

# INELASTIC BEHAVIOR OF ASYMMETRIC MULTISTORY BUILDINGS

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**ABSTRACT:** Studied in this paper is the inelastic seismic behavior and design of asymmetric multistory buildings emphasizing, primarily, the use of story shear and torque histories. The following six different structural characteristics and their effect on the torsional response of buildings are analyzed: strength of orthogonal resisting planes, stiffness asymmetry, strength asymmetry, planwise distribution of strength, number of resisting planes, and intensity of the ground motion component in the orthogonal direction. As a result of these analyses several techniques and conceptual guidelines are developed to correct the planwise unbalance in deformation demands typical of asymmetric structures. The two most important are to increase the torsional capacity of the system by introducing resisting planes in the orthogonal direction, and to modify the stiffness and strength distribution to localize yielding in selected resisting planes. Using these guidelines the undesirable earthquake response of a very asymmetric building is effectively corrected by changing slightly the strength of a few key resisting planes. Finally, it is concluded that the use of the story shear and response histories in conjunction with the corresponding story yield surfaces is a powerful tool for conceptual understanding of the earthquake behavior of asymmetric structures.

## INTRODUCTION

Most of the available results concerning the inelastic behavior of asymmetric buildings come from the interpretation of exhaustive parametric studies of the earthquake response of simplified models of asymmetric single-story structures [e.g., Rutenberg (1983); Tso (1985); Goel (1990)]. Although these results are an important step in trying to understand the behavior of asymmetric systems, they have two important drawbacks. First, because of the inherent complexity of the problem, it is very difficult to extract from them general trends that apply to structures other than those analyzed and to propose new design guidelines. Second, their extension to the practical case of multistory buildings is not obvious. In order to overcome the latter difficulty, recent studies have considered the behavior of code-designed asymmetric multistory buildings (Duan 1993). It is the writers' opinion, however, that these and other future studies will be better appreciated and understood once the inelastic seismic behavior of asymmetric-plan multistory systems is studied from a conceptual viewpoint. One possible conceptual framework, which is based on the interpretation of the building response in the story shear and torque space, has been developed for the inelastic behavior of single-story systems (De la Llera and Chopra 1995a). It is one of the objectives of this paper to extend this conceptual framework for understanding the inelastic seismic behavior of multistory systems.

Perhaps the most significant advantage of such a conceptual framework is the ability to answer, without complex nonlinear dynamic analyses, practical issues concerning the design and retrofit of asymmetric structures. Consider, for instance, the issue of inappropriate seismic resistance of an existing asymmetric building. What can be said about the torsional behavior of the structure before performing nonlinear dynamic analysis? How can we adjust the planwise distribution of stiffness and strength in the system in order to achieve a good performance? How can we localize or spread the damage among resisting

planes? What resisting planes should be stiffened or strengthened? What is the effect of the orthogonal component of ground motion on the design of resisting planes in the direction of the first component of ground motion? How is the system going to collapse? The ability to answer such questions will go a long way toward ensuring that a proposed retrofit strategy for a structure is technically and economically optimal. One may argue that a full three-dimensional nonlinear analysis of the structure could be performed to evaluate a retrofit solution. However, the large cost of such nonlinear analyses precludes experimentation of alternative solutions, especially if the structural configuration of the system changes between alternatives. Consequently, the second objective of this study is to provide generally applicable design guidelines for asymmetric buildings that enable us to answer the proposed questions.

The fundamental idea used in this investigation is to study the effects of plan asymmetry in light of the story shear and torque response histories of different structural configurations. These histories are represented in the force space spanned by the story shears  $V_x$  and  $V_y$  in the  $x$ - and  $y$ -directions, respectively; and story torque  $T$ ; the values of  $V_x$ ,  $V_y$ , and  $T$  at each time instant define one point in this space. These shear and torque combinations are bounded in this space by a surface defined by the set of story shear and torque combinations corresponding to the different collapse mechanisms that can be developed in the story; hereafter this surface is denoted as the SST (story shear and torque) ultimate surface. The reader is referred to a recent investigation (De la Llera and Chopra 1994) in which the construction and properties of these surfaces have been considered. Knowledge of such material proves very useful in the interpretation of the results presented next.

## SYSTEMS CONSIDERED AND ANALYSIS PROCEDURE

The systems analyzed are multistory buildings consisting of floor diaphragms that are rigid flexurally and axially, where all the story masses are lumped; lateral resistance is provided by resisting planes in the  $x$ - and  $y$ -directions (Fig. 1) composed of elasto-plastic resisting elements. As shown in the building plan, the  $i$ th resisting plane in the  $x$ -direction has lateral stiffness matrix  $k_x^{(i)}$  and is located at distance  $y^{(i)}$  from the center of mass (CM) of the  $j$ th floor ( $j = 1, \dots, n$ ); analogously, the stiffness matrix and location of the  $i$ th resisting plane of story  $j$  in the  $y$ -direction are defined by  $k_y^{(i)}$  and  $x^{(i)}$ , respectively. The resisting planes in the  $y$ -direction may have different stiffness matrices and lateral capacities, and may be unsymmetrically located about the  $y$ -axis, creating eccentricities between

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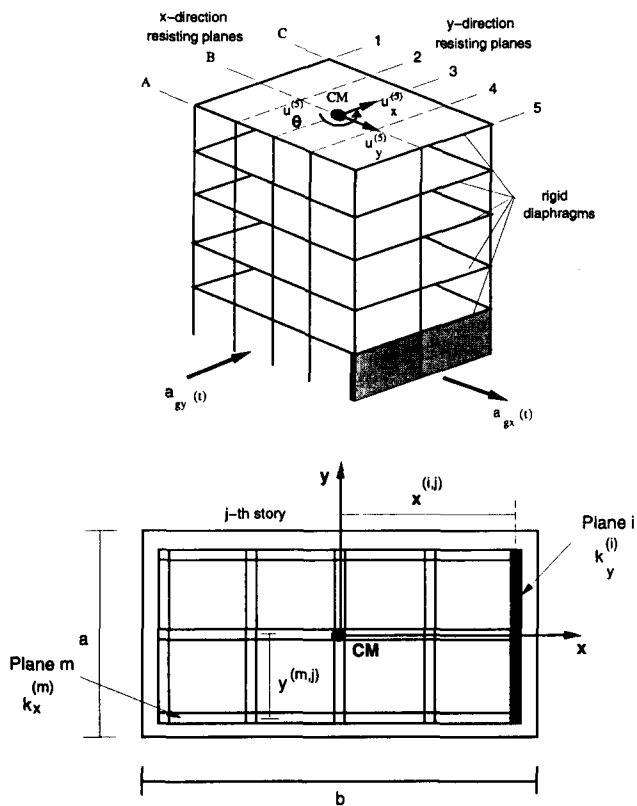


FIG. 1. Typical Five-Story Building Considered

the CM and center of stiffness in each story. On the other hand, the systems considered are symmetric in stiffness and strength about the  $x$ -axis. The structural model, although simple, is deemed adequate for the computation and study of global building responses, such as floor displacements and story shears and torque (Duan 1993).

The response quantities of interest in this study are as follows: (1) the combinations of  $V_x^{(j)}$ ,  $V_y^{(j)}$ , and  $T^{(j)}$ , the story shears in the  $x$ - and  $y$ -directions and story torque, respectively; (2) the displacement histories at the building edges and at the CM; and (3) the force-displacement histories in structural elements. For notational convenience, the principal direction of analysis always coincides with the  $y$ -direction (Fig. 1); because of that, the  $x$ -direction will be denoted hereafter as the orthogonal direction.

## RESPONSE OF INELASTIC SYSTEMS

The purpose of this section is to investigate, in light of the results derived earlier for single-story systems (De la Llera and Chopra 1995a), the effect of the following six characteristics controlling the behavior of multistory asymmetric structures: (1) strength of resisting planes in the orthogonal direction; (2) stiffness asymmetry; (3) strength asymmetry; (4) planwise distribution of strength; (5) number of resisting planes; and (6) bidirectional ground motion.

These characteristics are studied using five-story buildings with uncoupled lateral vibration periods ( $T_x = T_y = 0.7$  s) in the  $x$ - and  $y$ -directions, plan aspect ratio  $a/b$  of 1/2 where  $b$  is 120 ft (Fig. 1), and subjected to twice the N-S component of the El Centro ground motion in the  $y$ -direction. This amplification of the ground motion, which is not unrealistic in light of the recent earthquake data (Shakal 1994), was intended to drive the systems well into the inelastic range, which otherwise would have experienced little plasticity since their designs are essentially controlled by stiffness rather than strength code provisions. The vertical distribution of story strengths in the

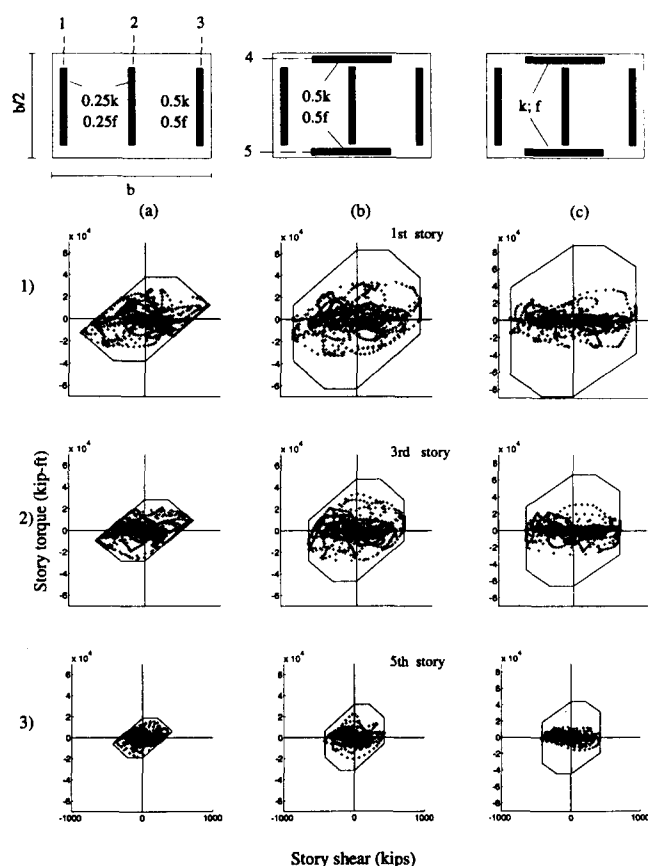


FIG. 2. Story Shear and Torque Response Histories for Buildings with Different Capacities in Orthogonal Resisting Planes: (a)  $V_x/W = 0$ ; (b)  $V_x/W = 0.15$ ; and (c)  $V_x/W = 0.3$  ( $e_x/b = e_y/b = 0.125$ )

building is as follows:  $V$  for the first two stories;  $3/4V$  for the third and fourth stories; and  $V/2$  for the fifth story. The story stiffnesses vary over the building height like the story strengths as follows:  $K$  in the first two stories;  $3/4K$  in the third and fourth stories; and  $K/2$  in the fifth story. This height-wise distribution of strength is consistent with the code distribution of lateral forces but also accounts for the fact that properties of structural elements do not vary usually in every story. Hereafter, the stiffnesses and strengths of structural elements are given in terms of the reference values  $K = 42,000$  kips/ft and  $V$  of 840 kips, respectively, corresponding to a realistic five-story structure.

The response results will be presented in a special format (see for example Fig. 2). Rows 1, 2, and 3 contain, respectively, the response results for the first, third, and fifth stories of buildings with plan configurations shown in columns (a), (b), and (c); identical stiffnesses and capacities in resisting planes are only shown once. Story shears and torque are used herein to interpret the earthquake response of different buildings; the reader is referred to De la Llera and Chopra (1994) for the corresponding building displacements and member forces.

It is important to note that in some cases changes in the building characteristics mentioned will not only imply changes in the inelastic response of the building but also in the elastic response. This distinguishes our study from traditional parametric studies where the goal is to keep identical elastic responses at the expense of adjustments to the structural system. In this study, the changes introduced to building characteristics are motivated by practical actions that the engineer may take to modify a building plan. For example, if more resisting planes are introduced in the direction orthogonal to the ground motion considered, not only the torsional capacity of the build-

ing will increase but also its torsional stiffness. Hence, both the elastic and inelastic parts of the building response will change.

### Strength of Resisting Planes in Orthogonal Direction

To study the effect of varying the strength of resisting planes orthogonal to the direction of ground motion, consider the three structural plans (a)–(c) presented in Fig. 2. The lateral capacity  $V_y$  of these three structures in the  $y$ -direction is the same and equal to  $0.15W$ , where  $W$  is the weight of the structure, but their base shear capacities  $V_x$  in the  $x$ -direction vary from zero to a maximum of  $0.3W$ . Each system has equal normalized stiffness eccentricity  $e_y/b$  of 0.125 and strength eccentricity  $e_x/b$  of 0.125 in all stories; these values have been chosen arbitrarily as the reference values about which changes will be considered in this study. Note that the torsional stiffness contributed by the resisting planes in the orthogonal direction differs among the three systems; however, as shown next, the differences in response among these systems can be explained mainly based on their different inelastic properties.

Let us consider the story shear and torque response histories for each system, which have been superimposed on the SST surfaces (polygons in solid lines) (Fig. 2). It is apparent that the SST surfaces for the third and fifth stories are scaled versions, by factors 3/4 and 1/2, respectively, of the first story SST surface due to the smaller lateral capacities in these stories. Further, the increase in the capacity of the orthogonal resisting planes leads to an increase in the length of the constant base shear branch and to an increase in the torsional capacity of the system (Fig. 2: 1a, 1b, 1c) (De la Llera and Chopra 1995a). The lengthening of the constant base-shear branch implies an increase in the number of story mechanisms that involve yielding of all  $y$ -direction planes. It is observed that the system with no orthogonal resisting planes (Fig. 2: 1a, 2a, and 3a) undergoes most of its inelastic behavior along the branches of the SST surface with positive slope. This implies that at many time instants the system develops torsional mechanisms about the strongest resisting plane (plane 3) in the plan (De la Llera and Chopra 1995a). Consequently, most of the inelastic behavior of the system is expected to occur in plane 1, the farthest from plane 3, due to the plan rotation. A corollary of this observation indicates that peak displacements of systems with no orthogonal resisting planes are associated with large rotations of the plan, as has been noted earlier in the context of single-story systems (Tso 1985).

As the strength of the orthogonal resisting planes increases, a larger proportion of the inelastic behavior of the system shifts from the branches of the SST surface with positive slope (Fig. 2: 1a, 2a, and 3a) to the constant base-shear branches (Fig. 2: 1c, 2c, and 3c). This implies that systems with strong orthogonal resisting planes will see in many of their inelastic excursions predominantly translational mechanisms involving yielding of all resisting planes in the  $y$ -direction. Thus, more uniform displacement demands are expected for the resisting planes in these systems; equivalently, systems with strong orthogonal planes will undergo smaller inelastic rotations of the plan as has been noted earlier in the context of single story systems (Goel 1990). Consistent observations are obtained from the displacement histories at different plan locations and force-displacement histories of the resisting planes (De la Llera and Chopra 1994).

### Stiffness Asymmetry

The effect of stiffness asymmetry in the inelastic response of asymmetric structures is studied considering the response of the three buildings with plans presented in Fig. 3(a)–(c). The lateral base shear capacities in the  $x$ - and  $y$ -directions for

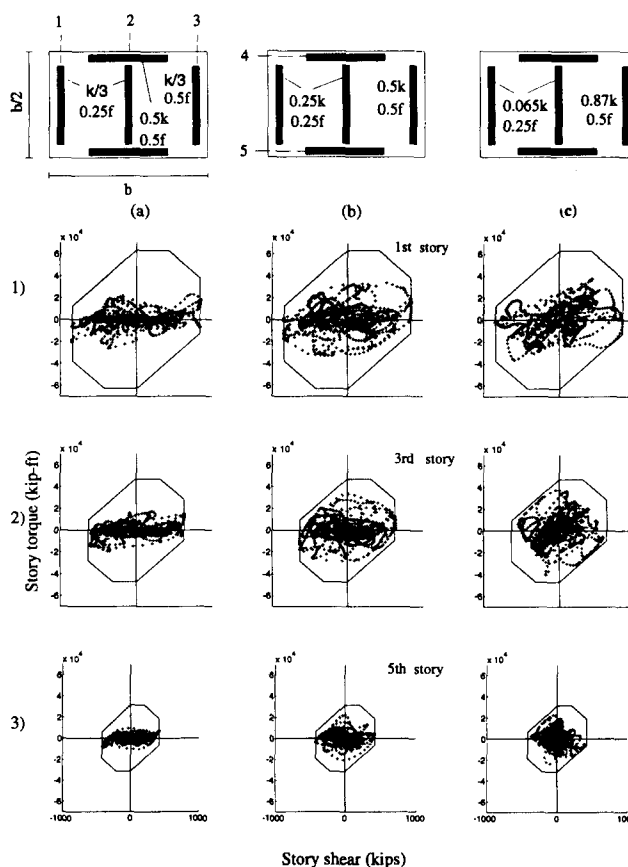


FIG. 3. Story Shear and Torque Response Histories for Buildings with Different Stiffness Eccentricity: (a)  $e_y/b = 0$ ; (b)  $e_y/b = 0.125$ ; and (c)  $e_y/b = 0.4$  ( $e_x/b = 0.125$ ;  $V_x/W = V_y/W = 0.15$ )

all the systems are  $0.15W$ . Their normalized strength eccentricity,  $e_y/b = 0.125$ , is also identical, but their normalized stiffness eccentricity  $e_x/b = 0, 0.125$ , and  $0.4$  for buildings (a), (b), and (c), respectively.

In single-story systems, stiffness eccentricity has shown to influence the story shear and torque response histories inside the SST surface (De la Llera and Chopra 1995a). This influence is also apparent from the shear and torque histories for multistory systems presented in Fig. 3. As the stiffness eccentricity increases, larger values of base torque are developed in the system (e.g., Fig. 3: 2a, 2b, 2c). If the stiffness eccentricity is large, as in building (c), the story shear and torque combinations produce a plot that is skewed toward the second and fourth quadrants of the SST surface (see Fig. 3: 3c), because we have assumed positive stiffness eccentricity (CS to the right of the CM), implying that positive story shears would frequently accompany clockwise (negative) base torques and vice versa. As a result, yielding in building (a), which takes place along the constant shear branches of the SST surface (Fig. 3: 1a, 2a, and 3a), tends to move to the positive slope branches for building (c) (Fig. 3: 1c, 2c, and 3c). This implies, as it should, that these systems with large stiffness eccentricity will experience inelastic behavior in mechanisms associated with important rotation of the building plan about the stiffest, and usually strongest, resisting plane. In other words, collapse of building (c) would likely occur in a torsional mechanism about plane 3. Similar observations are obtained considering the displacement histories at the edges and CM of the building and the force-displacement histories of the resisting planes (De la Llera and Chopra 1994).

### Strength Asymmetry

The effects of strength asymmetry are studied considering the three structural plans shown in Fig. 4. These systems are

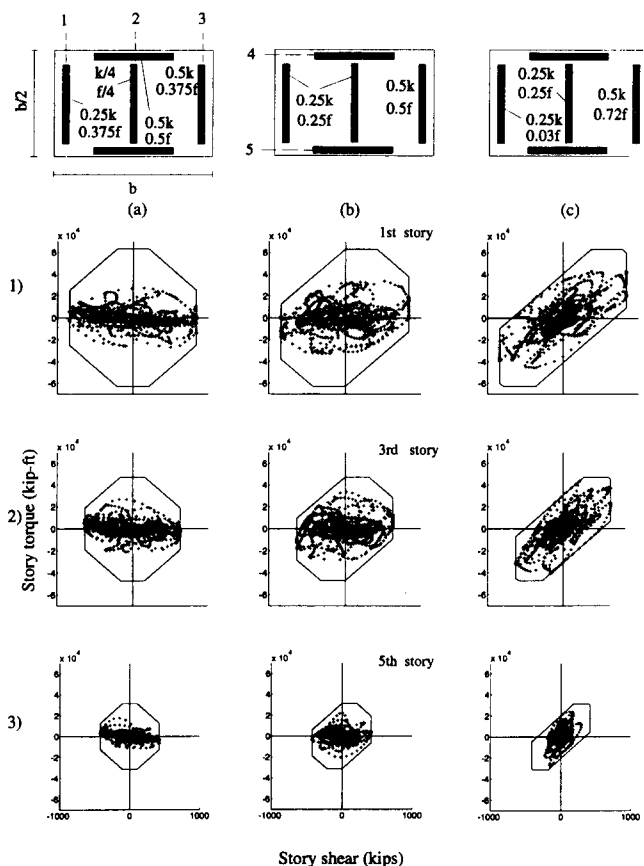


FIG. 4. Story Shear and Torque Response Histories for Buildings with Different Strength Eccentricity: (a)  $e_p/b = 0$ ; (b)  $e_p/b = 0.125$ ; and (c)  $e_p/b = 0.342$  [ $e_s/b = 0.125$ ;  $V_x/W = V_y/W = 0.15$ ;  $T_o/(Wb/2) = 0.1875$ ]

such that they all have the same normalized stiffness eccentricity  $e_s/b$  of 0.125, but variable normalized strength eccentricities— $e_p/b = 0, 0.125$  and  $0.342$ , respectively). Also, their lateral base shear capacities are  $0.15W$  in the  $x$ - and  $y$ -directions, respectively, and the three systems have the same base torsional capacity  $T_o$  of  $0.1875W(b/2)$  kips-ft. The elastic responses of the three buildings to the given ground motion are identical, and differences in their inelastic responses can be attributed principally to changes in the shape of the SST surface because of strength asymmetry.

Strength asymmetry has an important effect on the shape of the SST ultimate surface as shown by the results presented in Fig. 4. The SST surfaces for the strength symmetric building (a) (Fig. 4: 1a, 2a, and 3a) are symmetric about the shear and torque axes. This symmetry is lost gradually with increasing strength eccentricity in buildings (b) and (c), making the surfaces to narrow and lean toward the first and third quadrants (Fig. 4: 1c, 2c, and 3c) (De la Llera and Chopra 1995a).

The story shear and torque histories corresponding to building (a) (Fig. 4: 1a, 2a, and 3a) show that the inelastic behavior of this strength symmetric system is developed along the constant base-shear branches of the SST surfaces. However, the shear and torque combinations tend to lie off the center of these branches, implying that the system will undergo, besides its predominantly lateral yielding, some inelastic rotations as a result of its stiffness eccentricity. As the strength eccentricity increases, more base shear and torque combinations shift from the constant base-shear branches (Fig. 4: 2a) of the SST surface toward the branches with positive slope (Fig. 4: 2c); the latter being associated with torsional mechanisms leaving the strongest resisting plane (plane 3) elastic (De la Llera and Chopra 1995a). The practical consequences of this shift are im-

portant; large plastic rotations will tend to be produced about the strongest plane (plane 3), leading to large displacement demands on the planes farther from it. Thus, building (c), which represents large asymmetry in strength, demonstrates that the strength of the strongest plane 3 is of little help since the inelastic behavior of the system is such that a torsional mechanism about this plane is activated most of the time.

It is important to note that in some cases an increase in the strength asymmetry of a system may be beneficial or at least innocuous. A good example of this situation are buildings (a) and (b). We observe in building (b) that the change in the shape of the SST surface has not produced yielding along its inclined branches; therefore, the displacement and deformation demands for these buildings will be similar (De la Llera and Chopra 1994). In more general terms, unless the change in strength eccentricity is accompanied by a change in the region of the SST surface where the inelastic behavior takes place, the changes in response among different configurations will not be substantial. The preceding observations are confirmed when we analyze the displacement and force-deformation histories at different floors (De la Llera and Chopra 1994).

### Planwise Distribution of Strength

In order to study the changes in building response due to other changes in the planwise distribution of strength, we consider the three building plans shown in Fig. 5. All these buildings have the same stiffness eccentricity  $e_s/b$  of 0.125 but are symmetric in strength. The three systems also have identical lateral capacities,  $0.15W$ , in the  $x$ - and  $y$ -directions; their torsional capacities, however, vary from  $T_o$  of  $0.025W(b/2)$  kip-ft in building (a) to  $0.125W(b/2)$  kip-ft in building (c). This variation in strength is such that building (a) corresponds to a

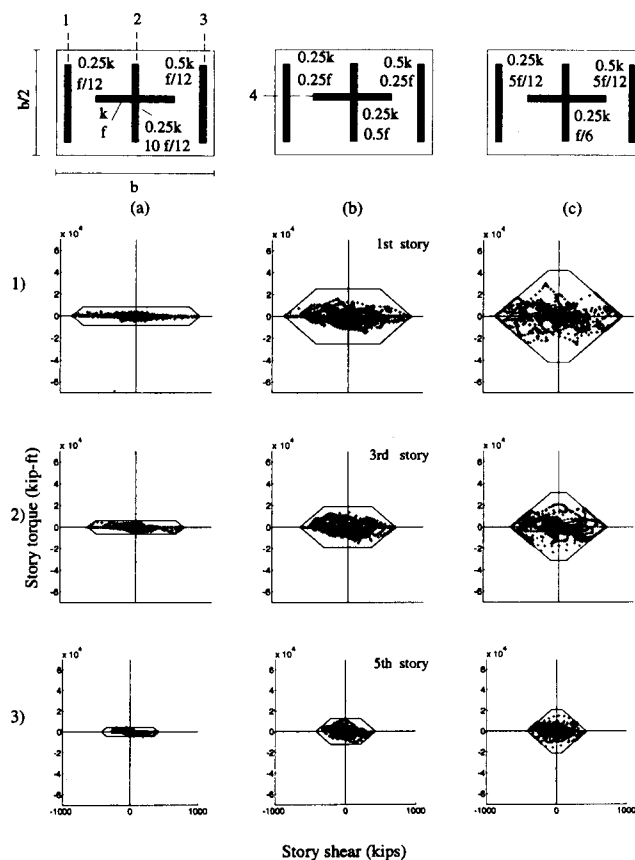


FIG. 5. Story Shear and Torque Response Histories for Buildings with Different Planwise Distribution of Strength: (a)  $T_o/(Wb/2) = 0.025$ ; (b)  $T_o/(Wb/2) = 0.075$ ; and (c)  $T_o/(Wb/2) = 0.125$  ( $e_s/b = 0.15$ ;  $e_p/b = 0$ ;  $V_x/W = V_y/W = 0.15$ )

system with a very strong central core and weak resisting planes along the two edges in the  $y$ -direction, and building (c) is just the opposite, that is, a system with very strong planes along the edges and a weak central core. Note that the elastic responses of all these systems are identical.

Shown in Fig. 5 are the SST surfaces for the three buildings considered. The surfaces for building (a) (Fig. 5: 1a, 2a, and 3a) are very flat as a result of the small torsional capacity of the system. On the other hand, the larger torsional capacity of buildings (b) and (c) implies a stretching of the surface along the torque axis and a reduction of the length of the constant torque branches (De la Llera and Chopra 1994).

In spite of the small torsional capacity of building (a), the story torques due to the selected excitation (Fig. 5: 1a, 2a, and 3a) remain below this capacity except at few time instants in the fifth story (Fig. 5: 3a). The system is unable to develop large story torques as a result of the small strength of resisting planes 1 and 3 and ends up responding primarily in translation as indicated by the story shear and torque combinations located in the zero-torque corner of the SST surfaces (e.g., Fig. 5: 1a). In other words, the torsional weakness of this system impedes the development of large rotations of the plan despite the stiffness asymmetry in the system. As the strength of the resisting planes at the edges increase, larger torques are developed and the buildings undergo yielding along the inclined branches of the SST surface [Fig. 5: (b) and (c)]. These mechanisms are predominantly torsional and lead, as expected, to uneven displacement demands at the stiff and flexible edges of the building (De la Llera and Chopra 1994). Therefore, although counter intuitive, these results show that the uneven torsional behavior of a structure with asymmetry in stiffness is reduced by decreasing the torsional capacity of the system. This does not imply, however, that buildings should be designed to be torsionally flexible and/or weak since excessive deformation demand may occur in these structures due to base rotational motion and other accidental factors. Further study is being conducted currently on this topic.

### Number of Resisting Planes

So far we have considered buildings with three resisting planes along the direction of ground motion. Let us consider now the effect of having more resisting planes by studying the response of the three buildings shown in Fig. 6. Buildings (a), (b) and (c) have 3, 5, and 7 resisting planes in the  $y$ -direction, respectively, and they all have the same stiffness eccentricity  $e_y/b$  of 0.125. Their lateral base shear capacities are the same and equal to  $0.15W$  in the  $x$ - and  $y$ -directions, but their base torsional capacities vary from  $0.1875W(b/2)$  kip-ft in building (a) to  $0.1125W(b/2)$  kip-ft in building (c). The systems also have identical strength eccentricities  $e_x/b$  of 0.125. Note that the elastic responses of these systems are not necessarily identical because their torsional stiffnesses decrease as the number of resisting planes increase. It will be shown next, however, that differences in elastic and inelastic responses among these systems are small.

Increasing the number of resisting planes of a building produces a rounding effect on the SST surfaces, especially for regions of large torque [compare Fig. 6: (a)–(c)]. However, since the inelastic behavior of the buildings does not take place in this region of the surfaces, we expect similar responses in all configurations. Consider, for example, the base shear and torque response histories on the first floor of buildings (a), (b), and (c). The inelastic behavior in all these systems is developed along the constant shear branches of the SST surface. Consequently, the displacements and deformation demands on the resisting planes of these systems will be similar (De la Llera and Chopra 1994). Other researchers have made this observation but on a different basis [e.g., Rutenberg (1983)].

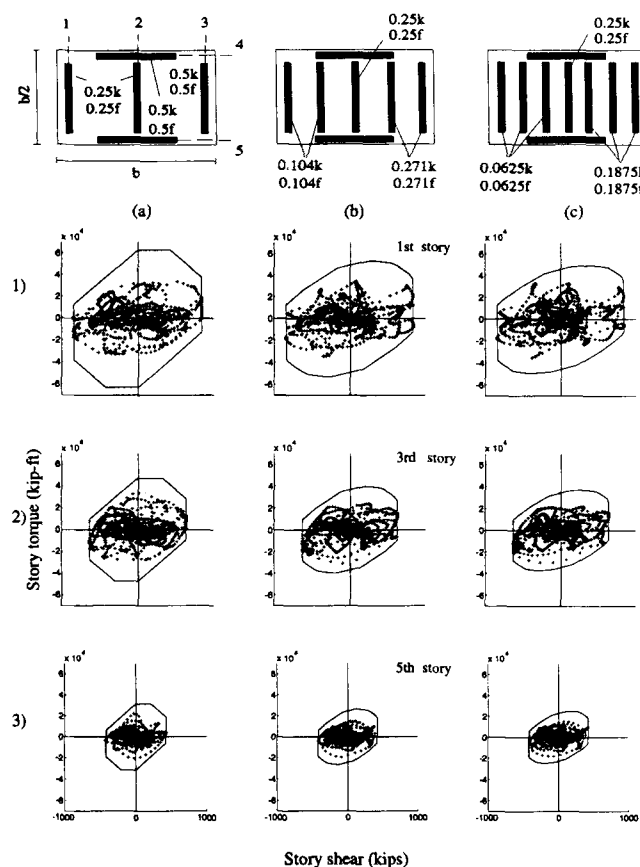


FIG. 6. Story Shear and Torque Response Histories for Buildings with Different Number of Resisting Planes: (a)  $n_y = 3$ ;  $T_o/(Wb/2) = 0.1875$ ; (b)  $n_y = 5$ ;  $T_o/(Wb/2) = 0.0844$ ; (c)  $n_y = 7$ ;  $T_o/(Wb/2) = 0.1125$  ( $e_x/b = e_y/b = 0.125$ ;  $V_x/W = V_y/W = 0.15$ )

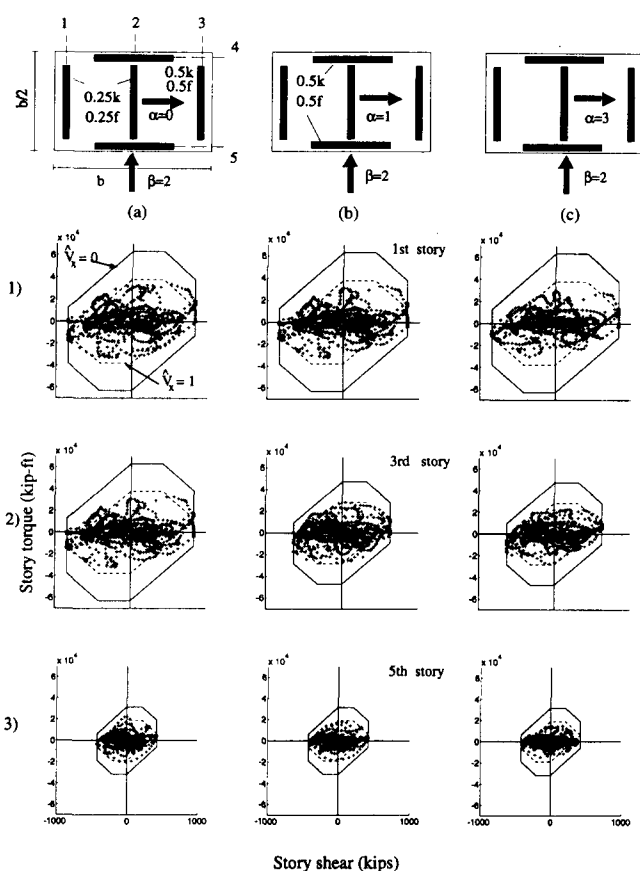
The results presented above are important in two respects. First, they justify the use of three-plane models in studying the response of asymmetric plane systems; an assumption that has been used by most previous researchers. Second, they suggest that, for simplified analysis and design purposes, most multiple resisting plane structures could be reasonably approximated by a three-plane model, if the latter matches the relevant elastic and inelastic properties of the actual building. Such a model has been developed in an earlier investigation (De la Llera and Chopra 1995b).

### Bidirectional Ground Motion

In the previous sections we have considered the response of asymmetric buildings subjected to a single component of ground motion. However, most buildings in practice are subjected to two horizontal components of ground motion. It is the objective of this section to study the response of asymmetric buildings when both ground motion components act simultaneously.

Consider the seismic response of the buildings shown in Fig. 7 subjected to twice the N-S component of the El Centro ground motion in the  $y$ -direction and three versions of its E-W component in the  $x$ -direction scaled by factors  $\alpha$  of 0, 1, and 3. The building considered has lateral capacities equal to  $0.15W$  in both principal directions, torsional capacity equal to  $0.1875W(b/2)$ , and identical stiffness and strength eccentricity ( $e_x/b = e_y/b = 0.125$ ).

The response of this system subjected only to ground motion in the  $y$ -direction ( $\alpha = 0$ ) has been already considered in Figs. 2(b), 3(b), 4(b), and 6(a); in this case, the orthogonal planes 4 and 5 contribute entirely to resist the story torque generated by the asymmetry of the system about the  $y$ -axis.



**FIG. 7. Story Shear and Torque Response Histories for Buildings Subjected to Bidirectional Ground Motion: (a)  $\alpha = 0$  (no  $x$ -direction ground motion); (b)  $\alpha = 1$ ; and (c)  $\alpha = 3$  [ $e_x/b = e_y/b = 0.125$ ;  $V_x/W = V_y/W = 0.125$ ;  $T_x/(Wb/2) = 0.1875$ ]**

However, as the intensity of the orthogonal component of ground motion increases, story shears in the  $x$ -direction increase and planes 4 and 5 yield, thus limiting their capacity to resist the story torque. In the limit, when these planes yield in the same direction at most time instants during the response, their contribution to resist the story torque will be very small, and the system will respond in the  $y$ -direction as if no orthogonal planes existed. To verify this concept, we have included in Fig. 7 two SST surfaces, one corresponding to no yielding in the orthogonal planes (solid line),  $\hat{V}_x$  of 0, and the other,  $\hat{V}_x = 1$ , to complete yielding in these planes (dashed line) (De la Llera and Chopra 1995a). It is clear that as the intensity of the orthogonal component increases from  $\alpha$  equaling 0 to  $\alpha$  of 3 (Fig. 7: 1a–1c; 2a–2c; and 3a–3c), more story shear and torque combinations move inside the SST surface corresponding to  $\hat{V}_x = 1$  (dashed lines).

A more subtle but relevant point is to study where the yielding takes place on each of the SST surfaces. For example, yielding in building (a) (Fig. 7: 1a, 2a, and 3a) occurs along the constant shear branches, implying that the resisting planes in this system experience relatively uniform deformation demands (De la Llera and Chopra 1994). On the other hand, when the intensity of the orthogonal component increases, more yielding occurs at the positive slope branches of the SST surface (see Fig. 7: 1c and 2c). This implies that the system responds at many time instants, as if it had no orthogonal resisting planes and, hence, it develops predominantly torsional mechanisms in most of its inelastic excursions. As a result, the deformation demands among the resisting planes will be uneven, that is, smaller for the strongest resisting plane and larger for the one farther from it. Displacements and force-deformation results presented in De la Llera and Chopra (1994) confirm these observations.

The increase in deformation demands on the resisting planes in the  $y$ -direction produced by yielding in the resisting planes in the orthogonal direction may be significant; for the example considered an increase of about 50% over the unidirectional case is expected for the flexible-edge planes (De la Llera and Chopra 1994). One possible alternative for incorporating this effect into the building design has been proposed in De la Llera and Chopra (1995b); the other possibility would be to bound the actual response of the building by two analysis cases, one assuming full capacity in the orthogonal resisting planes and the other ignoring that capacity.

## CONCEPTUAL DESIGN GUIDELINES

This section summarizes the most relevant observations of the inelastic behavior of asymmetric-plan buildings obtained in the previous section. These following observations may be used as conceptual guidelines for improving the design of asymmetric structures:

1. The responses of asymmetric-plan single and multistory buildings of the class considered, that is, with regular asymmetry in height [see Hejal (1989)], show trends that are very similar (De la Llera and Chopra 1995a). This suggests that, at least conceptually, results obtained by other researchers in single-story systems [e.g., Rutenberg (1983); Tso (1985); Goel (1990)] may be applicable to this wider class of multistory systems.
2. Stiffness asymmetry in a system influences the story shear and torque combinations inside the SST surface (Fig. 3: 2a, 2b, and 2c) and hence the elastic response of the system. However, changes in the stiffness eccentricity will affect the inelastic response of the system only if they lead to changes in the region (or branches) of the SST surface where the inelastic action is developed.
3. Strength asymmetry always produces concentration of deformation demand in resisting planes that are farther from the strongest plane in the plan (Fig. 4: 2c). Furthermore, buildings with strength asymmetry are prone to develop torsional mechanisms and, hence, uneven displacement demands among resisting planes.
4. The two observations above may be combined into an interesting point. Since stiffness asymmetry controls the behavior inside the SST surface and strength asymmetry the shape of the SST surface we can, theoretically speaking, adjust both to direct the inelastic behavior in any desired region of the SST surface. In particular, if we have, say positive strength eccentricity, the SST surface will lean toward the first and third quadrants (Fig. 4). Therefore, to get the shear and torque combinations to fall in the constant shear branches we must produce a tendency for them to lie on these quadrants by introducing a negative stiffness eccentricity (CS left of CM). Independently varying stiffness and strength may seem impractical for conventional steel and R/C buildings, but should be feasible, say, for a system using frictional devices.
5. As shown by preliminary research, a reduction in the torsional capacity of stiffness-asymmetric systems may lead, at the expense of larger plan displacements, to more uniform displacement demands among resisting planes (Fig. 5), that is, a translationally dominant response.
6. Equivalent three-plane models (Fig. 6: 2a, 2b, and 2c) of buildings with multiple resisting planes lead to sufficiently accurate estimations of the building displacements, story shears, and story torques. This is the basis for the simplified model developed in De la Llera and

Chopra (1995b) for preliminary analysis and design of buildings.

7. Increased strength in the resisting planes in the direction orthogonal to the ground motion always reduces the effects of torsion in an asymmetric structure. Their effectiveness must be assessed, however, in conjunction with the intensity of the ground motion along the orthogonal direction; if substantial yielding is expected in that direction the orthogonal planes should be ignored in the analysis, otherwise their contribution to resist the torsional motions of the system should increase with decreasing yielding. A simplified procedure to account for yielding in the orthogonal resisting planes has been suggested earlier (De la Llera and Chopra 1995b).
8. Any design or retrofit solution of an asymmetric building should consider the story shear and response histories together with the SST surfaces for each story. From direct analysis of these data, we have the ability to answer questions like the following: (1) what mechanism, translational or torsional, the building is likely to develop; (2) if the answer to one is a torsional mechanism, what resisting planes in the plan have larger/smaller displacement demands; (3) how does one modify the SST surfaces to achieve more uniform displacement demands on the resisting planes of each story; and (4) what the effect of yielding is in the orthogonal resisting planes, and so forth.

## RETROFIT DESIGN EXAMPLE

The purpose of this section is to apply the conceptual guidelines developed in the previous section to a hypothetical retrofit solution of a building. Let us assume that we have been asked to provide a retrofit solution for the five-story building shown in Fig. 8. The building has a rectangular plan of dimensions 200 ft by 100 ft for the first story and 100 ft by 50 ft for the second and upper stories; it also has infinitely rigid floor diaphragms where all the story masses are lumped, and

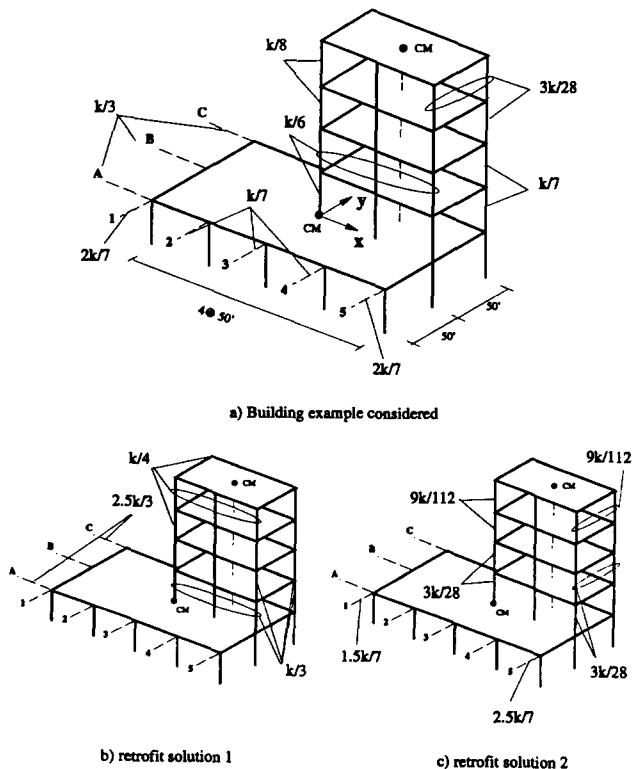


FIG. 8. Building Example Considered and Two Possible Retrofit Solutions to Improve Its Torsional Behavior

it is subjected to twice the E-W and N-S components of the El Centro earthquake in the x- and y-directions, respectively. The system has three and five resisting planes in the x- and y-directions in the first story, and two and three in these directions, respectively, in upper stories. Further, the stiffnesses of these planes are as shown and their yield deformations are all assumed to be equal to  $v_y = 0.02$  ft. Consequently, stiffnesses and strengths of different resisting planes are in the same proportion. Note also that the system has asymmetry in both directions as a result of the setback in the second story; besides, the building has resisting planes with lateral stiffness matrices which are not proportional. In that sense, this structure is different to the systems considered earlier in this study. This has been done intentionally to show that the conceptual design guidelines developed earlier for one-way unsymmetric systems with proportional resisting planes can also be applied effectively to more complex structures.

The most important feature of this structure is its irregularity in plan and height due to the setback, which produces an offset equal to  $0.25b$  between the centers of mass of the first and upper stories. Also, the building plan in the second and upper stories is asymmetric as a result of the larger stiffness and strength of resisting plane 5 compared to plane 3. Because of the setback and plan asymmetry, the building may develop significant torsional motions that may eventually lead to high demands on some resisting planes, thus, justifying the concern for its seismic safety.

The response results presented next are in a special format (see Fig. 9). Rows (1), (2), and (3) contain the response results for the first, second, and fifth stories of the building, respectively; columns (a), (b), and (c) present the floor displacements, force-deformation relations, and story shear and torque histories, respectively.

Before proposing different retrofit solutions for this structure it is necessary to understand its inelastic dynamic behavior. Shown in Fig. 9 is a summary of different response quantities in the building. It is apparent from parts 1a, 2a, and 3a that the floor displacements at the left and right edges of the CM are substantially different. This fact confirms our expectation that the system will undergo significant torsion. Observe that the ratio between peak displacements at the left and right edges ranges between 3 and 5 for different floors. Besides this

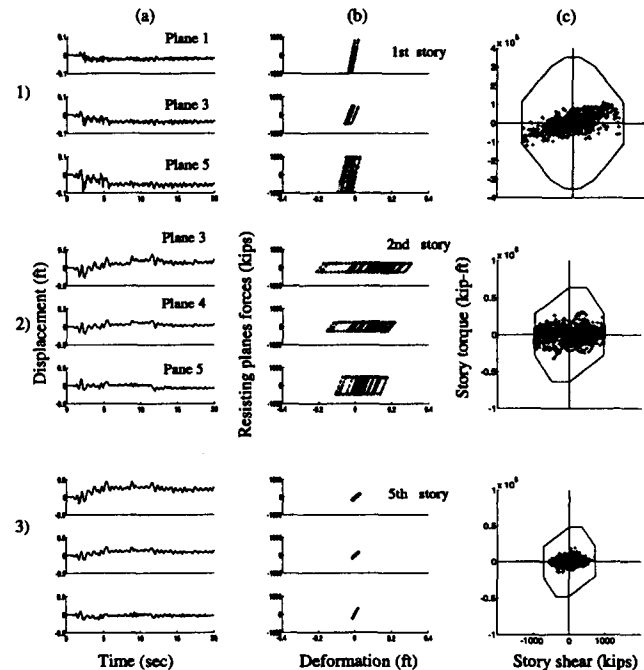


FIG. 9. Behavior of Building Example before Retrofit



undesirable discrepancy in edge displacements, the force-deformation histories presented in parts 1b, 2b, and 3b demonstrate that the peak deformation ductility demands for the most critical resisting plane in each story are 5, 15, and 1.5, approximately. The large ductility demand in the second story is accompanied by large differences among ductility demands on the resisting planes; these differences are close to 100% in the first story and 60% on the second story.

All the previous observations are verified by looking at the base shear and torque response histories superimposed on the SST surfaces (Fig. 9: 1c, 2c, and 3c). For example, Fig. 9: 1c shows a tendency of developing mechanisms at the upper and lower ends of the constant base shear branches in the first and third quadrants of the SST surface, respectively, associated with significant rotations of the building plan. Similarly, Fig. 9: 2c shows that yielding in the second story is quite extensive and spreads to the large-torque regions of the constant base shear branches where mechanisms become increasingly torsional. Therefore, there is clear evidence that the system is torsionally unbalanced and any proposed retrofit solution should aim to correct this unbalance in order to lead to more uniform deformation demands among resisting planes.

We will now consider two retrofit solutions of the system, which illustrate two important techniques for controlling the torsional behavior of a structure. They consider the possibility of (1) increasing the strength of resisting planes along the orthogonal direction ( $x$ -direction); and (2) modifying the stiffness and strength of key resisting planes in order to balance the torsional behavior.

### First Retrofit Solution

Recall from point seven under Conceptual Design Guidelines in the present paper that the effect of increasing the strength of the orthogonal resisting planes is to lead to more uniform deformation demands of the planes along the  $y$ -direction. Although this effect is reduced by the presence of an orthogonal component of ground motion (Fig. 7), increasing the strength of orthogonal planes will always reduce the plan rotations. Therefore, the first retrofit solution considered is to increase the strength and stiffness of these planes [Fig. 8(b)]; the strength of resisting planes A and C in the first story has been increased by 2.5, and by a factor of 2 in upper stories, such that the lateral capacity of each story in the  $x$ -direction has been doubled.

Shown in Fig. 10 is the earthquake response of the modified building. It is apparent by comparison with the responses of the original building (Fig. 9) that, as expected, the torsional unbalance in the system has been partially corrected. By this we mean that the displacements of different resisting planes become less different (Fig. 10: 1a, 2a, and 3a), their deformation ductility demands also become more similar (Fig. 10: 1b and 2b), and the mechanisms developed at different stories are less torsional (Fig. 10: 1c and 2c). For example, we have reduced the peak deformation ductility demands in the second story to about 10, similar for all resisting planes. The effect of the increased strength in orthogonal planes is most clearly seen in Fig. 10: 1c. The base shear and torque response history for this story shows a more symmetric concentration of shear-torque combinations about the zero torque axis than it did before (see Fig. 9: 1c). Also, the inelastic behavior of this story is now closer to the center of the constant base shear branches of the SST surface, implying that the system at collapse is likely to develop mechanisms that are predominantly translational in the first story.

Although the benefit of increasing the strength in the orthogonal planes is apparent, some aspects of the behavior of the new system are not completely satisfactory. First, given the large increase in strength of the orthogonal planes we

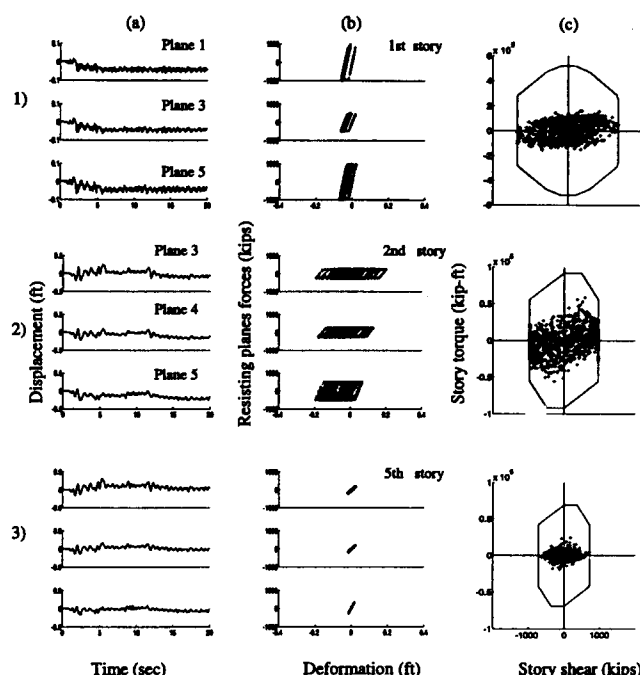


FIG. 10. Behavior of Building with Increased Strength in Orthogonal Resisting Planes (Retrofit Solution 1)

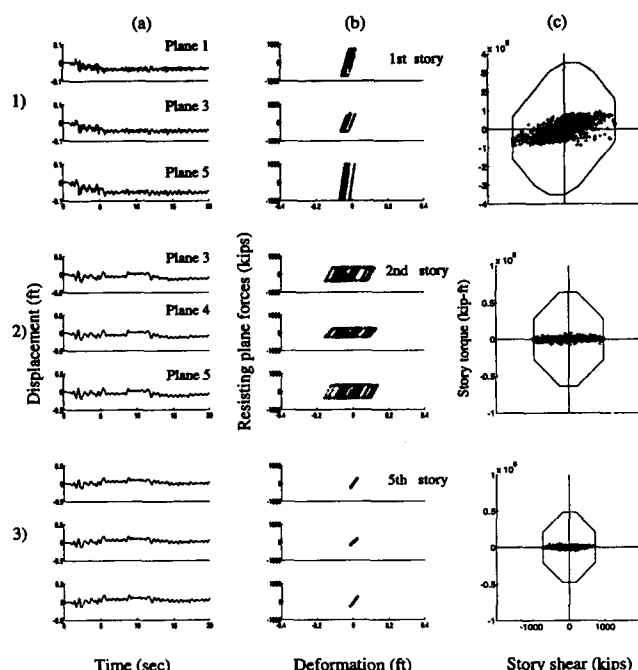
would have preferred a better agreement between the left and right edge displacement histories. Second, the force-deformation histories in the second story (Fig. 10: 2b) show that yielding in the resisting planes occurs asymmetrically about the force axis, indicating a residual drift in the structure. Third, the increase in strength in the orthogonal planes is accompanied by an increase in the story torques developed in the system (Fig. 10: 2c). Despite these deficiencies, the retrofit solution proposed accomplishes our goal of reducing differences in demands among resisting planes. However, as shown next, it is possible to achieve a much better performance by adjusting the stiffness and strength in the resisting planes.

### Second Retrofit Solution

We first recall from points 2, 3, and 4 under Conceptual Design Guidelines in this paper that by changing the stiffness and strength distribution in the system we may concentrate yielding in specific resisting planes of the structure. Thus, the strategy for this solution is to increase the yielding in those planes that are essentially elastic in the original system and reduce the yielding in planes that deform excessively in that system. More specifically we aim to (1) increase the yielding of resisting plane 1 and reduce the one of plane 5 in the first story (Fig. 9: 1b); (2) reduce the yielding in plane 3 in the second story (Fig. 9: 2b); and (3) improve the torsional behavior of the second story by inducing mechanisms that are predominantly translational in this story (Fig. 9: 2c).

Improving the torsional performance of a building has little to do with increasing the overall capacity of the system. It is indeed a problem of the planwise distribution of strength (and stiffness to a lesser degree). To emphasize this important concept, the retrofit solution proposed in Fig. 8(c) is such that it maintains the same lateral capacity as the original system. For this purpose the lateral capacity of resisting plane 1 in the first floor and 5 in the second and upper stories has been slightly reduced. These capacity reductions will not be introduced on the actual retrofit solution but the capacity of other planes will be increased slightly to ensure the relative capacity values presented in this figure. Note that the idea is to increase the first story capacity of plane 5 and simultaneously decrease the one





**FIG. 11. Behavior of Building with Modified Stiffness and Strength in Edge Resisting Planes (Retrofit Solution 2)**

of plane 1 by 25% so that the SST surface will lean toward the first and third quadrants, thus, forcing the story shear and torque combinations to touch this surface closer to the center of the constant base shear branches. This will even the ductility demands on planes 1 and 5. On the other hand, we have increased the strength of resisting plane 3 in the second floor by 50% and reduced the one of plane 5 by 25%, so as to increase the ductility demand in plane 5 and reduce it in plane 3 (Fig. 9: 2b).

Shown in Fig. 11 is the dynamic response of the system. The results are remarkable in many respects. First, the displacement histories at different locations of the building plan (Fig. 11: 1a, 2a, and 3a) are very similar, especially for the second and upper stories. Second, the force-deformation histories (Fig. 11: 1b, 2b, and 3b) show essentially identical peak deformation demands on the different resisting planes, as well as symmetric behavior about the force axis. Further, since all resisting planes are used more effectively the peak ductility demands have been reduced from 15 in the original system (Fig. 9: 2b) to about 8 in the new system (Fig. 11: 2b). Third, the story shear and torque histories have changed dramatically, especially those of the second and upper stories. Note that the second story shear and torque combinations lie now close to the zero torque axis (Fig. 11: 3b), showing the effectiveness of the retrofit scheme proposed. Although the base shear and torque history in the first story still goes predominantly in the first and third quadrants, the inelastic behavior takes place close to the center of the constant shear branches, implying that the mechanisms developed are mainly translational.

Therefore, as demonstrated by this example, the conceptual guidelines presented in the previous section concerning the inelastic behavior of asymmetric buildings provide a basis to develop practical solutions to improve the torsional behavior of an existing structure, even if that structure is highly asymmetric. Hereafter the retrofit solution can be tested further by inelastic analyses of the system. Such analyses, which seem very costly, can be greatly simplified using the simplified model developed in (De la Llera and Chopra 1995b).

## CONCLUSIONS

This study of the seismic behavior of asymmetric multistory buildings has led to the following conclusions.

The earthquake behavior of asymmetric single and multi-story buildings of the class considered in this investigation shows similar trends and is affected by the following same building characteristics: the strength of resisting planes and intensity of ground motion in the orthogonal direction, the stiffness and strength asymmetry in the system, and the distribution of strength between the core and edges of the building.

Stiffness and strength asymmetry may be effectively used to control the torsional performance of a structure. Stiffness asymmetry influences the story shear and torque combinations in elastic response, that is, inside the SST surface. Thus, by varying the stiffness eccentricity, story shear and torque combinations may be forced to fall into certain quadrants inside the surface. Further, strength asymmetry affects the shape of the SST surface, and, hence, it can be used to guide the inelastic behavior into certain desirable regions of the surface, such as the constant shear branches.

To create uniform inelastic deformation demands in the resisting planes of a given story, the story shear and torque combinations lying on the SST surface must not deviate significantly from the center of the constant shear branches. On the other hand, the worst behavior is achieved when these combinations lie at all time instants on one of the inclined branches of the surface, implying torsional mechanisms that leave the same resisting plane in the plan always elastic.

Increased strength in the resisting planes orthogonal to the direction of ground motion also leads to more uniform deformation demands among the resisting planes in the direction of ground motion. However, the influence of the orthogonal planes decreases as the intensity of the orthogonal ground motion component increases.

The building example considered has shown that it is even possible to correct the torsional unbalance of a very asymmetric system by manipulating the strength (and stiffness). Other retrofit situations may require additional tools, such as the use of stronger orthogonal resisting planes, or as suggested by preliminary results by lumping the strength close to the CM of the structure, or combinations of the aforementioned.

It has been demonstrated throughout this study that the knowledge of the SST surfaces in conjunction with the story shear and response histories is a powerful tool for conceptual understanding of the earthquake behavior of an asymmetric structure. Therefore, these concepts could be effectively used for preliminary analysis and design of asymmetric structures.

## ACKNOWLEDGMENTS

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