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SIMPLIFIED ANALYSIS OF ASYMMETRIC BUILDINGS SUBJECTED TO LATERAL LOADS

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Abstract—A simple procedure is presented for lateral load analysis of asymmetric buildings, taking into account the coupling between the lateral and torsional components of response. Bending rotations of the vertical members are taken into consideration and these are made proportional to the stiffnesses of the members resulting in an analysis with $5n$ degrees of freedom for a building with n storeys. The entire analysis is conveniently programmed and gives quick results on a microcomputer. Numerical examples pertaining to both static and free vibration analyses are presented and the results are in good agreement with those from a comprehensive package program.

NOTATION

A_r	bending rotation transformation matrix (2 × 2)
C_r	member displacement transformation matrix (10 × 10)
D_r	displacement transformation matrix (3 × 3)
E	Young's modulus
G	shear modulus
I	second moment of area (with appropriate subscripts)
J	polar moment of inertia (with appropriate subscripts)
K	stiffness matrix (with appropriate subscripts)
M	mass matrix (with appropriate subscripts)
L	length of member
T	kinetic energy
Q_k	generalized forces
X, Y	coordinate axes
a_{rx}, a_{ry}	ratio of second moment of area of member r to member at τ
k	stiffness matrix/elements of stiffness matrix (with appropriate subscripts)
m	mass of floor/roof slab
q_k	generalized displacements
w	vector of displacements (u, v, θ)
r, s	denote vertical members over entire height and those stopping at set-back level
u, v	displacements in X, Y directions
x, y	coordinates of vertical member
Δ	vector of nodal displacements of vertical member
α, β	bending rotations of vertical member about X, Y axes
γ	vector of bending rotations
θ	rotation about vertical axis
τ_0	reference point and origin of X, Y coordinates at ground level
τ_i	reference point at level i

1. INTRODUCTION

A multistorey building that is torsionally balanced has, at each floor level, coincident centres of mass and stiffness which lie on a common vertical axis. However, the conditions necessary for torsional balance are so restrictive as to never actually occur [1]. Hence, all buildings are torsionally unbalanced to some extent and display interaction between the lateral and torsional components of their response. When a building is asymmetric in its plan and/or elevation or when it has a 'non-symmetric' distribution of vertical members, torsional coupling becomes significant in the lateral load analysis of that building. In this paper, a simplified method is presented to treat such asymmetric buildings. The degrees of freedom considered in the analysis pertain to nodes on one vertical line, leading to a simple analysis which can be conveniently programmed to run on a microcomputer.

Analysis of multistorey buildings with and without torsional coupling has received extensive coverage in the literature. Extensive work pertaining to the response of torsionally coupled buildings, especially single-storey buildings, to various forms of loading, particularly earthquake loading, has been carred out by Hutchinson and co-workers [2, 3]. Some researchers have focused their attention on providing simplified procedures for analysing torsionally coupled buildings. Most of these methods seem to depend on either hand calculations or elaborate computer programs [4–7]. Practising engineers may not find these simplified methods convenient for use in their design office. There are also many general purpose programs which are capable of doing 'almost anything'. However, these programs require large computational effort and time, and hence may not be attractive for those with restricted computer facilities, budget and time. In an earlier paper a simple microcomputer analysis for torsionally coupled buildings was presented [8]. In that analysis all vertical members were assumed to be restrained at the floor levels, i.e. a 'shear beam model' was considered. This method was an extension to that for a single storey building [9]. However, when heavy cores and shear walls are present in a building, this assumption will not hold. The vertical members will suffer a rotation at the floor levels. This effect has been incorporated in the present analysis, which is yet simple and easily programmable on a microcomputer. Thus it will readily lend itself to be useful to practising engineers in their design office.

The present study can be outlined as follows: in Sec. 2, the analytical model is described and the assumptions used in the analysis are given. Potential and kinetic energies of the system are calculated and the stiffness and mass matrices derived using Lagrange's equations in Sec. 3, which highlights the contribution made by this study. The equations of motion are also presented in this section in a manner suitable for programming. Numerical examples are presented in Sec. 4 to illustrate the procedure and the results are discussed. Results are compared wherever possible with those from a commercial package. Finally, in Sec. 5 concluding remarks are made.

2. ANALYTICAL MODEL

Consider the building whose plan is shown in Fig. 1. Due to the presence of the core (at a location other than the centroid of the plane area), this building will display asymmetric behaviour for lateral loads parallel to the Y axis. Thus there will be torsional coupling in the building response even if the building has a prismatic cross-sectional plane. In order to be quite general, a set-back at an intermediate level is introduced. Vertical members such as s go upto the set-back level, while vertical members such as r continue right through the entire height of the building. $\tau_i (i = 1 \rightarrow n)$ denotes the intersections with the floor level of a common vertical axis originating at τ_0 which is taken as the origin of coordinates x, y. Thus the position P of a vertical member at any floor level is referenced by the x_r, y_r coordinates as shown in Fig. 2. The degrees of freedom (of the vertical members) at each floor level i will be related to the displacements at τ_i as shown in Fig. 3 and 4. This will enable the potential and kinetic energies to be expressed in terms of the displacements at τ_i. The following assumptions are made in the analysis:

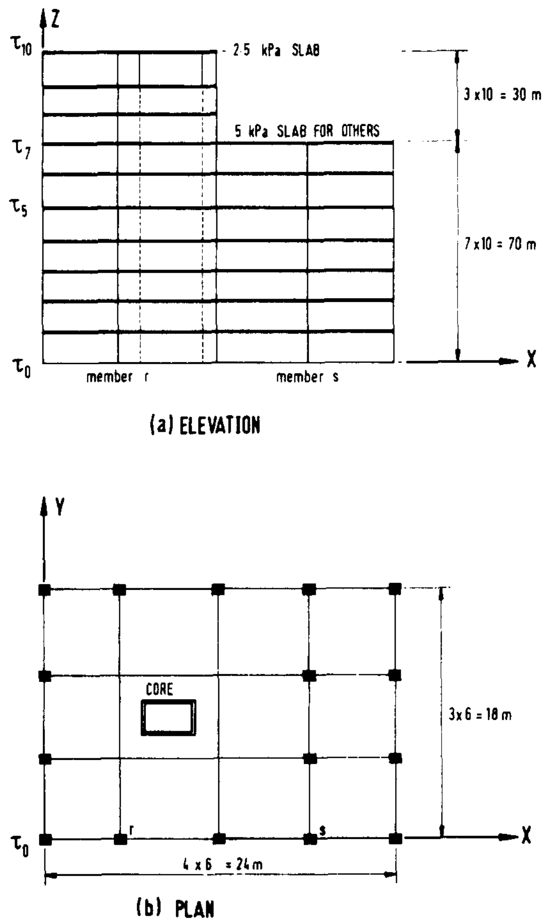


Fig. 1. Analytical model of asymmetric building—ten storeys with set-back and service core (example 3).

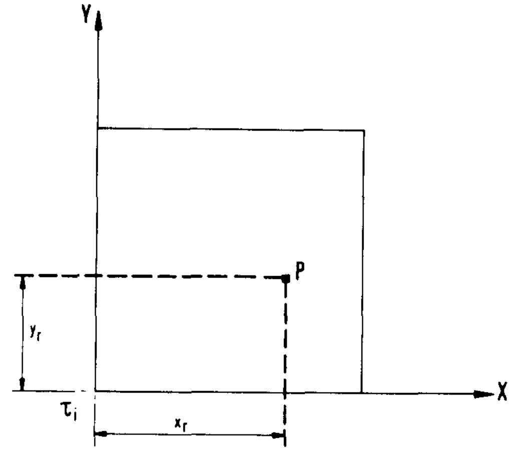


Fig. 2. Coordinate system.

(i) Floors are treated as rigid diaphragms which possess three degrees of freedom per floor, namely u, v and θ with respect to (the reference point) τ_i.
(ii) Kinetic energy of vertical members (columns, cores, shear walls, etc.) are neglected. However, we can smudge this by adding, say 10% to each floor inertia. Alternatively, for buildings with heavy cores, we can add in 'point' masses when working out the moment of inertia for floors.

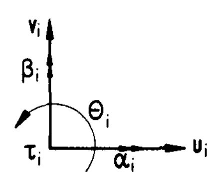
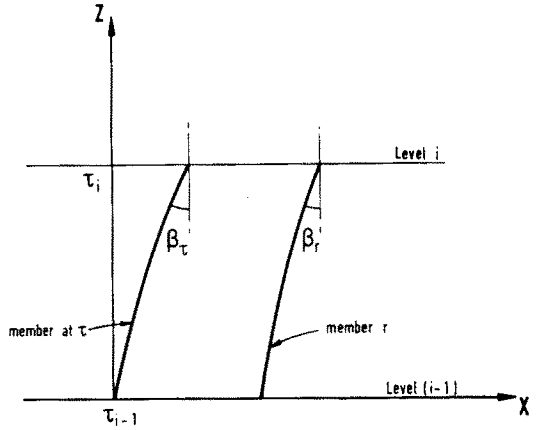


Fig. 3. Degrees of freedom.



$$\beta_r = \left(\frac{I_{ry}}{I_{\tau y}}\right)\beta_\tau \quad ; \quad \alpha_r = \left(\frac{I_{rx}}{I_{\tau x}}\right)\alpha_\tau$$

Fig. 4. Incorporation of bending rotations of vertical members in the analysis.

(iii) All vertical members are assumed to suffer rotations (α, β) about the x, y axes, respectively. Furthermore it is assumed that these rotations at any floor level are related to each other and are proportional to their stiffnesses (as shown in Fig. 4).

(iv) Principal axes of all vertical members are along the X–Y directions. This is a reasonable assumption as it is uncommon to have vertical members with axes in arbitrary directions.

(v) The system is closed coupled, i.e. effects at any floor level are not propagated beyond the levels just below and above that floor level.

An alternate choice for the degrees of freedom will be the centroidal displacements, u_{Gi}, v_{Gi}, θ_i, α_i, β_i at each floor level leading to a somewhat different analysis. In this case, the mass matrix will be diagonal but the derivation of the stiffness matrix will be more complicated. However, the authors prefer to choose the degrees of freedom with respect to τ_i, which has a distinct advantage in the case of buildings with set-backs. The stiffness matrix derivation is made considerably simpler now, with marginal increase of effort in computing the mass matrix which will not be diagonal now.

It is recognized that not all buildings need to be analysed according to the present method. In the absence of service cores and shear walls and when the vertical members are all 'light' columns, the earlier 'shear beam model' would probably suffice. The computer program developed (in Quick BASIC) has the capability of analysing by either procedure, with the option resting on the user. At present the effects of some parameters on the response are being studied. It is hoped that this study will enable the method of analysis to be chosen decisively by the user for a given building.

3. PROCEDURE

3.1. *Potential energy* V

Consider a vertical member such as r between levels i and $(i-1)$. Since rotations (α, β) about the X and Y axes are allowed, in addition to horizontal translations and rotation about the vertical axis, the potential energy of this member is given by

$$V_{r1} = \tfrac{1}{2}\Delta_{ri}^T k_{ri}\Delta_{ri}, \tag{1}$$

where

$$\Delta_{ri}^T = (w_{ri}^T, w_{ri-1}^T, \gamma_{ri}^T, \gamma_{ri-1}^T) = (u_{ri}, v_{ri}, \theta_i, u_{ri-1}, v_{ri-1}, \theta_{i-1}, \alpha_{ri}, \beta_{ri}, \alpha_{ri-1}, \beta_{ri-i}) \tag{2}$$

and

$$w_{ri}^T = (u_{ri}, v_{ri}, \theta_i), \quad w_{ri-1}^T = (u_{ri-1}, v_{ri-1}, \theta_{i-1})$$

$$\gamma_{ri}^T = (\alpha_{ri}, \beta_{ri}), \quad \gamma_{ri-1}^T = (\alpha_{ri-1}, \beta_{ri-1}) \tag{3}$$

and the 10×10 stiffness matrix k_{ri} is given in the Appendix. It is possible to incorporate shear effects into the matrix k_{ri}, if necessary. But in the present analysis, shear effects are not considered.

The degrees of freedom considered in the analysis are the displacements $w_{\tau i}$, $\gamma_{\tau i}$ and $w_{\tau i-1}$, $\gamma_{\tau i-1}$ for the levels i and $(i-1)$, respectively. With reference to Fig. 2, these are related to the end displacements of the vertical member r by

$$w_{ri} = D_r w_{\tau i}, \quad w_{ri-1} = D_r w_{\tau i-1} \tag{4a}$$

and with reference to Fig. 4

$$\gamma_{ri} = A_r\gamma_{\tau i}, \; \gamma_{ri-1} = A_r\gamma_{\tau i-1}, \tag{4b}$$

where the matrices D_r and A_r are given by

$$D_r = \begin{vmatrix} 1 & 0 & -y_r \\ 0 & 1 & x_r \\ 0 & 0 & 1 \end{vmatrix} \tag{5}$$

$$A_r = \begin{vmatrix} a_{rx} & 0 \\ 0 & a_{ry} \end{vmatrix} \tag{6}$$

with $a_{rx} = I_{rx}/I_{\tau x}$, $a_{ry} = I_{ry}/I_{\tau y}$.

In the above expressions a_{rx}, a_{ry} refer to the ratio of the second moment of area of the vertical member r to that of the vertical member at the reference point τ at the same level. Since all the members are assumed to be vertical prior to loading, D_r will be the same at all levels. From eqns (2) and (4)

$$\Delta_{ri} = C_r\Delta_{\tau i}, \tag{7}$$

where C_r is the 10×10 connectivity matrix given by

$$C_r = \begin{vmatrix} D_r & 0 & 0 & 0 \\ 0 & D_r & 0 & 0 \\ 0 & 0 & A_r & 0 \\ 0 & 0 & 0 & A_r \end{vmatrix}. \tag{8}$$

By using matrix A_r in eqn (4), all rotations $\gamma_r(=\alpha_r, \beta_r)$ are referred to those at 'τ' and made proportional to the stiffness of the vertical member. By doing this the number of degrees of freedom at each floor level is kept to the minimum possible value of five.

Using eqns (4) and (7) in (1), the potential energy of member r is now given by

$$V_{ri} = \tfrac{1}{2}\Delta_{\tau i}^T C_r^T k_{ri} C_r \Delta_{\tau i}. \tag{9}$$

The potential energy of all the vertical members between i and $(i-1)$ levels is therefore

$$V_i = \sum V_{ri} = \tfrac{1}{2}\Delta_{\tau i}^T K_{\tau i}\Delta_{\tau i}, \tag{10}$$

where

$$K_{\tau i} = \sum_{r=1} C_r^T k_{ri} C_r. \tag{11}$$

Equation (10) holds for the potential energy of vertical members between any two floor levels. If there are set-backs, as in the present case, then the summation over r will vary above the set-back level(s). The total potential energy of the structure is obtained by summing up the contributions V_i for i ranging from 1 to the numbers of floors (say n) in the form

$$V = \sum_{i=1}^{n} V_i = \frac{1}{2}\sum_{i=1}^{n} \Delta_{\tau i}^T K_{\tau i}\Delta_{\tau i}. \tag{12}$$

In the above equation w_{τ_0} and γ_{τ_0} will be taken as zero since the building is assumed to be restrained at its base. Equation (12) will be used to obtain the stiffness matrix for the buildings by using Lagrange's equations.

3.2. *Kinetic energy*

Kinetic energy of any floor (say at level i) which is assumed to be rigid and hence possessing three degrees of freedom is given by

$$T_i = \tfrac{1}{2}m_i(\dot{u}_{Gi}^2 + \dot{v}_{Gi}^2) + \tfrac{1}{2}J_{Gi}\dot{\theta}_i^2. \tag{13}$$

In the above expression u_{Gi}, v_{Gi}, θ_i are the displacements of the centroid G_i in X and Y directions and the rotation about the vertical, respectively, while m_i and J_{Gi} denote the mass and the polar moment of inertia of the slab about the centroid. The dot above any quantity indicates differentiation with respect to time. As the unknown displacements in the analysis pertain to τ_i, it will be necessary to express the velocities in eqn (13) in terms of those at τ_i. This is easily accomplished by

$$\dot{u}_{Gi} = \dot{u}_{\tau i} - y_{Gi}\dot{\theta}_i \tag{14a}$$

$$\dot{v}_{Gi} = \dot{v}_{\tau i} + x_{Gi}\dot{\theta}_i, \tag{14b}$$

where $\dot{\theta}_i$ is the same at τ_i and G_i due to the rigidity of the floor. Using eqns (14) in (13) gives

$$T_i = \tfrac{1}{2}m_i(\dot{u}_{\tau i}^2 + \dot{v}_{\tau i}^2 - 2\dot{\theta}_i(\dot{u}_{\tau i}y_{Gi} - \dot{v}_{\tau i}x_{Gi})\} + \tfrac{1}{2}J_{\tau i}\dot{\theta}_i^2, \tag{15}$$

where

$$J_{\tau_i} = J_{Gi} + m_i(x_G^2 + y_G^2) \tag{16}$$

is the polar moment of inertia about τ_i.

Kinetic energy T for the whole structure is obtained by summing up T_i in the form

$$T = \sum T_i = \frac{1}{2}\sum (m_i\{\dot{u}_{\tau i}^2 + \dot{v}_{\tau i}^2 - 2\dot{\theta}_i \times (\dot{u}_{\tau i}y_{Gi} - \dot{v}_{\tau i}x_{Gi}\} + J_{\tau i}\dot{\theta}_i^2). \tag{17}$$

Equation (17) will be used to derive the mass matrix of the structure with the aid of Lagrange's equations.

3.3. *Equations of motion*

Lagrange's equations (in the absence of non-conservative forces such as damping) are

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = Q_k, \quad k = 1, 2, 3, \ldots, n, \tag{18}$$

where Q_k are the generalized forces and q_k the generalized displacement ($w_{\tau i}^T = u_{ti}, v_{\tau i}, \theta_i$) at τ_i in the present analysis. Since the kinetic energy T is here not a function of the displacements, the second term in the above equation may be dropped.

Using eqns (12) and (17) together with (18), the equations of motion for the building are given by

$$M_\tau \ddot{w}_\tau + K_\tau \delta_\tau = Q_\tau, \tag{19}$$

where $\delta_\tau = (w_\tau^T, \gamma_\tau^T)$, M_τ and K_τ are the mass and stiffness matrices, respectively. The mass matrix M_τ is given by

$$M_\tau = \begin{bmatrix} M_1 & 0 & 0 & 0 & & & & & \\ 0 & M_2 & 0 & 0 & & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & M_i & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & M_n \end{bmatrix} \tag{20}$$

with

$$M_i = \begin{bmatrix} m_i & 0 & -m_i y_{Gi} \\ 0 & m_i & m_i x_{Gi} \\ -m_i y_{Gi} & m_i x_{Gi} & J_{Gi} \end{bmatrix}, \quad (21)$$

where x_{Gi}, y_{Gi} are the coordinates of the centroid of the ith floor with respect to the reference point τ_i in that level. The stiffness matrix K_τ is given by

$$K_\tau = \begin{bmatrix} (K_{\tau 1} + K_{\tau 2}) & -K_{\tau 2} & 0 & 0 \\ -K_{\tau 2} & (K_{\tau 2} + K_{\tau 3}) & -K_{\tau 3} & 0 \\ 0 & -K_{\tau 3} & (K_{\tau 3} + K_{\tau 4}) & -K_{\tau 4} \\ 0 & 0 & -K_{\tau 4} & K_{\tau 4} \end{bmatrix}. \quad (22)$$

The stiffness matrix is shown only for a building with four floors, as the pattern is repetitive. Its non-zero elements are given by

$$\begin{aligned} K_{i,i} &= K_{\tau,i} + K_{\tau,i+1} \\ K_{i+1,i} &= K_{i,i+1} = -K_{\tau,i+1}, \quad i = 1, 2, 3, \ldots, n. \end{aligned} \quad (23)$$

The generalized load vector Q_τ in eqn (19) pertains to lateral (horizontal) loads and couples applied at each point τ_i. From the actual loads acting at each floor level, the equivalent system at τ_i can easily be determined.

The order of the matrix K_τ is higher than that of M_τ by twice the number of floors in the building. The degrees of freedom pertaining to the rotations $\gamma_\tau (= \alpha_\tau, \beta_\tau)$ will be eliminated by condensation prior to solving eqns (19). If K_τ is partitioned and expressed as

$$K_\tau = \begin{bmatrix} K_\tau^{11} & K_\tau^{12} \\ K_\tau^{21} & K_\tau^{22} \end{bmatrix} \quad (24)$$

then eqns (19) can be expanded as

$$M_\tau \ddot{w}_\tau + K_\tau^{11} w_\tau + K_\tau^{12} \gamma_\tau = Q_\tau \quad (25)$$

$$K_\tau^{21} w_\tau + K_\tau^{22} \gamma_\tau = 0 \quad (26)$$

using eqns (26) in (25) will yield the reduced system

$$M_\tau \ddot{w}_\tau + K_\tau^{*} w_\tau = Q_\tau, \quad (27)$$

where the condensed matrix K_τ^* is given by

$$K_\tau^* = K_\tau^{11} - (K_\tau^{21})^T (K_\tau^{22})^{-1} K_\tau^{21} \quad (28)$$

once w_τ is determined from eqn (27), γ_τ can be determined, if necessary, from

$$\gamma_\tau = -(K_\tau^{22})^{-1} K_\tau^{21} W_\tau. \quad (29)$$

The equations of motion given in (27) and (29) above can be conveniently programmed on a microcomputer to obtain:

(a) response of the building to static loads, in which case $\ddot{w}_\tau = 0$;
(b) free vibration analysis of the building with $Q_T = 0$; and
(c) dynamic response of the building for applied dynamic loads Q_τ.

For case (c), the natural frequencies determined in case (b) can be used together with the made superposition method, or direct numerical integration of eqns (27) can be carried out using an assumed variation of acceleration over the time Δt.

4. NUMERICAL EXAMPLES AND DISCUSSION

Three numerical examples are presented in order to validate and illustrate the proposed method. In all three examples concrete buildings are treated with the concrete having a value of 30 GPa for Young's modulus E and a value of 0.20 for Poisson's ratio. The lateral load on the buildings is due to a uniform wind pressure of 0.7 kN/m^2. The results from the present analysis are given for the two cases: (i) with rotations of the vertical members at the slab levels made proportional to their stiffnesses and (ii) with these rotations made equal. These two cases are denoted by 5DOF and 5DOFA, respectively. Some of the present results are compared with those obtained from a comprehensive commercial package programme denoted by COMP and with those obtained from the analysis based on the earlier shear beam model which had three degrees of freedom (3DOF) per floor [8].

4.1. *Example* 1

The elevation and the plan of the building are shown in Fig. 5. All the columns are 0.35 × 0.35 m in cross-section. Under the action of the wind loading in the X direction, the deflected shape of the vertical line $\tau_0, \tau_1, \tau_1 . \tau_3$ (through τ_0), is shown in Fig. 6(a) together with those obtained from the 3DOF analysis and the commercial package (COMP). It can be seen that the shear beam model is too stiff and should therefore not be used unless one is certain that the building does have very rigid slabs and behaves as assumed in that model. In this example since all the vertical members have the same size, all of them suffer the same rotations at any slab level. The present results compare reasonably well with those from the package program. Perfect agreement cannot be expected as the details of the model employed in the package program are not known, especially, when the building is subjected to torsion.

Effect of column size on the lateral deflection of the building is shown in Fig. 6(b). The size of the square columns was varied from 0.25 to 0.40 m. As expected,

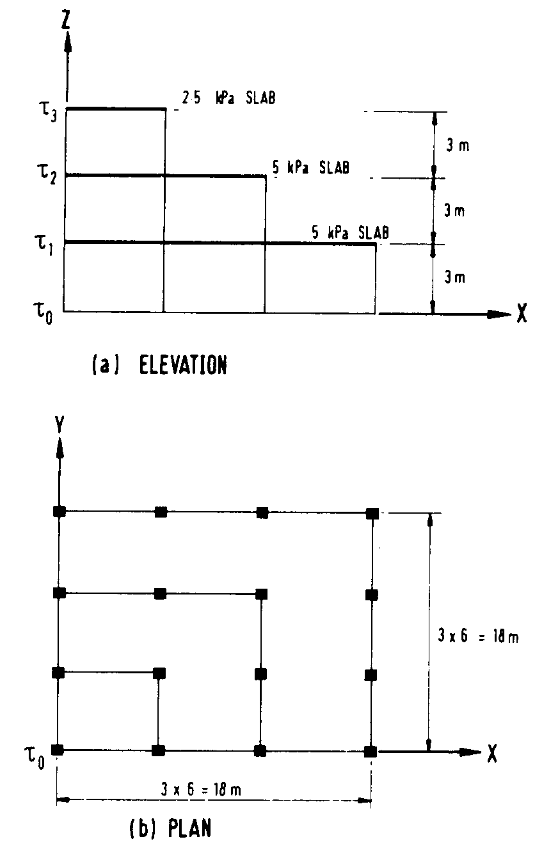


Fig. 5. Three-storey building with two set-backs (example 1).

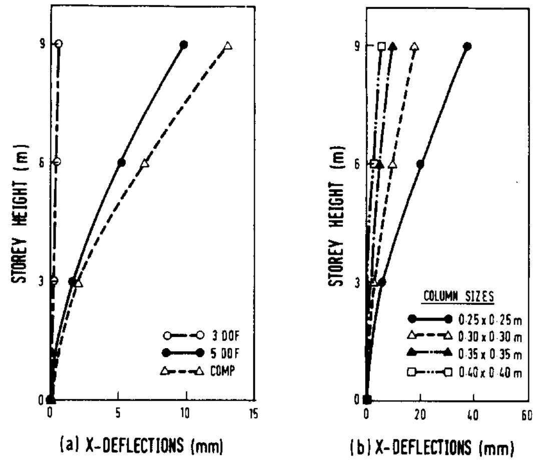


Fig. 6. (a) Lateral (X) deflection of τ in building of example 1. (b) Effect of column size on lateral deflection in example 1.

deflections decrease with increase in column size. There is approximately a 50% reduction in deflection when the column size was increased by 50 mm. The deflections of other vertical nodal lines can be obtained by using eqn (7), if necessary.

The periods corresponding to the first 9 modes of vibration of the building obtained from the present analysis together with those obtained from the commercial package are presented in Table 1. Results are given for buildings having columns of different sizes. The periods decrease with column size as expected. Since the building has axes of symmetry (at 45 and 135 to the X axis), there will be six pairs of periods having equal values, corresponding to six translational and six rotational modes of vibration. The present results show some of these six pairs of periods in Table 1. But the periods obtained from the commercial package did not display this feature, probably because the package program utilized several degrees of freedom as compared to the fifteen degrees of freedom in the proposed method. However, the periods obtained from both sources have the same order and some of the periods compare reasonably well.

4.2. *Example 2—ten-storey building with service core*

A ten-storey building whose elevation and plan are shown in Fig. 7 is considered. All columns are 0.30 × 0.30 m in section while the service core is 3 × 2.5 × 0.2 m thick and is located as shown in the plan. Due to the presence of the core, this building is not symmetrical about two axes in plan and is therefore torsionally coupled. The building is subjected to wind loading in the X direction. At first the building was analysed without the core and as expected, the lateral deflection were excessive implying

Table 1. Periods of vibration of three-storey building with two set-backs

Mode No.	Column size 0.25 × 0.25 m			Column size 0.30 × 0.30 m			Column size 0.35 × 0.35 m			Column size 0.40 × 0.40 m		
	3DOF	5DOF	COMP	3DOF	5DOF	COMP	3DOF	5DOF	COMP	3DOF	5DOF	COMP
1	0.443	1.542	1.805	0.307	1.071	1.253	0.226	0.786	0.921	0.173	0.602	0.705
2	0.427	0.539	1.584	0.297	1.069	1.100	0.218	0.785	0.808	0.167	0.601	0.619
3	0.289	0.411	0.784	0.201	0.286	0.544	0.148	0.209	0.400	0.113	0.161	0.306
4	0.211	0.407	0.440	0.147	0.283	0.306	0.108	0.207	0.225	0.083	0.159	0.172
5	0.202	0.276	0.409	0.140	0.191	0.284	0.103	0.141	0.208	0.279	0.108	0.160
6	0.172	0.221	0.349	0.119	0.154	0.242	0.087	0.113	0.178	0.067	0.086	0.136
7	0.133	0.214	0.249	0.093	0.149	0.173	0.068	0.109	0.127	0.052	0.084	0.097
8	0.124	0.159	0.219	0.086	0.111	0.152	0.063	0.081	0.112	0.049	0.062	0.085
9	0.080	0.080	0.182	0.056	0.056	0.126	0.041	0.041	0.092	0.031	0.031	0.071

Periods of vibration (sec)

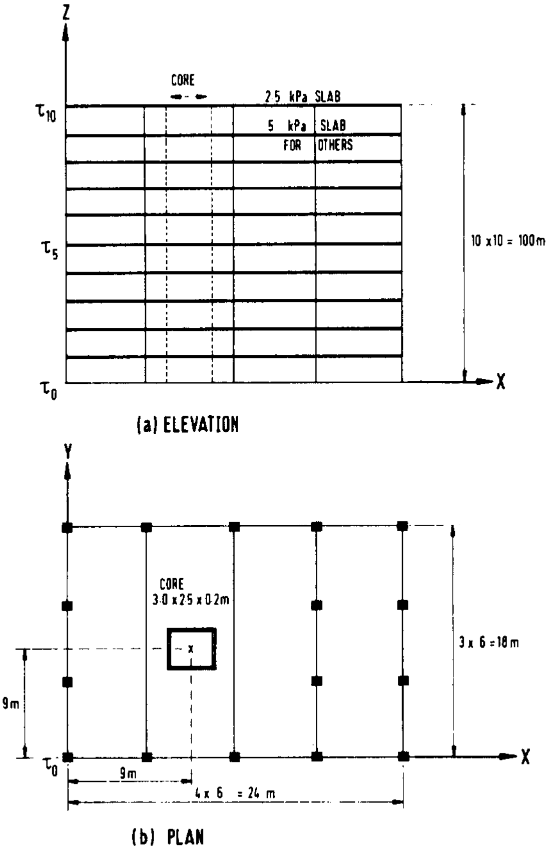


Fig. 7. Ten-storey building with service core (example 2).

the need for a shear wall or a core to restrict the deflections. Figure 8 shows the lateral deflection in the X direction, of the vertical line through 'τ' (5DOF) together with those obtained from the present analysis after setting the bending rotations of the vertical members to be equal at a slab level (5DOFA) and from the commercial package (COMP). From the figure it can be seen that the proposed model, where the bending rotations of the vertical members at a slab level were assumed to be proportional to their stiffnesses, has a smaller deflection and is therefore

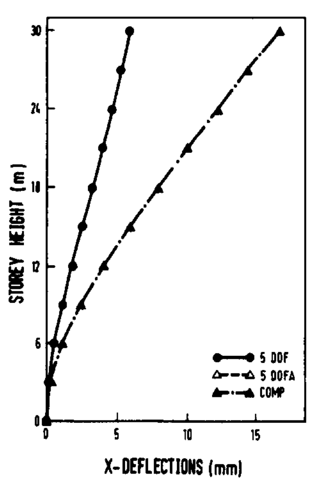


Fig. 8. Lateral (X) deflection of τ in building of example 2.

stiffer than the model considered in the commercial package. Moreover when the bending rotations of all the vertical members at a slab level were made equal by setting $a_{rx} = a_{ry} = 1$, the results compare exactly with those from the commercial package. It is therefore possible that the model used in the commercial program has a similar behaviour, at least in this case.

The first ten periods of vibration obtained from the present analysis, under both 5DOF and 5DOFA cases, together with those output by the package program (COMP) are presented in Table 2. Periods obtained under 5DOFA and COMP compare almost exactly and are higher than those obtained under 5DOF. This feature also indicates that the 5DOF model is stiffer than the 5DOFA model.

4.3. *Example 3—ten-storey building with a set-back and a service core*

The plan of the lower floors of this building is the same as that of the building treated in example 2. However beyond the seventh floor there is a set-back as shown in the elevation of this building in Fig. 1. The columns below the set back are all 0.25×0.25 m while the columns above the set-back are 0.20×0.20 m. There is also a service core of size $3 \times 2.5 \times 0.20$ m thick located at (9, 9) as shown in Fig. 1(b). The core continues upto the top of the building. Due to the set-back and the presence of the core, the building is not symmetrical both in the front elevation and in plan, making it torsionally coupled. In this example the wind pressure is assumed to act in the Y direction and hence a torsional moment is exerted on the building.

The deflection of the vertical line 'τ' in the Y direction is shown in Fig. 9 together with the deflections obtained under 5DOFA and by using the commercial program. As in the previous examples results for 5DOFA and COMP agree almost exactly, while results for 5DOF indicate smaller deflections and hence a stiffer model.

The first ten periods of vibration of the building are presented in Table 3, when it can be seen that results in 5DOFA and COMP agree well. Periods of vibration obtained from the stiffer model in 5DOF are smaller as expected.

Table 2. Periods of vibration of ten-storey building with service core

Mode No.	Periods of vibration (sec)		
	5DOF	5DOFA	COMP
1	1.042	1.851	1.893
2	0.959	1.578	1.500
3	0.800	0.801	1.129
4	0.296	0.320	0.423
5	0.230	0.255	0.279
6	0.220	0.248	0.255
7	0.175	0.175	0.239
8	0.129	0.131	0.176
9	0.103	0.104	0.136
10	0.094	0.098	0.112

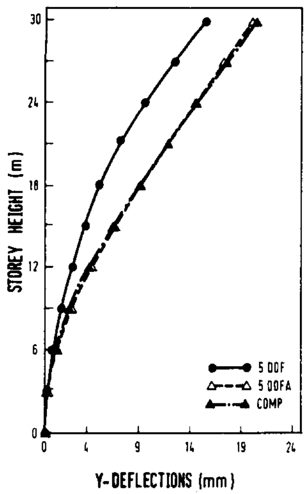


Fig. 9. Lateral (Y) deflection of τ in building of example 3.

From the results presented above, the model used in the commercial package seems to assume, at least in the absence of the torsional couples, that all the vertical members suffer the same rotations (about the X and Y axes) at the slab junctions, irrespective of their size. This is perhaps the most basic approximation which can take into consideration bending rotations in the vertical members, and is therefore quite a simplistic and somewhat an unrealistic assumption. The authors are convinced that the vertical member rotations need not be the same at any slab level. It is a much better approximation to assume that these rotations are proportional to the stiffness of the member. Even with this realistic approximation it was possible to drastically reduce the number of degrees of freedom to be treated, resulting in a simple analysis which was conveniently programmed.

5. CONCLUSION

A simplified procedure for lateral load analysis of asymmetric buildings, which are torsionally coupled, has been presented. This work is an improvement on the earlier work based on the shear beam model, by including the bending rotations of the vertical members. Due to the particular choice of the model, the analysis involves only $5n$ degrees of freedom, for an n storey building. This is achieved by assuming that the rotation of a vertical member (about the X and Y axes) at a slab level is proportional to its stiffness. The entire procedure can be conveniently programmed to run on a microcomputer and gives quick results.

Numerical examples have been presented to illustrate and validate the procedure. The results compare reasonably well, wherever possible, with those obtained from a comprehensive package program.

It is hoped that this simplified method will be useful to practising engineers who have access only to a microcomputer. The method yields approximate, but quick and reliable results.

Table 3. Periods of vibration of ten-storey building with a set-back and a service core

Mode No	5DOF	5DOFA	COMP
1	1.223	1.479	1.481
2	1.131	1.275	1.274
3	0.834	0.831	1.021
4	0.326	0.285	0.342
5	0.304	0.252	0.253
6	0.282	0.228	0.223
7	0.180	0.180	0.207
8	0.141	0.139	0.152
9	0.115	0.115	0.127
10	0.099	0.098	0.107

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APPENDIX

Non-zero elements of the stiffness matrix k_{ri} are given by

$$k_{ri}^{11} = k_{ri}^{66} = -k_{ri}^{61} = 12EI_y/L^3$$

$$k_{ri}^{22} = k_{ri}^{77} = -k_{ri}^{72} = 12EI_x/L^3$$

$$k_{ri}^{33} = k_{ri}^{88} = -k_{ri}^{83} = GJ/L$$

$$k_{ri}^{44} = k_{ri}^{99} = 2k_{ri}^{94} = 4EI_y/L$$

$$k_{ri}^{55} = k_{ri}^{10,10} = 2k_{ri}^{10,5} = 4EI_x/L$$

$$k_{ri}^{97} = -k_{ri}^{92} = -k_{ri}^{42} = -k_{ri}^{65} = 6EI_y/L^2$$

$$k_{ri}^{74} = k_{ri}^{51} = k_{ri}^{10,1} = -k_{ri}^{10,6} = 6EI_x/L^2$$

with

$$k_{ri}^{mn} = k_{ri}^{nm}.$$