

## THE STOCHASTIC RESPONSE OF ASYMMETRIC BASE ISOLATED BUILDINGS

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The coupled lateral–torsional stochastic response of an asymmetric one-storey base isolated building to earthquake excitation is presented. The stochastic model of the 1940 El-Centro earthquake, which preserves the non-stationary evolution of the amplitude and frequency content of the ground acceleration, is used as earthquake excitation. The base isolator consists of an array of elastomeric bearings between the base of the structure and its foundation. The root mean square (r.m.s.) response of the system is obtained under different parametric variations in order to investigate the effects of superstructure and isolator eccentricities on the stochastic response of a base isolated building. The system parameters include the eccentricity of the superstructure, the uncoupled torsional to lateral frequencies ratio of the superstructure, the eccentricity of the isolator, and the uncoupled torsional to lateral base isolation frequencies ratio of the isolator. It is shown that these parameters have considerable influence on the response of a base isolated structure and the effectiveness of base isolation in reducing the seismic forces in the superstructure.

### 1. INTRODUCTION

The traditional method of providing earthquake resistance to a structure is by increasing its strength as well as its energy absorbing capacity. However, during an earthquake many structures have suffered structural and non-structural damage due to inelastic deformation. An alternative method is to isolate the structure by the use of base isolators between the base of the structure and its foundation. These base isolators have two important characteristics: horizontal flexibility and energy absorbing capacity. Horizontal flexibility lowers the fundamental frequency of the structure below the range of frequencies which dominate in general earthquake excitation. Energy absorbing capacity reduces both the relative displacements and the seismic energy that are transmitted to the structure. The effectiveness of various types of base isolation in limiting the earthquake forces in buildings has been demonstrated both experimentally [1] and analytically [2–4]. An excellent review of the earlier works and recent investigations on base isolation is provided in references [5, 6].

A variety of base isolation devices, including laminated rubber bearing, frictional bearing and roller bearing, have been developed. Laminated rubber bearings are one of the most commonly used types. The main advantages of laminated rubber bearings are lower cost than the other options, simplification in layout and high damping at a smaller level of amplitude. Recently, friction type base isolators have been developed and studied. One of the most attractive features of this type of isolator is that the friction forces are natural and powerful energy dissipation devices. The simplest such device is the

pure-friction one, referred to as the P-F system. The more advanced devices involve pure-friction elements in combination with laminated rubber bearings. These devices include the "Electricité de France" (EDF) system [7], the resilient-friction base isolator (R-FBI) system [8] and a new device—the sliding resilient-friction (SR-F) base isolation system [9]. A comparative study of the performance of different base isolators has been carried out by several authors [3, 4, 9–12].

Most of the analytical works on base isolation of buildings deal with 2-D idealization, which is strictly valid for symmetric buildings, or buildings with very small eccentricity, or buildings that are torsionally very rigid [2, 4, 9–12]. These investigations are primarily aimed at studying the parametric behaviour of the base isolated building and provide useful information on the design and optimization of base isolation systems. Very few analytical works have been reported on the seismic base isolation of 3-D building models. Lee [13] has shown that the base isolation reduces the structural torque drastically, even if the structural eccentricity is large. Pan and Kelly [14] studied the effect of eccentricity on the elastic response of rigid mass supported on a base isolator. Nagarajaiah, Reinhorn and Constantinou [15, 16] studied the non-linear response of a 3-D base isolated building to El-Centro earthquake motion. Recently, Jangid and Datta [17–19] conducted extensive parametric studies on the response behaviour of a torsionally coupled base isolated system under random ground motion, using simulation analysis.

Probabilistic analysis of base isolated structures has attracted considerable attention in recent years. The response of a rigid block sliding on a randomly moving foundation was studied by Ahmadi [20], Constantinou and Tadjbakhsh [21], Crandall *et al.* [22] and Su and Ahmadi [23]. The stochastic response of a shear beam type structure isolated by different base isolation devices was carried out by Su and Ahmadi [11, 12]. Chen and Ahmadi [24] have studied the stochastic response of a secondary system in a shear beam type structure to a non-stationary earthquake excitation. These studies have been confined only to transitional response analysis of 2-D idealized models of the buildings.

In what follows here a stochastic response analysis of a torsionally coupled base isolated system to non-stationary random ground motion is presented. The statistics of responses are obtained for a set of important parametric variations of the asymmetric building. The objectives of the parametric study are (i) to find the effects of torsional coupling produced due to superstructure and isolator eccentricities, and (ii) to investigate the influence of the important parameters on the response, especially in identifying the optimum isolator characteristics.

## 2. ASSUMPTIONS AND IDEALIZATION

In Figure 1 is shown the structural system considered, which is an idealized one-storey asymmetric-plan building model mounted on the base isolator. The rigid deck mass is supported by massless inextensible columns. The base isolator consists of an array of elastomeric bearings arranged between the base of the building and its foundation. The isolator and columns are assumed to remain within the elastic range during the earthquake excitation. The stiffness distribution of the columns and bearings are symmetric about the  $y$ -axis but not about the  $x$ -axis; as a result, the system displays torsional effects when excited in the lateral direction— $x$ . The four degrees of freedom in which coupling occurs are lateral displacement of the deck ( $u_x$ ), torsional displacement of the deck ( $u_\theta$ ), lateral displacement of the base isolator ( $u_{bx}$ ) and torsional displacement of the isolator ( $u_{b\theta}$ ). Furthermore, the isolator is assumed to carry the vertical load without undergoing any vertical deformation.

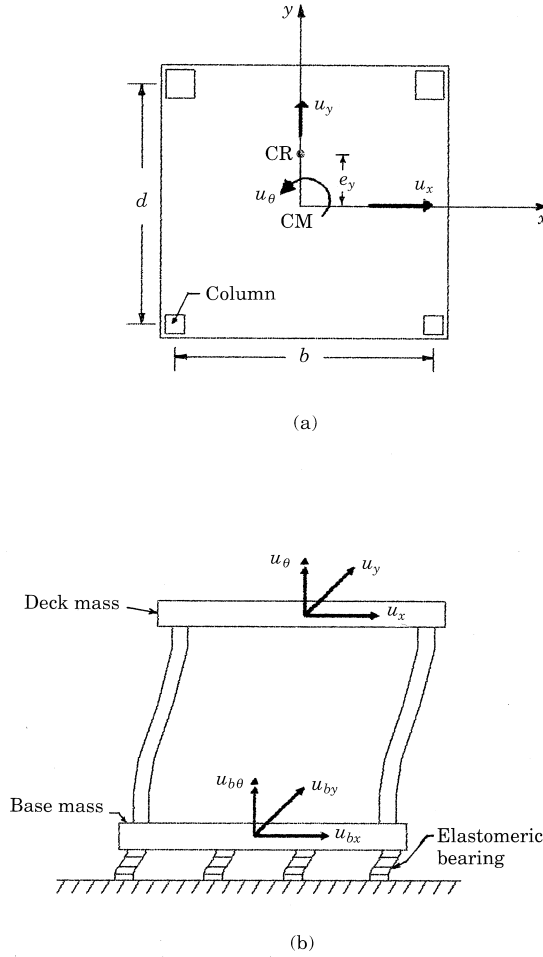


Figure 1. The structural model; (a) plan; (b) section.

Let  $e_y$  represent the superstructure eccentricity as shown in Figure 1(a). The isolator eccentricity  $e_{by}$  is defined similarly at the base. The eccentricities normalized with respect to lateral dimension of the structure are expressed as

$$\epsilon = e_y/d \quad \text{and} \quad \epsilon_b = e_{by}/d, \quad (1)$$

in which  $d$  is the lateral dimension of the plan of the building (see Figure 1(a)) (a list of notation is given in the Appendix).

For characterizing the dynamic properties of the superstructure and isolator, four uncoupled frequency parameters of the system are defined as follows:

$$w_x = \sqrt{K_x/m}, \quad w_\theta = \sqrt{K_\theta/mr^2}, \quad (2)$$

$$w_b = \sqrt{K_b/(m + m_b)}, \quad w_{b\theta} = \sqrt{K_{b\theta}/(mr^2 + m_br_b^2)}. \quad (3)$$

Here  $m$  and  $m_b$  are the masses of the superstructure (including additional lumped masses), and the base mass, respectively;  $r$  and  $r_b$  are the radius of gyration of the deck and of the base mass about the vertical axis through the CM, respectively;  $K_x$  and  $K_\theta$  are the uncoupled lateral and torsional stiffnesses of the superstructure, respectively; and  $K_b$  and

$K_{b\theta}$  are the uncoupled lateral and torsional base isolation stiffnesses, respectively. The frequencies  $w_x$  and  $w_\theta$  may be interpreted as the natural frequencies of the fixed base system if it were torsionally uncoupled: i.e., a system with  $e_y = 0$ , but with the mass ( $m$ ),  $K_x$  and  $K_\theta$  the same as in the coupled system.  $w_b$  and  $w_{b\theta}$  are referred to as the uncoupled translational and torsional frequencies of the base isolated structure for rigid superstructure conditions, respectively. The torsional mass of the deck and base mass are varied to provide various uncoupled torsional to lateral frequencies ratios. This can be achieved by varying the position of the lumped mass with respect to the CM. Two ratios of uncoupled torsional to lateral frequencies are defined as

$$\Omega_\theta = w_\theta/w_x \quad \text{and} \quad \Omega_{b\theta} = w_{b\theta}/w_b. \quad (4)$$

### 2.1. GROUND EXCITATION

Earthquake ground motions are inherently random and multi-dimensional. To describe such ground motions, a multi-variate random process model has been proposed. The generalized non-stationary Kanai-Tajimi model of the 1940 El-Centro earthquake, which was developed by Fan and Ahmadi [25], is considered for the present study. The ground acceleration  $\ddot{u}_g(t)$  is expressed as

$$\ddot{u}_g(t) = -[2\xi_g w_g(t)\dot{u}_f(t) + w_g^2(t)u_f(t)]a(t), \quad (5)$$

with

$$\ddot{u}_f(t) + 2\xi_g w_g(t)\dot{u}_f(t) + w_g^2(t)u_f(t) = f(t), \quad (6)$$

in which  $u_f(t)$  is the filter response,  $\xi_g$  is the damping of the filter,  $w_g(t)$  is the time dependent ground filter frequency and  $a(t)$  is the deterministic amplitude envelope function;  $f(t)$  is the zero mean Gaussian white noise process with the statistics

$$E[f(t)f(t+\tau)] = 2\pi S_0 \delta(\tau), \quad (7)$$

where  $E$  is the expectation operator,  $\delta(\cdot)$  is the Dirac delta function and  $S_0$  is the constant power spectrum of the white noise process  $f(t)$ . The parameters suggested for the El-Centro earthquake are

$$a(t) = 9.44t^3 \exp(-1.1743t) + 3.723 \quad (8)$$

and

$$w_g(t) = \pi\{19.01[\exp(-0.0625t) - \exp(-0.15t)] + 3.0\}. \quad (9)$$

The damping constant of the filter is  $\xi_g = 0.42$  and the intensity of the white noise  $S_0 = 1.0 \text{ cm}^2/\text{s}^3$ .

### 2.2. EQUATIONS OF MOTION

The equations of motion of the system under consideration are written as

$$[M](\ddot{U} + \ddot{U}_b) + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{T_g\}\ddot{U}_g(t), \quad (10)$$

$$[M_b]\{\ddot{U}_b\} + [C_b]\{\dot{U}_b\} + [K_b]\{U_b\} - [C]\{\dot{U}\} - [K]\{U\} = -[M_b]\{T_g\}\ddot{U}_g(t), \quad (11)$$

in which  $[M]$ ,  $[K]$  and  $[C]$  are the lumped mass, stiffness and damping matrices corresponding to the degrees of freedom (DOF) at the deck,  $\{U\} = [u_x, u_\theta]^T$  is the vector of displacements at the deck relative to the base mass,  $[M_b]$  and  $[C_b]$  are the lumped mass and damping matrices corresponding to the DOF at the base mass,  $[K_b]$  is the stiffness matrix of the isolator,  $\{U_b\} = [u_{bx}, u_{b\theta}]^T$  and  $\{T_g\} = [1, 0]^T$  is the influence coefficient matrix.

The equations (5), (6), (10) and (11) can be re-written as a system of first order stochastic differential equations,

$$(d/dt)\{Y\} = [H]\{Y\} + \{F\}, \quad (12)$$

in which

$$\{Y\} = \{u_f, \dot{u}_f, u_x, u_\theta, u_{bx}, u_{b\theta}, \dot{u}_x, \dot{u}_\theta, \dot{u}_{bx}, \dot{u}_{b\theta}\}^T \quad (13)$$

and

$$\{F\} = \{0, f(t), 0, 0, 0, 0, 0, 0, 0, 0\}. \quad (14)$$

The augmented response vector  $\{Y\}$  is a Markov process. The corresponding covariance matrix  $[V]$  satisfies [26] the equation

$$(d/dt)[V] = [H][V]^T + [V][H]^T + [P], \quad (15)$$

where the elements of the matrices  $[V]$  and  $[P]$  are given by

$$v_{ij} = E[Y_i Y_j], \quad P_{ij} = E[f_i f_j], \quad i, j = 1, 2, \dots, 10, \quad (16, 17)$$

and  $[V]^T$  denotes the transpose of  $[V]$ . Also

$$P_{ij} = 0 \quad \text{except for} \quad P_{22} = 2\pi S_0. \quad (18)$$

The response of the system (i.e., the  $[V]$  matrix) is obtained by solving the moment equation (15) numerically based on a step-by-step integration method. The fourth order Runge–Kutta method has been employed for this study.

### 3. PARAMETRIC STUDY

The non-stationary response of the torsionally coupled base isolated system has been investigated with respect to the following parameters:  $\epsilon$ ,  $\epsilon_b$ ,  $\Omega_\theta$  and  $\Omega_{b\theta}$ . The torsional mass of the deck and base raft is varied to provide various values of  $\Omega_\theta$  and  $\Omega_{b\theta}$  ratios. With the assumed set of values for the parameters chosen, the inputs for the numerical study are selected as follows: the uncoupled lateral base isolation frequency  $w_b = \pi$  rad/s, which corresponds to a time period of 2.0 s of the isolator;  $m_b/m = 1.5$ ; the time period of the superstructure is taken as 1 s which corresponds to  $w_x = 2\pi$  rad/s; modal damping for the superstructure is taken as 5% for all modes; the viscous damping in the elastomeric bearing is taken as 10% of critical. Responses have been obtained for both asymmetric and corresponding symmetric ( $\epsilon = \epsilon_b = 0$ ) systems. The symmetric and asymmetric systems have the same values of the uncoupled frequency parameters as described in equations (2) and (3). The ground excitation is taken to be a non-stationary model of the El-Centro earthquake of duration 25 s as described before. The evolutionary r.m.s. values of response quantities have been determined and the maximum values of the r.m.s. responses (referred to henceforth as r.m.s. values of response) have been used for the parametric study.

#### 3.1. THE EFFECT OF THE SUPERSTRUCTURE ECCENTRICITY ( $\epsilon$ )

In Figure 2 is shown the variation of the r.m.s. values of the responses  $u_x$ ,  $u_\theta$ ,  $u_{bx}$  and  $u_{b\theta}$  against the eccentricity ratio,  $\epsilon$ , for  $\Omega_\theta = \Omega_{b\theta} = 1$ . The response  $u_x$  increases with the increase in eccentricity. This indicates that the torsional coupling increases the lateral response of the superstructure; as a result, the effectiveness of base isolation for asymmetric system is overestimated if the eccentricity is ignored. The deck torsion  $u_\theta$  increases, the base displacement  $u_{bx}$  decreases and the base torsion  $u_{b\theta}$  increases with increase in  $\epsilon$ . Furthermore, the displacement responses are less and torsional responses are more for an

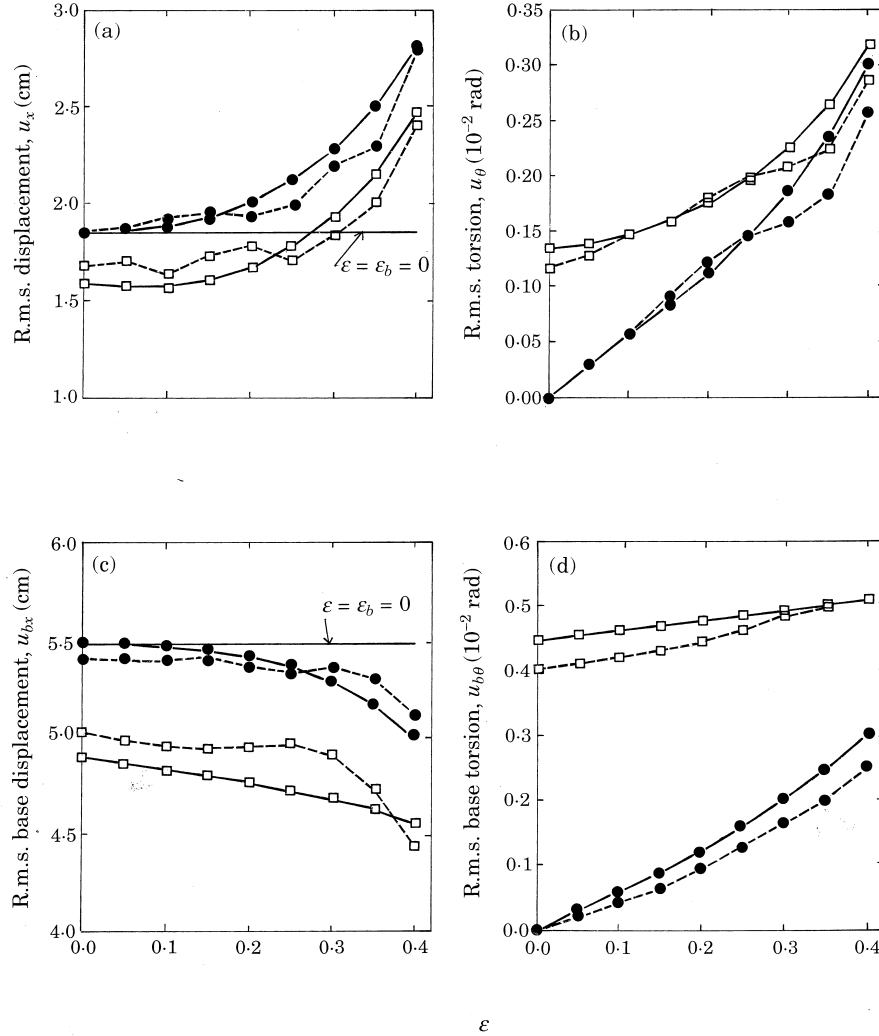


Figure 2. The effect of eccentricity ( $\epsilon$ ) on the r.m.s. response of a base isolated structure ( $\Omega_\theta = \Omega_{b\theta} = 1$ ). —, NOOW El-Centro; —, theoretical; —●—,  $\epsilon_b = 0$ ; —□—,  $\epsilon_b = 0.15$ .

asymmetric base isolator (i.e.,  $\epsilon_b = 0.15$ ) compared to those for a symmetric base isolation ( $\epsilon_b = 0$ ). Also, the isolator eccentricity influences the base responses more than the superstructure responses.

In Figure 2, the dashed line values are one-third of the peak values of the responses  $u_x$ ,  $u_\theta$ ,  $u_{bx}$  and  $u_{b\theta}$  obtained for the acceleration record of the NOOW component of the 1940 El-Centro earthquake (for which the non-stationary model excitation has been used in this study). It is observed that the stochastic responses are generally in good agreement with those obtained from the earthquake accelegram if one assumes that the maximum response is nearly equal to three times the r.m.s. response (5).

### 3.2. THE EFFECT OF THE TORSIONAL TO LATERAL FREQUENCY RATIO ( $\Omega_\theta$ ) OF THE SUPERSTRUCTURE

In Figure 3, the variation of the r.m.s. responses  $u_x$ ,  $u_\theta$ ,  $u_{bx}$  and  $u_{b\theta}$  as functions of the torsional to lateral frequency ratio  $\Omega_\theta$  of the superstructure are shown for a symmetric

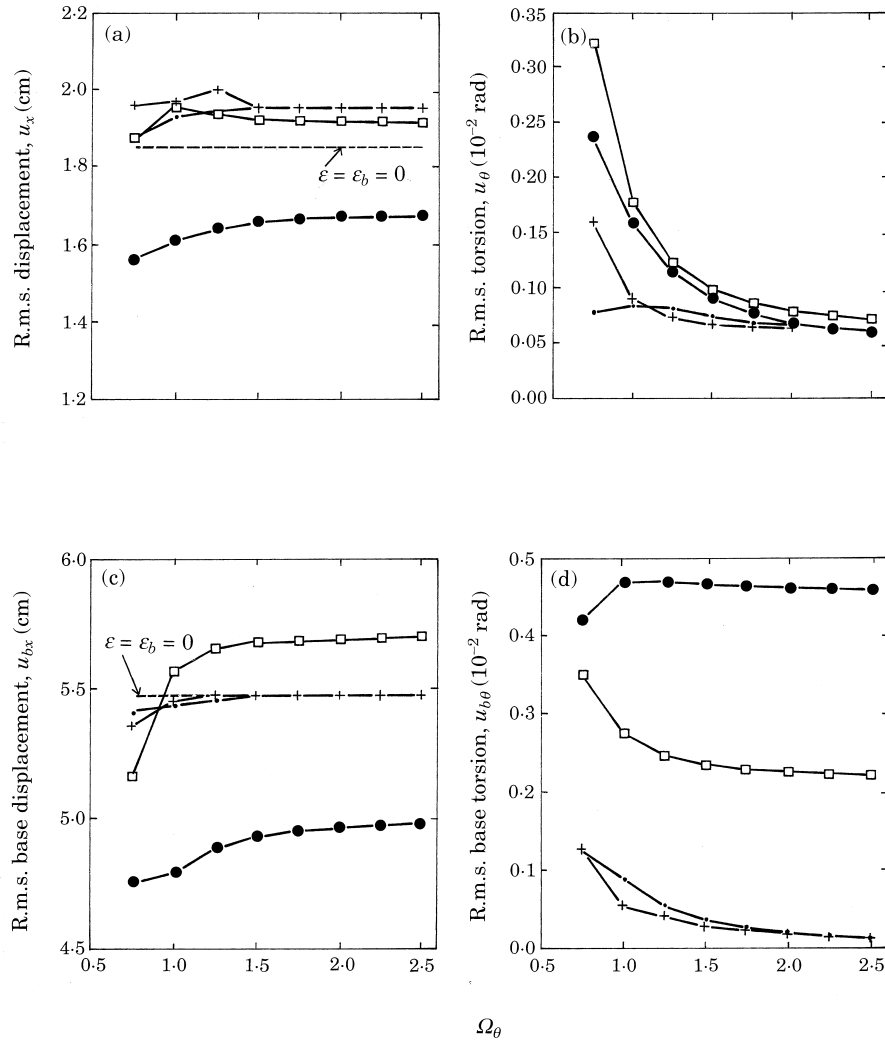


Figure 3. The effect of the  $\Omega_\theta$  ratio on the r.m.s. response of a base isolated structure ( $\epsilon = 0.15$ ).  $\epsilon_b$ ,  $\Omega_{b0}$  values:  $\bullet$ —, 0.0, 1;  $+$ —, 0.0, 2;  $\times$ —, 0.15, 1;  $\square$ —, 0.15, 2.

( $\epsilon_b = 0$ ) and an asymmetric isolator ( $\epsilon_b = 0.15$ ). The increase of  $\Omega_\theta$  beyond 1.5 does not significantly influence the responses of the system. Furthermore, the figure indicates that  $\Omega_\theta$  has a greater influence on the responses of the system for an asymmetric isolator ( $\epsilon_b = 0.15$ ). Also, there is a significant difference between the responses obtained with symmetric ( $\epsilon_b = 0$ ) and asymmetric isolators.

These results indicate that the isolator eccentricity and the uncoupled torsional to lateral frequency ratio of the isolator significantly influence both the superstructure and the isolator responses. The variations of the responses of the system with respect to these two parameters, therefore, have been investigated in order to arrive at an optimum combination of these parameters for effective reduction of the superstructure response. This information may be of much use in the practical design of isolators.

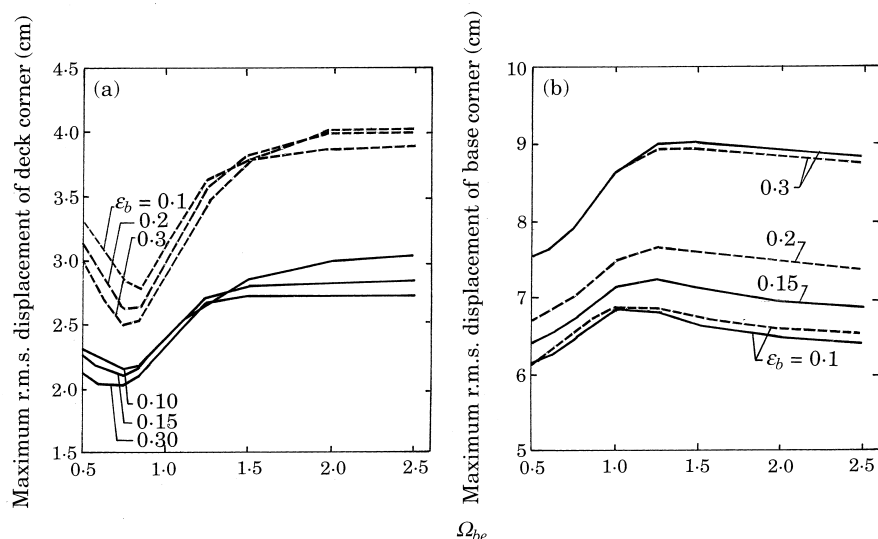


Figure 4. The effect of  $\Omega_{b\theta}$  ratio on the r.m.s. response of a base isolated structure ( $\Omega_\theta = 1$ ).  $\epsilon$  values: —, 0.15; —, 0.3.

### 3.3. THE EFFECT OF THE ISOLATOR ECCENTRICITY ( $\epsilon_b$ ) AND FREQUENCY RATIO ( $\Omega_{b\theta}$ )

In order to illustrate the effects of these two parameters on the response of the system, the r.m.s. value of the  $x$ -displacement of the corner of the deck and the corner base displacement are plotted against these parameters in Figures 4 and 5. The corner displacement provides the magnitude of deformation due to the combined effect of translation and torsion, and is crucial for design purposes. It is indicated in Figure 4 that there exists a definite minima for the variation of the deck response with the frequency ratio  $\Omega_{b\theta}$  for a given set of superstructure characteristics ( $\epsilon$  and  $\Omega_\theta$ ) and

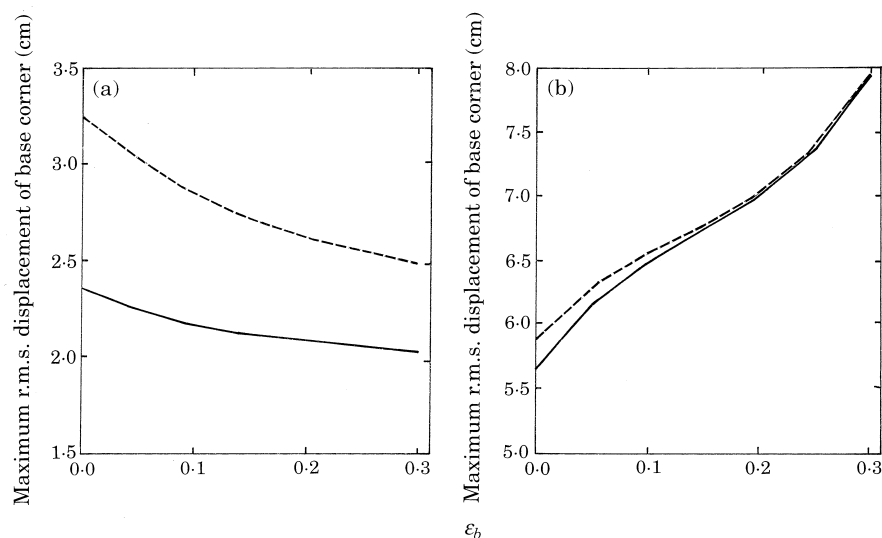


Figure 5. The effect of eccentricity ( $\epsilon_b$ ) on the r.m.s. response of a base isolated structure: ( $\Omega_\theta = 1$  and  $\Omega_{b\theta} = \Omega_{b\theta, \min}$ ). Key as Figure 4.



$\epsilon_b$ . It is seen from the figure that the values of  $\Omega_{b\theta}$  for which the minima response occurs range between 0.5 to 1 (for  $0.1 \leq \epsilon_b \leq 0.3$ ). An opposite trend is observed for the base displacement: i.e., there exists a definite maximum for the variation of the base displacement with the frequency ratio  $\Omega_{b\theta}$ . However, the value of  $\Omega_{b\theta}$  for which the maximum base response occurs is far away from that for which the minimum response occurs for the superstructure. As a result, the base displacement corresponding to  $\Omega_{b\theta}$  (i.e.,  $\Omega_{b\theta,min}$ ), for which the minimum superstructure response is achieved, is much lower than that for the maximum value of the base displacement. Therefore, the value of  $\Omega_{b\theta,min}$  may be considered as one of the optimum isolator parameters.

In Figure 5, the variations of responses with the isolator eccentricity  $\epsilon_b$  are shown for the optimum values of  $\Omega_{b\theta}$  (i.e.,  $\Omega_{b\theta,min}$ ) for a set of given superstructure characteristics. In obtaining the responses, the  $\Omega_{b\theta,min}$  corresponding to each value of  $\epsilon_b$  was used in the analysis. It is seen from the figures that the deck displacement decreases as  $\epsilon_b$  increases and an opposite trend is observed for the base displacement. Therefore, the optimum value of the isolator eccentricity should be selected as large as possible subject to the constraint on the permissible design value of the isolator displacement.

#### 4. CONCLUSIONS

The stochastic response of an asymmetric base isolated structure to random excitation has been described. The stochastic model of the 1940 El-Centro earthquake incorporating the non-stationary characteristics of the ground motion was used as ground excitation. The effects of the torsional coupling on the r.m.s. response of the system was then investigated with the help of a parametric study. In particular, optimum values of the isolator parameters for effective reduction of the superstructure response were identified. The results of the parametric study lead to the following conclusions.

- (1) The lateral response  $u_x$  of the superstructure increases with the increase in the superstructure eccentricity. Thus, the effectiveness of base isolation is overestimated if the superstructure eccentricity is ignored and the system is modelled as 2-D system.
- (2) The torsional deformation of the superstructure increases with the increase in the superstructure eccentricity, showing that the effectiveness of base isolation in reducing torsional response is reduced with the increase in superstructure eccentricity.
- (3) The superstructure eccentricity reduces the translational response and increases the torsional response of the base.
- (4) An increase in the isolator's eccentricity tends to reduce the translational response and increase the torsional response of the system. The effect of the isolator eccentricity on the base responses is more than that on the superstructure responses.
- (5) The uncoupled torsional to lateral frequency ratio  $\Omega_\theta$  of the superstructure has a considerable influence on the responses of the system up to range of  $\Omega_\theta \approx 1.5$ . This ratio has greater effect on the responses of a system with an asymmetric isolator.
- (6) For given superstructure characteristics, there exists a value of isolator frequency ratio  $\Omega_{b\theta}$  for which the superstructure response becomes minimum. For this optimum value of  $\Omega_{b\theta}$ , the superstructure response decreases and the base displacement increases with an increase in isolator eccentricity. Thus, for reducing the superstructure response, the isolator eccentricity should be as large as possible, subject to the constraints imposed on the base displacement. In the optimum design of an isolator for a given superstructure one should, therefore, consider these two factors.

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## APPENDIX: LIST OF NOTATION

$a(t)$	modulating function	$S_0$	spectral density of input white noise
$b$	lateral dimension of the deck in $x$ -direction	$t$	denotes time
CM	center of mass	$\{T_g\}$	influence coefficient matrix
CR	center of resistance	$u_f$	response of the filter
$[C]$	damping matrix of superstructure	$u_x$	lateral displacement of the deck
$[C_b]$	damping matrix of isolator	$u_\theta$	torsional displacement of the deck
$d$	lateral dimension of the deck in $y$ -direction	$u_{b\theta}$	torsional displacement of isolator
$[H]$	augmented matrix	$u_{bx}$	lateral base displacement of isolator
$e_y$	eccentricity of superstructure	$u_g(t)$	ground displacement
$e_{by}$	eccentricity of isolator	$\{U\}$	displacement vector of superstructure
$f(t)$	stationary white noise ground acceleration	$\{U_b\}$	displacement vector of isolator
$\{F\}$	excitation vector	$w_g(t)$	predominant ground frequency
$K_x$	lateral stiffness of superstructure in $x$ -direction	$w_x$	uncoupled lateral frequency of superstructure in $x$ -direction
$K_\theta$	torsional stiffness of superstructure	$w_\theta$	uncoupled torsional frequency of superstructure
$K_b$	total lateral stiffness of isolator	$w_b$	uncoupled lateral frequency of isolator
$K_{b\theta}$	total torsional stiffness of isolator	$w_{b\theta}$	uncoupled torsional frequency of isolator
$[K]$	stiffness matrix of superstructure	$[V]$	covariance matrix of vector $\{Y\}$
$[K_b]$	stiffness matrix of isolator	$\{Y\}$	vector of response quantity
$m$	mass of floor	$\epsilon$	ratio of superstructure eccentricity to plan dimension
$m_b$	base mass	$\epsilon_b$	ratio of isolator eccentricity to plan dimension
$[M]$	mass matrix of superstructure	$\Omega_\theta$	ratio of torsional to lateral frequencies of superstructure
$[M_b]$	base mass matrix	$\Omega_{b\theta}$	ratio of torsional to lateral frequencies of isolator
$[P]$	covariance matrix	$\zeta_g$	damping parameter of K-T spectrum
$r$	radius of gyration of deck mass		
$r_b$	radius of gyration of base mass		