# Parametric earthquake response of torsionally coupled buildings with foundation interaction

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A study is made of the effect of soil-structure interaction on the coupled lateral and torsional responses of asymmetric buildings subjected to a series of historical free-field earthquake base motions. It is shown that for particular classes of actual buildings the equivalent rigid-base responses are significantly increased for structures founded on medium-stiff soils, and hence the assumption of the major building codes that a conservative estimate of response is obtained by considering the structure to be fixed rigidly at its base is shown to be inconsistent with the presented dynamic results. It is shown that foundation interaction produces greatest amplification of torsional coupling effects for structures subjected to a particular class of European strong-motion earthquake records, identified by similarities in their spectral shape, for which the vibrational energy of the ground motion is distributed approximately uniformly over the range of frequencies which are of interest for real structures. It is recommended that provision be made in the torsional design procedures of building codes for the increase in the coupled torsional response due to soil-structure interaction as indicated in this study. Such provision should be based on the results of comprehensive parametric studies employing a wide selection of earthquake records and accounting for expected variations in localized soil conditions.

Key Words: torsional coupling, soil-structure interaction, time history earthquake response, earthquake building codes.

## EARTHQUAKE INTERACTION OF ASYMMETRIC BUILDINGS

Most parametric studies of torsional coupling, in the linear[1-4] and nonlinear[5,6] range of earthquake response, have been limited to cases in which the structure is supported by an infinitely rigid foundation medium. However, observations on the dynamic response of actual buildings[7,8], and intensive research work on the influence of foundation flexibility on the earthquake response of structures[9-11], have revealed that there are many cases in which the effects of soil-structure interaction may be significant and should therefore be accounted for in earthquake analyses if realistic results are to be obtained (for an extensive discussion of the literature, see Ref. 12). A variety of numerical and analytical techniques have been employed to model the compliance of the foundation medium[13,14] and to compute the dynamic response of both foundation and superimposed structural models[10,15,16]. In the analysis of a rigidly based structure, the earthquake input applied at the base is identical to the free-field ground motion. Flexibly based structures, however, generally experience ground motion which is significantly different from the free-field recording[9]. In determining whether significant interaction is developed in a particular situation, the controlling factor is the relative properties of structure and soil, and in particular their relative stiffness. The most significant effects of soil-structure interaction are[17]: (1) an increase in the fundamental period compared with the rigidly based structure, sometimes by as much as 20 to 30 per cent, and (2) a change (increase or decrease) in the peak structural response.

Whilst a large number of parametric and case studies have investigated the effects of soil-structure interaction for single and multi-storey building models[9,13,15,16], little research to date has been carried out into its effects on the earthquake response of torsionally coupled building systems with eccentricity between the centres of mass and resistance at one or more storey levels. Recent work by Balendra *et al.*[10], Tsicnias and Hutchinson[11] and Sun and Tang[18] has, however, made some progress in investigating the qualitative effects of torsional coupling in interacting systems under steady-state and earthquake loadings. In a previous parametric study of single-storey torsionally coupled building models supported on an elastic foundation[11], Tsicnias and Hutchinson concluded that, for a particular range of values of the controlling parameters of lateral-torsional coupling (identified in earlier studies of rigidly based models[1,3]), their qualitative effect on the coupling of lateral and torsional responses is not affected by increases in the flexibility of the foundation medium. This conclusion is based on the effects of

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harmonic loading and requires verification for buildings subjected to free-field earthquake motions. Some attempt has recently been made by Wu and Leyendecker[19] to investigate these effects for a particular ground motion model. Further research is required to extend these results and hence allow more generalized conclusions and design recommendations to be made. In this paper, the most important effects of foundation flexibility are studied, in relation primarily to its influence on torsional coupling in the structural response to earthquake base loadings. Both qualitative and quantitative assessments are made; in particular, the ranges of the controlling parameters (defining the structure-foundation system) for which interaction is significant are identified. The ground motion characteristics associated with individual earthquake records and their influence on the effects of soil–structure interaction are also discussed.

## DEVELOPMENT OF ANALYTICAL PROCEDURES

### *Model of building-foundation system*

The model chosen in this study to represent the structure-foundation system consists of a single-storey torsionally coupled building founding on an elastic half-space. Comparisons are made with the results of earlier work relating to rigidly based buildings[1,4]. Hence the model is parametrically described with emphasis on the identification of those additional parameters required in order to account for interaction effects. The effect of these parameters on torsional coupling is then assessed, the values chosen in the analysis being representative of a range of typical structure-foundation systems.

The controlling parameters for the response of a rigidly based torsionally coupled single-storey building model with one eccentricity component have been identified in Ref. 12. They consist of the eccentricity ratio $e_r(=e/r)$, the uncoupled natural frequency ratio $\lambda_T(=\omega_\theta/\omega_v)$ and the modal damping ratio $\xi$ (assumed equal in both vibration modes). Additionally, in the study of response to steady-state base loading at frequency $\omega$, the frequency parameter $f=\omega/\omega_1$ is introduced ($\omega_1$ is the fundamental coupled natural frequency), this term being replaced by the uncoupled translational natural period $T_v$ of the system when consideration is given to the response to earthquake excitation[12]. This study evaluates the effect of the parameters $e_r$, $\lambda_T$, $\xi$ and $T_v$ on the time-history response to earthquake ground motion of the same building model supported on a flexible foundation. The soil medium is assumed to consist of an elastic, homogeneous, isotropic half-space defined by its Poisson's ratio $v$, mass density $p$ and the speed $V_s$ with which shear waves propagate through the medium. The building model is cylindrical in shape with height $h$ and radius $r$ (Fig. 1). The mass of the building is distributed at two levels, namely ground (base) level and floor level. The floor diaphragm and the foundation mat are idealized rigid circular discs of negligible thickness and masses $m$ and $m_0$ respectively. Further assumptions relating to the structural stability, and the nature and distribution of the vertical structural elements which support the floor disc, are identical to those discussed in Ref. 11. Similarly, the eccentricity $e$ between the centres of mass and resistance (the latter coinciding with the centre of the floor disc) arises due to different mass densities $p_a$ and $p_b$ ($p_a > p_b$) in the two halves of the floor disc (Fig. 1). The range of values of $e$ is therefore[12] $0 \leq e \leq 4r/3\pi$ ($0.4244r$). The horizontal axes of reference, $X$ and $Y$, have directions coincident with the principal axes of resistance and the vertical axis of reference ($Z$) passes through the centre of resistance.

The dynamic response of the building model to free-field ground displacement, $v_g$, in the $X$-direction is completely defined by the following geometric co-ordinates (i.e., degrees of freedom):

(1) translation, $v$, of the floor mass in the $X$-direction due to deformation of the structural elements;
(2) rotation, $\theta$, of the floor mass about the $Z$-axis due to deformation of the structural elements;
(3) translation, $v_0$, of the foundation mass in the $X$-direction (i.e., sliding) due to the deformability of the half-space;
(4) rotation, $\theta_0$, of the foundation mass about the $Z$-axis (i.e., twisting) due to the deformability of the half-space; and
(5) rotation, $\phi$, of the whole building about the $Y$-axis (i.e., rocking) due to the deformability of the half-space.

The building-foundation system is analysed as two interacting subsystems[12] (building and soil); the response of the building is calculated by replacing the soil subsystem with the elastic interaction forces, i.e., horizontal shear $V$, twisting moment $T$ and rocking moment $M$ developed at the building-soil interface due to the interaction displacements, $v_0$, $\theta_0$ and $\phi$. If $V=\tilde{V}\exp(i\omega t)$, $M=\tilde{M}\exp(i\omega t)$ and $T=\tilde{T}\exp(i\omega t)$ are respectively the harmonic horizontal force, rocking moment and torque acting on a massless, rigid circular disc supported on an elastic half-space, the steady-state response of the disc at frequency $\omega$ is given by the harmonic displacements $v_0=\tilde{v}_0\exp(i\omega t)$, $\phi=\tilde{\phi}\exp(i\omega t)$, and $\theta_0=\tilde{\theta}_0\exp(i\omega t)$. The force-displacement relationship is then expressed as

$$\begin{Bmatrix} \tilde{V} \\ \tilde{M} \\ \tilde{T} \end{Bmatrix} = \begin{bmatrix} K_{VV}(\omega) & K_{VM}(\omega) & 0 \\ K_{MV}(\omega) & K_{MM}(\omega) & 0 \\ 0 & 0 & K_{TT}(\omega) \end{bmatrix} \begin{Bmatrix} \tilde{v}_0 \\ \tilde{\phi} \\ \tilde{\theta}_0 \end{Bmatrix} \quad (1)$$

The numerical approximations of the impedence functions $K_{VV}(\omega)$, $K_{MM}(\omega)$, $K_{TT}(\omega)$ and $K_{VM}(\omega)$ ($=K_{MV}(\omega)$ by reciprocity) used in this study are taken from Veletsos and Wei[20] (for lateral and rocking vibration) and Veletsos and Nair[21] (for torsional vibration). Assuming $K_{VM}(\omega)=K_{MV}(\omega)=0$ (which in practice introduces negligible errors[20]) the impedences are expressed as

$$K_{VV}(\omega)=(k_1+ia_0c_1)K_{SV} \quad (2a)$$

$$K_{MM}(\omega)=(k_2+ia_0c_2)K_{SM} \quad (2b)$$

$$K_{TT}(\omega)=(k_3+ia_0c_3)K_{ST} \quad (2c)$$

where $a_0$ is Reissner's dimensionless frequency parameter ($=\omega r/V_s$); $k_j$, $c_j$ are dimensionless dynamic stiffness and damping coefficients respectively, which depend on $a_0$ (for $j=1,2,3$) and on the Poisson's ratio $v$ (for $j=1,2$); $K_{SV}$, $K_{SM}$ and $K_{ST}$ are static stiffnesses of the half-space for

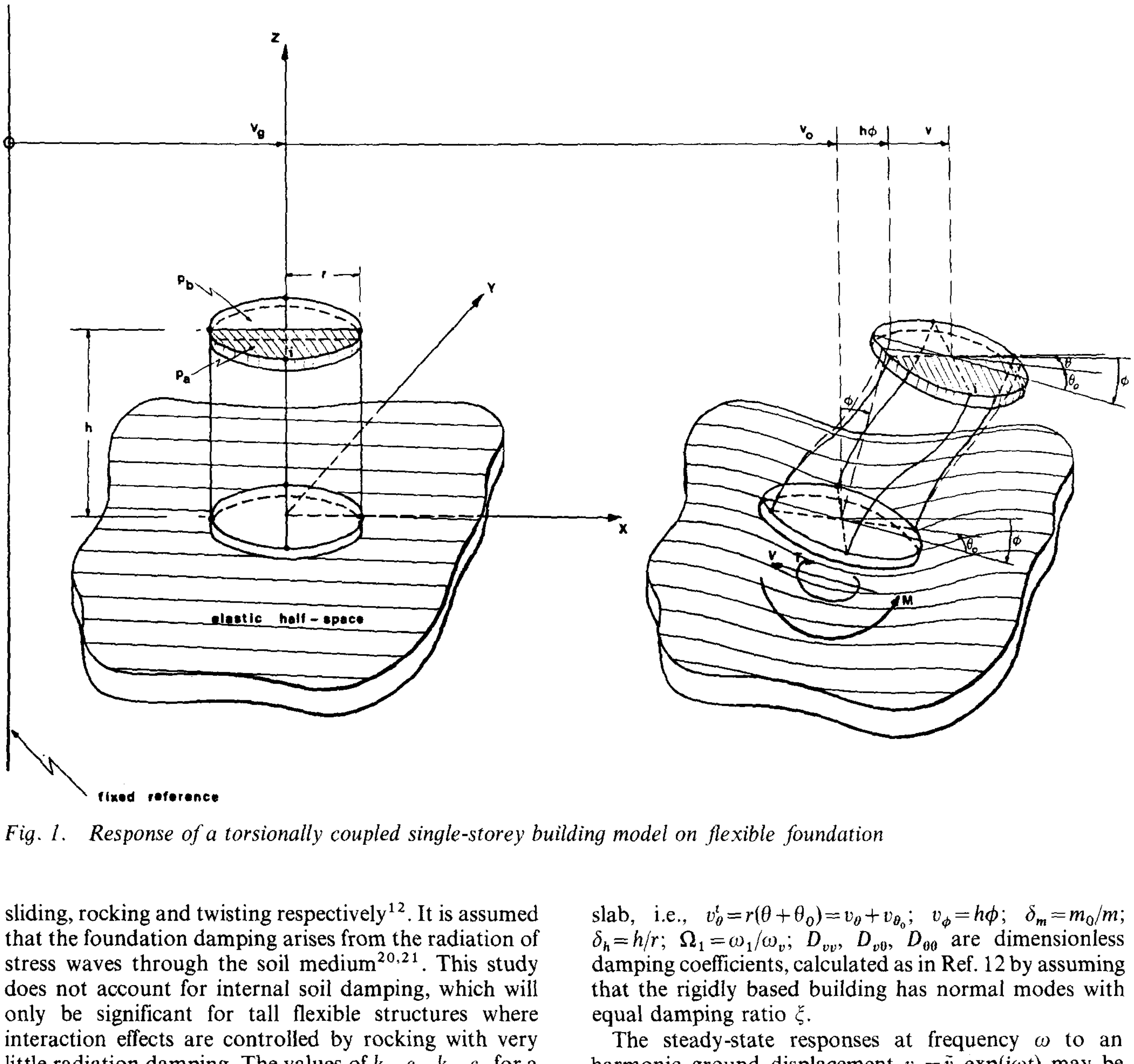


*Fig. 1. Response of a torsionally coupled single-storey building model on flexible foundation*

sliding, rocking and twisting respectively[12]. It is assumed that the foundation damping arises from the radiation of stress waves through the soil medium[20,21]. This study does not account for internal soil damping, which will only be significant for tall flexible structures where interaction effects are controlled by rocking with very little radiation damping. The values of $k_1$, $c_1$, $k_2$, $c_2$ for a half-space with $\nu = \frac{1}{3}$ (the value used in this study) have been tabulated in Ref. 11. The coefficients $k_3$, $c_3$ are calculated approximately as[21] $k_3 = 1 - A$ and $c_3 = 0.687A$, where $A = 0.425(0.687a_0)^2/[1 + (0.687a_0)^2]$.

*Response to steady-state ground motion*

The equations of motion expressing dynamic equilibrium in each degree of freedom are written in the parametric form[12]

$$\ddot{v}^t + e_r\ddot{v}_\theta^t + 2\xi\omega_1 D_{vv}\dot{v} + 2\xi\omega_1 e_r D_{v\theta}\dot{v}_\theta + (\omega_1/\Omega_1)^2 v = 0 \tag{3a}$$

$$e_r\ddot{v}^t + 0.5\ddot{v}_\theta^t + 2\xi\omega_1 e_r D_{v\theta}\dot{v} + \xi\omega_1 D_{\theta\theta}\dot{v}_\theta + 0.5(\omega_1/\Omega_1)^2\lambda_T^2 v_\theta = 0 \tag{3b}$$

$$\ddot{v}^t + e_r\ddot{v}_\theta^t + \delta_m(\ddot{v}_g + \ddot{v}_0) + V/m = 0 \tag{3c}$$

$$e_r\ddot{v}^t + 0.5\ddot{v}_\theta^t + 0.5_m\ddot{v}_{\theta_0} + T/mr = 0 \tag{3d}$$

$$\delta_h\ddot{v}^t + \delta_h e_r\ddot{v}_\theta^t + [0.25(1+\delta_m)/\delta_h]\ddot{v}_\phi + M/mr = 0 \tag{3e}$$

where $v^t$ is the total translational displacement of the floor slab in the $X$-direction, i.e., $v^t = v_g + v_0 + h\phi + v$; $v_\theta^t$ is the total torsional displacement at the periphery of the floor slab, i.e., $v_\theta^t = r(\theta + \theta_0) = v_\theta + v_{\theta_0}$; $v_\phi = h\phi$; $\delta_m = m_0/m$; $\delta_h = h/r$; $\Omega_1 = \omega_1/\omega_v$; $D_{vv}$, $D_{v\theta}$, $D_{\theta\theta}$ are dimensionless damping coefficients, calculated as in Ref. 12 by assuming that the rigidly based building has normal modes with equal damping ratio $\xi$.

The steady-state responses at frequency $\omega$ to an harmonic ground displacement $v_g = \tilde{v}_g \exp(i\omega t)$ may be expressed as $v = \tilde{v}\exp(i\omega t)$, $v_g = \tilde{v}_\theta \exp(i\omega t)$, $v_0 = \tilde{v}_0 \exp(i\omega t)$, $v_{\theta_0} = \tilde{v}_{\theta_0}\exp(i\omega t)$ and $v_\phi = \tilde{v}_\phi \exp(i\omega t)$. By substituting appropriate expressions into equation (3), and using the force-displacement relationships of the force excited disc problem (equations (1) and (2)), the following set of linear algebraic equations with complex coefficients are yielded[12]

$$[(1/f^2\Omega_1^2) - 1 + 2i\xi D_{vv}/f]\tilde{v} + e_r[2i\xi D_{v\theta}/f - 1] \times \tilde{v}_\theta - \tilde{v}_0 - e_r\tilde{v}_{\theta_0} - \tilde{v}_\phi = \tilde{v}_g \tag{4a}$$

$$e_r[2i\xi D_{v\theta}/f - 1]\tilde{v} + [0.5\lambda_T^2(1/f^2\Omega_1^2) - 0.5 + i\xi D_{\theta\theta}/f] \times \tilde{v}_\theta - e_r\tilde{v}_0 - 0.5\tilde{v}_{\theta_0} - e_r\tilde{v}_\phi = e_r\tilde{v}_g \tag{4b}$$

$$-\tilde{v} - e_r\tilde{v}_\theta + [C_1 - (1+\delta_m)]\tilde{v}_0 - e_r\tilde{v}_{\theta_0} - \tilde{v}_\phi = (1+\delta_m)\tilde{v}_g \tag{4c}$$

$$-e_r\tilde{v} - 0.5\tilde{v}_\theta - e_r\tilde{v}_0 + [C_3 - 0.5(1+\delta_m)]\tilde{v}_{\theta_0} - e_r\tilde{v}_\phi = e_r\tilde{v}_g \tag{4d}$$

$$-\tilde{v} - e_r\tilde{v}_\theta - \tilde{v}_0 - e_r\tilde{v}_{\theta_0} + [C_2 - 1 - 0.25(1+\delta_m)/\delta_h{}^2]\tilde{v}_\phi = \tilde{v}_g \tag{4e}$$

where $C_1$, $C_2$ and $C_3$ are dimensionless coefficients related to the dynamic stiffnesses of the foundation medium. Expressions for $C_1$, $C_2$ and $C_3$ are given in Ref. 12 in terms of the parameters $\delta_h$, f, $\alpha$ and $\delta_p$, where $\alpha$ is a measure of the relative stiffness of the foundation and the structure, i.e., $\alpha = V_s/(h\omega_1/2\pi)$ and $\delta_p$ is a measure of the relative density of the structure and soil, i.e., $\delta_p = (m/\pi r^2 h)/p$. Solution of equation (4) yields the complex frequency response functions $\tilde{v}$, $\tilde{v}_\theta$, $\tilde{v}_0$, $\tilde{v}_{\theta_0}$ and $\tilde{v}_\phi$, normalized with respect to the amplitude $\bar{v}_g$ of the free-field ground displacement. Ref. 12 contains a discussion of the physical significance of the set of dimensionless parameters which affect the response of the structure-foundation system, as identified in this study.

In addition to the responses obtained from equation (4), a further response $v_i = v + v_\theta$ is of interest when assessing the combined effects of torsional coupling on the $X$-translation of a point $i$ located on the periphery of the floor disc at a distance $r$ from the $X$-axis (Fig. 1), due to the structural deformation. This response relates directly to the primary building code allowances for torsional coupling[2,22], which make provision for the amplification of dynamic torque as well as specifiying an appropriate value for the dynamic shear. The comparison of the dynamic edge displacement $v_i$ with the corresponding design values of major building codes has been previously studied[22], based on analysis of the response of the equivalent rigidly based torsionally coupled building model.

## STEADY-STATE EFFECTS OF TORSIONAL COUPLING

The frequency response characteristics of the building-foundation system have been studied in detail in Ref. 12. The effects of foundation flexibility on lateral-torsional coupling under steady-state vibration are clearly demonstrated by plotting the maximum values of the response amplitudes against $\lambda_T$ (Figs 2–4). The results shown in these figures have been obtained assuming the following set of values for the dimensionless parameters: $\xi = 0.05, \delta_h = 1.0, \delta_m = 0.0, \delta_p = 0.15, v = \frac{1}{3}$. Realistically the range of $V_s$ is from 100 to 500 m/sec[12] and the range of building heights $h$ from 5 to 70 m (the latter values corresponding approximately to single and twenty storey buildings respectively). Assuming that the fundamental coupled frequency $f_1$ $(= \omega_1/2\pi)$ is inversely proportional to $h$, such that $hf_1$ is constant at 35 m/sec, the structure-foundation stiffness ratio $\alpha$ $(= V_s/hf_1)$ takes values in the range 3 to 14. Throughout this study results have been presented for $\alpha = 3$, 6, 10 and where appropriate infinity, which corresponds to a rigid foundation[22]. Although there is some difference between curves for $\alpha = 10$ and $\alpha = \infty$, these are not large; consequently curves for values of $\alpha$ intermediate between 10 and infinity are not presented.

Fig. 2 shows that for all nonzero values of eccentricity, the maximum amplitude of the translational displacement, $v$, of the floor is always less that the value obtained in the case of a building with no eccentricity (that is, torsionally uncoupled). As the foundation stiffness increases relative to the structure (i.e., $\alpha$ increases), and for values of $\lambda_T$ close to unity, there is a sharp reduction of the maximum amplitude of $v$. The reduction is accentuated for structures with large eccentricity. As the value of $\alpha$ decreases, however, the sharp reduction of max $\bar{v}$ becomes less distinguishable and the response amplitudes tend to fall continuously for $\lambda_T < 1$.

The effect of $e_r$ and $\lambda_T$ on the maximum steady-state amplitudes of the torsional and combined displacements, $v_\theta$ and $v_i$, is qualitatively unaltered by increases in the foundation flexibility (Figs 3 and 4). Thus, for small values of eccentricity and $\lambda_T$ close to unity, there is a significant increase in the value of max $\bar{v}_\theta$. Torsional coupling also increases the combined response $v_i$ relative to uncoupled values when $\lambda_T > 1.0$, especially for large eccentricities. The value of max $\bar{v}_i$ decreases significantly for $\lambda_T < 1.0$, due to the cancellation arising from the combined phase effects of the individual translational and torsional floor responses. Since all real buildings have frequency ratios $\lambda_T > 0.5$[23], the values of the responses have been omitted for $\lambda_T < 0.5$ in Figs 2–4.

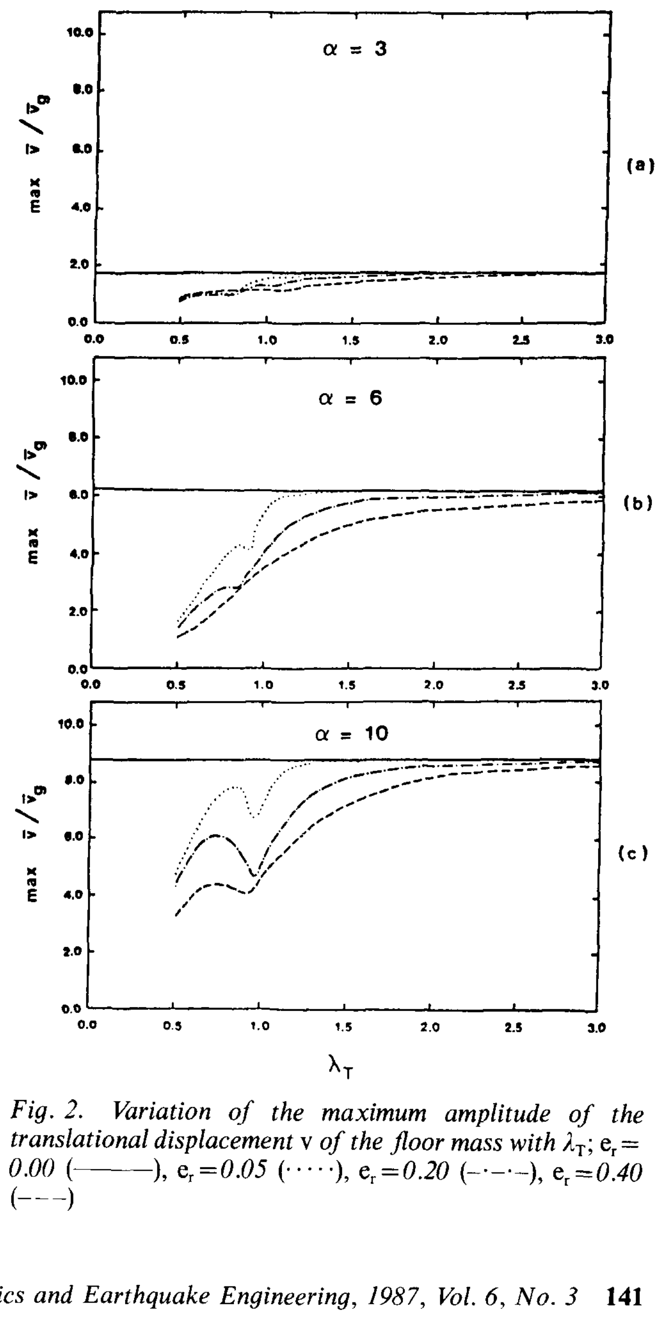


*Fig. 2. Variation of the maximum amplitude of the translational displacement* v *of the floor mass with* $\lambda_T$; $e_r = 0.00$ (———), $e_r = 0.05$ (····), $e_r = 0.20$ (–·–·–), $e_r = 0.40$ (– – –)

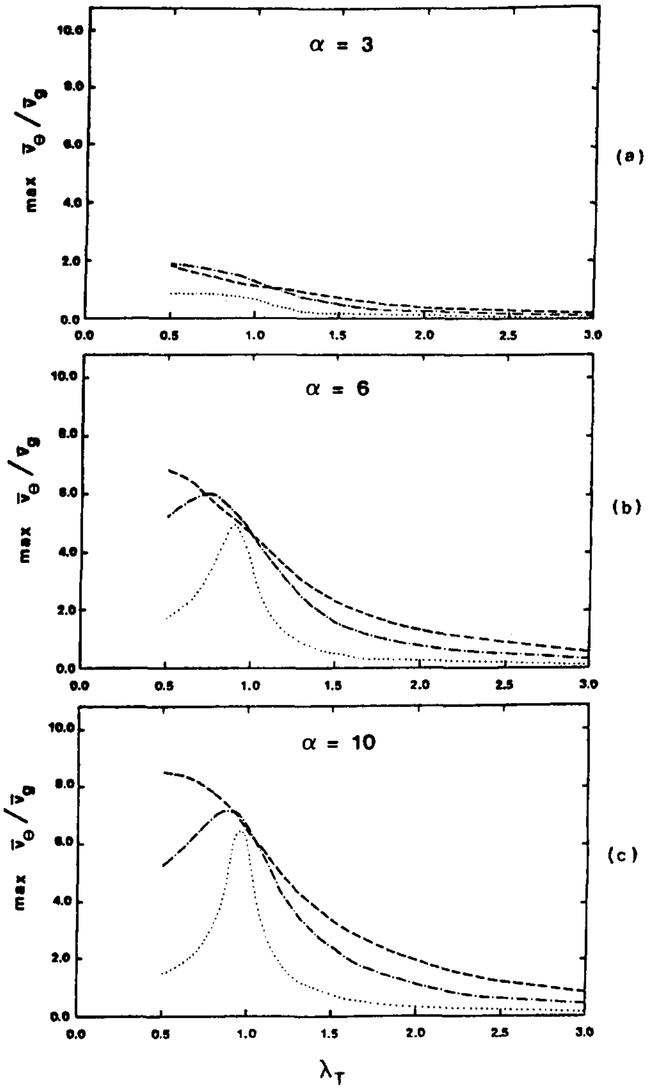


*Fig. 3. Variation of the maximum amplitude of the torsional displacement* $v_\theta$ *of the floor mass with* $\lambda_T$; $e_r=0.05$(·····), $e_r=0.20$ (–·–·–), $e_r=0.40$ (– – –)

## TIME HISTORY EARTHQUAKE RESPONSE

It has been demonstrated in an earlier study[4] that the effects of the parameters $e_r$ and $\lambda_T$ on the steady-state response of torsionally coupled, rigidly based buildings are similar to those produced by earthquake ground excitations. In this study a further comparison is made, based on the time-history response of equivalent flexibly based systems to a series of free-field strong motion earthquake records with peak ground accelerations between 0.20 and 0.61 g. The motions have been classified broadly according to their spectral shape[12,24]; the two main categories (designated Sets A and B) include, respectively, earthquakes with 'peaking' spectra exhibiting dominant frequencies over a well-defined range, and 'broad-band' spectra with approximately uniform amplitudes over the relevant frequency range. Examples of these spectral types are given in Figs 5(a), 5(b) for the Leukas, Greece (1973) and Patras, Greece (1974) earthquakes respectively. Other Set A earthquakes considered include the Romanian (1977) and Thessaloniki, Greece (1978) records, whilst the Set B earthquakes additionally include 3 separate recordings from the Ancona-Rocca (Italy) earthquake of 1972. Finally, the El Centro, California (1940) record has been included as a comparison with the selected European records (Fig. 5(c)). In all cases, the component with greater peak ground acceleration has been employed in the analysis of response to uni-directional ground motion.

The transient response in each degree of freedom to specified ground motion has been evaluated through the frequency domain using the fast Fourier transform technique[25]. The basis of these calculations is the solution of equation (4) in discrete form for the response functions $\tilde{v}, \tilde{v}_\theta, \tilde{v}_i\,(=\tilde{v}+\tilde{v}_\theta), \tilde{v}_0, \tilde{v}_{\theta_0}$ and $\tilde{v}_\phi$. Records of time-history response are then computed for each of the 8 selected ground acceleration records; in each case it is the maximum, or peak response which is of interest for design purposes. Since the earthquake ground acceleration records are specified in a dimensioned form, the natural frequencies $\omega_1, \omega_2$ of the rigidly based building, on which the calculation of the complex frequency response functions of the flexibly based model depend[12], are evaluated by specifying the translational natural period

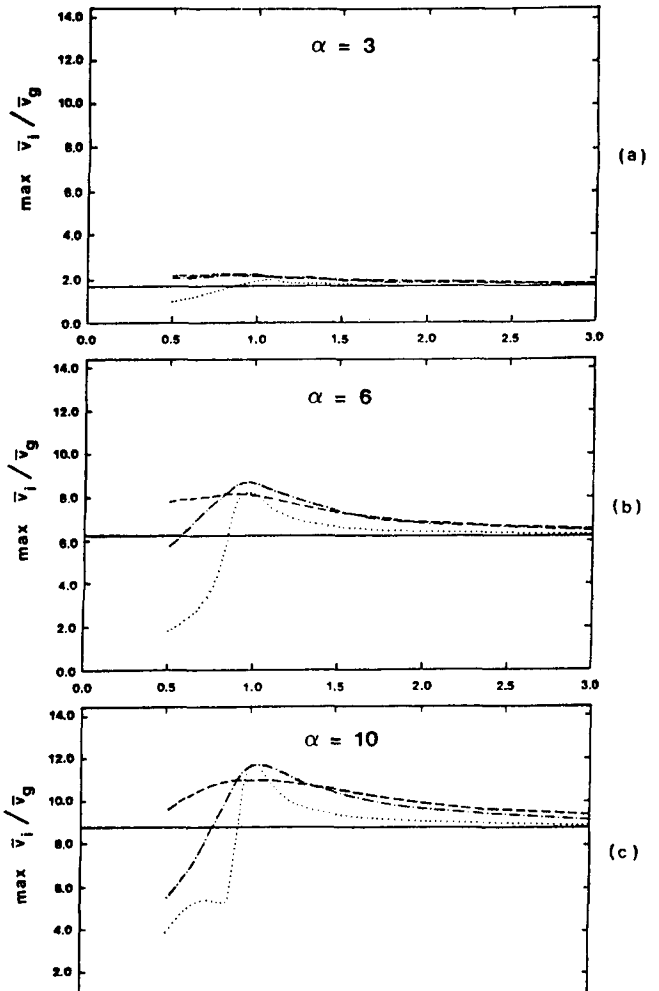


*Fig. 4. Variation of the maximum amplitude of the combined displacement* $v_i$ *of the floor mass with* $\lambda_T$; $e_r=0.00$ (——), $e_r=0.05$ (·····), $e_r=0.20$ (–·–·–), $e_r=0.40$ (– – – –)

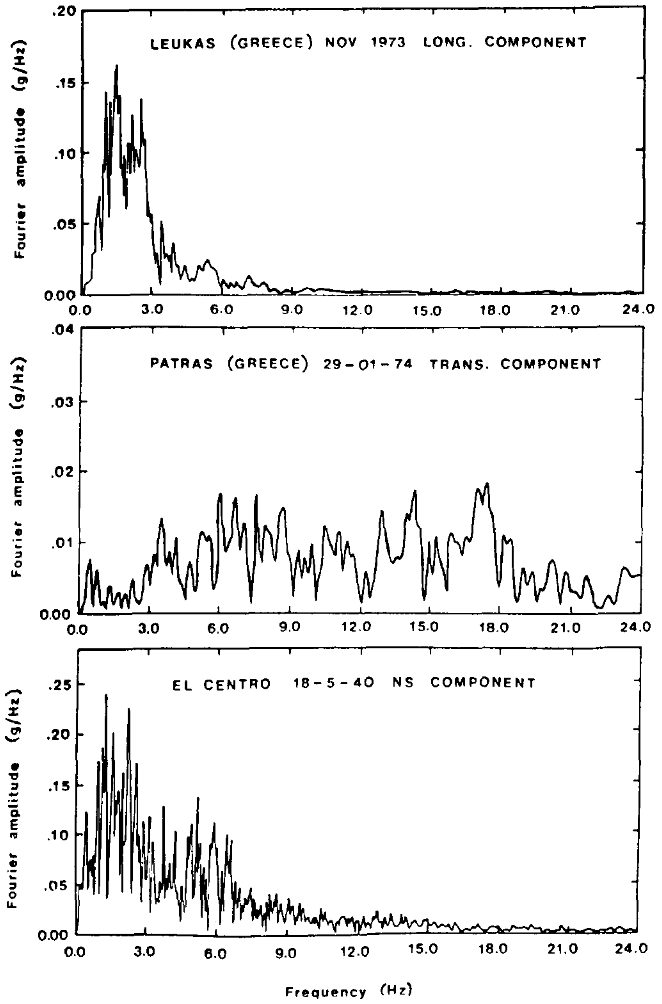


*Fig. 5. Fourier amplitude spectra for the Leukas Long. (a), Patras Trans. (b) and NS El Centro (c) earthquake records*

$T_v$ of the torsionally uncoupled, rigidly based building (SDOF).

## SOIL–STRUCTURE INTERACTION EFFECTS ON TORSIONAL COUPLING

The results presented in Figs 6 to 12 correspond to the following fixed parametric values in cases where their influence on the steady-state response trends has been observed to be negligible[11]: $\xi=0.05$, $\delta_p=0.15$, $\delta_m=0.0$, $v=\frac{1}{3}$. Hence the key parameters, whose influence on the nature and magnitude of torsional coupling effects is to be investigated, are $e_r$, $\lambda_T$, $\alpha$, $\delta_h$ and $T_v$. Since the natural period of many structures, especially those of simple reinforced concrete or steel framed construction, is related approximately to the building height $h$[26], it is possible to derive an empirical relationship between the height ratio $\delta_h$ and the period $T_v$. If it is assumed that $T_v \simeq 0.1N$ seconds, where $N$ is the number of storeys, and furthermore if the inter-storey height $h_s$ is assumed to be a fixed proportion of the building plan dimension $2r$ such that typically $h_s=r/2$, then $\delta_h \simeq 5T_v$. It should be noted that when consideration is given to multi-storey structures ($N>1$), the total mass $m$ of the building is assumed to be concentrated at a height $h$ above foundation level, thus enabling the use of the simplified building model illustrated in Fig. 1.

Fig. 6 shows the variation of the normalized maximum floor displacement $v$ of the uncoupled flexibly based building model, as a function of the natural period $T_v$ plotted on a logarithmic scale in the range $0.1 < T_v < 2.0$ s, corresponding approximately to rigidly based buildings from 1 to 20 storeys high. Graphs (a) to (c) correspond, respectively, to the response to the El Centro, European Set A and European Set B earthquake records as defined earlier, where the maximum ground displacement has been used as the normalizing quantity. The uncoupled, flexibly based building model has responses constrained to three degrees of freedom, namely the translational displacements of the floor and base masses, $v$ and $v_0$, together with the rocking displacement $v_\phi$ of the whole building. In Fig. 6, the effect of foundation flexibility, defined by the structure-foundation stiffness ratio $\alpha$ in the range $3<\alpha<\infty$, on the magnitude of the structural displacement $v$ is studied in order to assess the assumption of the major building codes[27] that conservative estimates of response, for design purposes, are obtained by neglecting interaction effects. Whilst this

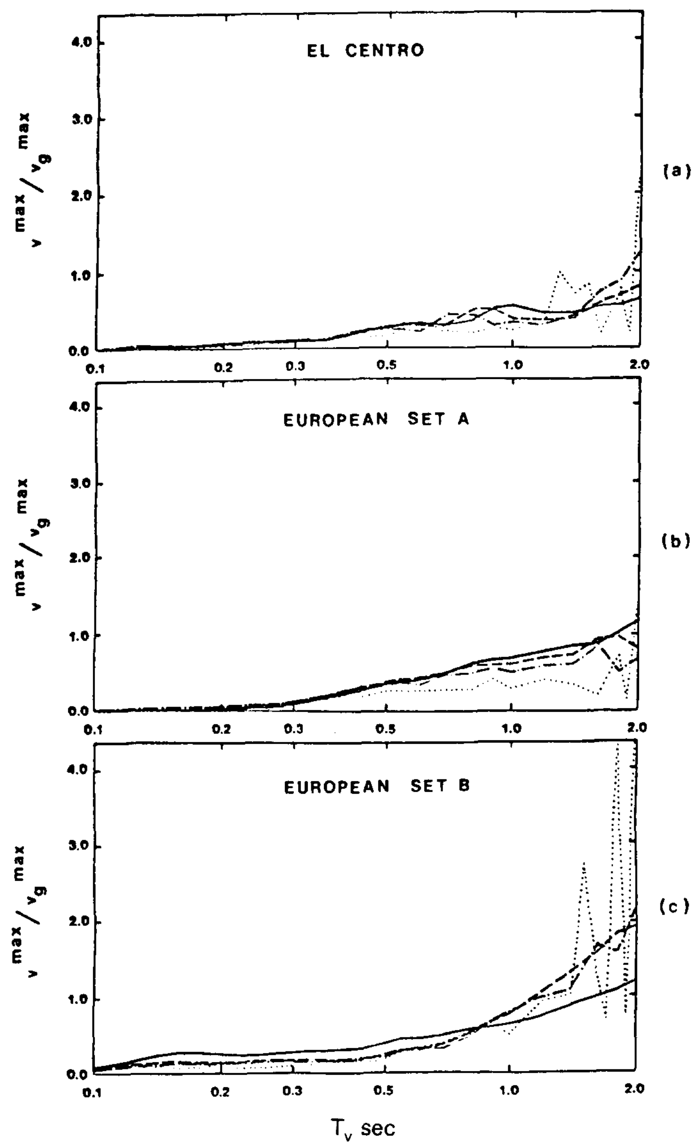


*Fig. 6. Normalized and averaged displacement response spectra for the translational floor response v of the torsionally uncoupled building ($e_r=0$); $\alpha=3.0$ (·····), $\alpha=6.0$ (–·–·–), $\alpha=10.0$ (– – – –), $\alpha=\infty$ (——); $\xi=0.05$*

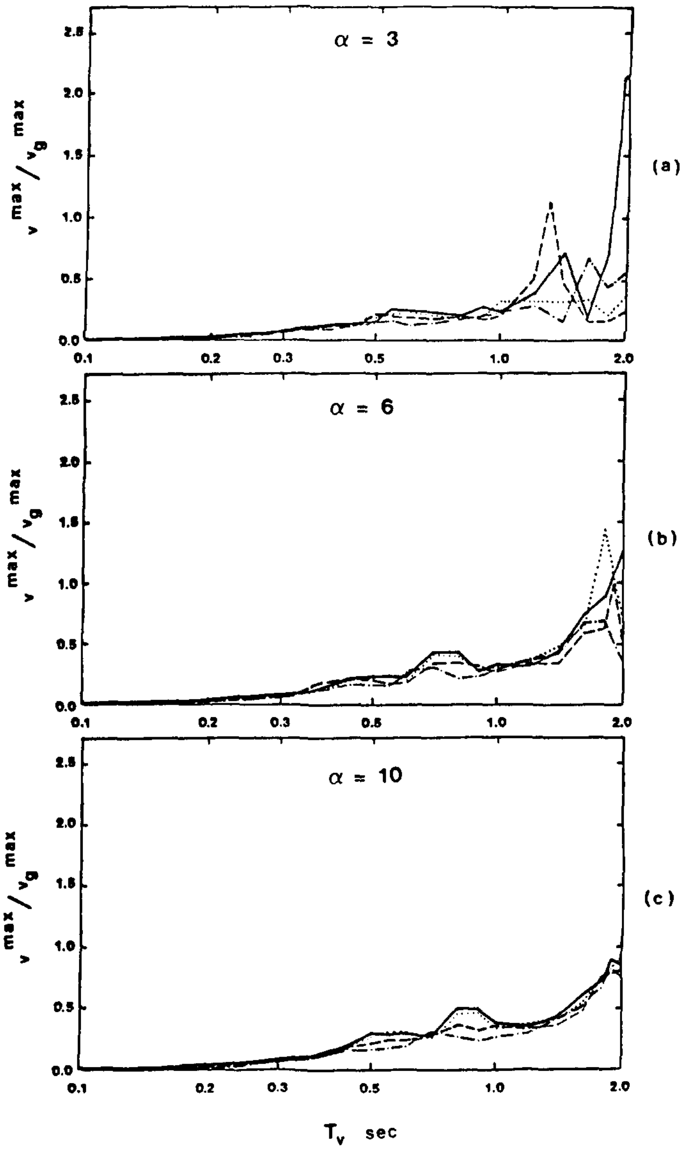


*Fig. 7. Normalized displacement response spectra for the translational floor response* v *of the torsionally coupled building subjected to the El Centro earthquake record;* $e_r = 0.00$ (——), $e_r = 0.05$ (·····), $e_r = 0.15$ (– – –), $e_r = 0.30$ (–·–·–)

appears to be a reasonable assumption for buildings with vibration periods less than 0.5 s, the results presented in Fig. 6 indicate that the response of structures with longer vibration periods can be significantly increased by soil-structure interaction effects. This is especially important for the responses to the El Centro and European Set B earthquake records, where the response of structures founded on relatively flexible foundations ($\alpha \leq 6.0$) is shown to be increased by factors exceeding 2 for periods $T_v$ approaching 2.0 s. The increase in structural response in this range confirms the results of a corresponding study[12] for tall structures under steady-state base loadings, in which similar amplification was observed in conjunction with large-amplitude rocking motions $v_\phi$.

Hence, it is concluded that some care is necessary in predicting the effects of soil-structure interaction, which may increase or decrease the structural response depending on the value of $T_v$ and hence the height $h$ of the building. For taller structures the amplification of response may be especially significant, and allowance for this increase should be made in obtaining the corresponding values of the design storey and base shear forces, in some cases involving a dynamic analysis using modal analysis or time history methods in order to evaluate the effect.

Figs 7 to 9 show the variation of the normalized maximum displacement responses $v$, $v_\theta$ and $v_i$ of the building-foundation system with nonzero eccentricity, as a function of $T_v$. In all cases the responses shown are those produced by the El Centro earthquake record, and correspond to $\lambda_T = 1.0$ which is the value at which the rigidly based model exhibits greatest coupling of the lateral and torsional responses[1-4]. The results presented in Fig. 7 confirm that the greatest reduction in the translational response $v$ occurs for systems with large eccentricity. With some minor exceptions this effect is unaltered by increases in the foundation flexibility. A corresponding similarity is observed in the variation of the torsional and combined responses for various values of $\alpha$ (Figs 8 and 9). The increase in floor torsional response

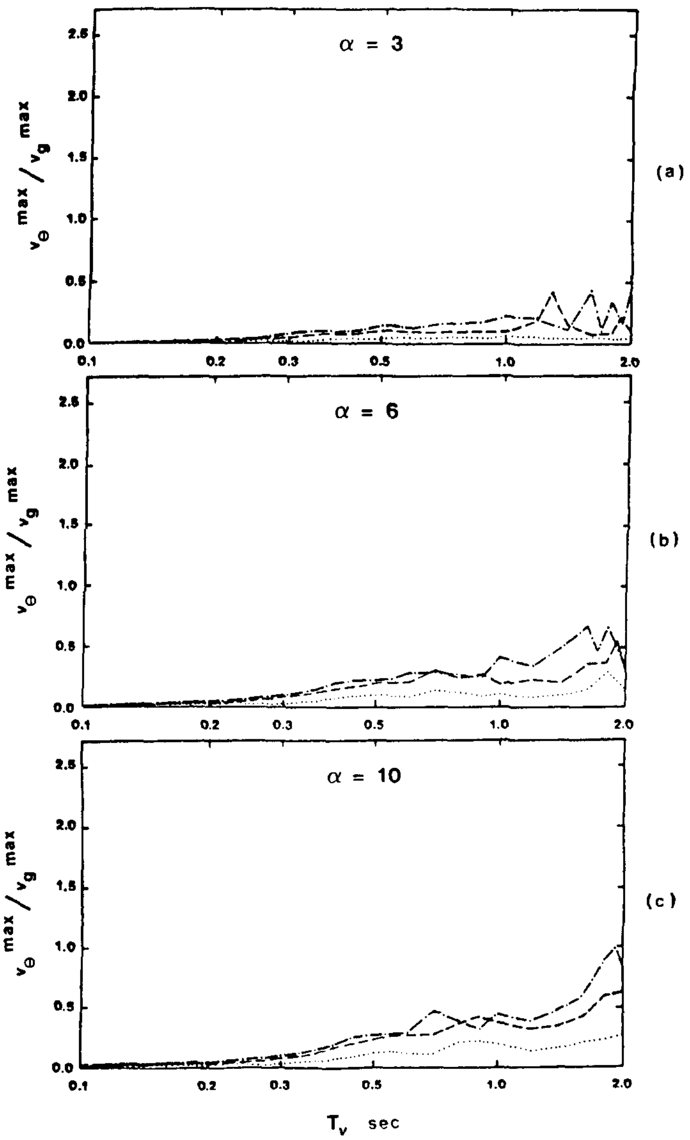


*Fig. 8. Normalized displacement response spectra for the torsional floor response* $v_\theta$ *of the torsionally coupled building subjected to the El Centro earthquake record;* $e_r = 0.05$ (·····), $e_r = 0.15$ (– – –), $e_r = 0.30$ (–·–·–)

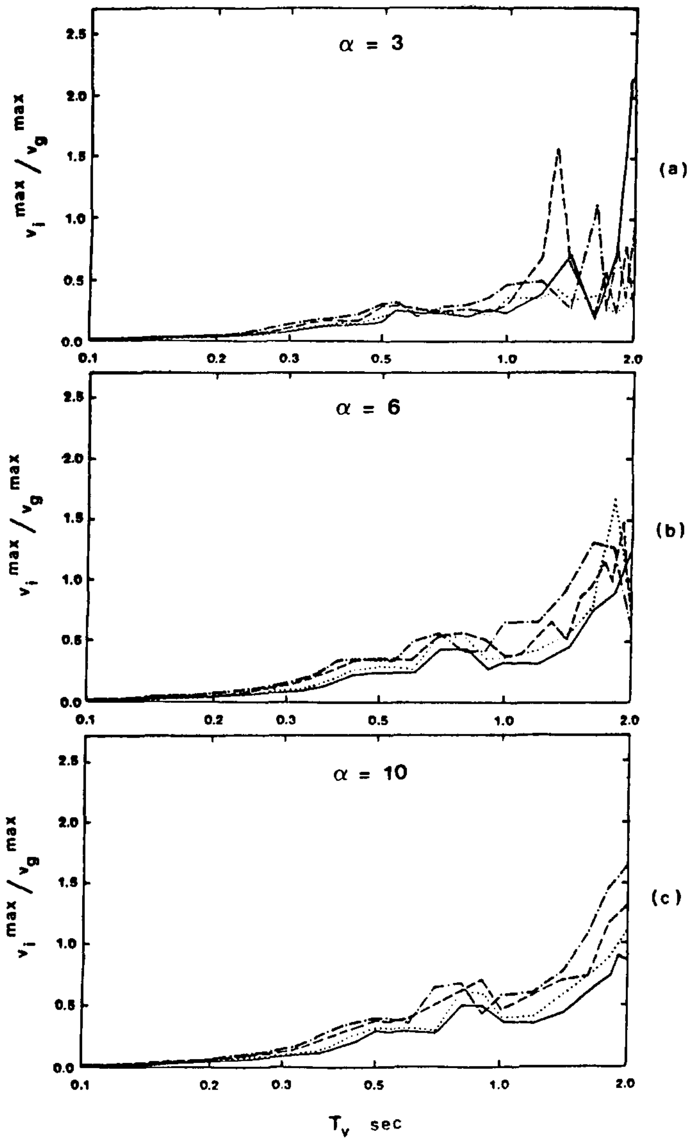


*Fig. 9. Normalized displacement response spectra for the combined floor response* $v_i$ *of the torsionally coupled building subjected to the El Centro earthquake record;* $e_r = 0.00$ (——), $e_r = 0.05$ (·····), $e_r = 0.15$ (– – – –), $e_r = 0.30$ (– · – · –)

for systems with larger eccentricity $e_r$ is qualitatively consistent for systems with foundation flexibility in the range shown. Similarly, Fig. 9 shows that the amplification of edge displacement response $v_i$, compared with uncoupled values, increases with the magnitude of the eccentricity and that this trend is unaltered by soil-structure interaction.

Figs 10 to 12 show the normalized relative displacement responses $v$, $v_\theta$ and $v_i$ of the building in response to the selected earthquake records; in each case the parameters $e_r$ and $\lambda_T$ have values 0.15 and 1.0 respectively. The effect of soil-structure interaction for tall slender structures with high natural vibrarion periods, $T_v$, is to produce some significant increase of the translational displacement response $v$ when subjected to the El Centro and European Set B earthquake records (Fig. 10); in the latter case the response is increased for all buildings with $T_v > 1.0$ sec founded on moderately stiff soils, $6 \leqq \alpha \leqq 10$. In contrast, the European Set A records induce a reduced translational displacement response over the full range of $T_v$ shown (Fig. 10(b)), the reduction being greater as the foundation flexibility increases. In Fig. 11, only the European Set B records for $T_v > 1.0$ sec and $\alpha = 10.0$ induce any increase of torsional displacement response $v_\theta$ as a result of soil-structure interaction. In all other cases torsional coupling is significantly reduced by interaction effects such that negligible torsional displacement response is observed in buildings founded on very flexible foundations, $\alpha = 3.0$, corresponding to shear wave velocities of around 100 m/s [12]. The combined effects of translational and torsional building response on the displacement of a typical edge element $i$ are shown in Fig. 12. Especially significant is the amplification of response induced by the European Set B records for $T_v > 0.8$ sec (corresponding approximately to buildings with greater that 8 storeys) as a result of interaction, with an increase in this range relative to rigid-base values of 50 per cent on average for $\alpha = 10.0$, corresponding approximately to $V_s = 350$ m/s [12]. It should be noted that the set of European Set B recordings (specified earlier) were obtained from moderately stiff soil sites[28] consisting of stiff clays, sand or gravel with $V_s > 300$ m/s for a depth $< 50$ m.

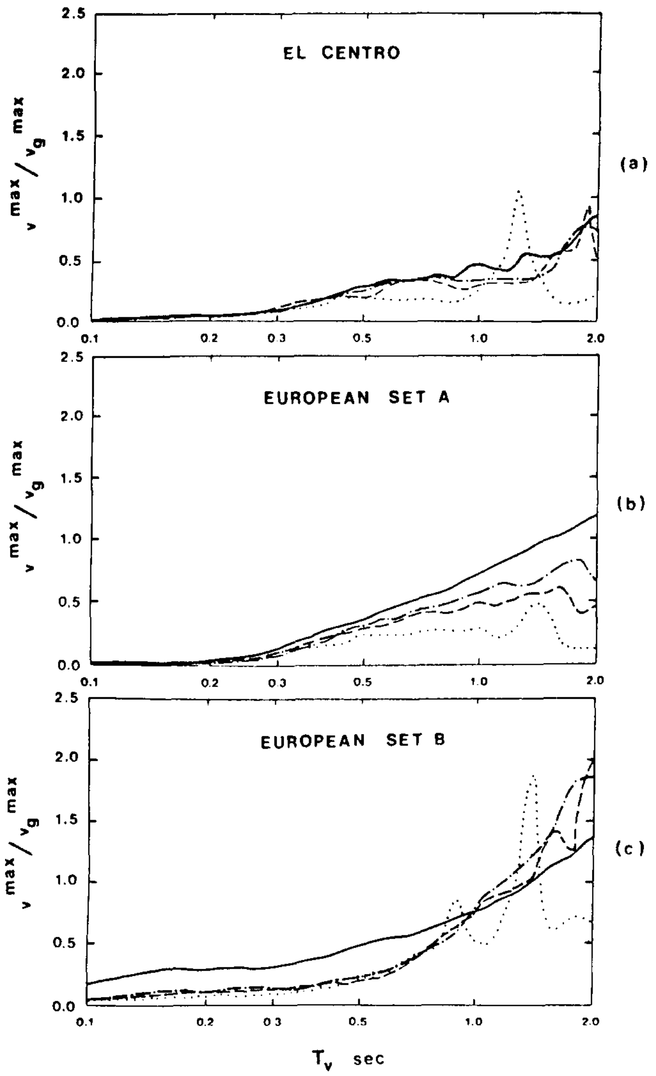


*Fig. 10. Normalized displacement response spectra for the translational floor response* v *of the torsionally coupled building;* $\alpha = 3.0$ (····), $\alpha = 6.0$ (– – – –), $\alpha = 10.0$ (– · – · –), $\alpha = \infty$ (——)

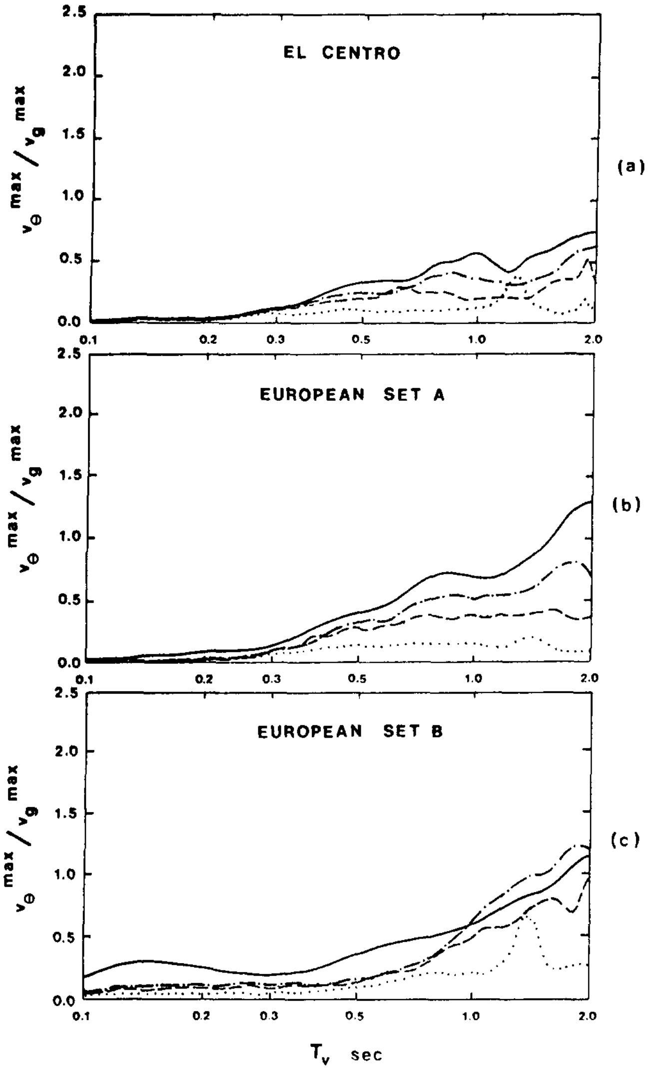


*Fig. 11. Normalized displacement response spectra for the torsional floor response* $v_\theta$ *of the torsionally coupled building;* $\alpha = 3.0$ (·····), $\alpha = 6.0$ (– – –), $\alpha = 10.0$ (–·–·–), $\alpha = \infty$ (——)

The shear wave velocities at the sites concerned therefore correlate well with those deduced from dynamic analysis for peak interaction effects, resulting directly from the increases of both translational and torsional displacement responses in this range as discussed earlier. Some increase of response is also observed for the El Centro record (Fig. 12(a)), with increases of around 5–10 per cent for $\alpha$ in the range $6 \leq \alpha \leq 10$ over virtually the whole range of $T_v$ shown. Soil-structure interaction in all cases reduces the combined lateral and torsional displacement response of buildings to the European Set A records (Fig. 12(b)), and hence it is conservative to base their design on rigid-base calculations as recommended in most major building codes[27]. It should be noted that most codes[27] specify reduced design values for base shear when soil-structure interaction effects are considered.

It is concluded that design recommendations for torsionally coupled buildings should include reference to soil-structure interaction effects, and in particular should identify those cases where the combination of local soil conditions and the frequency content of appropriate localized earthquake records (which may be employed in time-history analyses for computing the design forces in the structure[29]) induces an amplification of coupled lateral and torsional displacement responses as identified in this study. Further research is needed to assess the effect in more detail and to provide appropriate recommendations for incorporation in building codes.

## CONCLUSIONS

The study presented in this paper has enabled some evaluation of the changes in torsional coupling effects brought about by soil-structure interaction in the earthquake response of asymmetric buildings founded on flexible foundations. The following conclusions have been drawn:

(1) The effect of soil-structure interaction on the coupled lateral and torsional displacement responses of structures subjected to historical earthquake records has been shown to be qualitatively similar to that identified in a study of responses to steady-state base loadings.

(2) Torsional coupling effects exhibited in the attenuation or amplification of the individual

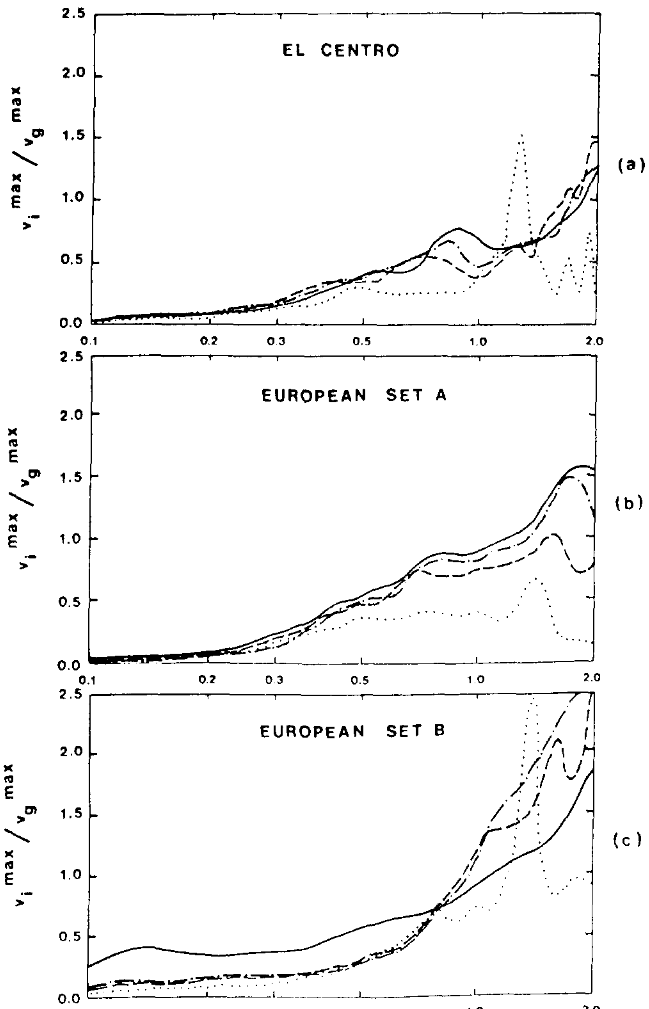


*Fig. 12. Normalized displacement response spectra for the combined floor response* $v_i$ *of the torsionally coupled building;* $\alpha = 3.0$ (·····), $\alpha = 6.0$ (– – –), $\alpha = 10.0$ (–·–·–), $\alpha = \infty$ (——)

translational and torsional displacement response components of the structure are not qualitatively affected by changes in the flexibility of the foundation medium.

(3) The most influential parameters which control the nature and magnitude of the dynamic earthquake responses of the structure-foundation system are the eccentricity and uncoupled frequency ratios, $e_r$ and $\lambda_T$, and the uncoupled lateral period, $T_v$, all of which were previously employed in the analysis of rigidily based torsionally coupled buildings, together with the structure-foundation stiffness ratio $\alpha$ and the building height ratio $\delta_h$, the latter particularly influencing the magnitude of the rocking motions of the stucture.

(4) The magnitude of changes in the combined edge displacement of the structure induced by interaction effects is influenced strongly by the frequency content of the earthquake record employed in the analysis. It is concluded from the present study that the European Set A records produce on average a significant attenuation of coupled lateral-torsional displacement response when employed in the analysis of flexibly based buildings, in comparison with the corresponding response to rigid base loadings. In contrast, the El Centro record and in particular the European Set B records induce significant amplification of the coupled displacement response when analysing flexibly based structural systems with moderately stiff foundations.

(5) It is clear from comments made in (4) above that the varying frequency content of earthquake records employed in dynamic analysis has a significant effect on the influence of soil-structure interaction in the response of flexibly based buildings. Further research is needed in order to analyse the effect in more detail. In particular, adequate allowance should be made in the torsional provisions of earthquake design codes for those cases where foundation interaction may accentuate coupling of the lateral and torsional displacement responses.

## NOMENCLATURE

| | |
|---|---|
| $a_0$ | Reissner's dimensionless frequency parameter |
| $C_1, C_2, C_3$ | dimensionless damping coefficients for the half-space |
| $D_{vv}, D_{v\theta}, D_{\theta\theta}$ | dimensionless damping coefficients for the structure |
| $e$ | static eccentricity between the centres of mass and resistance |
| $e_r$ | eccentricity ratio $(e/r)$ |
| $f$ | normalized harmonic excitation frequency $(\omega/\omega_1)$ |
| $G$ | shear modulus of elasticity of the foundation medium |
| $h$ | height of the building |
| $h_s$ | inter-storey height $(h=Nh_s)$ |
| $i$ | vertical element at the periphery of the structure |
| $J_0$ | mass polar moment of inertia of the floor mass about the $Z$-axis |
| $k_1, k_2, k_3$ | dimensionless stiffness coefficients for the half-space |
| $K_v$ | translational stiffness of the rigidly based building |
| $K_\theta$ | torsional stiffness of the rigidly based building |
| $K_{SV}, K_{SM}, K_{ST}$ | static stiffnesses of the half-space for sliding, rocking and twisting of the disc, respectively |
| $K_{VV}(\omega), K_{MM}(\omega), K_{TT}(\omega)$ | frequency-dependent dynamic stiffnesses of the half-space for sliding, rocking and twisting of the disc, respectively |
| $K_{VM}(\omega), K_{MV}(\omega)$ | frequency-dependent dynamic stiffnesses of the half-space expressing the coupling between the sliding and rocking motions of the disc |
| $m, m_0$ | mass of the floor slab and foundation (base) slab of the building, respectively |
| max | maximum value of a response quantity |
| $M, \tilde{M}$ | time history and complex frequency response of the interaction overturning moment |
| $N$ | number of building storeys |
| $p$ | mass density of the half-space |
| $p_a, p_b$ | mass density of the $a$ and $b$ parts of the floor disc $(p_a > p_b)$ |
| $r$ | radius of the floor and base discs |
| $t$ | time |
| $T, \tilde{T}$ | time history and complex frequency response of the interaction torque |
| $T_v$ | translational natural period of the uncoupled rigidly based building $(=2\pi/\omega_v)$ |
| $v, \tilde{v}, \bar{v}$ | time history, complex frequency response and amplitude of the translational displacement of the floor disc, arising from structural deformation |
| $v_g, \bar{v}_g$ | time history and amplitude of the ground displacement |
| $v_i, \tilde{v}_i, \bar{v}_i$ | time history, complex frequency response and amplitude of the combined displacement at a point $i$ on the periphery of the floor disc, arising from structural deformation $(v_i = v + v_\theta)$ |
| $v_0, \tilde{v}_0$ | time history and complex frequency response of the translational displacement of the base disc relative to the free-field ground displacement |
| $v_\theta, \tilde{v}_\theta, \bar{v}_\theta$ | time history, complex frequency response and amplitude of the torsional displacement at the periphery of the floor disc, arising from structural deformation $(v_\theta = r\theta)$ |
| $v_{\theta_0}, \tilde{v}_{\theta_0}$ | time history and complex frequency response of the torsional displacement at the periphery of the base disc $(v_{\theta_0} = r\theta_0)$ |

| | |
|---|---|
| $v_\phi$, $\tilde{v}_\phi$ | time history and complex frequency response of the rocking displacement of the floor disc ($v_\phi = h\phi$) |
| $v^t$ | time history of the total translational displacement of the floor disc in the $X$-direction ($= v_g + v_0 + v_\phi + v$) |
| $v_\theta^t$ | time history of the total torsional displacement at the periphery of the floor disc ($= v_\theta + v_{\theta_0}$) |
| $V$, $\tilde{V}$ | time history and complex frequency response of the interaction base shear |
| $V_s$ | velocity of propagation of shear wave in the half-space ($=\sqrt{G/p}$) |
| $X, Y, Z$ | Cartesian axes of reference ($X, Y$ horizontal) |
| $\alpha$ | structure-foundation stiffness ratio |
| $\delta_h$ | height ratio ($h/r$) |
| $\delta_m$ | mass ratio ($m_0/m$) |
| $\delta_p$ | structure-foundation density ratio |
| $\xi$ | viscous damping ratio, expressed as a fraction of critical damping |
| $\lambda_T$ | uncoupled natural frequency ratio ($\omega_\theta/\omega_v$) |
| $v$ | Poisson's ratio of the half-space |
| $\omega$ | harmonic excitation frequency |
| $\omega_n$, $n = 1,2$ | natural frequencies of the torsionally coupled rigidly based building |
| $\omega_v$ | translational natural frequency of the torsionally uncoupled rigidly based building ($=\sqrt{K_v/m}$) |
| $\omega_\theta$ | torsional natural frequency of the torsionally uncoupled rigidly based building ($=\sqrt{K_\theta/J_\theta}$) |
| $\Omega_1$ | $\omega_1/\omega_v$ |
| $\theta$, $\tilde{\theta}$ | time history and complex frequency response of the rotation of the floor disc about the $Z$-axis, arising due to structural deformation |
| $\theta_0$, $\tilde{\theta}_0$ | time history and complex frequency response of the rotation of the base disc about the $Z$-axis |
| $\phi$, $\tilde{\phi}$ | time history and complex frequency response of the rotation of the whole building about the $Y$-axis |

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