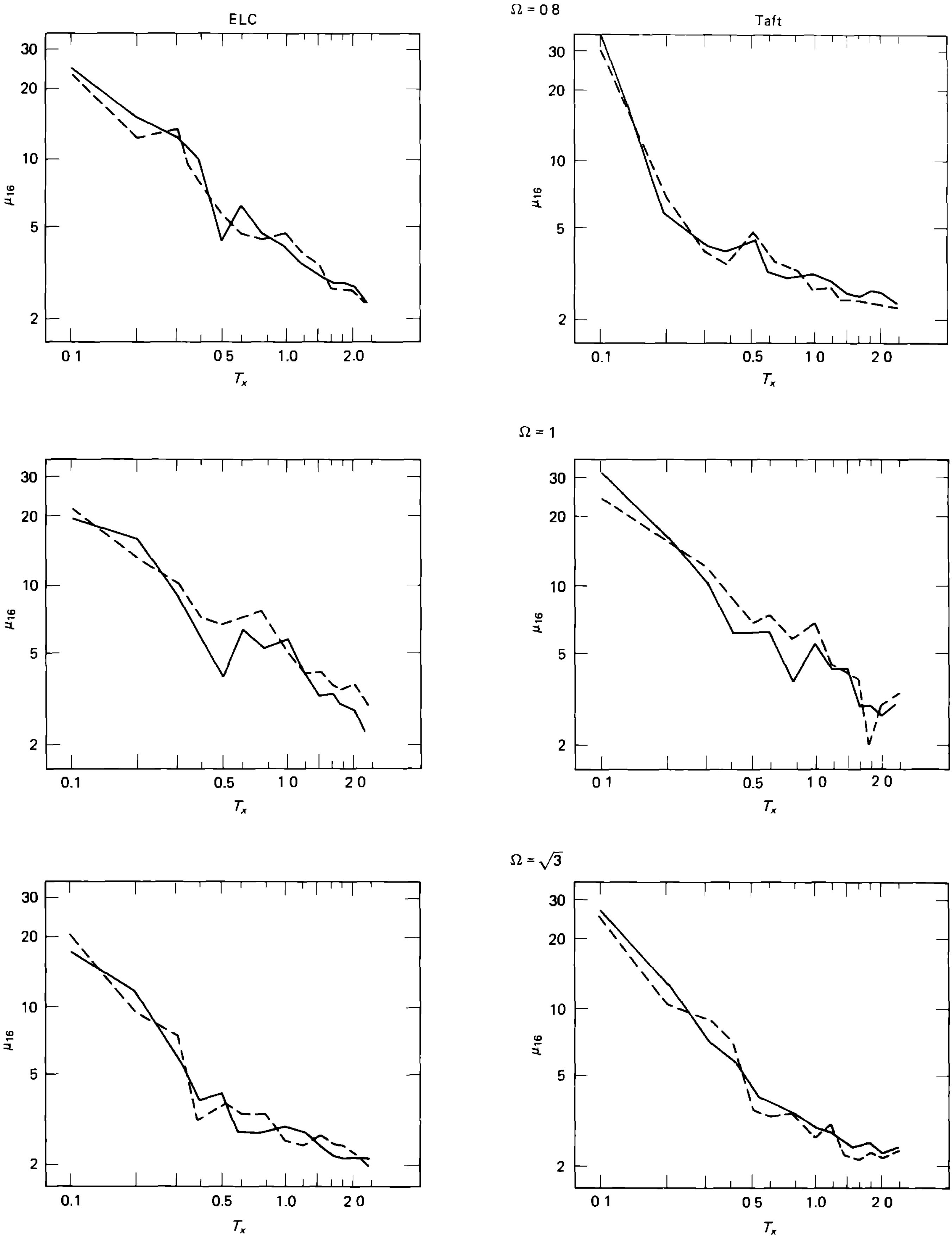


Figure 13 Comparison of ductility responses for (——), 16-element and (– – –) single-element models (e/D = 0.2, R = 5)

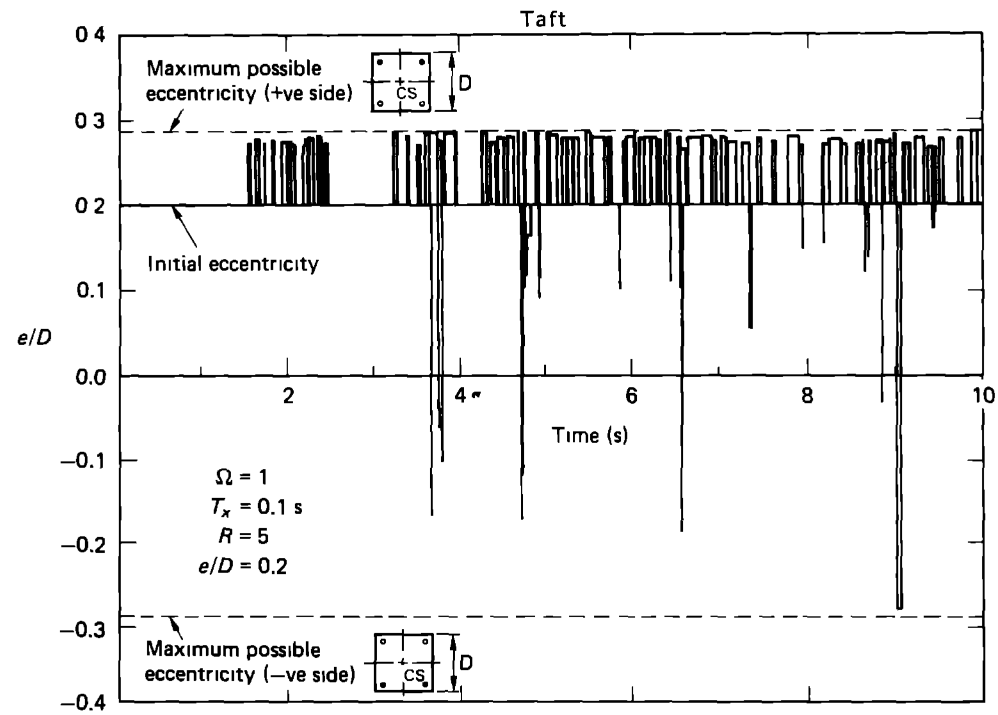


Figure 14 Variation of actual eccentricity with time for 4-element model

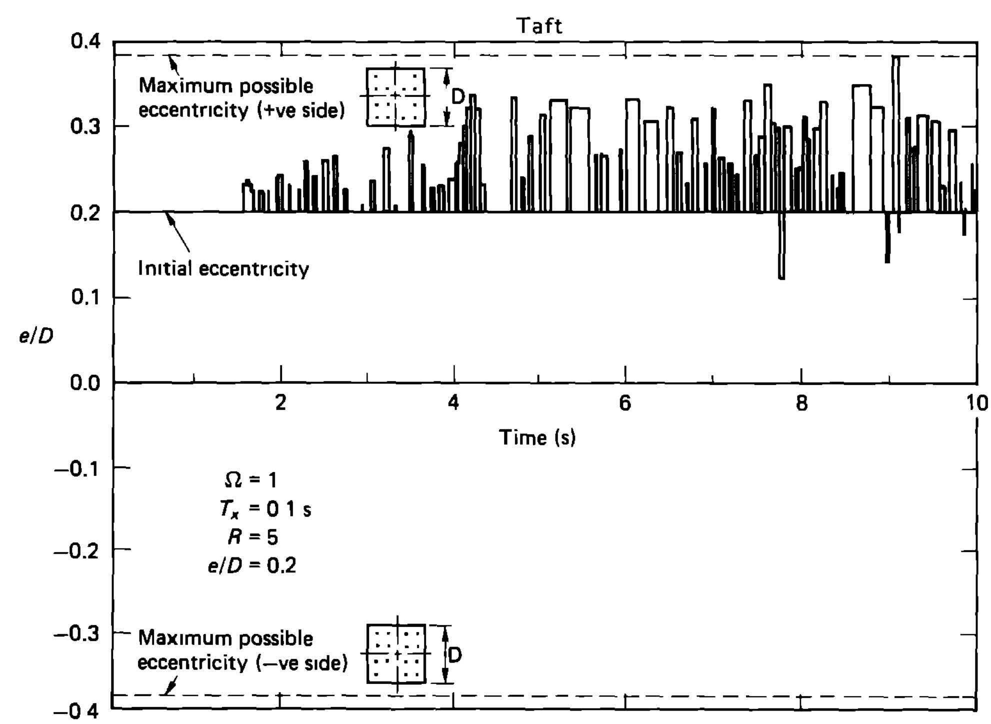


Figure 15 Variation of actual eccentricity with time for 16-element model

number of resisting elements are typically large. To examine the effect of a number of resisting elements in the original structure on the predictions of the companion single element model, a sixteen-element model (see *Figure 12*) is used. *Figure 13* shows the accurate and approximate estimates of the ductility demand on the most stressed column, μ_{16} of such a model. Other system parameters are $\Omega = 0.8$, 1, and $\sqrt{3}$, $e/D = 0.2$, and $R = 5$. A noticeable improvement in agreement between the estimates of the original structural model and the companion single element model as compared to the previous case with a lesser number of resisting elements can be observed. It can be seen that the accuracy of the single element model increases as the number of resisting elements in the original structure increases.

An attempt is made below to clarify this observation. In the single element model, eccentricity is assumed to remain invariant having a value equal to the initial

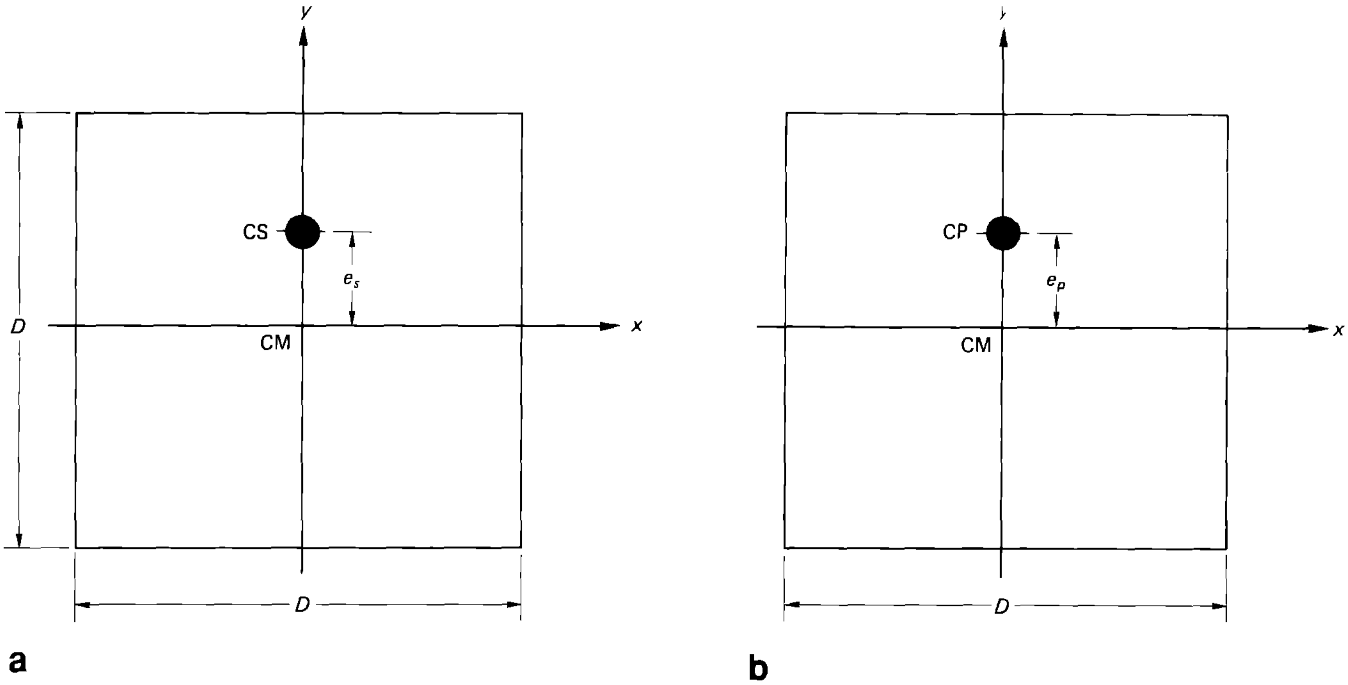


Figure 16 Single and modified single-element models (a) SEM, (b) proposed modified SEM

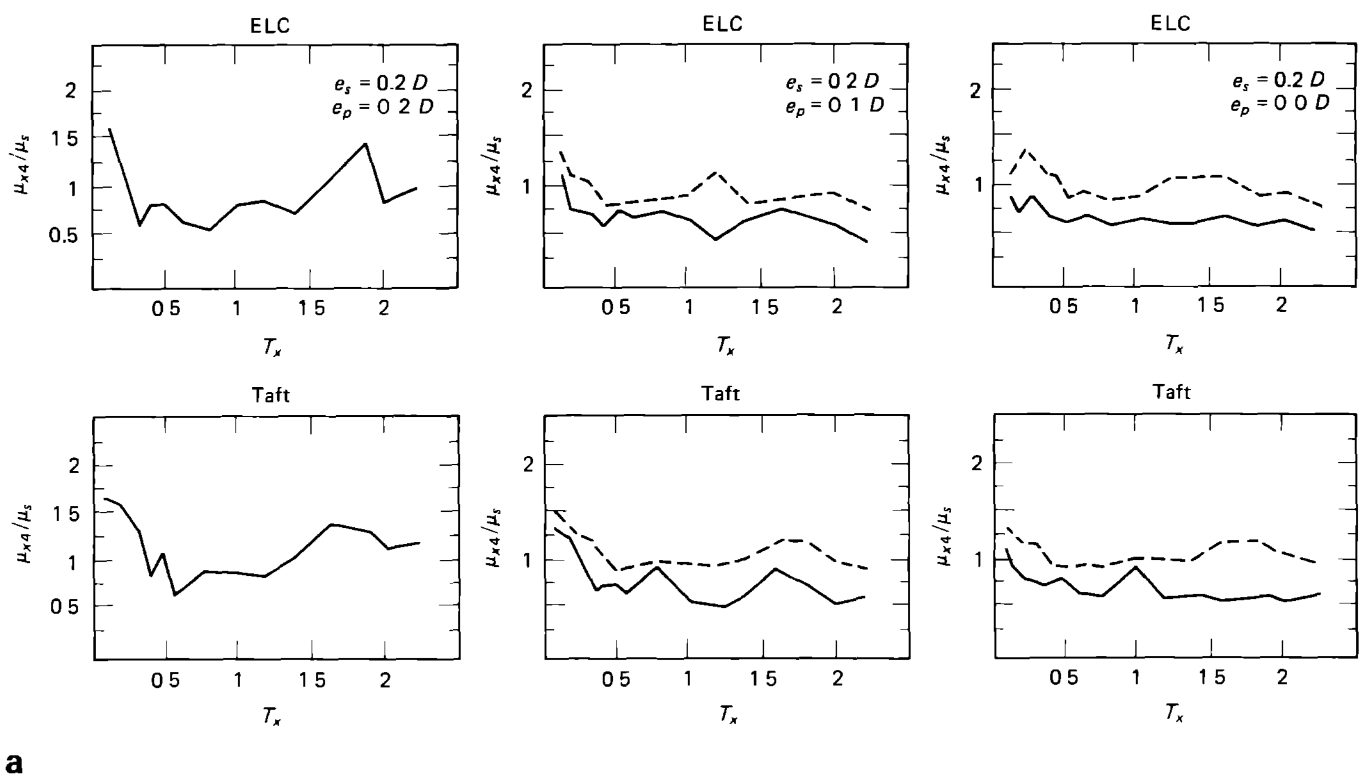
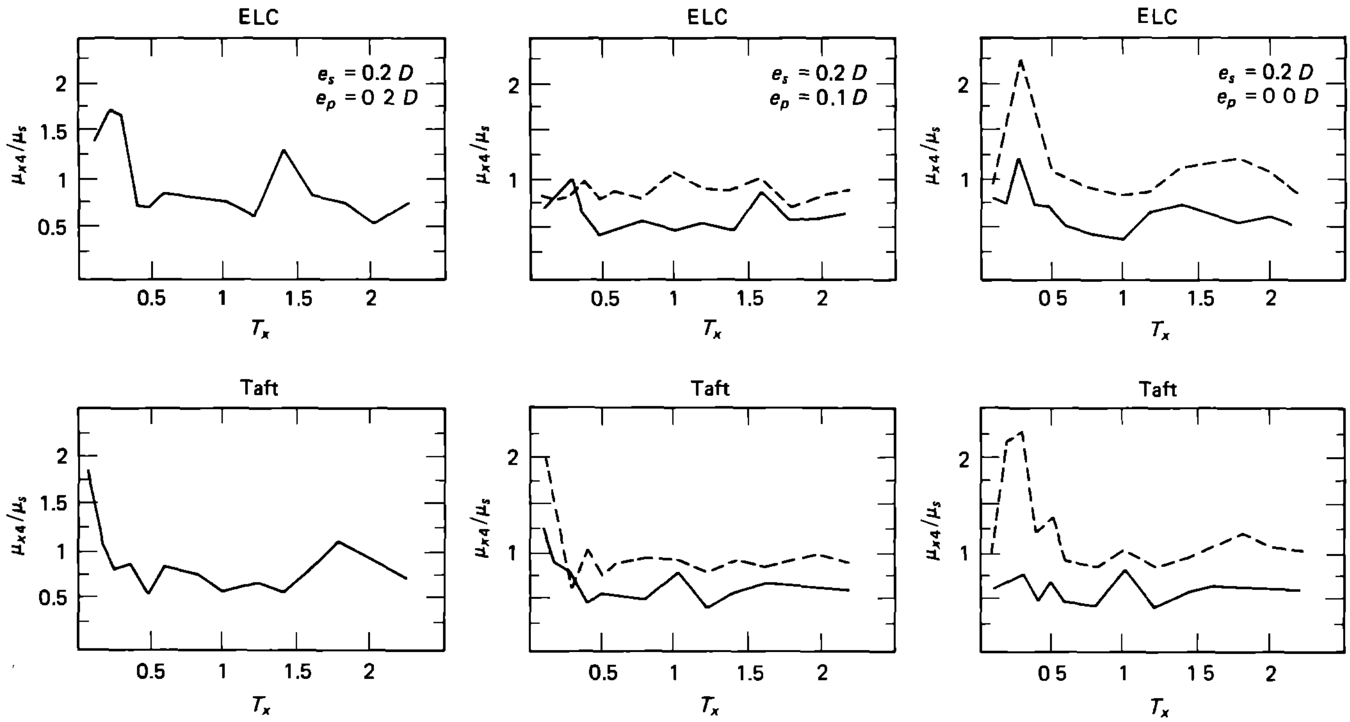
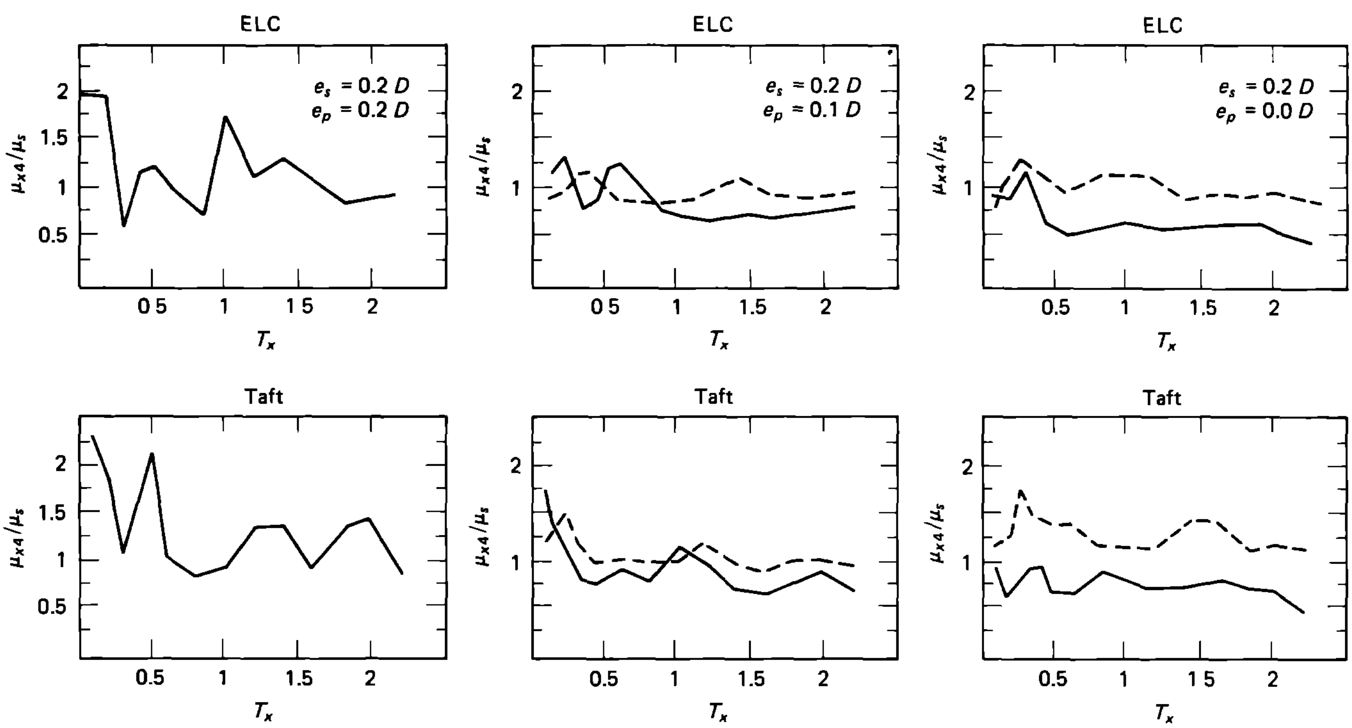


Figure 17 Comparison of ratio μ_{x4} for single and modified single element models (a) $R = 5$, $\Omega = 0.8$

eccentricity e. On the other hand, in the original structure, the location of the effective centre of rigidity changes due to yielding of resisting elements. Consequently the effective eccentricity, defined as the distance from the instantaneous centre of stiffness to CM, will vary from its elastic value. The time variation of the effective eccentricity in the 4- and 16-element models are shown in *Figures 14* and *15*, respectively. The maximum possible eccentricities on positive and negative sides are also shown in *Figures 14* and *15*. At the beginning of the excitation, the models are in the elastic range and the effective eccentricity is equal to the elastic value. The onset of yielding induces variations in the value of the actual eccentricity. In the 4-element model, the effective eccentricity varies considerably in magnitude as well as in direction. However, in the 16-element model the effective eccentricity also varies but to a lesser extent. Therefore it can be said that the assumption of the fixed location of the single element at the initial CS will be more justified as the number of



b



c

Figure 17 Comparison of ratio μ_{x4}/μ_s for single and modified single element models (b) $R = 5$, $\Omega = 1$, (c) $R = 5$, $\Omega = \sqrt{3}$, (——) SEM, (– – –) modified SEM

resisting elements increases and consequently the use of the approximate method becomes more appropriate.

Modified single element model

An alternative definition of eccentricity has been introduced[3] and is based on the strength distribution of resisting elements rather than on their stiffness distribution. The strength eccentricity is defined as the offset of the centre of yield strength CP from the centre of mass CM. The centre of strength, CP can be found by taking the first moment of the yield strength values of resisting elements about CM. The ordinates of (P, x_p, y_p are then given by $x_p = \Sigma v_{iy} x_i / \Sigma v_{iy}$, $y_p = \Sigma v_{ix} y_i / \Sigma v_{ix}$.

In the light of the findings[3] that the inelastic torsional deformations correlate well with the strength eccentricity e_p, a modified single element model is proposed here. The modification is basically that the single element is to be located at the centre of strength CP as shown in *Figure 16*. The proposed model assumes the eccentricity to be fixed but equal to the system strength eccentricity on the basis of its influential effect on inelastic torsional responses.

The modified single element model is assessed and compared with the single element model as follows. The ratio of ductility demand is accurately evaluated to that obtained using each of the two approximate models is shown for three structural systems The first structure has equal values of stiffness and strength eccentricities $e_s = e_p = 0.2\ D$ In this case both models are identical and that will give identical results. The other two structures have e_p values different from $e_s = 0.2\ D$ namely $e_p = 0.1\ D$ and 0 On this account the two approximate models will give different estimates. *Figure 17* shows the variation of ductility ratio μ_{s4}/μ, with period T for the single and modified single element models. Compared with models with $\Omega = 0.8$, 1 and $\sqrt{3}$ one can observe that, excluding the short period range, the modified model gives better estimates as indicated by the ductility ratios being closer to unity. For the short period range, both models generally fail to predict the ductility responses

Conclusions

The accuracy of calculating the inelastic earthquake response of simple asymmetric structures via the single element approximation has been examined, and the following conclusions can be drawn.

Laterally-stiff torsionally-flexible structures with low yield strength are vulnerable to the combined effect of asymmetry and inelasticity Their ductility demand responses may very well be exceptionally large. For such structures, the single element approximation is found to be inaccurate and hence an accurate analysis is essential

Laterally flexible structures can be reasonably approximated using the single element model in time history analyses. The approximate method tends to overestimate the ductility demand responses

A modified single element model is proposed, by locating the single element at the inelastic centre of strength introduced by Sadek[3] rather than at the elastic centre of stiffness. The strength centre can be easily found by taking the first moment of yield strength values of resisting elements about an arbitrary point The proposed modified model is found to be superior to the earlier single element model[1] as it provides better estimates of ductility responses of inelastic structures with long or intermediate vibrational periods.

The accuracy of the estimates provided by the single element approximation improves considerably as the number of resisting elements in the original structure increases.

References

1 Kan, C L and Chopra, A K 'Simple model for earthquake response studies of torsionally coupled buildings', *J Eng Mech Div , ASCE*, 1981, **107**, (EM5) 935–951

2 Bozorgnia, Y and Tso, W K 'Inelastic earthquake response of asymmetric structures', *J Struc Div , ASCE*, 1986, **112**, (2) 383–399

3 Sadek, A. W. 'Seismic response to inelastic structures subjected to bidirectional excitations', *PhD Thesis*, McMaster University, Hamilton, Ontario, Canada, 1985

4 Newmark, N M and Hall, W J 'Earthquake spectra and design', Earthquake Engineering Research Institute, Berkeley, California, 1982