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PLANE FRAME ANALYSIS OF LATERALLY LOADED ASYMMETRIC BUILDINGS—AN UNCOUPLED SOLUTION

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Abstract—It is shown that for a class of asymmetric building structures, a lateral load analysis can be performed by means of analogous plane frames rather than by a three-dimensional procedure. Based on a co-ordinate transformation, the technique is applicable to structures with two types of framing systems each comprising several vertical planar assemblages having similar stiffness properties and a common variation thereof along the height.

INTRODUCTION

The analysis of tall building structures under wind and earthquake loading is conventionally carried out by considering planar models for the structure in each of the two orthogonal directions. This procedure is valid when lateral (translational) and torsional displacements are uncoupled, i.e. for symmetric buildings and for those having an elastic axis. For a dynamic analysis it is also required that the mass axis coincides with the elastic axis.

It appears that unless these requirements are satisfied it may be necessary to perform a full three dimensional analysis for an asymmetric building. In such cases the size of the problem is often larger than can be handled by in-house computers or too costly to be analyzed by the three dimensional options of standard general purpose computer programs. Therefore, it is often necessary to reduce the size of the problem by neglecting less significant degrees of freedom (e.g. torsion of frame members and their normal bending) and by taking advantage of the particular geometric and structural features usually found in tall buildings.

The structural system of many tall buildings is typically composed of several planar assemblages such as shear walls (simple or coupled) and frames. This fact permits a rapid evaluation of the stiffness matrix from the individual properties of the planar assemblages, provided the following assumptions are reasonably satisfied: (1) the floor is rigid in its own plane; (2) the stiffness contribution of each assemblage is in its own plane only, i.e. when a column pertains to more than one frame, the errors resulting from allocating its cross-sectional area among the frames are not significant. A detailed description of the method is given by Clough[1].

When these assemblages are symmetrically arranged, lateral as well as torsional behaviour can be adequately modeled by an equivalent plane frame, as demonstrated by Coull and Stafford Smith[2] and Rutenberg[3]. It can be easily shown that the procedure described therein is also applicable to asymmetric structures provided a vertical elastic axis exists. Evidently uncoupled lateral and torsional behaviour is amenable to planar solutions.

It may appear that the coupled lateral-torsional response of asymmetric buildings with no elastic axis always requires a three-dimensional analysis. However, this is not the case for an important class of asymmetric structures, and it is the purpose of this note to illustrate how a plane frame representation of such structures may be achieved. For simplicity, the details developed herein are for a floor plan with one axis of symmetry so the coupling is confined to one axis only, but the procedure is also applicable to the more general case of no symmetry.

PLANAR MODEL

The model analyzed in this note is a building with two types of families of framing systems. Each family comprises several vertical planar assemblages, having similar stiffness properties, such as frames or flexural walls, and a common variations thereof along the height of the building. The elastic centers of the two systems are located on two different vertical axes, i.e. points CK and CS in Fig. 1. From the foregoing description it follows that in each family all members can be lumped into an equivalent assemblage, and the torsional stiffness matrix is proportional to the lateral stiffness matrix. The equation of lateral and torsional equilibrium may thus be given by:

$$\left(\begin{bmatrix} \mathbf{K}_{yy} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{yy} \end{bmatrix} + \begin{bmatrix} \mathbf{S}_{yy} & \dfrac{a}{r_k}\mathbf{S}_{yy} \\ \dfrac{a}{r_k}\mathbf{S}_{yy} & \dfrac{\bar{r}_s^2}{r_k^2}\mathbf{S}_{yy} \end{bmatrix}\right)\begin{Bmatrix} \underline{u}_y \\ r_k\underline{u}_\theta \end{Bmatrix} \begin{Bmatrix} \underline{F}_y \\ \dfrac{1}{r_k}\underline{F}_\theta \end{Bmatrix} \tag{1a}$$

or

$$(\mathbf{K}+\mathbf{S})\underline{u} = \underline{F} \tag{1b}$$

in which K_{yy} and S_{yy} are respectively the reduced stiffness matrices in the y direction of the K and S families of assemblages, $\underline{u}_y$, $\underline{u}_\theta$ = lateral and rotational displacement vectors respectively, $\underline{F}_y$, $\underline{F}_\theta$ = lateral and torsional force vectors, a = distance between CS and CK. The radii of inertia of K and S are given by

$$r_k^2 = \Sigma k_j\rho_j^2/\Sigma k_j;\quad r_s^2 = \Sigma s_l\rho_l^2/\Sigma s_j;\quad \bar{r}_s^2 = r_s^2 + a^2 \tag{2}$$

where k_j and s_l are respectively the representative values of the stiffness of the assemblages in each family, and ρ_j and ρ_l are their distances from CK and CS.

Now it is possible to transform the system of equations in (1) into [4, 5]:

$$\begin{bmatrix} \mathbf{K}_{yy}+p_1^2\mathbf{S}_{yy} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{yy}+p_2^2\mathbf{S}_{yy} \end{bmatrix}\begin{Bmatrix} \underline{v}_1 \\ \underline{v}_2 \end{Bmatrix} = \frac{1}{\sqrt{1+\gamma^2}}\begin{bmatrix} \mathbf{I} & -\gamma\mathbf{I} \\ \gamma\mathbf{I} & \mathbf{I} \end{bmatrix}\begin{Bmatrix} \underline{F}_y \\ \dfrac{1}{r_k}\underline{F}_\theta \end{Bmatrix} = \begin{Bmatrix} \underline{F}_{v1} \\ \underline{F}_{v2} \end{Bmatrix} \tag{3}$$

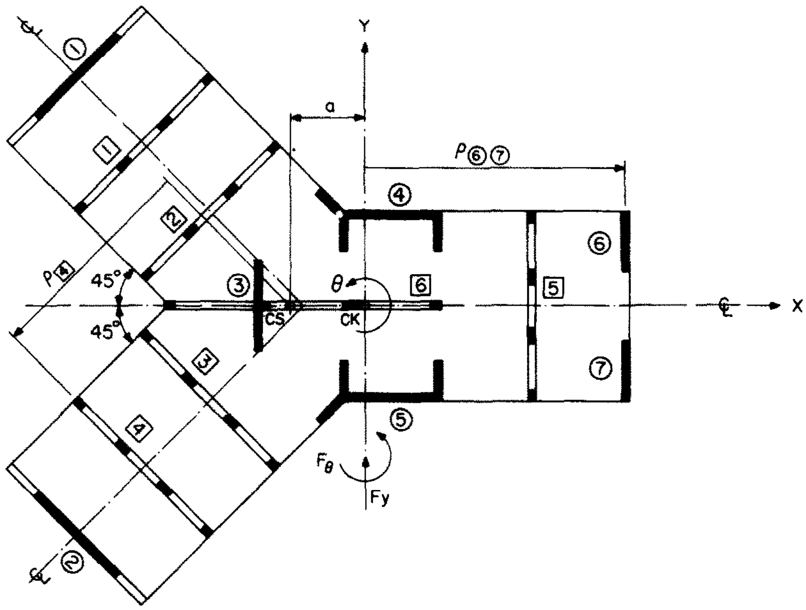

Fig. 1. Typical floor plan, asymmetric wall-frame structure (free adaptation from Ref. [6]).

where:

$$p_{1,2}^2 = \frac{1}{2}\left[1 + \frac{\bar{r}_s^2}{r_k^2} \pm \sqrt{\left(1 - \frac{\bar{r}_s^2}{r_k^2}\right)^2 + 4\frac{a^2}{r_k^2}}\right] \tag{4}$$

and

$$\gamma = (1 - p_1^2)\; r_k/a \tag{5}$$

by means of the following transformation:

$$\begin{Bmatrix} u_y \\ r_k u_\theta \end{Bmatrix} = \frac{1}{\sqrt{1+\gamma^2}} \begin{bmatrix} \mathbf{I} & \gamma\mathbf{I} \\ -\gamma\mathbf{I} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \tag{6a}$$

or

$$\underline{u} = \Gamma \underline{v}. \tag{6b}$$

The uncoupled system in eqn (3) can be represented by two plane frames, as is illustrated in Fig. 2 for the structure shown in Fig. 1. In the first set of equations S_{yy} is multiplied by p_1^2 and in the second by p_2^2. Therefore all member properties of the equivalent frame (representing the S family) should first be thus factored, and then combined with the equivalent cantilever representing the K family. Each of the resulting two analogous frames in Fig. 2(a) and Fig. 2(b) is then analyzed for its respective loading $\underline{F}_{v1}$ and $\underline{F}_{v2}$ as given by the right hand side of eqn (3).

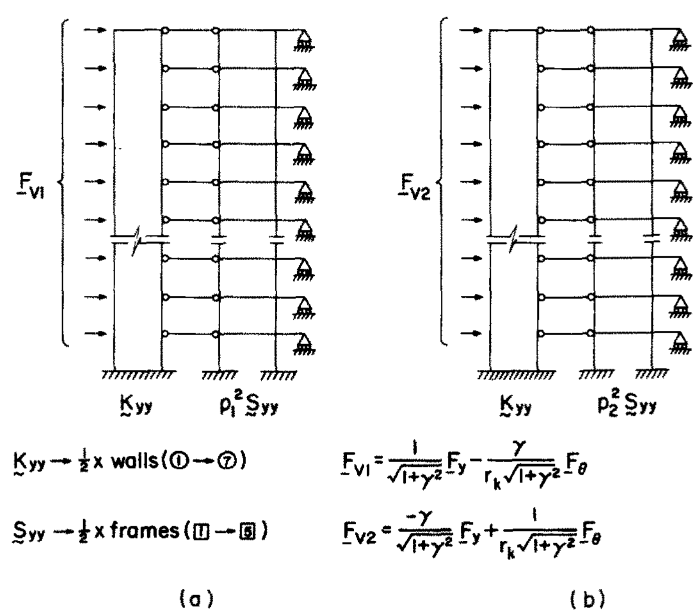

Fig. 2. Analogous plane frames (a) Frame in v_1 direction; (b) Frame in v_2 direction.

Once the response values for the two analogous frames in Fig. 2 have been computed by any standard plane frame computer program, eqn (6) is used to evaluate the internal forces and displacement in the real system[4]. Thus the generalized forces Q_y and r_kQ_θ and displacements d_y and r_kd_θ are obtained from their counterparts in the two analogous frames as follows:

$$\left\{\begin{matrix} \underline{Q}_y, \underline{d}_y \\ r_k\underline{Q}_\theta, r_k\underline{d}_\theta \end{matrix}\right\} = \Gamma\left\{\begin{matrix} \underline{Q}_{v1}, \underline{d}_{v1} \\ \underline{Q}_{v2}, \underline{d}_{v2} \end{matrix}\right\}. \tag{7}$$

For example:

$$\underline{u}_y = \frac{1}{\sqrt{1+\gamma^2}}(\underline{v}_1 + \gamma\underline{v}_2)$$

$$r_k\underline{u}_\theta = \frac{1}{\sqrt{1+\gamma^2}}(-\gamma\underline{v}_1 + \underline{v}_2)$$

since all the assemblages were lumped for the purpose of computations into two families (represented respectively by the equivalent frame and equivalent cantilever in Fig. 2), the forces acting on a given assemblage have to be calculated from the lumped responses of the two analogous frames. Thus for member j of the K family, the generalized forces Q are given by:

$$Q_j^k = Q_y^k \frac{k_j}{\Sigma k_j} + r_k Q_\theta^k \frac{k_j\rho_j}{r_k^2 \Sigma k_j} \tag{8}$$

where the superscript indicates that the Q values pertain to the K family. For members of the S family it is necessary first to transfer the forces to CS:

$$\left\{\begin{matrix} Q_y^s \\ Q_\theta^s \end{matrix}\right\}_{CS} = \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix}\left\{\begin{matrix} Q_y^s \\ Q_\theta^s \end{matrix}\right\}_{CK}. \tag{9}$$

The internal forces in the assemblages of S are then obtained from:

$$Q_i^s = Q_{yCS}^s \frac{s_i}{\Sigma s_i} + r_k Q_{\theta CS}^s \frac{s_i\rho_i}{r_s^2 \Sigma s_i}. \tag{10}$$

The foregoing procedure can be easily carried out by hand or programmed for an in-house computer.

SUMMARY AND CONCLUSIONS

A simple lateral load analysis for asymmetric structures comprising two types of planar assemblages has been described. The procedure is based on a co-ordinate transformation yielding three uncoupled systems of equations (two for the case of one axis of symmetry). After forces and displacements have been computed for the analogous frames representing the uncoupled systems of equations, they are transformed into the real three-dimensional system by linearly combining the results. The analysis of the analogous frames may be performed by standard plane frame programs, and the member forces then computed by hand. However, the procedure is particularly suitable for engineering offices with in-house computer facilities where reduction in the size of the problem is a major consideration, and minor modifications of existing programs are routinely carried out.

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