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# Response of pure-friction sliding structures to three components of earthquake excitation

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#### Abstract

This study has been conducted to evaluate the effects of three components of earthquake excitations on the response of the pure-friction base isolated structures. The structure is idealized as a three-dimensional single-storey building resting on the sliding support. The frictional forces mobilized at the sliding support are assumed to have the ideal Coulomb-friction characteristics. The response of this idealized system subjected to three components of El Centro 1940, Tabas 1978 and Northridge 1994 earthquakes is investigated. The effect of base vertical stresses on the lateral response of the system is illustrated by comparing the behaviour of the system subjected to three-component (including vertical component), two-component (excluding vertical component) and single-component (excluding vertical component and no interaction between the orthogonal directions) earthquake excitations. It is observed that the three-component earthquake can significantly affect the response of low period structures with the sliding support. Furthermore, it was found that the different input motions could highly influence the response of the system under three-component earthquake.

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# 1. Introduction

The transmission of ground motion of a structure can be effectively controlled through isolation of the structure at its base. This involves a specially designed foundation system that limits the intensity of the ground motion transmitted to the superstructure. Assimic base isolation of structures has attracted considerable attention in recent years, owing to its significant capability in protecting structures from earthquake excitations. One main concept in base isolation is to reduce the fundamental frequency of structural vibration to a value lower than the energy containing frequencies of earthquake ground motions. The other purpose of an isolation system is to provide means of energy dissipation and,

thereby, reducing the transmitted accelerations to the superstructure. Accordingly, by using base isolation devices in the foundation, the structure is essentially decoupled from ground motion during earthquakes. Several base isolation systems including laminated rubber bearing, frictional bearing and roller bearing have been developed. A significant number of recent researches on base isolation has focused on the use of frictional elements to concentrate flexibility of the structural system and to add damping to the isolated structure. The most attractive feature of the frictional base-isolated system is its effectiveness for a wide range of frequency inputs. The other advantage of a frictional type system is that it ensures the maximum acceleration transmissibility equal to maximum limiting frictional force. The simplest friction type device is the pure-friction referred to as P-F system.

Sliding system with an oscillating single DOF superstructures subjected to harmonic support motion has

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been studied by Mostaghel et al. [1] and Westermo and Udwadia [2]. Mostaghel and Tanbakuchi [3] also studied a similar sliding system, using a semi-analytical solution procedure to compute the response of earthquake ground motions. Younis and Tadjbakhsh [4] employed a 2-DOF system to study the condition of relative slip between the rigid bodies. Qumaroddin et al. [5] proved experimentally the effectiveness of sliding supports in the protection of masonry buildings from the seismic loading. Li et al. [6] also studied the behaviour of the sliding masonry building by an experimental model on a shaking table to verify the analytical result obtained from the system subjected to sinusoidal excitations. The problem of sliding structures is discontinuous because of the different number of equations of motion for sliding and non-sliding phases. As a result, there is inconvenience in solving the differential equations of motion. To overcome this, Yang et al. [7,8] adopted a fictitious spring to represent the frictional effect of the sliding device. Lu and Yang [8] also studied the dynamic behaviour of single DOF equipment mounted on a sliding primary structure subjected to harmonic and earthquake ground motions. With the problem formulated in a state space form, they propose an incremental numerical scheme capable of dealing with multi-DOF sliding structure system for solving the time history responses.

The necessity of considering the vertical component of earthquake in the design of buildings with sliding support is pronounced by Liaw et al. [9]. They stated that the frictional stress is a function of vertical reaction, which is produced by the supporting element on the foundation mat. Hence, both vertical and frictional forces vary when there is vertical motion on the sliding system. As an example, they used the El Centro earthquake records on a two-dimensional structural model, to verify the effect of the vertical component on the lateral response of the system. Lin and Tadjbakhsh [10] also evaluated the effect of vertical ground motion on the horizontal response of a two-dimensional P-F system. They indicated that the effect of vertical motion is only significant in the cases of harmonically excited foundation. Evaluation of the effect of the vertical component of ground excitation on the response of the resilientfriction base isolator (R-FBI) system is studied by Mostaghel and Khodaverdian [11]. They demonstrated that in the case of El Centro 1940 earthquake, the contribution of the vertical excitation to the horizontal response quantities were generally of less than 1%. However, as they had used a two-dimensional system they could not incorporate the effect of interaction between the stresses in the principal directions.

The interaction between the orthogonal components of the frictional forces mobilized at the sliding interface are investigated experimentally by Mokha et al. [12]. Jangid [13,14] also studied the response of a structure with sliding support to bi-directional (i.e. two horizontal

components) earthquake ground motion. He incorporated the coupling effects due to circular interaction between the frictional forces and stated that the design sliding displacement may be underestimated if the bidirectional interaction of frictional forces is neglected.

The frictional stress is a function of the vertical reaction and it varies when there is the vertical motion on the sliding system. As a result, the governing equations of motion of the sliding structure in the three directions are coupled. Therefore, it is advisable to investigate the effects of three directional interactions of the frictional stresses on the response of the sliding structures under three-component earthquake excitations.

In this paper the behaviour of the idealized building resting on a sliding support (with pure-friction) is investigated under the three components of the earthquake excitations. The effect of base vertical stresses on the lateral response of the system is illustrated by comparing the behaviour of the system subjected to three-component (including vertical component), two-component (excluding vertical component) and single-component (excluding vertical component and no interaction between the orthogonal directions) earthquake excitations. The effect of the vertical component of earthquakes on the response of the system is also evaluated for different input motions such as El Centro, Tabas and Northridge earthquakes.

# 2. Structural system and modelling

Fig. 1 represents the assumed structural system, which is an idealized three-dimensional single-storey building model, mounted on a sliding support. The top mass  $m_{\rm s}$  and base mass  $m_{\rm b}$  are rigid decks supported on axially inextensible mass-less columns. The superstructure is assumed to be linear elastic. This is a reasonable assumption, since the purpose of the base isolation is to reduce the earthquake forces on the structure. The centre of mass (CM) of the top deck and the base deck are assumed to be vertically aligned. The structure is symmetric with respect to two orthogonal and vertical directions. As a result, there is no torsional coupling. The frictional stresses mobilized at the sliding support have the ideal Coulomb-friction characteristics. That is the coefficient of friction of the sliding support remains constant and independent of pressure and velocity. The sliding support is isotropic and it is modelled by the spring-damper element. The base deck is idealized as a rigid plane resting on spring-damper element. Under static condition the supporting spring-damper element is subjected to the initial vertical reactive forces resulting from the gravity force of the building.

The dynamic behaviour of the investigated system subjected to earthquake excitation can be described by the following six degrees of freedom:  $u_x$ ,  $u_y$  and  $u_z$  are the

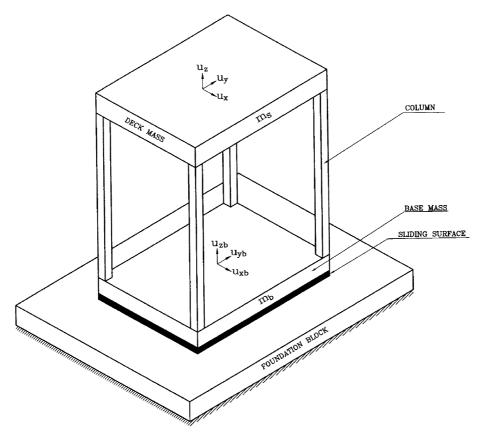


Fig. 1. Idealized three-dimensional single-storey structure with sliding support.

displacement of the superstructure at the center of top deck and  $u_{xb}$ ,  $u_{yb}$  and  $u_{zb}$  are the base displacement at the center of base deck relative to the ground in x-, y- and z-directions, respectively.

The stiffness matrix of the three-dimensional superstructure will be of the form:

$$[K_{\text{sup}}] = \begin{bmatrix} [k_{\text{bb}}] & [k_{\text{bs}}] \\ [k_{\text{sb}}] & [k_{\text{ss}}] \end{bmatrix}$$

$$= \begin{bmatrix} k_x & 0 & 0 & -k_x & 0 & 0 \\ k_y & 0 & 0 & -k_y & 0 \\ k_z & 0 & 0 & -k_z \\ & k_x & 0 & 0 \\ \text{sym} & & k_y & 0 \\ & & & k_z \end{bmatrix}$$

$$(1)$$

in which  $k_x$ ,  $k_y$  and  $k_z$  are the stiffness of the superstructure in x-, y- and z-directions, respectively. The diagonal mass matrix of the superstructure will be of the form:

$$[M_{\text{sup}}] = \begin{bmatrix} m_{\text{b}} & & & & & & & \\ & m_{\text{b}} & & & & & & \\ & & m_{\text{b}} & & & & & \\ & & & m_{\text{s}} & & & & \\ & & & & m_{\text{s}} & & & \\ & & & & m_{\text{s}} & & & \\ & & & & m_{\text{s}} & & & \\ & & & & & m_{\text{s}} & & \\ & & & & & & m_{\text{s}} & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & \\ & & & & \\ & & &$$

in which  $m_b$  is the mass of base deck and  $m_s$  is the mass of the superstructure deck.

For the spring-damper element, the following relationship between tractions acting on the sliding support and relative displacements is defined:

$$\{t\} = [D]\{\delta\} \tag{3}$$

The above equation can be written in the form:

$$\begin{bmatrix} t_{s1} \\ t_{s2} \\ t_n \end{bmatrix} = \begin{bmatrix} k_{s1} & 0 & 0 \\ & k_{s2} & 0 \\ sym & k_n \end{bmatrix} \begin{bmatrix} u_{xb} \\ u_{yb} \\ u_{zb} \end{bmatrix}$$
(4)

where  $\{t\}$ ,  $\{\delta\}$ , [D] are the tractions, relative displacement vectors and elasticity matrix, respectively.  $t_{s1}$  and  $t_{s2}$  are the shear tractions,  $t_n$  is the contact pressure and  $k_{s1}$ ,  $k_{s2}$  and  $k_n$  are shear and normal stiffnesses. The stiffness matrix of spring-damper element is  $[K_b] = [D]A$ , in which A is the area of foundation. In this study it is

assumed that there is no separation. If the shear strength of the sliding support is exceeded, irreversible slip occurs. The yield function is

$$F_{\rm s} = t_{\rm s} + \kappa \tag{5}$$

in which  $t_s$ 

$$t_{\rm s} = \sqrt{(t_{\rm s1}^2 + t_{\rm s2}^2)} \tag{6}$$

and

$$\kappa = \mu t_{\rm n} \tag{7}$$

in which  $\mu$  is the coefficient of friction. A circular interaction curve between stresses in two orthogonal directions has been assumed. When the stresses are on yield function,  $k_{s1}$  and  $k_{s2}$  are assumed to be zero, and the vertical stress in base is affected by vertical component of earthquake. For computation of stress increments because of ideal plasticity, where the yield surface form a meaningful limit, a proportional scaling of stresses (or radial return procedures) have been used to obtain stresses [15].

## 3. System dynamic equilibrium

The governing differential equations of motion for the whole system (superstructure and sliding support) can be stated in the matrix form as

$$[M]\{\ddot{a}\} + [C]\{\dot{a}\} + \{P(a)\} = -[M][I]\{\ddot{a}_g\}$$
(8)

in which [M] is the lumped mass matrix; [C] is the viscous damping matrix;  $\{P(a)\}$  is the non-linear internal force vector which is indicated as a function of the nodal parameters  $\{a\}$ ; for the linear problem,  $\{P(a)\} = [K]\{a\}$  (in which [K] is the elastic stiffness matrix);  $\{a\}^T = \{u_x, u_y, u_z, u_{xb}, u_{yb}, u_{zb}\}^T$  is a vector of displacement of the system;  $\{\ddot{a}_g\}$  is the vector of the components of earthquake excitation, i.e.  $\{\ddot{u}_{xg}, \ddot{u}_{yg}, \ddot{u}_{zg}\}^T$ ; and [I] is a unit diagonal matrix. The damping matrix is considered to be proportional to the mass and stiffness matrices.

The differential equations of motion (Eq. (8)) are solved in incremental form by employing the Newmark- $\beta$  method assuming a constant-average acceleration over a short time interval. In convergent process, the New-

ton-Raphson method is used for the solution of the nonlinear problem and the iteration continues to a specified number of iteration or when the tolerance reaches an acceptable value. An 'energy' tolerance is used to terminate the iteration if the convergence is achieved.

# 4. Numerical study

The response of the three-dimensional structure with the sliding support subjected to three-component earth-quake excitations has been investigated. Response quantities of interest for the system under consideration are the absolute acceleration of the superstructure in the x- and y-directions ( $\ddot{u}_{xa} = \ddot{u}_x + \ddot{u}_{xg}$  and  $\ddot{u}_{ya} = \ddot{u}_y + \ddot{u}_{yg}$ ) and the relative sliding displacement of the base mass ( $u_{xb}$  and  $u_{yb}$ ). The absolute acceleration is directly proportional to the forces (shear force and bending moments) exerted in the superstructure due to the earthquake excitations. The latter is a measure of displacement between the isolated structure and the ground, which is crucial in the design of sliding system.

In order to study the effects of three-component, two-component and single-component earthquake excitations on the lateral response of the sliding system, the response of frictional stresses with and without vertical components is compared with those corresponding to no interaction. Three real earthquake records are considered and applied to the sliding structure. The characteristics of these earthquake records are shown in Table 1. El Centro 1940 earthquake is chosen as it has been used widely in the previous investigations of pure-friction base-isolated structures. Tabas 1978 and Northridge 1994 earthquakes are selected as they have the strong vertical component.

The response of the system with and without vertical component is referred to as the response to three-component and two-component earthquakes, respectively. The response of the system is also obtained for two orthogonal directions (i.e. *x*- and *y*-directions) acting independently in each direction. In this case there is not any interaction between the frictional stresses in the two orthogonal directions and this condition can be referred to as single-component earthquake. In the present study, the time period of the superstructure as a fixed base is considered to be equal in the *x*- and *y*-directions (i.e.

Table 1 Earthquake characteristic records

Eurorquake characteristic records							
	Item	Name of earthquake record	Name of station	Peak acceleration in <i>x</i> -direction ( <i>g</i> )	Peak acceleration in <i>y</i> -direction ( <i>g</i> )	Peak acceleration in vertical direction (g)	Ratio of peak acceleration of vertical to horizontal direction
	1	Imperial Vally 1940	El Centro	0.313	0.205	0.215	0.686
	2	Tabas 1978	Tabas	0.937	0.88	0.746	0.796
	3	Northridge 1994	Renaldi	0.842	0.48	0.846	1.005

 $T_x/T_y=1.0$ ); and the ratio of time period in z-direction to x- or y-directions is selected as 0.5. The damping ratio of superstructure is assumed to be 5% of the critical damping. The friction coefficient ( $\mu$ ) of the sliding support is considered to be equal to 0.1, and the mass ratio  $m_b/m_s$  is assumed to be unity. In the sliding surface, shear and normal stiffnesses are taken to be very high during the non-sliding state (inside yield surface).

The sensitivity of the starting times of the sliding and non-sliding phases of the system response is much more for three-component earthquake excitations compared to the system response for two-component earthquake excitations [14]. After many trials, it was found that a time interval of  $\Delta t = 0.0005$  s for El Centro 1940 earthquake would yield accurate results. Smaller time interval ( $\Delta t = 0.0002$  and  $\Delta t = 0.00002$  in the

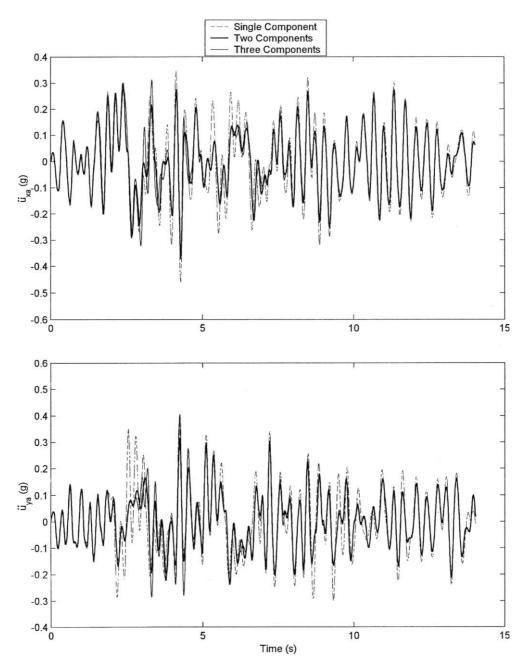


Fig. 2. Time history of the absolute acceleration of the superstructure subjected to the Northridge 1994 earthquake excitation  $(T_s = 0.35)$ .

neighbourhood of transition of phases) is employed for Tabas 1978 and Northridge 1994 earthquakes. The convergence tolerance taken for iteration is  $|E(i)|/|E(1)| = 1.0 \times 10^{-18}$ , and the numbers of iteration in each time step is selected as 15.

In Figs. 2 and 3, the time history variation of the absolute acceleration of the superstructure ( $\ddot{u}_{xa}$  and  $\ddot{u}_{va}$ ) and the sliding base displacement in x- and y-directions are plotted for single, two and three components of Northridge 1994 earthquake excitations. Fig. 2 indicates that the nature of variation of the absolute acceleration is almost the same for all of the three cases. The absolute acceleration of the superstructure is less for two- and three-component ground motion as compared to singlecomponent ground motion. In Fig. 3, there is a significant difference in the base displacement for two-component and three-component earthquakes. This is due to the fact that for the three-component earthquake excitation, the system starts sliding at a relatively lower value of the frictional forces mobilized at the sliding support since the frictional stress is a function of the vertical reaction. Thus, the sliding base displacement may be underestimated if the three components of earthquake excitations are not considered simultaneously for designing the sliding support.

In order to study the effectiveness of sliding support for different earthquakes as the input motions the peak

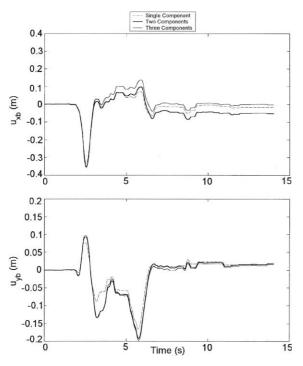


Fig. 3. Time history of the peak base displacement of the sliding system subjected to the Northridge 1994 earthquake excitation ( $T_s = 0.35$ ).

absolute acceleration spectra of the superstructure  $\left(\text{i.e. }\sqrt{\left(\ddot{u}_{xa}\right)_{max}^{2}+\left(\ddot{u}_{ya}\right)_{max}^{2}}\right)$  with and without sliding support under the three components of El Centro, Tabas and Northridge earthquake excitations (Table 1) are shown in Fig. 4. The figure clearly indicates that the sliding support is quite effective in reducing the seismic lateral response of the superstructure subjected to three components of the earthquake excitations. Furthermore, the input motions considerably affect the peak absolute acceleration of the superstructure with low structural period ( $T_s = 0.7$ ). It is interesting to note that this effect is more pronounced for the fixed base structures.

Figs. 5–7 show the variation of the peak absolute acceleration of the superstructure (i.e.  $\sqrt{\left(\ddot{u}_{xa}\right)_{max}^2+\left(\ddot{u}_{ya}\right)_{max}^2}$ )

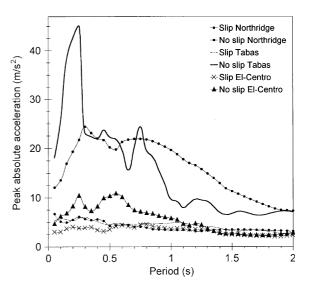


Fig. 4. The comparison of peak absolute acceleration spectra of the superstructure versus time period of the structure with and without sliding.

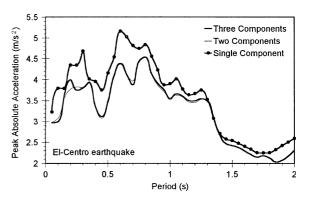


Fig. 5. The variation of the peak absolute acceleration of the superstructure versus time period of the structure to the El Centro earthquake excitation.

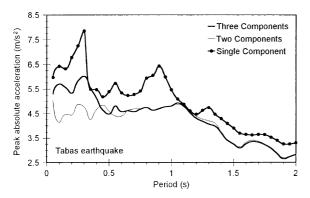


Fig. 6. The variation of the peak absolute acceleration of the superstructure versus time period of the structure to the Tabas earthquake excitation.

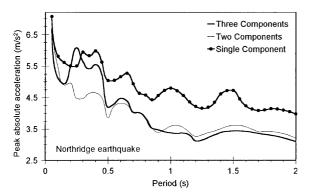


Fig. 7. The variation of the peak absolute acceleration of the superstructure versus time period of the structure to the Northridge earthquake excitation.

versus the time period of the structure for single, two and three components of El Centro, Tabas and Northridge earthquake excitations, respectively. The peak absolute acceleration of the superstructure is not affected considerably when the system is subjected to three-component El Centro earthquake in comparison with its two-component one (Fig. 5). As it can be observed in Figs. 6 and 7 the three-component Tabas and Northridge earthquakes highly influence the response quantity of the low period structures (i.e.  $T_{\rm s} < 0.7$ ).

In Figs. 8–10 the variation of peak base displacement of the sliding system is plotted against the structural period  $(T_s)$  for single, two and three components of El Centro, Tabas and Northridge earthquake excitations, respectively. The curves of Fig. 8 show that the peak base displacements of the sliding system are significantly higher for three and two components of El Centro earthquake excitations in comparison with its single-component counterpart. However, the peak base displacements that resulted from the three-component and

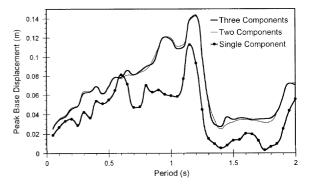


Fig. 8. The variation of the peak base displacement versus time period of the structure to the El Centro earthquake excitation.

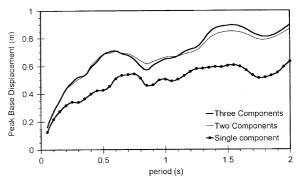


Fig. 9. The variation of the peak base displacement versus time period of the structure to the Tabas earthquake excitation.

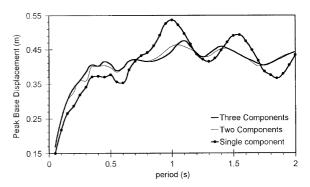


Fig. 10. The variation of the peak base displacement versus time period of the structure to the Northridge earthquake excitation.

two-component earthquakes are almost the same in this case. Almost, the same pattern of variation could be seen for Tabas earthquake (Fig. 9). The peak base displacement of the sliding system subjected to three components of Northridge earthquake (Fig. 10) is higher than single- and two-component earthquakes in low period structures ( $T_s = 0.7$ ). For moderate and high

structural periods, generally the single-component case has higher values.

## 5. Conclusion

The response of the three-dimensional structure with sliding support subjected to three components of El Centro, Tabas and Northridge earthquakes is investigated, to verify the effects of simultaneous components of earthquakes on the lateral response of the system. It is observed that the sliding support is quite effective in reducing the seismic lateral response of the structure subjected to three components of the earthquake excitations. This effect is strongly dependent on the system period as well as the input motions. The peak absolute acceleration and peak base displacement of the low period superstructures ( $T_s < 0.7$ ) may be underestimated if the three-component earthquake is not considered and the structures with sliding support are designed merely on the basis of single- and two-component earthquakes. However, in moderate and high period structures there is no significant difference in the peak base displacement of the sliding system for two and three components of earthquake excitations.

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