Base Isolation and Control of an Asymmetric Building*

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Abstract

The paper considers a seismic protection system using base isolation augmented with control devices which provide additional, variable damping. The test structure is a four-story building model which is symmetric in the x-direction but has asymmetry in the y-direction; hence a three-dimensional model is needed. The model for the building includes states representing movement in the horizontal plane and states representing torsional movement. A simple base isolation system is designed; and three paradigms to control the augmenting control devices are studied—LQG control, Risk Sensitive (RS) control, which manages the value of a denumerable linear combination of the cumulants of the LQG performance index, and dissipative-type semi-active control which only absorbs energy from the protected system and hence guarantees stability. The closed-loop systems, including the structure with base isolation and augmenting device controllers, are simulated under four different earthquakes. Using a group of eight non-dimensionalized evaluation criteria, the results show that active control of the augmenting devices offers broad improvement over optimally designed base isolators acting alone, with the RS approach outperforming the LQG method. The semi-active design offers an even broader, but less pronounced, improvement over the isolators alone. Finally, it is found that control designs which fail to account for the asymmetry of the structure can cause degradation of the torsional behavior.

1. Introduction

Protecting a structure from damage due to severe earthquakes is a challenging topic in civil engineering. Among protection methods considered to date, base isolation, whose principle is to decouple the horizontal movement of the structure from the movement of the ground during the earthquake—by means of a base isolation layer—is being extensively applied in designing new structures as well as in retrofitting existing structures. However, there are intrinsic disadvantages in base isolation methods. For example, they are passive in nature; and the parameters are hard to change after the initial design is installed. With the base isolation system augmented by additional control devices, some of these disadvantages can be overcome. The paper addresses a seismic protection system using a base isolation system together with control devices which provide additional, variable damping. The test structure is a four-story building model given by De La Llera *et al.* (1995). The systems, of asymmetric structure with

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base isolation and different controllers, are simulated under four different earthquakes as input excitations: El Centro 1940 and Hachinohe 1968 representing the "design" earthquake category, and Northridge 1994 and Kobe 1995 representing the "severe near-field" earthquake. A group of eight non-dimensionalized evaluation criteria—of the type used in recent months for ASCE benchmark structure-earthquake problems (Spencer *et al.* 1999)—are defined to evaluate the efficacy of control strategies.

2. Problem Formulation

The test structure in this paper is a four-story building model given by De La Llera *et al.* (1995), which is a four-story building symmetric in x-direction but with asymmetry in y-direction. The first two floors of the four-story building are symmetric in mass and stiffness. There is a setback on the third floor, therefore the centers of mass of the third and fourth floor of the four-story building have a shift with respect to the first two floors, hence the eccentricity. The values of the masses and the moments of inertia of each floor are: $m_1 = m_2 = 145.72 \times 10^3\ kg$, $m_3 = m_4 = 116.57 \times 10^3\ kg$, $I_{p1} = I_{p2} = 21659.79 \times 10^3\ kg - m^2$, $I_{p3} = I_{p4} = 16244.84 \times 10^3\ kg - m^2$. The stiffness in y-direction of the building is provided by five resisting planes. The stiffness giving by each resisting plane is calculated as: $k_y^{(i)} = 3(12EI/h^3) = 5932.2KN/m$, where $i = 1, 2, \ldots, 5$, and E, I and h are the Young's modulus of steel, the second moment of area of the column section, and the story height, respectively. The governing equations of motion for the asymmetric four-story building are given by

$$M_u \ddot{x}_u + C_u \dot{x}_u + K_u x_u = -M_u L \ddot{x}_g \tag{1}$$

where x_u represents both planar movement and torsional movement, M_u is the mass matrix of the building, K_u is determined by the property and location of each resisting plane, and C_u is designed in such way that the damping ratio $\zeta_i = 1\%$ for each mode of the building, and $\ddot{x}_g$ is the earthquake input. To get the base isolated system model, we have to design the parameters of the base isolation system. We first assume that the mass ratio between the super structure and the base isolation slab is 10. Next, we fix the effective period of the isolated structure as 4 second. The damping ratio is set to be 1%. By combining the model of the four-story building with the model of the base isolators and by re-arranging the rows and the columns of the mass matrix M, the stiffness matrix K and the damping matrix C, we can redefine the degree of freedom as $q = [x_b,\ x_1,\ x_2,\ x_3,\ x_4,\ \theta_b,\ \theta_1,\ \theta_2,\ \theta_3,\ \theta_4]$, where x_b and θ_b are the displacement and the torsional movement of the isolation base with respect to the ground, and x_i and θ_i are the displacement and the torsonal movement of the i-th story with respect to the ground, respectively.

Finally, by defining the state vector as $x = [q,\ \dot{q}]$, we can write the state space equations for the base-isolated system as:

$$\dot{x} = Ax + Bu + E\ddot{x}_g \tag{2}$$

$$y = Cx + Du + v \tag{3}$$

where

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{pmatrix},\ B = \begin{pmatrix} 0 \\ M^{-1}P \end{pmatrix},\ E = \begin{pmatrix} 0 \\ -L \end{pmatrix},$$

$$C = \begin{pmatrix} \Delta & 0 \\ -M^{-1}K & -M^{-1}C \end{pmatrix}, \; D = \begin{pmatrix} 0 \\ M^{-1}P \end{pmatrix}$$

in which Δ defines the measured outputs such as relative displacements and interstory displacements, $u = [f_1, \; f_2]^T$ is a vector of two forces given by supplemental dampers in addition to the base isolators, P represents the location of controllers (in our case, the controllers and base isolators are combined together along the edges of walls parallel to y-direction), and v is the sensor noise.

3. Risk Sensitive Control Strategies

Attach to (2) the cost functional

$$J = x^T(t_f)Q_f x(t_f) + \int_{t_0}^{t_f} [x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau)]d\tau \tag{4}$$

where Q, Q_f and R are symmetric positive semidefinite and R positive definite. The basic idea of risk sensitive control is to minimize

$$J_{RS} = -\frac{1}{\theta} ln\Big(E_K\{\exp(-\theta J)\}\Big) \tag{5}$$

over admissible control laws K, and E_K denotes the expectation conditioned on the control law K generating the control action $u(t)$ from the state $x(t)$. The full-state feedback gain (Won 1994) is given by $K(t) = -R^{-1}B^T F(t)$, where $F(\cdot)$ is the solution of the design Riccati matrix differential equation

$$\dot{F}(t) = -F(t)A - A^T F(t) + F(t)(BR^{-1}B^T + 2\theta EWE^T)F(t) - Q. \tag{6}$$

where W is the noise covariance of the earthquake excitation, and θ is a small scalar parameter. The terminal condition $F(t_f)$ is equal to Q_f. Furthermore, there are some interesting cases associated with the value of θ: risk seeking ($\theta > 0$), risk aversive ($\theta < 0$), and risk neutral ($\theta = 0$, LQG). Note that the optimal controller is time-varying. However, when applied to this study, the time-invariant risk sensitive controller is calculated by solving the algebraic Riccati equation. The resulting controller is understood as the approximation to (6) for large t_f. In our case, for the risk sensitive controller, θ is set to be -3.5046e-05. Notice that, for $\theta = 0$, one gets the LQG case. Thus, in a certain sense, when we compare RS and LQG designs, we are really comparing two instances of RS design.

4. Semi-Active Control

Semi-active control belongs to the fourth category of control method according to Housner *et al.* (1997). Spencer and Sain (1997) introduced one special type of semi-active actuator — the magnetorheological (MR) fluid damper. Ramallo *et al.* (1999) discussed the seismic protection of structures using MR dampers. Spencer *et al.* (1998) developed a full-scale experiment for the MR damper which is of such a size as to be useful in certain of today's structures. Semi-active control can dynamically change its damping property and yet still keep its property of being a passive damper, i.e., it will not add energy into the structure it protects. One simple model of the MR damper is the switch model. When the velocity of the shaft of the MR damper is in the “correct” direction, the MR damper produces damping force; when the velocity of the shaft of the MR damper is in the “incorrect” direction, the MR damper produces no force:

$$f = \begin{cases} f_d & f_d\dot{x} < 0 \\ 0 & otherwise \end{cases} \tag{7}$$

where f_d is the desired force, $\dot{x}$ is the velocity of the shaft of the MR damper and f is the damper force output. In this paper, f_d is generated by an LQG design.

5. Numerical Results

In this section, some of the evaluation criteria presented by Spencer *et al.* (1999) are introduced. First is the non-dimensionalized measure of the floor displacement relative to the ground, $J_1 = (\max_{t,i} |x_i(t)|/x_{max})^*$; second is the inter-story displacement $J_2 = (\max_{t,i} |d_i(t)|/d_{max})^*$; and third is the peak absolute acceleration of all the four stories $J_3 = (\max_{t,i} |\ddot{x}_{ai}(t)|/\ddot{x}_{a-max})^*$, where superscript $*$ represents maximizing over El Centro, Hachinohe, Kobe, Northridge earthquakes, x_i is the relative displacement of each story, d_i is the inter-story displacement of each story relative to the story below, $\ddot{x}_{ai}$ is the absolute acceleration of each story, $i = 1, 2, 3, 4$ represent the four stories of the building, and x_{max}, d_{max} and $\ddot{x}_{a-max}$ are the peak displacement relative to ground, peak inter-story displacement and peak absolute acceleration, respectively, for the no control case for each earthquake. The next three evaluation criteria are the RMS versions of the above three evaluation criteria: $J_4 = (\max_i ||x_i(t)||/||x_{max}||)^*$, $J_5 = (\max_i ||d_i(t)||/||d_{max}||)^*$, $J_6 = (\max_i ||\ddot{x}_{ai}(t)||/||\ddot{x}_{a-max}||)^*$, where $||x_i(t)|| = \sqrt{\int_0^{t_f} x_i^2(t)dt}$, $||d_i(t)|| = \sqrt{\int_0^{t_f} d_i^2(t)dt}$ and $||\ddot{x}_i(t)|| = \sqrt{\int_0^{t_f} \ddot{x}_i^2(t)dt}$. The last two evaluation criteria are the non-dimensionalized measure of the peak and RMS value of the isolation base: $J_7 = (\max_t |x_b(t)|/x_{b-max})^*$, $J_8 = (||x_b(t)||/||x_{b-max}||)^*$, where x_b is the displacement of the isolation base relative to ground. These two criteria represent the gap needed for base isolator construction.

Five different cases are studied: (1) no-control with base isolation at damping ratio 1%; (2) optimal passive (Chi 2000) base isolation, with damping ratio 20%;(3) base-isolation at damping ratio 1% with controlled damping device attached to each base isolator, using LQG control methods; (4) base-isolation at damping ratio 1% with controlled damping device attached to each base isolator, using RS control methods; (5) base-isolated at damping ratio 1% with controlled damping devices attached to each base isolator, using semi-active control methods having f_d generated by LQG methods.

Peak responses and RMS responses for the five control cases are given in Table 1. Underlined values represent responses of the three controlled systems which are greater than those of the optimal passive case. Of the 96 entries involved, 86 show improvement over the optimal passive case. Moreover, 8 of the 10 exceptions are associated with base drift. But it must be remembered that the base drift is acceptable as long as it is within the design value range. It is the structure responses that we want to improve. For these structural responses, only 2 of 72 fail to out-perform the optimal passive damper. The results for various J_i are given in Table 2. From the table we can obtain the following observations. First, the evaluation criteria for the optimal passive case are better than those for no control case, except J_3, the peak absolute acceleration, for which the optimal passive case is worse a little bit. Second, the evaluation criteria for the LQG control case are better than those for optimal passive case, except J_8, the RMS value of the isolation base drift. Third, the evaluation criteria for the risk sensitive control case are better than those for the LQG control case, except J_7 and J_8. Using the same argument, we consider risk sensitive control case as better than the LQG control case. Finally, the semi-active control case is not as good as the LQG control case, but it is better than optimal passive case in every evaluation criterion, except J_7. But it is better than the no control case in every evaluation criterion.

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Table 1: Peak Responses and RMS Responses

Cases	El Centro	Hachinohe	Kobe	Northridge	El Centro	Hachinohe	Kobe	Northridge
	Peak Base Drift [mm]				RMS Base Drift [mm]			
No Control	192	224	446	530	78.9	93.0	204	233
Optimal Passive	124	154	235	400	32.3	52.3	50.7	96.7
LQG Control	143	151	183	362	44.6	61.6	31.3	61.9
Risk Sensitive	146	155	185	364	46.7	64.6	31.5	62.2
Semi Active	124	150	278	438	31.0	43.8	42.0	96.1
	Peak Base (Absolute) Acceleration [g]				RMS Base (Absolute) Acceleration [g]			
No Control	0.047	0.057	0.108	0.137	0.020	0.023	0.050	0.058
Optimal Passive	0.053	0.053	0.077	0.159	0.009	0.015	0.016	0.029
LQG Control	0.030	0.031	0.076	0.114	0.006	0.008	0.011	0.016
Risk Sensitive	0.029	0.030	0.073	0.109	0.005	0.008	0.011	0.015
Semi Active	0.040	0.040	0.074	0.128	0.007	0.011	0.012	0.027
	Peak Structure Drift[mm]				RMS Structure Drift[mm]			
Fixed Base	143	181	518	464	45.8	51.0	179	152
No Control	8.14	8.92	18.9	19.9	3.37	3.78	8.55	9.22
Optimal Passive	5.26	6.85	8.29	17.3	1.54	2.23	2.12	3.59
LQG Control	4.26	3.98	7.70	11.4	1.01	1.39	1.12	1.60
Risk Sensitive	4.11	3.82	7.37	10.9	0.99	1.37	1.07	1.53
Semi Active	4.45	5.28	7.80	16.4	1.18	1.61	1.25	3.20
	Peak Structure (Absolute) Acceleration [g]				RMS Structure (Absolute) Acceleration [g]			
Fixed Base	0.660	0.579	2.037	1.958	0.152	0.161	0.518	0.453
No Control	0.049	0.062	0.115	0.145	0.020	0.024	0.052	0.059
Optimal Passive	0.052	0.062	0.104	0.166	0.010	0.016	0.019	0.031
LQG Control	0.052	0.041	0.116	0.138	0.007	0.009	0.016	0.019
Risk Sensitive	0.050	0.040	0.111	0.132	0.007	0.009	0.015	0.018
Semi Active	0.047	0.040	0.081	0.147	0.009	0.011	0.015	0.029
	Peak Damping/Control Force [KN]				RMS Damping/Control Force [KN]			
No Control	9.1	8.6	15.4	25.4	2.46	3.12	6.14	7.55
Optimal Passive	157.8	128.5	296.6	447.8	26.7	41.8	52.2	85.1
LQG Control	127.4	129.0	304.2	392.9	30.3	40.8	38.6	55.8
Risk Sensitive	126.1	127.2	293.4	383.7	32.0	43.7	37.3	54.7
Semi Active	112.6	113.3	184.3	247.7	23.3	30.6	29.0	48.8

Table 2: The Js for Different Controllers

Cases	No Control	Passive	LQG	Risk Sensitive	Semi Active
J1	0.056890	0.037959	0.029783	0.028721	0.035354
J2	0.082289	0.075939	0.059912	0.057726	0.060939
J3	0.106897	0.106970	0.078513	0.075454	0.075042
J4	0.074185	0.043773	0.027181	0.026829	0.031666
J5	0.105287	0.064445	0.038575	0.037502	0.050026
J6	0.146795	0.097619	0.055265	0.053101	0.075612
J7	1.000000	0.754469	0.746718	0.764567	0.826964
J8	1.000000	0.562352	0.662578	0.694709	0.471180