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MODE COUPLING IN THE VIBRATION RESPONSE OF ASYMMETRIC BUILDINGS

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Abstract—Coupling of the lateral and torsional modes in the vibration response of asymmetric buildings is treated using simple three-dimensional computer models. This study will be useful in predicting the torsional coupling in the earthquake response of the building. The computer models are based on energy methods and assume that each floor is a rigid diaphragm with three degrees of freedom (two lateral and one rotational) and can analyse buildings which are unsymmetrical in plan and/or elevation. To treat various types of asymmetry, buildings which are square, rectangular and L-shaped in plan are considered. In these buildings, the position of the core is varied to obtain different degrees of asymmetry and the consequent coupling of the modes of vibration. Numerical examples are presented where the natural periods of vibration and associated mode shapes are obtained for different degrees of asymmetry. The mode shapes demonstrate the torsional coupling inherent in asymmetric buildings. The relationship between the coupled mode shapes obtained in the free vibration analysis and the anticipated dynamic response is discussed.

1. INTRODUCTION

A multistorey building that is torsionally balanced has at each floor level coincident centres of mass and stiffness which lie on a common vertical axis. In practice this is unlikely to occur, as buildings are unsymmetrical either in elevation and/or plan or in the mass distribution of the floors, i.e. they are asymmetric. Asymmetric buildings are torsionally unbalanced and display complex interaction between the lateral and torsional components of their response.

Structural asymmetry has been identified as a major contributor to the failure or poor performance of multistorey buildings during earthquakes [1, 2]. Much of this poor torsional performance can be attributed to increases in effective eccentricity due to the dynamics of the system. It has been shown by Tso and Dempsey [3] that effective eccentricity can be magnified six-fold if the first uncoupled lateral and torsional periods of vibration are equal.

In the design of buildings subjected to earthquake loadings the simplest design technique is to use either a quasi-static code approach or a response spectra method to determine equivalent static loads. Both these methods assume that the fundamental (or the dominant) mode excited by the earthquake is lateral and require the building's fundamental period of vibration to determine the lateral loads to be applied. It is interesting to note that most seismic design codes give simplified formulae for this purpose, based on building height and sometimes on building height and width [4–6]. These will not be satisfactory as the code formulae do not give consideration to the distribution of mass and stiffness in the building and are certainly most unsatisfactory for asymmetric high-rise buildings. It is therefore very unlikely that the resulting designs of asymmetric buildings, which exhibit torsional coupling, will respond satisfactorily when subjected to earthquakes.

If the dynamics of the structure are considered, the inertia forces acting at the centres of mass at each storey level and the resisting forces acting at the centres of resistance of the vertical elements form dynamic couples which interconnect the translational and torsional response of the building. This is most significant in buildings with abnormal load and stiffness distributions such as hotels (with their myriad of pools, shops, ballrooms and restaurants etc.) and buildings with heavy plant rooms at unusual levels. This problem will become increasingly prevalent as architects employ new and unusual features into the buildings they design (such as set-backs, unusual column shapes and distribution and unusual floor plans). Simultaneously engineers are becoming increasingly cunning in their load resisting systems using lightweight floors and transfer girders etc. For these reasons, assumption of the dominant mode to be lateral and then basing the fundamental period of vibration on just the height of the building may be too simplistic. This will result in inadequate provision for dynamic torsional amplifications and lead to failure or severe damage of the building. It is therefore necessary to arm practising engineers with a knowledge of free vibration characteristics in general and a knowledge of mode coupling in particular. It is also necessary to provide them with new analytical tools that can take these factors into account while remaining simple enough to give them a good feel for the behaviour of the building they are designing.

Free vibration characteristics of a building play a significant role in determining its dynamic response. When a building is asymmetric due to any one of the reasons stated above, the coupling between the lateral and torsional components of its response becomes an important consideration in the earthquake response of the building. A knowledge of the natural periods (or frequencies) of vibration and the associated mode shapes will be useful in evaluating the degree of coupling between the modes and the consequent dynamic torsional coupling. This is most important in the case of asymmetric high-rise buildings in which the uncoupled, translational and torsional natural frequencies of the building are close. Relatively large torsional motions may be generated under such circumstances even in buildings that are nominally asymmetric. With this in mind, this paper treats the free vibration analysis of asymmetric buildings, using three-dimensional computer models. There are finite element packages which can do almost anything. However, they will require considerable computational effort and time and for this reason may not be attractive to practising engineers.

In Section 2 of this paper, the two computer models used in the free vibration analysis are discussed. In the next section, numerical examples are treated and the results discussed. In particular, the coupling of the modes of vibration due to the asymmetrical nature of the building, is illustrated. Section 4 discusses the relevance of the free vibration characteristics of asymmetric structures to their dynamic response. Some relevant comments are presented in Section 5, which concludes the paper.

2. COMPUTER MODELS

Two computer models, based on energy considerations, are used to perform the free vibration analysis. Theory underlying the two models is presented briefly.

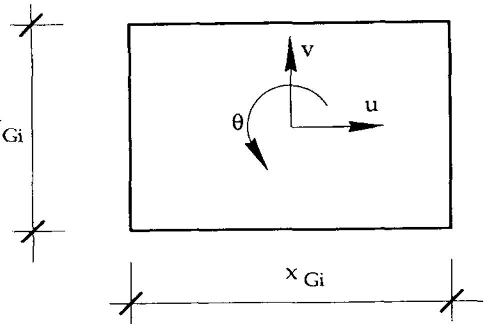

Fig. 1. Degrees of freedom of slab.

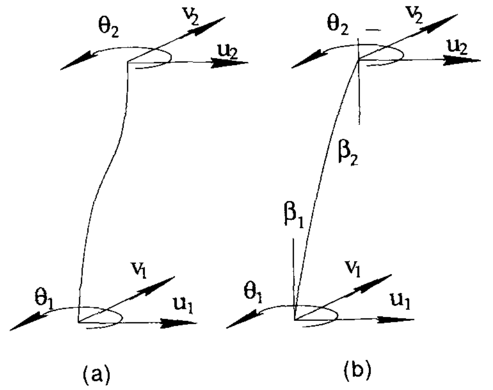

Fig. 2. Vertical members in the model. (a) 3 DOF; (b) 5 DOF.

2.1. *Model (a)—three degrees of freedom at ends of vertical members (3 dof)*

The major assumptions in the first model [7] are:

(1) floors are rigid diaphragms possessing three degrees of freedom (two lateral and one rotational) as shown in Fig. 1;

(2) kinetic energies of vertical members are neglected, but, if necessary, may be lumped, with the slab into the mass matrix;

(3) all vertical members are restrained at floor levels (i.e. a shear beam model), see Fig. 2a;

(4) the principal axes of all vertical members are assumed to lie along the x–y axes.

Assumption (3) results in the following stiffness matrix for a vertical element r between levels i-1 and i:

$$k_{ri} = \begin{bmatrix} \frac{12EI_y}{L^3} & 0 & 0 \\ 0 & \frac{12EI_x}{L^3} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{bmatrix}, \quad (1)$$

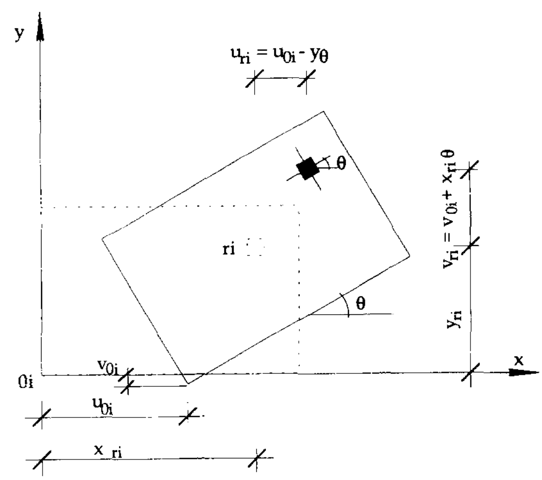

Fig. 3. Transformation between column r and origin.

where I_x and I_y are the moments of inertia, about the x and y axes, respectively, of the vertical element r. J is the polar moment of inertia of the vertical element and L is the storey height between levels i-1 and i.

Figure 3 shows how assumption (1) allows the displacement of any column to be related to the displacement of an origin column by the relationship

$$w_{ri} = C_r w_{0i}, \tag{2}$$

where $\langle w_{ri}\rangle^{\mathrm{T}} = \langle u_{ri}, v_{ri}, \theta_i\rangle$ is the displacement vector of the end of the vertical element r at level i and $\langle w_{0i}\rangle^{\mathrm{T}} = \langle u_{0i}, v_{0i}, \theta_{0i}\rangle$ is the displacement vector of the origin of level i. The transformation matrix C_r is given by:

$$C_r = \begin{Bmatrix} 1 & 0 & -y_r \\ 0 & 1 & x_r \\ 0 & 0 & 1 \end{Bmatrix}. \tag{3}$$

$$K_0 = \begin{Bmatrix} (K_{01}+K_{02}) & -K_{02} & 0 & . & 0 \\ -K_{02} & (K_{02}+K_{03}) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & (K_{0n-1}+K_{0n}) & -K_{0n} \\ 0 & . & . & -K_{0n} & K_{0n} \end{Bmatrix} M_0 = \begin{Bmatrix} M_1 & 0 & . & . & . & 0 \\ 0 & M_2 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & . & . & . & . & M_n \end{Bmatrix}, \tag{9}$$

In this case the origin column is chosen at the intersection of the x and y axes. An equivalent stiffness matrix (relative to the origin displacements) for all columns at a particular level may be obtained by the summation:

$$K_{0i} = \sum_{r=1} C_r^{\mathrm{T}} k_{ri} C_r, \tag{4}$$

where K_{0i} is the equivalent storey stiffness at the origin and is obtained by the summation from $r = 1$ to the number of columns between level i-1 and level i. This effectively reduces the problem to three degrees of freedom per storey. The potential energy of all vertical members is obtained from

$$V = \sum_{i=1}^{n} V_i = \frac{1}{2}\sum_{i=1}^{n} \{w_{0i}\}^{\mathrm{T}} K_{0i} \{w_{0i}\}. \tag{5}$$

The relationship between column displacements and origin displacements holds for slab velocities. Using this and the transformation matrix in eqn (3), the kinetic energies of all slabs can be computed by:

$$T = \sum_{i=1}^{n} T_{0i} = \frac{1}{2}\sum_{i=1}^{n} \{m_i[\dot{u}_{0i}^2 + \dot{v}_{0i}^2 - 2\dot{\theta}_i(\dot{u}_{0i} y_{Gi} - \dot{v}_{0i} x_{Gi})] + J_{oi}\dot{\theta}_i^2\}, \tag{6}$$

where J_{0i} is the mass polar moment of inertia of slab i about the origin, i.e.

$$J_{0i} = j_{Gi} + M_i(x_{Gi}^2 + y_{Gi}^2) \tag{7}$$

and T = kinetic energy of the system, m_i = the mass of slab i, J_{Gi} = the mass polar moment of inertia of slab i about the centroid of the slab, x_{Gi} and y_{Gi} = distance along the x and y axes, respectively, of the centroid of slab i.

Finally, using Lagrange's equation together with eqns (5) and (6), the equation of motion (in the absence of damping) is:

$$M_0\ddot{w}_0 + K_0 w_0 = Q(t), \tag{8}$$

where w_0 = the origin displacement vector (u_0, v_0, θ) and $\ddot{w}_0$ is the origin acceleration vector, $Q(t)$ is the forcing function (note: $Q(t) = 0$ for free vibration analysis), M_0 is the structure mass matrix and K_0 is the structure stiffness matrix.

The forms of the mass and stiffness matrices for an n storey building are:

where K_{0i} and M_i are the mass and stiffness matrices of level i and are defined in eqn (4) and in eqn (10) below:

$$[M_i] = \begin{Bmatrix} m_i & 0 & -m_i y_{Gi} \\ 0 & m_i & m_i x_G i \\ -m_i y_{Gi} & m_i x_{Gi} & J_{Gi} \end{Bmatrix}. \tag{10}$$

Further details of this method can be found in Ref. [7].

2.2. *Model (b)—five degrees of freedom at ends of vertical members (5 DOF)*

The second model is an extension of the first model [8]. The major assumptions of this model are the same as in the previous model, but the third assumption is replaced by

(3) the vertical members are assumed to suffer rotations (α and β) about the x and y axis and these rotations are proportional to their stiffness.

This third assumption increases the degrees of freedom at the ends of the vertical members from three to five; as shown in Fig. 2b (note the rotation into the page has been left off Fig. 2b for clarity). The stiffness matrix for this type of element makes it more convenient to formulate the problem using the full 10×10 stiffness matrix for each vertical element. The problem will be reduced to five degrees of freedom per floor when the structural stiffness matrix is assembled.

The stiffness matrix for each vertical member between level i-1 and i is given by:

$$k_{ri}=\begin{bmatrix}
\frac{12EI_Y}{L^3} & 0 & 0 & 0 & \frac{6EI_Y}{L^2} & \frac{-12EI_Y}{L^3} & 0 & 0 & 0 & \frac{6EI_Y}{L^2} \\
0 & \frac{12EI_x}{L^3} & 0 & \frac{-6EI_x}{L^2} & 0 & 0 & \frac{-12EI_x}{L^3} & 0 & \frac{-6EI_x}{L^2} & 0 \\
0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 \\
0 & \frac{-6EI_x}{L^2} & 0 & \frac{4EI_x}{L} & 0 & 0 & \frac{6EI_x}{L^2} & 0 & \frac{2EI_x}{L} & 0 \\
\frac{6EI_Y}{L^2} & 0 & 0 & 0 & \frac{4EI_Y}{L} & \frac{-6EI_Y}{L^2} & 0 & 0 & 0 & \frac{2EI_Y}{L} \\
\frac{-12EI_Y}{L^3} & 0 & 0 & 0 & \frac{-6EI_Y}{L^2} & \frac{12EI_Y}{L^2} & 0 & 0 & 0 & \frac{-6EI_Y}{L^2} \\
0 & \frac{-12EI_x}{L^3} & 0 & \frac{6EI_x}{L^2} & 0 & 0 & \frac{12EI_x}{L^3} & 0 & \frac{6EI_x}{L^2} & 0 \\
0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & \frac{-6EI_x}{L^2} & 0 & \frac{2EI_x}{L} & 0 & 0 & \frac{6EI_x}{L^2} & 0 & \frac{4EI_x}{L} & 0 \\
\frac{6EI_Y}{L^2} & 0 & 0 & 0 & \frac{2EI_Y}{L} & \frac{-6EI_Y}{L^2} & 0 & 0 & 0 & \frac{4EI_Y}{L}
\end{bmatrix}. \tag{11}$$

The displacement transformation relationship is now:

$$\Delta_{ri}=C_r\Delta_{0i}, \tag{12}$$

where the 5 × 1 displacement vector will be:

$$\langle\Delta_{ri}\rangle^{\mathrm{T}}=\langle u_{ri-1}, v_{ri-1}, \theta_{i-1}, \alpha_{i-1}, \beta_{i-1}, u_{ri}, v_{ri}, \theta_{ri}, \theta_i, \alpha_i, \beta_i\rangle \tag{13}$$

and the transformation matrix becomes:

$$C_r=\begin{bmatrix}
1 & 0 & -y_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & x_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{rx} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{ry} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -y_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & x_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{rx} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{ry}
\end{bmatrix} \tag{14}$$

where

$$a_{rx}=I_{rx}/I_{0i} \quad \text{and} \quad a_{ry}=I_{ry}/I_{0i}. \tag{15}$$

The equivalent stiffness matrix for all vertical members between levels i-1 and i can be obtained by using the transformation matrix of eqn (14) and the summation in eqn (4).

In this model assumptions (1) and (2) are unchanged. This renders the stiffness matrix (where all degrees of freedom contribute) to be of order $2n$ (where n is the number of storeys) higher than the mass matrix (where the column rotational inertias are, reasonably, neglected). To overcome this incompatibility the stiffness matrix obtained from eqn (4) is partitioned and condensed prior to solution.

Decomposing the 10×10 floor stiffness matrix of eqn (4) into four, 5×5 submatrices i.e.

$$K_{0i} = \left\{ \begin{array}{c|c} K_{0i}^{11} & K_{0i}^{12} \\ \hline K_{0i}^{21} & K_{0i}^{22} \end{array} \right\} \tag{16}$$

the structure stiffness matrix, K_0, can be assembled as shown:

$$K_0 = \left\{ \begin{array}{cccc} (K_{01}^{22} + K_{02}^{11}) & K_{02}^{12} & 0 & 0 \\ K_{02}^{21} & (K_{02}^{22} + K_{03}^{11}) & K_{03}^{12} & 0 \\ 0 & K_{03}^{21} & (K_{03}^{22} + K_{04}^{11}) & K_{04}^{12} \\ 0 & 0 & K_{04}^{21} & K_{04}^{22} \end{array} \right\}. \tag{17}$$

The structure stiffness matrix is rearranged so that the degrees of freedom common to both the mass and stiffness matrices (i.e. terms pertaining to u, v and θ) appear first, i.e. the corresponding displacement vector becomes:

$$\Delta_0^{\mathrm{T}} = [u_{01}, v_{01}, \theta_{01}, u_{02}, \ldots, \theta_{0n}, \alpha_{01}, \beta_{01}, \alpha_{02}, \ldots, \beta_{0n}] \tag{18}$$

$$= [w_{01}^{\mathrm{T}}, \ldots, w_{0n}^{\mathrm{T}}, \gamma_{01}^{\mathrm{T}}, \ldots, \gamma_{0n}^{\mathrm{T}}], \tag{19}$$

where

$$w_{01}^{\mathrm{T}} = [u_{01}, v_{01}, \theta_{01}] \quad w_{0n}^{\mathrm{T}} = [u_{0n}, v_{0n}, \theta_{0n}] \tag{20}$$

$$\gamma_{01} = [\alpha_{01}, \beta_{01}] \quad \gamma_{0n}^{\mathrm{T}} = [\alpha_{0n}, \beta_{0n}] \tag{21}$$

and the rows and columns of the structure stiffness matrix are rearranged to correspond to their respective degrees of freedom, i.e.

$$K_0 = \left[\begin{array}{c|c} K_0^{ww} & K_0^{w\gamma} \\ \hline K_0^{\gamma w} & K_0^{\gamma\gamma} \end{array} \right]. \tag{22}$$

Using this rearranged stiffness matrix, eqn (8) can be expanded to

$$M_0 \ddot{w}_0 + K_0^{ww} w_0 + K_0^{w\gamma} \gamma_0 = Q_t \tag{23}$$

$$K_0^{\gamma w} w_0 + K_0^{\gamma\gamma} \gamma_0 = 0. \tag{24}$$

Combining eqns (23) and (24) and setting the force vector to zero results in the reduced system:

$$M_0 \ddot{w}_0 + K_0^* w_0 = 0, \tag{25}$$

where the condensed matrix K_0^* is calculated from

$$[K_0^*] = [K_0^{ww}] - [K_0^{\gamma w}]^{\mathrm{T}} [K_0^{\gamma\gamma}]^{-1} [K_0^{\gamma w}]. \tag{26}$$

Once w_0 is determined from eqn (25), γ_0 can be determined from

$$\gamma_0 = -[K_0^{\gamma\gamma}]^{-1} [K_0^{\gamma w}] W_0. \tag{27}$$

3. NUMERICAL EXAMPLES AND DISCUSSION

To illustrate mode coupling in asymmetric buildings, three numerical examples, pertaining to three different types of buildings, are treated using the simple procedure and the two computer models, discussed in the previous section. The periods of vibration and the associated mode shapes of these buildings have been obtained by using both the computer models. Despite earthquake damage of several buildings, engineers continue to use the simple methods given in codes of practice, which assume the dominant mode excited during earthquakes to be lateral. To emphasize the need for a three-dimensional model for vibration and dynamic analysis and to point out the inadequacies of the code formulae, results for the fundamental period are compared with those obtained from the formulae given in codes of practice.

Table 1. Natural periods of vibration

	Example 1 T_1(s)				Example 2 T_1(s)				Example 3 T_1(s)			
Method	$e = 0$	$e = 2.7$	$e = 5.5$	$e = 7.3$	$e = 0$	$e = 5.9$	$e = 11.9$	$e = 17.8$	$e = 0.6$	$e = 5.4$	$e = 6.5$	$e = 10.1$
Rayleigh		0.51				1.22				1.27		
AS2121		0.64				0.64				0.95		
		1.0				1.0				1.49		
3 DOF	0.38	0.39	0.43	0.45	1.11	1.16	1.24	1.31	1.97	1.79	2.06	1.67
3 DOFV	0.43	0.44	0.47	0.5	1.16	1.20	1.28	1.33	2.01	1.83	2.10	1.73
5 DOF	0.51	0.51	0.51	0.51	1.11	1.20	1.28	1.33	2.05	1.95	2.09	1.87
5 DOFV	0.56	0.56	0.56	0.56	1.15	1.24	1.32	1.38	2.10	2.01	2.13	1.93

Note: 3 and 5 DOFV include the effects of vertical members.

The codes used for this purpose are [4–6]: Uniform Building Code (1985); Australian Standard AS 2121 (1979); New Zealand Standard NZS 4203 (1984). A more accurate method which is also employed, in some form, by some codes is the Rayleigh method. Fundamental periods of vibration of the three buildings obtained by the proposed models are presented in Table 1 and are compared with those obtained from the code formulae and the Rayleigh method. It should be noted that, since the Australian Standard, AS 2121-1979 [5] is a metric version of the Uniform Building Code, 1985 [4], the results obtained from the formulae in these two documents will be identical and are presented under the heading of AS 2121. Of the two formulae in AS 2121, eqn 5.2(1) is generally considered more precise as it takes into account building height and width and not just the number of storeys (as is the case with the second formula). The New Zealand Standard [6] only allows designers to calculate the fundamental period of vibration using the Rayleigh method. The fundamental periods of vibration according to the New Zealand code have been presented under the heading of Rayleigh method.

In all the buildings, floor slabs have a weight density of 5 Pk except the top storey which has a roof loading of 2.5 Pk. The material properties of these concrete building are: Young's modulus, $E = 30$ MPa and Poisson's ratio $\nu = 0.2$.

3.1. *Example* 1—10 *storey square building*

A 10 storey building with the prismatic, square cross-section and constant storey height of 3 m, shown in Fig. 4, is considered. All columns in this building are 0.50 × 0.50 m in cross-section. The outer dimensions of the central core are 2 × 2 m with a wall thickness of 0.15 m and the core is placed, in turn, at four different locations, as shown in the figure.

Fundamental periods of vibration of this building obtained from the two computer models are presented in Table 1, for various locations of the core, and compared with those obtained from code formulae. The periods of vibration using the Rayleigh method were obtained by analysing a plane frame on a commercially available structural analysis package. Results labelled 3 DOFV and 5 DOFV pertain to those obtained from the computer models in which the masses of the vertical members had been lumped with the slabs. The two results presented under the heading AS 2121 were calculated using eqns 5.2(1) and 5.2(2), respectively, in the Australian Standard [5].

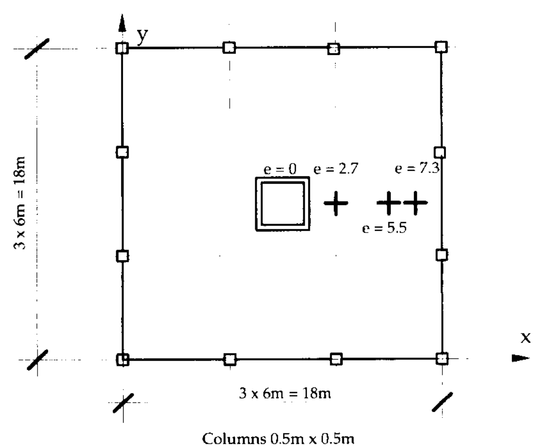

Fig. 4. Example 1—square building.

Results obtained from the 3 DOF computer model show that the fundamental period of vibration varies with the position of the core, which in turn represents the degree of asymmetry. This variation is smaller when the masses of the vertical members are lumped. The fundamental periods of vibration obtained with the 5 DOF and 5 DOFV models remain constant for all positions of the core. This is because, in this model, the core dominates the behaviour with considerable bending rotation (about a horizontal axis), irrespective of its position. The AS 2121 formulae over-estimate the fundamental period and will result in significant reduction in the acceleration factor to be used in seismic design. This reduction can be as much as 100% if the period is taken as 1.0 s, and will obviously result in under-design of the building.

At the time of completing this paper, the new Australian Standard "Minimum Design Loads On Structures—Part 4: Earthquake Loads", AS 1170.4-1193, [9] has been just released. If this standard is used, the fundamental period will be further over-estimated to approx. 2 s, and the design acceleration will be further reduced.

Table 1 also shows that the result obtained from the second formula of AS 2121 differ by nearly 100% when compared to that from the Rayleigh method and more than 100% when compared with that from some of the computer models. The results obtained from the 5 DOF model and the Rayleigh method are in very good agreement; however, when the masses of the vertical elements are considered in the analysis, the periods of vibration increase (as would be expected) by 9%. The Rayleigh method could also lump the mass of vertical elements with the slab, but it cannot account for variation in the distribution of mass. The three-dimensional models introduced in this paper can cater for variations in the distribution of masses and therefore can be expected to give more accurate results. It can be seen that the results obtained from the stiffer 3 DOF model are lower than those obtained from all other sources.

The various code formulae merely calculate the fundamental period of vibration, but cannot account for the effects of asymmetry. For this reason the first three periods of vibration (T_1, T_2, T_3) of the building in example 1 have been calculated using the 3 and 5 DOF models, and their variations with

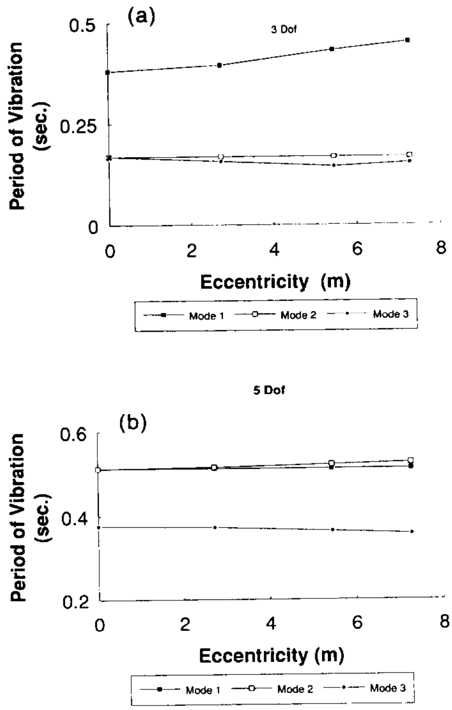

Fig. 5. Example 1—periods of vibration vs eccentricity. (a) 3 DOF model, (b) 5 DOF model.

eccentricity (e) are shown in Fig. 5a and b, respectively. These figures show that, for this case, the fundamental periods of vibration increase slightly with eccentricity, demonstrating the general trend that the fundamental period of vibration of a building increases with eccentricity. Periods of vibration (T_2, T_3) in the second and third modes seem to decrease with eccentricity. But in the 3 DOF model, T_3 reduces first and then increases, as seen in Fig. 5a. This interesting feature is discussed later on.

Even if periods of vibration obtained from the code formulae and the Rayleigh method agree with those obtained from the three-dimensional models, the mode shapes can be quite different. It is possible that torsional modes (which cannot be identified in plane frame analysis) dominate. Only three-dimensional models can account for these torsional modes, which are important in the earthquake analysis of asymmetric buildings. It is also noted, in Table 1, that the difference in results between the 3 and 5 DOF models is not as large as may be expected. Since the two models have identical mass matrices and in a static analysis the 3 DOF system is approximately 10 times stiffer than the 5 DOF system, one would expect the period of vibration using the 3 DOF model to be of the order of $10^{1/2}$ less than the period calculated using the 5 DOF model. However, this is not so, and the reason for this discrepancy is that, before the equation of motion is solved for the eigenvalues, the stiffness matrix in the 5 DOF model is condensed from 5 DOF at the ends of vertical members to 3. This has the effect of removing the rotational stiffness elements from the matrix and, for a building dominated by the flexure of a core, only marginally reducing the (shear) stiffness of the remaining elements. Thus, when the eigenvalue solution is performed, the values obtained are of similar order.

To understand some of the effects of torsional coupling on the free vibration response, the mode shapes (of the 3 DOF model at the centre of mass) at various eccentricities are presented in Fig. 6. While the 5 DOF model is probably more appropriate for this type of building the results of the 3 DOF model have been plotted because they demonstrate an interesting feature. It should also be noted that this figure does not show the actual mode shapes, but the relative magnitudes of each degree of freedom (u, v, θ). Figure 6 shows that, when the eccentricity is zero (i.e. the building is "nominally" symmetric), the problem is uncoupled and as such there is no interaction between the various components (i.e. u, v, and θ) of the modal response, as shown earlier [10]. Each mode will have only one component, either u, v or θ in its free vibration response. The first mode is purely torsional, while the second and third modes are purely lateral in the x and y directions, respectively. As the problem is uncoupled, all eigenvector shapes are identical for the first three modes. The eigenvalue only changes between modes 1 and 2 as the second and third modes are basically the same, except that the deflections are in the x and y directions, respectively.

As eccentricity is increased, T_1 increases and the coupling between the v and θ components of the modes occurs. This coupling becomes more pronounced as the eccentricity increases, as shown in Fig. 6b–d. Moving the core of the building to the right makes the building asymmetric about the y axis only. As the building is still symmetrical about the x axis, the response in that direction is still uncoupled and the second period, T_2, and the second mode shape are unaffected by the change in eccentricity.

The behaviour of the third mode shape is not so straightforward. Increasing eccentricity at first reduces T_3 and then increases it. The coupling between v and θ components of this mode displays a similar trend to T_3; that is, the coupling increases at first and then reduces. At the greatest eccentricity, T_3 and the coupling of v and θ components of the modes reduce because, as can be seen from Fig. 6c and d, the third mode shape has a changed form.

3.2. *Example* 2—10 *storey rectangular building*

A 10 storey building with a prismatic, rectangular cross-section and constant storey height of 3 m, as shown in Fig. 7 is considered. All columns in this building are 0.30 × 0.30 m in cross-section. The outer

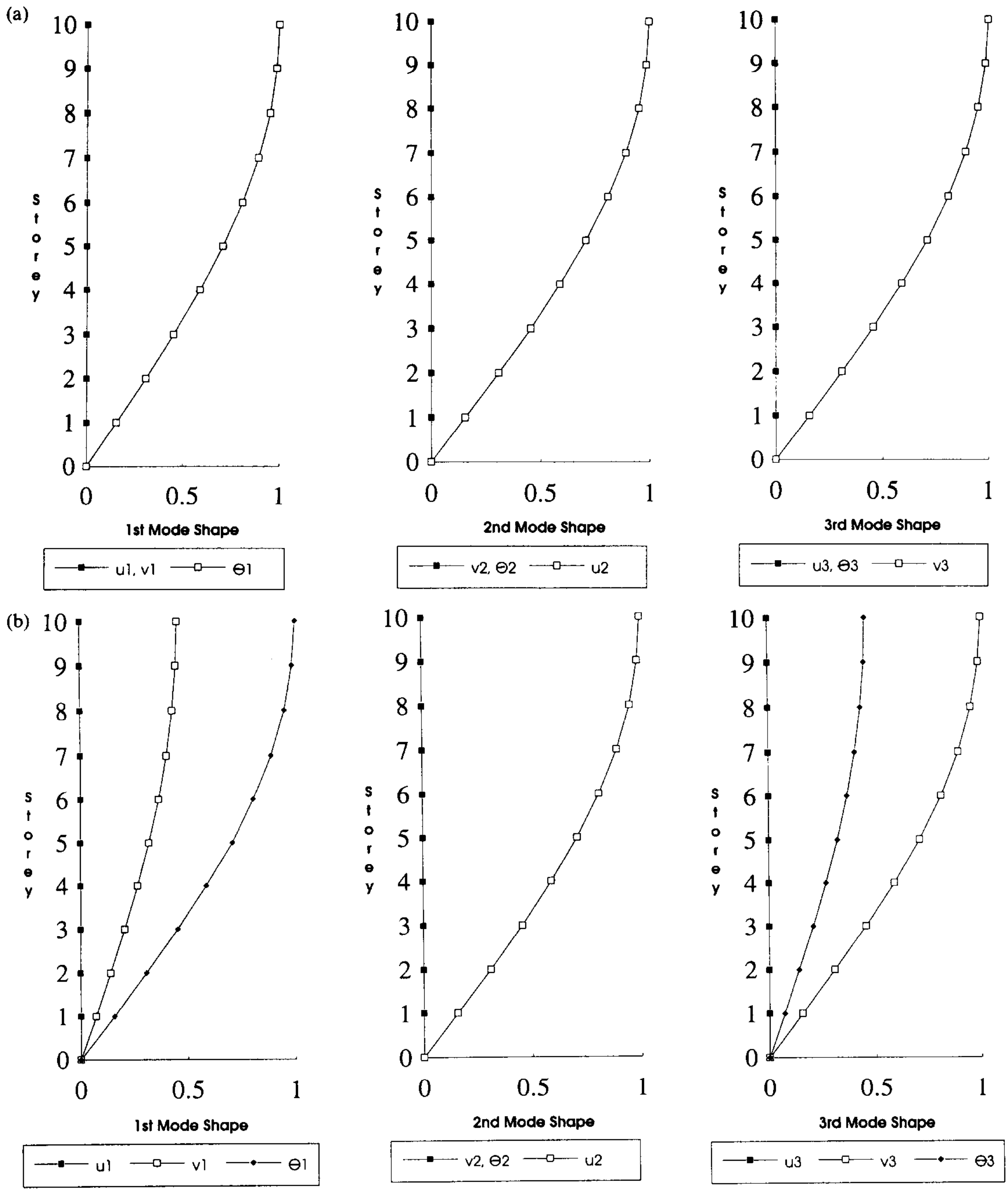

Fig. 6(a and b). (*Caption opposite*).

dimensions of the central core are 3 × 2.5 m with a wall thickness of 0.20 m and the core placed in four different locations, as shown in the figure.

The fundamental periods of vibration of this building obtained from the two computer models for the various positions of the core are presented in Table 1, together with the results obtained from the formulae in the codes of practice and the Rayleigh method. The results obtained from both the 3 and 5 DOF models at intermediate eccentricities, when the masses of vertical members are not included, are in good agreement with the fundamental period of vibration calculated by the Rayleigh method. The 5 DOF gives a 20% increase in fundamental period of vibration for the most eccentric building in comparison with that for the symmetric case. AS 2121 formulae underestimate the fundamental period. When the masses of the vertical members are included in the analysis, the fundamental periods of vibration obtained with both the three-dimensional models are increased at all eccentricities, as expected. For this case, code provisions will yield considerable

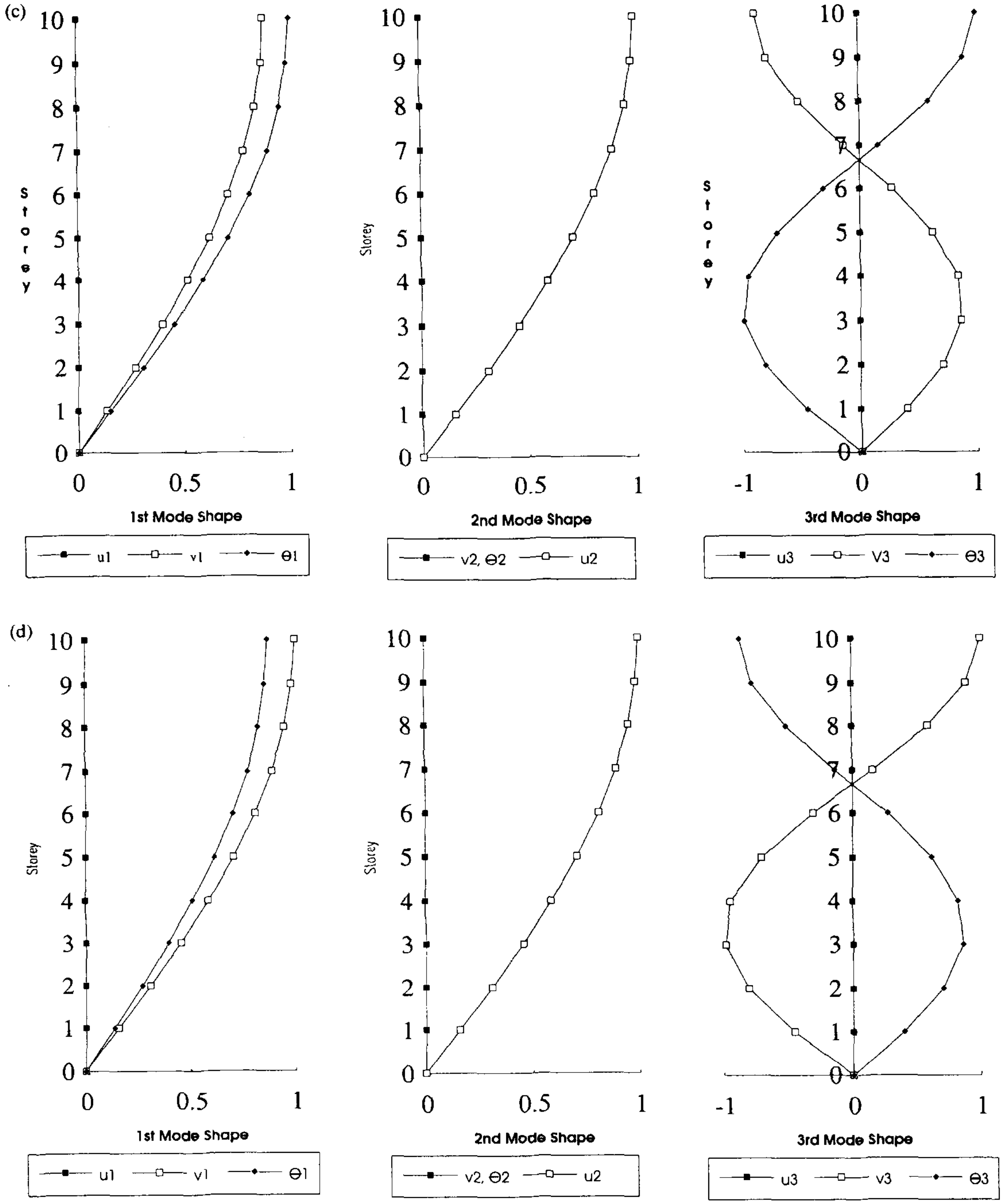

Fig. 6. Example 1—mode shapes in 3 DOF model. (a) $e = 0.0$ m, (b) $e = 2.7$ m, (c) $e = 5.5$ m, (d) $e = 7.3$ m.

deviations from the true response. It is also noteworthy that, in this particular example, the results obtained from both the three-dimensional models agree very well when the masses of the vertical members are included in the analysis.

As this building has a very large slab (in plan) and relatively small columns and core, the ratio of mass polar moment of inertia of the slab to torsional stiffness of the vertical elements is much greater than the ratio of mass of the slab to lateral stiffness of the vertical elements. For this reason the fundamental mode is torsional and is, presumably, ignored by designers as the codes are unable to predict this mode. The first lateral period of vibration is the second mode and is 1.07 s. The first three mode shapes for the 5 DOF model are shown in Fig. 8. If in the seismic

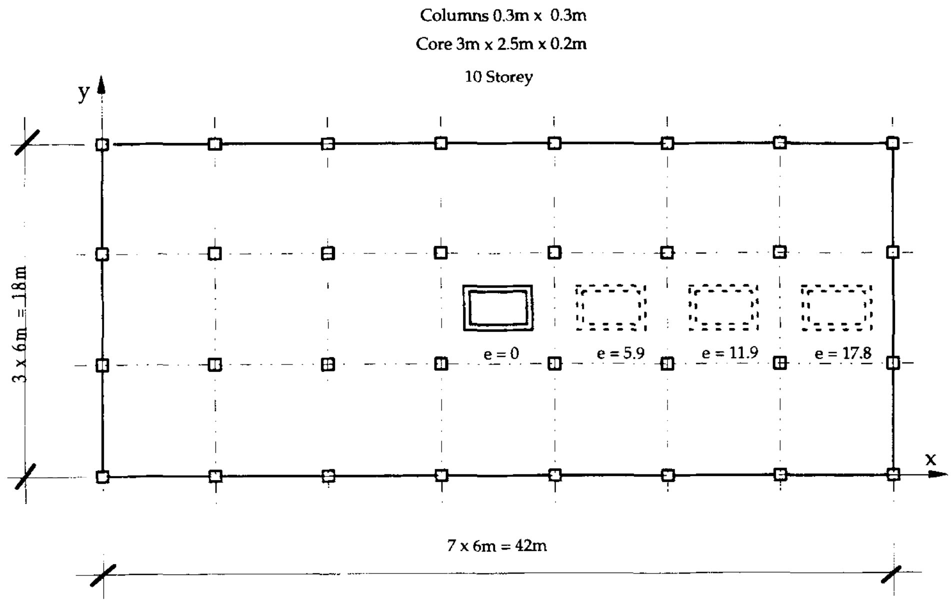

Fig. 7. Example 2—rectangular building.

design of this building, the fundamental period is calculated by using one of the code formulae, and the analysis done by the quasi-static code approach, which assumes the fundamental mode to be lateral, it will not be a surprise if this building did not survive a moderate earthquake. The design would not have provided for the significant torsional effects experienced by the building when subjected to lateral earthquake forces.

It is interesting to note that the Rayleigh method still gives an accurate value for the period of vibration, but as the vibration mode is assumed to be lateral, the type of mode is in error. To ensure the first lateral mode is in fact the second mode and not an error due to inaccuracies in the eigenvalue subroutine, a time history analysis was performed on the symmetric case of example 2 subjected to an impulse load. The model used for the time history response was the 5 DOF. The period of vibration of the structure after the impulse load had passed was measured. As the problem is symmetric the lateral and torsional responses are uncoupled. This means a lateral force resulted in only lateral displacements, while torsional forces applied about the symmetrical vertical axis of the building resulted in twisting about the vertical axis of the building with no lateral deflections. The period of the lateral displacements was 1.07 s, while the period of the rotations about the

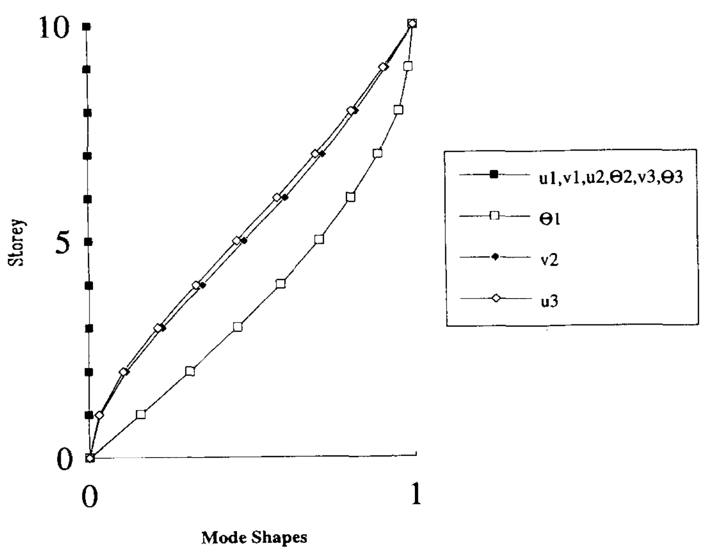

Fig. 8. Example 2—mode shapes in 5 DOF of model at $e = 0.0$ m.

vertical axis was 1.11 s. This provided confirmation of the mode shapes obtained in the free vibration analysis.

3.3. *Example* 3—15 *storey L-shaped building*

A 15 storey building with the prismatic, L-shaped cross-section and constant storey height of 3 m, shown in Fig. 9, is considered. All columns in this building are 0.30 × 0.30 m in cross-section. The outer dimensions of the central core are 2.5 × 2.0 m with a wall thickness of 0.20 m and the core is placed in four different locations, as shown in the figure.

The fundamental period of vibration of this building obtained from the two computer models is presented in Table 1, together with those obtained from the codes of practice and the Rayleigh method. In this example, the code formulae and the Rayleigh method under-estimate the fundamental period (at all eccentricities). Use of the Rayleigh method on a structure like the one in this example is particularly difficult, as the building is asymmetric about both axes. The use of a plane frame analysis on a problem like this leaves the designer with a range of uncertainities, such as effective slab width, degree of torsion on the building and three-dimensional effects. The plane frame model used in this example resulted in a fundamental period of vibration of 1.27 s; however, other assumptions on the behaviour of the building could be made and different frames could be analysed, resulting in different periods of vibration—such is the nature of asymmetric buildings. Therefore, as mentioned earlier, the mode shapes pertaining to the fundamental mode will be model-dependent. In this example, both the three-dimensional models gave similar results at all eccentricites. Both AS 2121 formulae significantly underestimate the period and

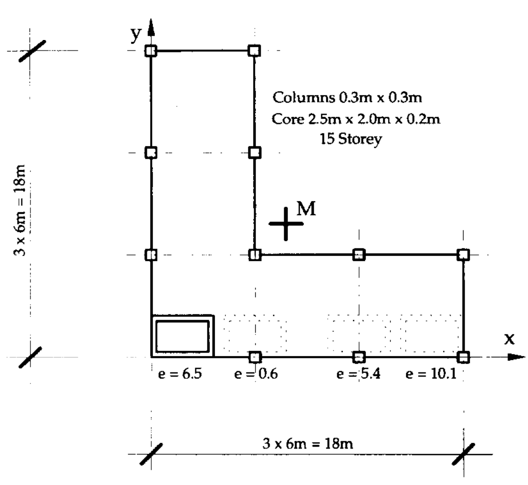

Fig. 9. Example 3—L-shaped building.

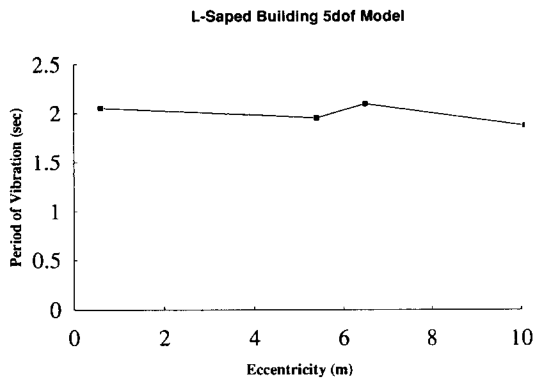

Fig. 10. Example 3—period of vibration vs eccentricity.

will result in over-design of the building for all core positions.

There is an interesting trend in the variation of the fundamental period T_1 with core position. As the core is moved to the right, the periods obtained from both the 3 and 5 DOF models reduce at first and then increase. The maximum and minimum values of T_1 occur at core positions 3 and 4, respectively. Inclusion of masses of vertical members increase the periods of vibration, as expected, but without altering this trend.

The buildings in the two previous examples were symmetric about one (x) axis; however, in the present problem the structure is asymmetric about both (x and y) axes. This results in the coupling of the response in both lateral directions as well as torsionally. To the authors' knowledge all simplified relationships, given in the literature, consider only coupling between torsion and one lateral direction response [2, 3, 11]. Figure 10 shows the relationship between eccentricity and period of vibration (for the 5 DOF model), from where it could be seen that the relationship between eccentricity and period of vibration is not straightforward. The effect of coupling in both lateral directions has increased the complexity of the free vibrations analysis. This will be reflected in the dynamic response of the building.

4. DYNAMIC RESPONSE

In order to relate the results of free vibration analysis to the dynamic response, the following discussion is presented.

The usefulness of response spectra in the design and analysis of buildings subjected to earthquake loads is linked to the fact that two single degree of freedom oscillators with the same natural period of vibration and damping ratio subjected to the same loading will show identical responses regardless of the dimensions of the two systems. For the same system, if eccentricity is introduced, only the torsional natural period of vibration (or more correctly the ratio of torsional lateral natural period of vibration) and the eccentricity will be the extra parameters relevant to the dynamic response. As shown in Ref. [3], if the uncoupled lateral and torsional periods of vibration of a 2 DOF system are known, the dynamic eccentricity can be calculated. Moreover, as discussed previously and shown in the earlier examples, it is not always easy to represent a complex building as a 1 or 2 DOF system. It is also very difficult or impossible to calculate parameters such as static or dynamic eccentricities for a general asymmetric building. Therefore the results obtained with simple models cannot be extrapolated for general cases and the need for three-dimensional models is evident.

Consider the two simple systems shown in Fig. 11. They have identical total mass and stiffness, but the slab in the second system is twice the size of that in the first model. The second system has twice the eccentricity and twice the radius of gyration of the first model. If a free vibration analysis is performed, the two eigenvectors will be the same, except that the torsional component of the eigenvector of the second model will be half of that in the first model. If a time history analysis is performed, the twisting rotations will also be half for the model with the larger slab. This means that, while the static eccentricity for the larger model is double that of the smaller model, the dynamic eccentricity is effectively half. If the eccentricities are normalized by dividing by the radii of gyration of the respective slabs, the static eccentricities will be equal for both cases, while the normalized dynamic eccentricity of the larger model will be one-quarter of that of the smaller model.

From the basis of the method superposition it can be said that two similar systems with the same mode shapes and periods of vibration will have identical dynamic response when subject to the same loading. In other words the torsional component of the dynamic response is proportional to the magnitude of the torsional component of the free vibration response.

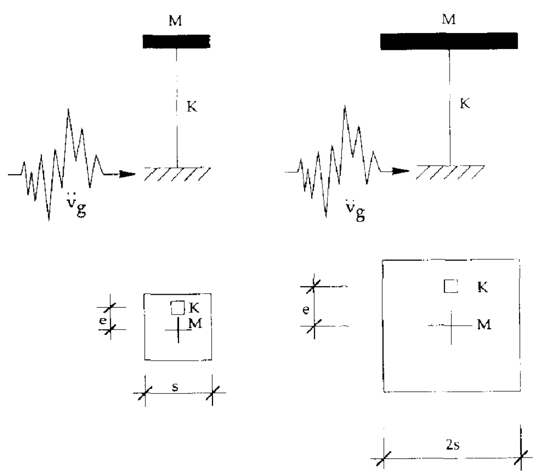

Fig. 11. Two degrees of freedom systems.

5. CONCLUSIONS

Free vibration analysis of three buildings with varying degrees of asymmetry has been treated by using three-dimensional analyses and the results discussed. To demonstrate the inadequacies of the code procedures, results for T_1 are compared with those obtained from formulae given in the codes of practice. Dynamic analysis of some simple systems has also been treated in order to complement and supplement the results of the free vibration analysis and to enhance the understanding of the total dynamic behaviour of the buildings. From the results obtained, the following conclusions may be drawn.

A symmetric building is torsionally uncoupled and will show no interaction between the components of its modal response. As the degree of asymmetry is increased (for example, by increasing the eccentricity of the core) the various modes share the response with the other components that they are coupled with. A change in eccentricity may completely change the character of the higher modes by bringing into play different mode shapes.

The various codes of practice may not accurately calculate the fundamental or first period of vibration as they take no account of mass and/or stiffness or their distributions, and, hence, unusual buildings may have significantly different periods of vibration to that predicted by the codes. Moreover current design procedures may miss the true first period of vibration as it may be torsional. The computer models (3 and 5 DOF) used in the present study can easily and accurately predict the natural periods of vibration for complex structures. The period of vibration calculated by the 5 DOF model is in agreement with the Rayleigh method for structures where the mass distribution is regular, but the mode shapes may be different. Unlike the Rayleigh method, the computer models can easily account for non-uniform distributions in the mass of a building.

The torsional components of the dynamic response of two similar structures with the same period of vibration will be proportional to the torsional components of their free vibration response. Torsional coupling can have a pronounced effect on the free vibration response and hence on the dynamic response of a building, as it can characteristically change the nature of the vibrating/deflected shapes.

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