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Simplified procedure for seismic analysis of asymmetric buildings

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Abstract

A simplified procedure, which is a modification of the shear beam model, has been developed and applied to treat the seismic analysis of asymmetric buildings. This three dimensional procedure accounts for torsional coupling and the bending rotations at beam–column junctions and it can be used with a personal computer to give fast and reasonably accurate results, which compare well with those from comprehensive finite element analysis. This procedure will therefore be useful for preliminary seismic analysis and design of buildings. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

For a building to be symmetric it must have, at each floor level, coincident centres of mass and stiffness which lie on a common vertical axis. The centre of stiffness can be defined as the centroid of the stiffness coefficients (EI values, E is the Young's modulus and I; the second moment area) of all the vertical members about a horizontal axis perpendicular to the load. In practice, this is unlikely to occur and most buildings are unsymmetrical to some extent due to either (i) an unsymmetrical plan, or (ii) unsymmetrical elevation, or (iii) unsymmetrical distribution of vertical members or (iv) unsymmetrical mass distribution on the floors. Such buildings are called asymmetric buildings and when they are subjected to lateral loads such as earthquakes or winds, there is a complex interaction between the lateral and torsional components of the building response, called torsional coupling, which can result in significant response amplifications. Torsional coupling has been identified as one of the major causes of poor performance or failure of multi-storey buildings during recent earthquakes [1,2].

Several researchers have treated the lateral load response of buildings—but their investigations have been model specific, often using simple models, and hence the results are not applicable to a vast majority of "real" buildings. The finite element method can be used to treat all types of buildings. But a comprehensive finite element model will require extensive input and will take a long time to analyse, with the levels of accuracy often not justifying the time required for the analysis. This will render the finite element method unsuitable for the analyses of asymmetric buildings in most practical cases.

There has also been a great deal of research in the fields of seismic response of asymmetric buildings and the resulting torsional coupling. This research, unfortunately, has mainly been on simple and mostly single

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storey building models and has identified the uncoupled torsional/lateral frequency ratio to be an important parameter. It also attempted to extrapolate the results from the simple building models to multi-storey buildings. Recent research developments have shown that it is not possible to extrapolate the results from these simple models to realistic multi-storey buildings. In particular, the parameter torsional/lateral frequency ratio is difficult or impossible to determine in real buildings.

To address these concerns and to develop a simplified procedure for the seismic analysis of asymmetric buildings, a three dimensional analytical model with rigid slabs having three degrees of freedom per floor level and vertical members with three degrees of freedom at their ends was first developed [3]. In the second stage, the bending rotations about the two horizontal axes were also included as additional degrees of freedom at the ends of vertical members, increasing this to five, in order to accommodate more building types [4]. These extra degrees of freedom, however, can be condensed out prior to solution stage which still had only three degrees of freedom per floor. But the proportioning of the bending rotations at the ends of vertical members had remained questionable [5]. In the final stage of development, these bending rotations, which had been found to significantly influence the response [6,7] received special attention and the entire procedure has been refined and fine tuned and the analytical model calibrated [5]. It is intended that this final version of the procedure will be simple with effectively three degrees of freedom per floor and yet capable of modelling and analysing buildings which have both frames and a core (i.e. flexure and shear actions). This paper describes this important phase of development of the procedure and its application to the analysis of some typical buildings under lateral seismic loads. Lateral deflections, bending moments and shear forces are presented and the effects of asymmetry and the consequent response amplifications are demonstrated. Results indicate that the seismic response of asymmetric buildings depends on a number of factors such as the mass and stiffness distributions, configuration (or geometry) of the system, degree of asymmetry, natural periods and the earthquake loading. These findings emphasise the need for a realistic three dimensional analytical model, which has been the objective of the present paper.

At this stage the model is restricted to linear elastic analysis, which can be used in the quasi-static procedure proposed in many seismic design codes. The model can also carry out modal analysis which can be coupled to the response spectrum method and seismic time history analysis, all in the linear elastic range. It is expected to extend the procedure to non-linear analysis and the outcomes will be reported in another paper. The authors believe that the proposed procedure is simple and hence attractive to practising engineers and it will be especially useful for preliminary analysis and checking.

2. Past research

Many researchers have studied the lateral load analysis of buildings [3,4,8–12]. Some of these methods (models) were very simple and made no proper account of asymmetry while others used simplifying assumptions, which are not valid in all cases, and yet others used the finite strip method and must be considered on a case by case basis. Heidebrecht and Stafford Smith [8] considered a coupled frame–shear wall system to simplify the analysis, while Cheung and Tso [9] treated a certain type of setback buildings. Balendra et al. [10] and Swaddhiwudhipong et al. [11] used techniques similar to the finite strip method to model uniform multi-storey buildings. Stafford Smith and Cruvellier [12] developed a special model consisting of plane frames and bents to account for asymmetry. Thambiratnam and Irvine [3] and Thambiratnam and Thevendran [4] used a shear beam model and a modified shear beam model having three degrees of freedom per floor, to develop analytical procedures for treating certain classes of asymmetric buildings.

The response of asymmetric buildings to earthquake loading has also been treated by many researchers who have also studied the effects of torsional magnifications [1,13–21]. Most of these studies were based on analysis of single storey two degrees of freedom (one lateral and one torsional) models or special types of models. This allowed the researchers to identify important parameters related to torsional coupling, but were too simple to describe the full range of behaviour associated with a multi-storey asymmetric structures and therefore of limited use in their design. Kan and Chopra [13,14] treated a single storey structure consisting of a rigid deck supported by six columns and the rigid deck supported by an equivalent single column, while Hejal and Chopra [15,16] treated framed buildings. Chandler and Hutchinson [17,18] also studied the response of single storey building models under earthquake loadings and along with most of the other researchers identified the uncoupled torsional/lateral frequency ratio Ω and the static eccentricity e_S, defined as the distance between the centres of mass and stiffness at any floor level, to be the most important parameters and concluded that peak responses were obtained when Ω was close to unity, for small values of e_S. Rady and Hutchinson [19] used a flat response spectrum to analyse a single storey model consisting of a rigid deck supported by a four columns and a shear wall and observed the natural period of the structure to be an important system variable. Thambiratnam and Corderoy [1] used a shear beam model to treat asymmetric buildings and demonstrated the effect of asymmetry on the response. More recently Chandler and Duan [20] studied torsional coupling in a single storey model with three vertical elements (shear walls) and (among other things) showed that the natural period of vibration influenced the response of the stiffer elements and amplification was more pronounced when the natural period was high. Tso and Smith [21] studied the torsional provisions evaluated by adjusting the strength of lateral load resisting systems and compared the results with code provisions.

The ability of single storey models to represent accurately the response exhibited by multi-storey buildings subjected to seismic loads has remained questionable. Yoon and Stafford-Smith [22] concluded that "the single storey analogy for multi-storey buildings is correctly applicable to only proportionate structures, whose ratios between the stiffnesses of the lateral load resisting elements are constant throughout the height". Duan and Chandler [23] proposed a simplified procedure for analysing a proportionate structure. It will be clearly evident that this requirement cannot be met in most of the multi-storey buildings of today.

Most of the above findings have been confirmed by these and several other researchers and are now well understood. In summary, past research has indicated that there is difficulty in predicting response using simple models and based on simple parameters such as the uncoupled frequency ratio Ω. Other factors such as the building configuration, fundamental period T_1, earthquake loading and degree of asymmetry significantly influence the seismic response. The use of results from single storey models to predict the response of multi-storey structures seems questionable. The need for a three dimensional analytical model, which is comprehensive, but simple to use, can therefore be well appreciated.

3. Need for a realistic analytical model

Today there are many computer techniques and computer packages available to analyse structures. One of the most significant of these techniques is the finite element method. More recently, with new generations of supercomputers, sophisticated programming languages and new numerical techniques the finite element method has emerged as a tool with almost limitless versatility, but in spite of this, most practising engineers do not use full finite element analysis in the design of tall buildings. One of the problems associated with achieving this is that the computations involved in the dynamic analysis (required for earthquake loads) of high-rise buildings can be too intensive for the computer resources available.

For the realistic design of buildings, engineers use simplified finite element analyses or even plane frame analyses and these are generally limited to static analysis or modal analysis because of restricted computer resources. Practising engineers then rely on earthquake codes to account for the dynamic and torsional coupling effects in structures.

Cruz and Chopra [6,7] have shown that bending rotations at the ends of the columns of a symmetric building significantly influence its response and identified the beam-to-column stiffness ratio as one of the parameters to be considered. Later Hejal and Chopra [15,16] showed that for asymmetric structures the dynamic eccentricity is also influenced by this parameter. Prior to these studies, investigations on simple asymmetric systems used mostly single storey shear beam models. The present paper extends this investigation towards the development of a simple, but comprehensive analytical model, which is fast and capable of running on a microcomputer and hence useful to practising engineers.

4. Development of model

4.1. Basic theory and earlier models

Thambiratnam and Irvine [3] proposed a simple "shear beam model" with three degrees of freedom at the end of each vertical member and which resulted in three degrees of freedom per floor due to the assumption of rigid floor slabs. This three dimensional model, referred to as model 1, included the effects of torsional coupling and was suitable for analysing buildings with columns, but without heavy service cores or shear walls. Later on this model was extended by Thambiratnam and Thevendran [4] to have five degrees of freedom at the ends of the each vertical member. Bending rotations of the vertical members about the two horizontal axes were the extra degrees of freedom in this model and these were assumed to be proportional to the second moments of area about the appropriate axes. This model will be referred to a model 2. During the solution stage, these bending rotations were condensed reducing the problem to three degrees of freedom per floor as before. Recently, to address the shortcomings in the previous models, a third model, called model 3, was developed in which the bending rotations at the ends of vertical members received special attention. The entire procedure has been refined, fine tuned and the analytical model calibrated. All three models use Lagrange's equations to derive mass and stiffness matrices in terms of degrees of freedom located on an imaginary column at an arbitrary origin.

This paper will describe the development of the procedure for model 3 and its application to the analysis of some typical buildings under lateral loads.

The major assumptions of the models are

1. Floors are rigid diaphragms possessing three degrees of freedom (two lateral translations and a rotation about the vertical axis).
2. Kinetic energies of vertical members are either ignored or the column masses are lumped onto the floors.
3. The vertical members possess three (model 1) or five (models 2 and 3) degrees of freedom at each end.
4. The principal axes of all vertical members are assumed to lie along the horizontal x–y axes.
5. The lateral stiffness of any floor depends on the stiffness of the vertical members (columns and shear walls) just below and just above that floor level.
6. In the second model, the rotations at the ends of the vertical members are proportional to their respective second moments of area. These bending rotations are more accurately modelled in the present method.

Since the model treated in this paper is an extension of model 2, it is appropriate to commence with the basic theory of that model in which vertical elements have five degrees of freedom at each end as shown in Fig. 1. The degrees of freedom at a typical end "i" are: u_i, v_i, θ_i, α_i and β_i, where u_i, v_i, are the translations along the x and y axes, θ_i the rotation about the vertical axis and α_i and β_i are the bending rotations about the x and y axes respectively. The rotation α_i into the page has been omitted in Fig. 1 for clarity. Since the slabs are assumed to be rigid, the horizontal translations u_i, v_i and the rotation θ_i (and the corresponding velocities), at the end of any vertical member can be easily related to those at the same level on the reference column at the origin. In addition to this, in model 2, the bending rotations α_i and β_i are related to those at the reference column in proportion to the ratios of stiffness of the vertical member to that of the reference column.

4.2. Condensation of stiffness matrix and solution

In this model (model 2) the slabs have three degrees of freedom while the vertical elements have five degrees of freedom and hence the sizes of the stiffness and mass matrices will be different. The stiffness matrix is therefore condensed to yield three degrees of freedom per floor prior to solution. Further details are given in the paper by Thambiratnam and Thevendran [4]. The equations of motion can be solved to obtain:

1. static response of the structure,
2. free vibration response of the structure,
3. dynamic response of the structure to applied dynamic loads.

The dynamic response of the structure is obtained by numerical integration of the equation of motion using Newmark's constant acceleration method.

5. Present model

5.1. Proportioning bending rotations at the ends of vertical members

Wilkinson [5] has shown that portioning bending rotations according to the relative stiffness of the columns without considering the restraint of the slab would be inaccurate. The present approach, therefore, aims to model these bending rotations (about horizontal axes) in a more realistic manner.

The refinements carried out to develop the present analytical model can be understood by considering the three simple structures shown in Figs. 2–4. It will be shown later that structures in Figs. 2 and 3 will be

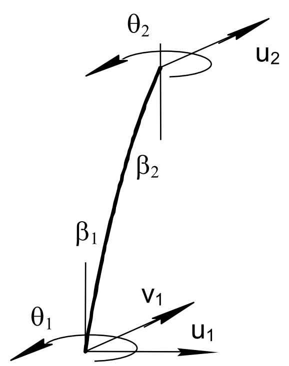

Fig. 1. Degrees of freedom at ends of vertical elements.

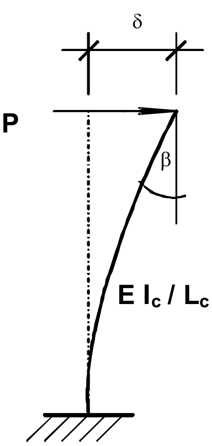

Fig. 2. Two degrees of freedom beam element.

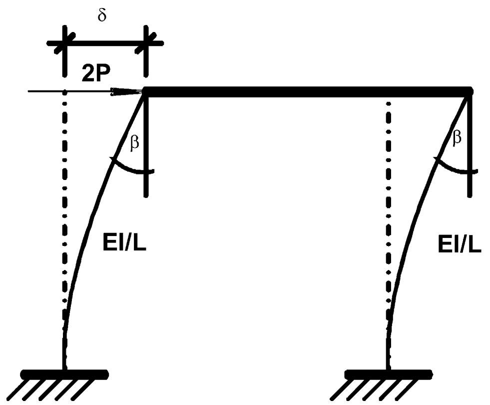

Fig. 3. Analogous plane frame.

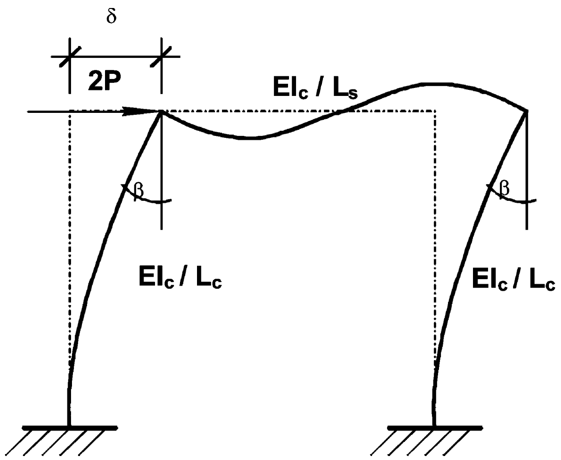

Fig. 4. True plane frame.

special (or limiting) cases of the structure in Fig. 4 with appropriate adjustment factors. Considering first the vertical element with two degrees of freedom, subjected to the lateral load P in Fig. 2, the 2×2 stiffness matrix will contain the terms corresponding to the appropriate degrees of freedom as seen in the governing equation below.

$$\begin{bmatrix} \frac{12EI_c}{L_c^3} & \frac{6EI_c}{L_c^2} \\ \frac{6EI_c}{L_c^2} & \frac{4EI_c}{L_c} \end{bmatrix} \begin{Bmatrix} \delta \\ \beta \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix} \tag{1}$$

In the simple frame shown in Fig. 3, the load has been doubled since there are now two columns. Since both columns in the frame have equal stiffness they will have the same rotations at the top. The stiffness matrix obtained for the frame from the procedure in model 2 [4] is identical to the stiffness matrix in Eq. (1) obtained directly for the cantilever structure in Fig. 2. The deflection and rotation at the level of the horizontal member should therefore be identical to those of the cantilever (i.e. the column tops are effectively free to rotate). In reality the slab will provide some restraint to even the heaviest of vertical members and therefore the model is more flexible than reality.

This shows the shortcoming in model 2 which provides either no rotational restraint if the columns have identical stiffness and, if the columns have different stiffness, the rotational restraint provided will be independent of any horizontal member. This has happened as the potential energy due to beam–slab deformation was not considered in the derivation of the stiffness matrix in model 2, which considers only the potential energy of the vertical members. By replacing the stiffness of the slab with a torsional spring located at the ends of the vertical elements, the potential energy of the system can be increased to match the actual case. This can be conveniently done by modifying term (2,2) in the stiffness matrix of Eq. (1), which corresponds to the bending rotation in the above case, or by modifying the appropriate diagonal terms in the stiffness matrix of any other structure to account for the influence of the slab. These modification factors represent the stiffness of an equivalent torsional spring and involve the beam to column stiffness ratios. For more complex structures, the position of the column and the influence of the other members in the building will also be of concern.

Considering the stiffness matrix of the structure, if the diagonal terms corresponding to the rotational stiffness at the top of a vertical element are set to infinity then this will effectively set the rotations to zero and convert the system to model 1 which is the shear beam model. Obviously the rotational stiffness at the ends of vertical members can be modified to any degree by multiplying the diagonal elements by a rotational stiffness modification factor, K_r, which will vary between one and infinity.

5.2. Rotational stiffness modification factor (K_r)

The value of the rotational stiffness modification factor K_r will be determined by comparing the response of the plane frame shown in Fig. 4 with that of the plane frame modelled as shown in Fig. 3. The deflection and rotation at the top of the column of the frame in Fig. 4 can be calculated by the stiffness method, or other technique, and are given in Eqs. (2) and (3) respectively.

$$\delta = \frac{PL_c^3}{3EI_c}\left[\frac{2I_cL_s + 3I_sL_c}{2I_cL_s + 12I_sL_c}\right] \tag{2}$$

$$\beta = \frac{PL_c^2 L_s}{2EI_c L_s + 12EI_s L_c} \tag{3}$$

Both these values approach the limiting values for a cantilever (Fig. 2) or a shear column (with $\beta = 0$ in Fig. 3) as the second moment of area of the slab approaches zero and infinity respectively. In order to control the rotation at the top of the vertical elements in Fig. 4, the diagonal term corresponding to the rotational degree of freedom (in the stiffness matrix corresponding to the structure in Fig. 3) can be multiplied by the factor K_r as shown in Eq. (4).

$$\begin{bmatrix} \frac{24EI_c}{L_c^3} & \frac{12EI_c}{L_c^2} \\ \frac{12EI_c}{L_c^2} & K_r\left(\frac{8EI_c}{L_c}\right) \end{bmatrix} \begin{Bmatrix} \delta \\ \beta \end{Bmatrix} = \begin{Bmatrix} 2P \\ 0 \end{Bmatrix} \tag{4}$$

Solving the above system of equations for the deflection or the rotation and comparing them with the respective values in Eq. (2) or Eq. (3) will yield a value for K_r as

$$K_r = 1 + 1.5\frac{I_s L_c}{I_c L_s} \tag{5}$$

5.3. Proportioning rotations in multi-storey buildings

To determine the value of K_r for a multi-storey building, the two storey building shown in Fig. 5a is considered. If the left- and right-hand columns have the same stiffness, a point of contra-flexure would exist in the middle of the slabs. Considering an equivalent frame as shown in Fig. 5b, the slope deflection equations for this frame are given, in matrix form, in Eq. (6).

$$\begin{bmatrix} \frac{24EI_c}{L_c^3} & 0 & \frac{-12EI_c}{L_c^3} & \frac{6EI_c}{L_c^2} \\ 0 & \left(\frac{8EI_c}{L_c} + \frac{6EI_s}{L_s}\right) & \frac{-6EI_c}{L_c^2} & \frac{2EI_c}{L_c} \\ \frac{-12EI_c}{L_c^3} & \frac{-6EI_c}{L_c^2} & \frac{12EI_c}{L_c^3} & \frac{-6EI_c}{L_c^2} \\ \frac{-6EI_c}{L_c^2} & \frac{2EI_c}{L_c} & \frac{-6EI_c}{L_c^2} & \left(\frac{4EI_c}{L_c} + \frac{6EI_s}{L_s}\right) \end{bmatrix} \times \begin{Bmatrix} \delta_1 \\ \beta \\ \delta_2 \\ \beta_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ 0 \\ P_2 \\ 0 \end{Bmatrix} \tag{6}$$

The corresponding stiffness matrix using model 2 (with five degrees of freedom at the ends of vertical members) described above with the rotational potential energies of the first and second storey columns multiplied by the factors K_{r1} and K_{r2} respectively is given in Eq. (7).

$$\begin{bmatrix} \frac{24EI_c}{L_c^3} & 0 & \frac{-12EI_c}{L_c^3} & \frac{6EI_c}{L_c^2} \\ 0 & \left(\frac{8EI_c}{L_c}\right)K_{r1} & \frac{-6EI_c}{L_c^2} & \frac{2EI_c}{L_c} \\ \frac{-12EI_c}{L_c^3} & \frac{-6EI_c}{L_c^2} & \frac{12EI_c}{L_c^3} & \frac{-6EI_c}{L_c^2} \\ \frac{-6EI_c}{L_c^2} & \frac{2EI_c}{L_c} & \frac{-6EI_c}{L_c^2} & \left(\frac{4EI_c}{L_c}\right)K_{r2} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \beta \\ \delta_2 \\ \beta_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ 0 \\ P_2 \\ 0 \end{Bmatrix} \tag{7}$$

These two matrices are identical except for the diagonal terms pertaining to rotation. Setting these elements equal to each other gives the relationships

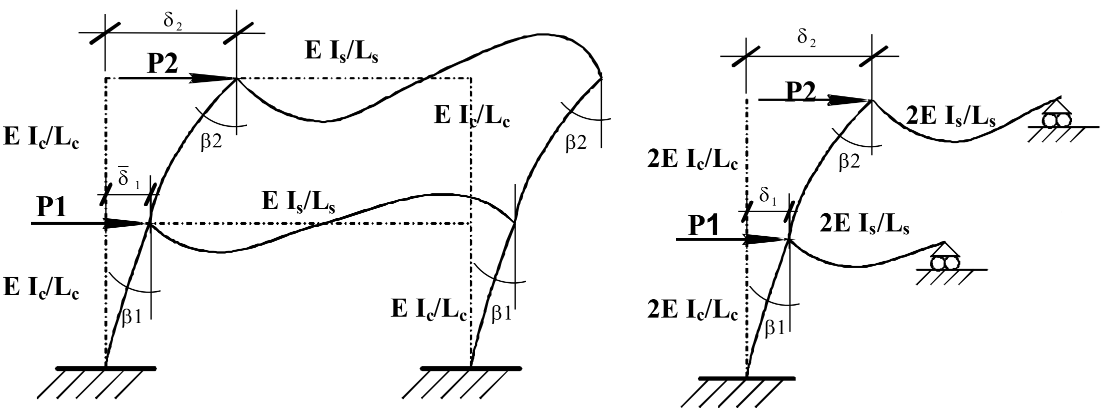

Fig. 5. Frames with columns having different stiffness.

$$K_{r1} = 1 + 0.75\frac{I_s L_c}{I_c L_s} \tag{8}$$

$$K_{r2} = 1 + 1.5\frac{I_s L_c}{I_c L_s} \tag{9}$$

In multi-storey buildings, there are vertical members above and below all intermediate floor slabs, except the top floor slab (or roof slab) and the ground floor slab. Hence buildings with more than two storeys can be treated as described above and K_r values for vertical members in all stories between the ground floor and the top storey floor can be calculated from Eq. (8), while the K_r value for the top storey members can be calculated from Eq. (9).

5.4. Proportioning rotations in columns with different stiffness

While a straightforward exact solution for K_r exists for the simple frames treated above, it will not be the case for frames in which the columns have different stiffnesses or when the number of bays is increased. Wilkinson [5] has clearly demonstrated the inadequacy of model 2 to provide realistic rotational stiffness at the column ends by treating a simple frame (Fig. 6) in which one column was "n times" ($n > 1$) stiffer than the other, and studied the effect of n. When n is more than 10, the behaviour of the frame is dominated by the stiffer column. He has also pointed out that this is mainly because model 2 does not consider the rotational restraint at the column end imparted by the slab/beam. The net rotational restraint at a column end depends on the stiffness of the column and the stiffness of the slab and/or beam and the limiting cases of a cantilever column or a shear column result when the slab is very flexible or very stiff relative to the column.

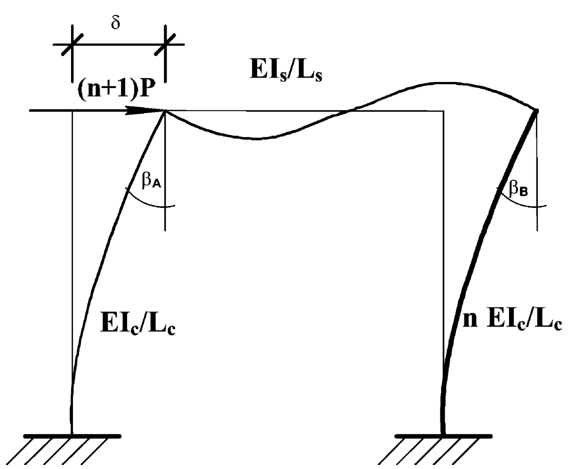

Fig. 6. Two storey frame and simpler equivalent frame.

To obtain an K_r factor for the simple frame shown in Fig. 6 where the columns have different stiffnesses, a method similar to what was used earlier is employed, but with the assumption,

$$\beta_B = C_2 \beta_A \tag{10}$$

Using the usual stiffness method, the governing equations are

$$\begin{bmatrix} \frac{12EI_c}{L_c^3}(n+1) & \frac{6EI_c}{L_c^2} & \frac{6EI_c}{L_c^2}n \\ \frac{6EI_c}{L_c^2} & \left(\frac{4EI_s}{L_s} + \frac{4EI_c}{L_c}\right) & \frac{2EI_s}{L_s} \\ \frac{6EI_c}{L_c^2}n & \frac{2EI_s}{L_s} & \left(\frac{4EI_s}{L_s} + \frac{4EI_c}{L_c}n\right) \end{bmatrix} \times \begin{Bmatrix} \delta \\ \beta_A \\ \beta_B \end{Bmatrix} = \begin{Bmatrix} P(n+1) \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

The corresponding stiffness matrix from the theory of model 2 (five degrees of freedom model) with the term controlling the rotation multiplied by the factor K_r is given in Eq. (12).

$$\begin{bmatrix} \frac{12EI_c}{L_c^2}(n+1) & \frac{6EI_c}{L_c^2}(n+1) \\ \frac{6EI_c}{L_c}(n+1) & \left(K_r\frac{4EI_c}{L_c}\right)(n+1) \end{bmatrix} \begin{Bmatrix} \delta \\ \beta \end{Bmatrix} = \begin{Bmatrix} P(n+1) \\ 0 \end{Bmatrix} \tag{12}$$

Eq. (12) pertains to a two degrees of freedom system while Eq. (11) in its present form pertains to a three degrees of freedom system. To overcome this incompatibility, β_A in Eq. (11) is replaced by $C_2\beta_B$ and the last two rows are added together to result in Eq. (13).

$$\begin{bmatrix} \frac{12EI_c}{L_c^2}(n+1) & \frac{6EI_c}{L_c^2}(nC_2+1) \\ \frac{6EI_c}{L_c}(n+1) & \left(\frac{6EI_s}{L_s}(1+C_2) + \frac{4EI_c}{L_c}\right)(nC_2+1) \end{bmatrix} \times \begin{Bmatrix} \delta \\ \beta_A \end{Bmatrix} = \begin{Bmatrix} P(n+1) \\ 0 \end{Bmatrix} \tag{13}$$

Equating the first rows of Eqs. (12) and (13) gives:

$$(1 + nC_2)\beta_A = (n+1)\beta \tag{14}$$

Equating the second rows of Eqs. (12) and (13) and using Eq. (14) to replace and eliminate the rotations gives:

$$K_r = 1 + 1.5\frac{I_s I_c}{I_c L_s}\left(\frac{1 + C_2}{1 + nC_2}\right) \tag{15}$$

For a multi-storey building the constant of 1.5 in the above equation will become 0.75 for storeys between the ground floor and the top storey. Using the stiffness method (or any comparable technique) the relative rotation of the columns (i.e. $\beta_B = C_2\beta_A$) is given by,

$$\beta_B = \left[\frac{2n\ I_c L_s + I_s L_c\ (2n-1)}{2n\ I_c L_s + I_s L_c\ (2-n)}\right]\beta_A \tag{16}$$

Eq. (16) can be used to calculate the ratio of column rotations (i.e. C_2) which may be substituted into Eq. (15) to calculate K_r. Finally, Eq. (14) can be used to proportion the rotations at the tops of the columns. The present model is developed by using Eqs. (12), and (14)–(16) and will yield identical values for the deflection and rotation of a building frame to those obtained by a plane frame analysis.

As mentioned earlier, when the stiffness of one column is of the order of ten times stiffer than the other column, the stiffer column tends to dominate the overall behaviour of the frame. It is therefore possible to approximate the behaviour of such a frame by calculating K_r using Eq. (5), in which I_c should correspond to the stiffer column.

5.5. Proportioning bending rotations in multi-bay structures

The procedure for this case is similar to the one discussed above. The rotations at the ends of the columns are assumed to be proportional to that of a reference column in the frame, with proportionality factors β_i where necessary. The equations for the frame obtained from the traditional stiffness method are compared with those obtained from the simplified five DOF model (model 2). By simplifying the former equations (as done before) and comparing appropriate rows from the two sets of equations, expressions for proportionality factor(s) β_i and the rotation modification factor K_r are obtained. Further details of this procedure can be found in Ref. [5].

5.6. Adjustment factor for torsion

Three dimensional asymmetric buildings twist about a vertical axis and this will couple the deflections and bending rotations to the twist of the building. The bending rotations of vertical members, about horizontal axes, have thus far been controlled by modifying appropriate elements in the stiffness matrix by the factor K_r. This factor only modifies the diagonal elements of the stiffness matrix and therefore will not affect the terms which couple the deflections and bending rotations of the vertical members to the twist of the slab. This when a building twists, there will be no bending rotations generated as these two phenomena are not coupled. There is no simple method of including this coupling effect directly and the model will therefore be not very accurate for cases of significant torsion, which are fortunately, very rare. To improve the accuracy of the model for analysing three dimensional asymmetric buildings, a torsion adjustment factor has been incorporated in the proposed model [5]. The model now yields results which compare very well with available solutions.

To determine the torsion adjustment factor, a number of numerical examples were analysed using both the present simplified model and comprehensive finite element analysis. When the results for deflections and rotations were compared, it was found, as expected that the simple model was too stiff by about 2.5 times. A torsion adjustment factor of 2.5 was introduced by dividing the appropriate leading diagonal term by 2.5. The results obtained with this adjusted model compared well with those from the finite element analysis. The above results have been incorporated into the analytical procedure described and used in this paper. Further details on this procedure are available from Ref. [5].

The theory described above has been used to develop a computer based procedure capable of running on a personal computer. This computer model will achieve the following objectives:

1. To be comprehensive but be simple to use and yield fast results.
2. It will be applicable to most building configurations, but in order to keep it sufficiently simple it does not consider structures with sophisticated lateral load resisting mechanisms, such as buildings with transfer girders.
3. To be used in the preliminary design of multistorey buildings.
4. To be used to analyse earthquake and other lateral load effects on a range of buildings and provide simplified methods for designing multi-storey buildings.
5. To identify important parameters and study the influence of these on the response of buildings.
6. To be used to analyse existing buildings that may need retrofitting to achieve a desired response to a design earthquake.

6. Applications

Several numerical examples have been treated to demonstrate the versatility of the proposed analytical model, which is equally capable of analysing both regular and less regular structures [5]. In this paper three such examples are presented and the results have been compared with those obtained from a finite element analysis, using a commercially available program. In the finite element analysis all vertical elements were modelled with beam elements and the slab was modelled as a grillage of beam elements with matching properties. Convergence was established. This finite element model had earlier been calibrated using some experimental testing.

In determining the factor K_r, simple assumptions, on the relative rotations of the columns and the effective second moment of area of the slab, have been made. This is to demonstrate how reasonably accurate answers may be obtained from such simple assumptions. For even more accurate answers, simple, single storey, finite element models of the structure have been analysed. K_r and T_r factors have been obtained by comparing the response of these single storey models with the response of a single storey model using the proposed method.

6.1. Example 1—10 storey square building with a core

A 10 storey building with the prismatic, square cross-section and constant storey height of 3 m, shown in Fig. 7 is considered. All columns in this building are $0.50\ \text{m} \times 0.50\ \text{m}$ in cross-section, while all floor slabs have a weight density of 8 kPa. The outer dimensions of the core are $2\ \text{m} \times 2\ \text{m}$ with a wall thickness of 0.15 m. The material properties of this concrete building are: Young's modulus, $E = 30,000$ MPa and Poisson's ratio $v = 0.2$. For the static analysis, the building is given an equivalent earthquake loading as shown in the Fig. 7. K_r was chosen by comparing the deflections of two single storey models (having the plan shown in Fig. 7), analysed using the finite element method and the proposed procedure, and adjusting the factor K_r to match the deflections. A value of $K_r = 1.021$ was obtained. The deflections and moments in the core are shown in Figs. 8 and 9 respectively, and compared with those obtained from the finite element analysis. Deflections and moments calculated by the proposed model are shown to be in excellent agreement with the finite element analysis. Fig. 8 also shows the results obtained with the earlier models, i.e., models 1 and 2, and the improvement in results is clearly evident.

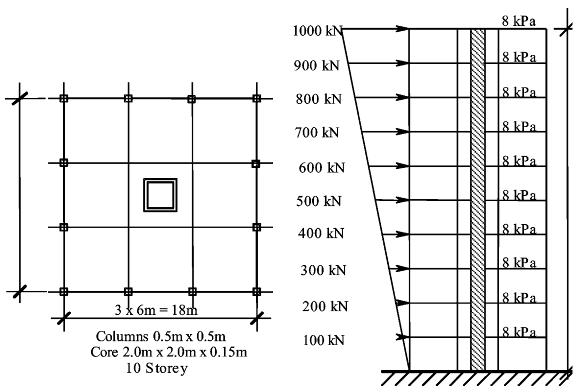

Fig. 7. Ten storey building with central core and seismic loading.

The core was next moved to the extreme right so as to occupy a cental position on the right-hand edge of the

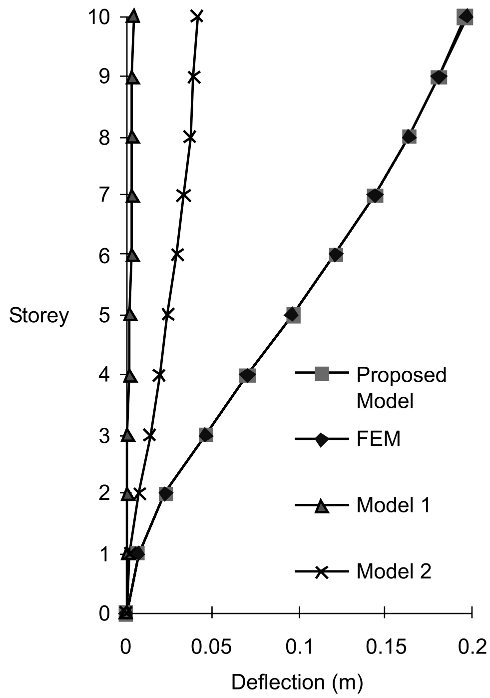

Fig. 8. Lateral deflections of core.

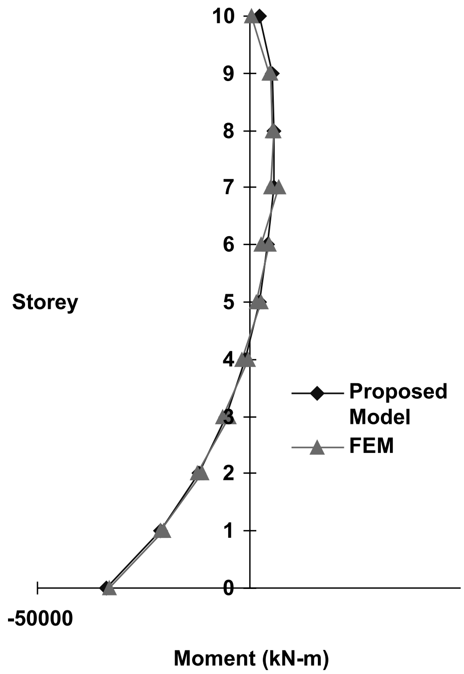

Fig. 9. Bending moments in core.

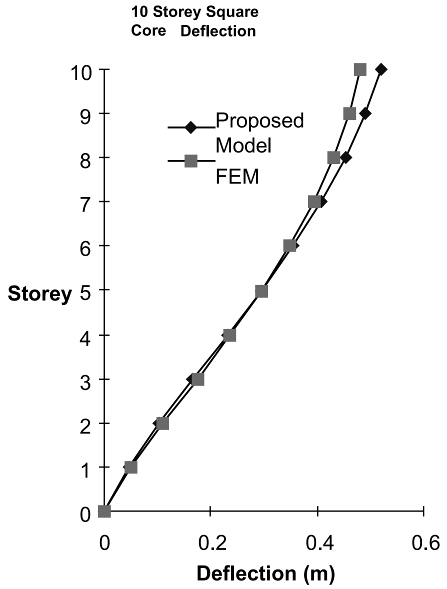

Fig. 10. Deflections of column at lower left-hand corner.

building, which was then analysed as before. Since there was torsional coupling in this structure, it was necessary to determine the torsion adjustment factor T_r. In order to do so, a single storey model was analysed using a commercially available structural analysis package and the twists of the slab matched to the twists obtained by the proposed model and the value of T_r was determined as 0.125. K_r was kept at its previous value [5]. Fig. 10 shows the displacement of the column situated at the lower left-hand corner, compared with that obtained from the analysis of a finite element model. It can be seen that there is excellent agreement in the results from the two analyses.

6.2. Example 2—15 storey L-shaped building

A 15 storey building with the prismatic, L-shaped cross-section and constant storey height of 3 m, shown in Fig. 11 is considered. All columns in this building are 0.30 m × 0.30 m in cross-section. The outer dimensions of the core are 2.0 m × 2.0 m with a wall thickness of 0.15 m. All floor slabs have a weight density of 8 kPa. The material properties of this building (E and v) are as in the previous example, while the equivalent seismic loading is shown in the figure. The value of K_r used in this example is 1.026. The assumptions and calculations used to derive the value of K_r can be found in Ref. [5]. Since this building has fairly slender columns and is taller than the previous 10 storey example the value of T_r will be smaller than that used earlier. T_r has been given a value of one (i.e. no adjustment to the torsional stiffness). Deflections of column 1 and the core and the bending moments in the core, obtained from the present analysis, are shown in Figs. 12–14 respectively, and compared with those obtained from a finite element analysis. It can be seen that the results compare quite favourably. Figs. 12 and 13 also show the results obtained by using models 1 and 2 and the improvements in the results obtained with the present model are again clearly evident.

The above results include the effects of torsional coupling in this asymmetric building, where the load applied at the centre of mass (6.6 m from the lower left-hand corner in Fig. 11 and indicated with '+'), and the resultant resisting force acting at and the centre of stiffness located close to the core form a couple causing the structure to twist about a vertical axis.

6.2.1. Natural periods

The first five natural periods of vibration of the L-shaped building obtained from the present procedure and the finite element analysis are presented in Table 1.

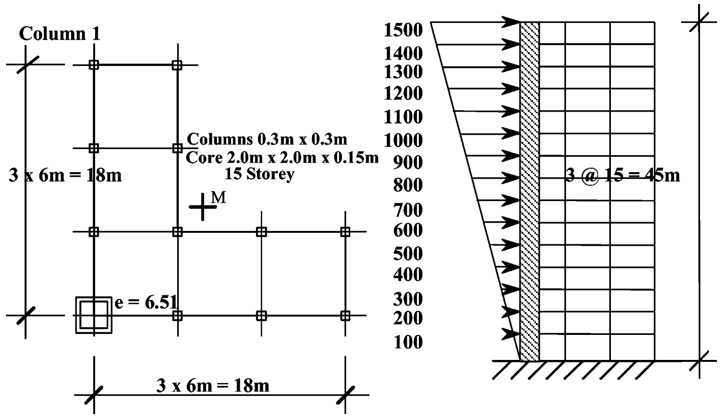

Fig. 11. 15 Storey L-shaped building and seismic loading.

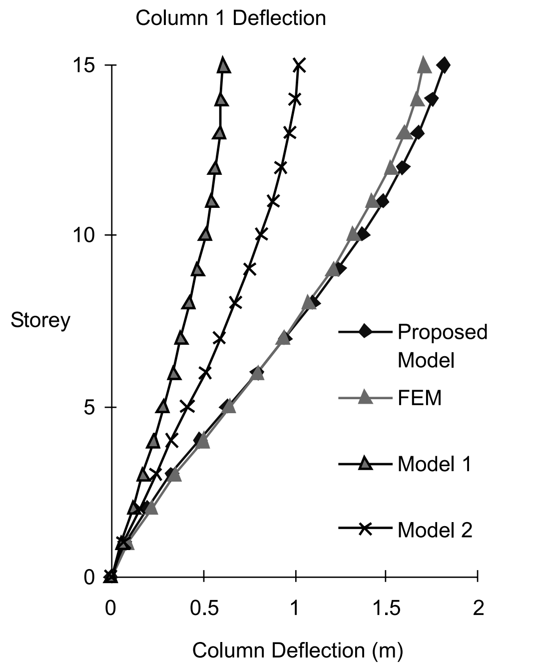

Fig. 12. Deflections of column 1 in 15 storey L-shaped building.

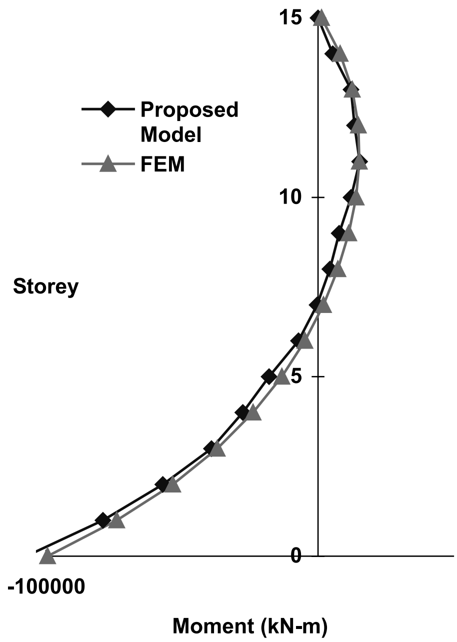

Fig. 14. Core moments for 15 storey L-shaped building.

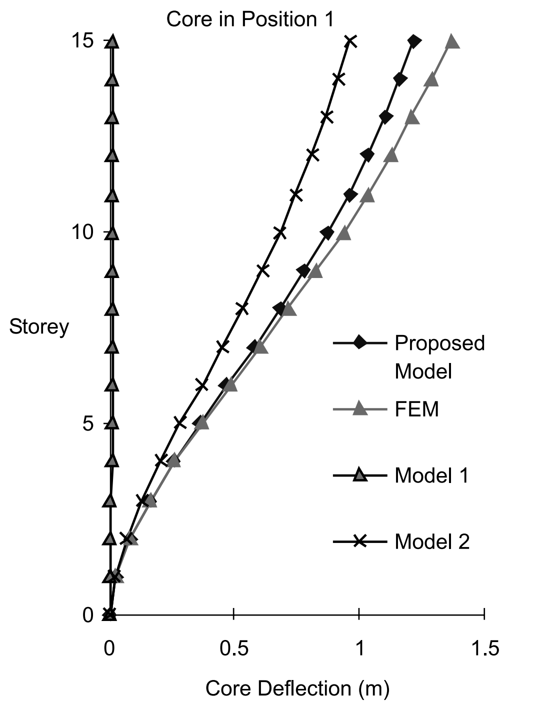

Fig. 13. Deflections of core in 15 storey L-shaped building.

Table 1
Natural periods in L-shaped building

Mode	Proposed method	Finite element method
1	2.67	2.60
2	2.21	2.34
3	0.94	1.65
4	0.76	0.82
5	0.62	0.65

From this table it can be seen that the periods of the first two modes, which are in the lateral direction in both analyses, compare well. In seismic response, usually the most significant contribution is made by the lower modes and hence the proposed model will yield reasonably accurate results. In any case, the finite element model has more degrees of freedom and direct match across all five periods may not be justified. When the core was moved to the uppermost position along the left-hand edge of the building, there were no significant changes in the results. This is because there was no change in the lateral stiffness of the two structures and no change in the eccentricity or lever arm between the centre of mass and centre of stiffness, the latter being almost at the core due to its very high stiffness compared to the columns.

6.3. Example 3—3 storey set back building

A three storey building, similar to the example treated by Thambiratnam and Irvine (1989), with setbacks at all levels and constant storey height of 3 m, shown in

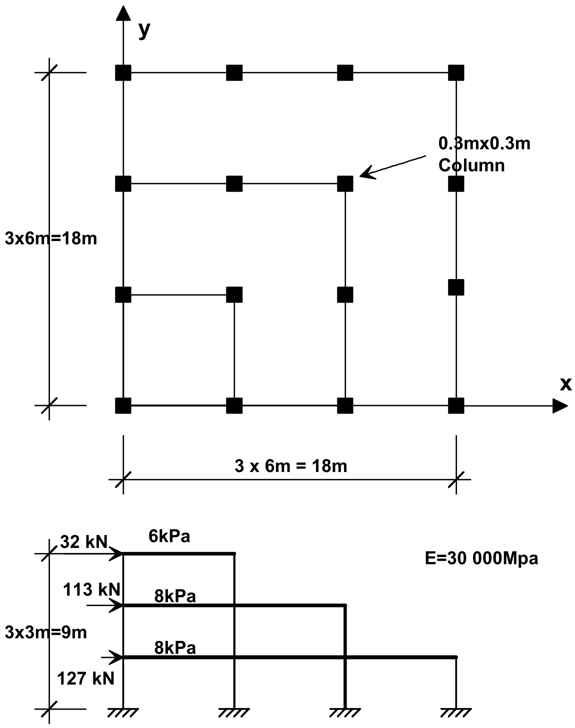

Fig. 15. Three storey setback building.

Fig. 15 is considered. All columns in this building are 0.35 m × 0.35 m in cross-section. The material properties of this building (E and v) are as in the previous example. The building is subjected to a quasi-static earthquake load as shown in the figure. The building was analysed using the proposed model and a three dimensional finite element model using beam elements. In the finite element analysis the columns were modelled with beam elements. The slab was also modelled by equivalent beam elements running between these columns. Since each storey has a different number of columns each storey will have a different K_r factor. The values of K_r used in the analysis are $K_r = 2.6$, 2.9 and 2.2 for levels 1, 2 and 3 respectively. The calculations used to obtain K_r are given in Ref. [5]. For this example the value of T_r was set to 1.

The deflections in the direction of the loading at the lower left-hand column are presented in Table 2, from which it can be seen that the results compare favourably. Deflections and rotations at other column positions also compared in an analogous manner. The shear forces in each of the columns, in the front elevation of the building, in the direction of loading are shown in Fig. 16, together with those from the finite element analyses (within brackets). It is evident that these results too compare favourably and are sufficiently accurate for design purposes.

Table 2
Lateral deflections (mm) of column in setback building

Level	Proposed method	Finite element method
1	1.64	1.55
2	3.44	3.25
3	4.55	4.28

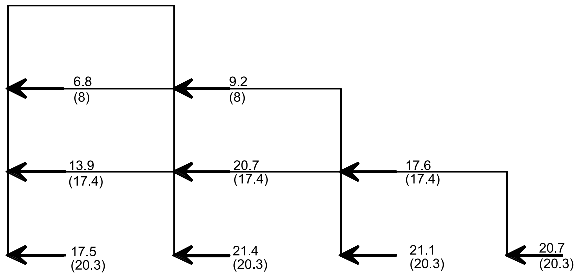

Fig. 16. Shear forces in the columns.

7. Conclusions

Presently buildings in many earthquake prone areas of the world are built to design codes and yet many still suffer failures during earthquakes. This may mean that there are deficiencies in design codes. Earthquakes cannot be avoided and hence structural engineers need to design buildings to respond favourably during earthquakes. Most buildings are asymmetric due to one reason or another and the torsional coupling in such asymmetric buildings has been identified as one of the major reasons for the failure or poor performance of buildings during recent earthquakes, including the recent Turkey earthquake. Past research has shown that results from seismic analysis have been model dependent and hence indicate the need for an adequate three dimensional analytical model. This paper presented the development and application of such a model, which is an extension of earlier basic models. It has three degrees of freedom at each floor at the solution stage. Using simple assumptions to calculate the factor K_r, required to model the bending rotations between the vertical and horizontal members, the proposed model gives results with sufficient accuracy for preliminary design purposes. For significant torsion an adjustment factor T_r was introduced at the appropriate location in the stiffness matrix to modify the torsional stiffness in the proposed model. For more accurate results, the values of the adjustment factors K_r and T_r are first determined by comparing the responses of simple single storey models obtained by the proposed method and finite element analysis. The K_r and T_r so determined are then used in the subsequent analysis by the proposed method.

The overall behaviour of buildings, including those with flexural and shear load resisting mechanisms, are modelled well by the proposed model. Results obtained from the analysis of three different buildings compare reasonably well with those from a comprehensive finite element analysis, which requires a much larger (>50) time compared to analysis by the proposed method. The natural periods of the structure treated in Section 6.2.1 agreed well with those from the finite element model. Past research has also shown that column and core size, stiffness and its distribution, slab stiffness and eccentricity were important parameters in determining the response of a building. The examples presented have demonstrated that all these influencing factors can be taken into account by the proposed model. In conclusion, the proposed model meets the needs of researchers and designers since it can:

- Run on a microcomputer (PC).
- Achieve levels of accuracy approaching that of a comprehensive finite element analysis, but in a much shorter time.
- Calculate the response of a structure by all three analysis method, viz, the quasi-static, response spectrum and time history methods, allowed in seismic codes of practice.
- Be sufficiently accurate for preliminary analysis and design.
- Adequately describes the behaviour of the building, including shear and flexural actions of the vertical members.

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