Demonstration of Lateral-Torsional Coupling in Building Structures

A PROJECT DEVELOPED FOR THE UNIVERSITY CONSORTIUM ON INSTRUCTIONAL SHAKE TABLES

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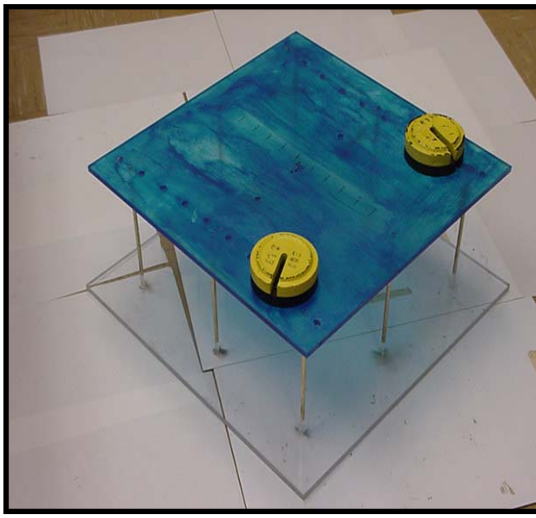

Required Equipment:

- Shake Table
- One-Story Plexiglas Model
- 2-Pound Circular Weights
- Four Single-Channel Accelerometers
- Dynamic Data Acquisition Board
- Computer
- Matlab
- Data Acqusition Program
- SAP2000

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Student's Manual

University of California, Los Angeles

Objectives:

This experiment serves two main objectives:

(1) To demonstrate the dynamic lateral-torsional coupling phenomenon exhibited by base-isolated building structures due to non-coincident center of mass and center of stiffness.

(2) To compare experimental simulation with numerical simulation of a specialized structural dynamic experiment.

1. INTRODUCTION

Base-isolated buildings can be idealized as single-story structures with cylindrical columns. In this experiment, you will be directed to build a single-story, two-bay by two-bay, Plexiglas model with adjustable level of mass eccentricity. Upon completion of the model, you will perform static stiffness tests and free vibration tests to determine the actual properties of the model such as stiffness, damping ratio, and natural vibration frequencies. You will then compare the system properties identified experimentally with those predicted by the theory or simulated numerically. Next, you will operate the shaking table to simulate earthquake excitations and observe the seismic response of the physical model for different levels of eccentricity between the center of mass and the center of stiffness. To further enrich your background in the use of finite element programs, you will simulate numerically the seismic response of the physical model using a finite element model created in SAP2000. You will then compare the seismic response observed experimentally with that predicted numerically and explain the possible sources of discrepancy.

2. BACKGROUND

Before beginning the experiment, it is necessary to understand the concepts and rationale behind the model idealizations.

What is an eccentric system?

An eccentric system is defined as a system with non-coincident center of mass and center of stiffness. When such a system is subjected to dynamic excitations (e.g., earthquakes, wind, ocean waves), the inertia forces can be modeled as acting through the center of mass, while the resultants of the resisting forces respond through the center of stiffness. This creates a moment between the two opposing forces, resulting into a torsional effect coupled with the lateral motion. An eccentric system is depicted in Figure 1.

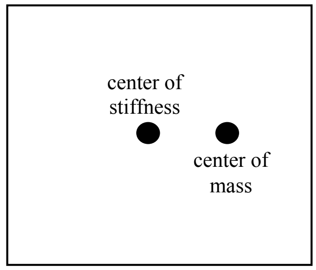

Figure 1: Plan View of an Eccentric System

In real life, eccentricity in a building is impossible to eliminate. Even if the design is nominally symmetric, accidental eccentricity is unavoidable due to imperfection in construction and uncertainty about the spatial distribution of dead and live loads. Therefore, structural engineers must understand its effects on the dynamic response of the building and account for these effects at the design stage.

How does a single-story building structure represent a multi-story base-isolated building?

Base isolators are used as structural devices to lessen the damage done to a building during earthquakes. Base isolators are typically cylindrical laminated-rubber bearings designed to take large shear deformations. They are interposed between the base of the building and its foundation. During an earthquake, the base isolators act essentially as shock absorbers, so that the building above undergoes less deformation. Consequently, the building suffers much less damage and the risk of injury to the occupants is decreased.

Figure 2: Typical Base Isolator

Since deformations localize mainly in the base-isolators, the structure above can be assumed to displace as a rigid unit. This explains why a multi-story base-isolated building structure can be idealized as a single story structure. In the physical model used here, the base-isolators are represented by flexible, cylindrical columns supporting the roof.

3. THEORY

3.1 Multiple-Degree-of-Freedom System

In this experiment, the model under investigation is a one-story structure composed of a rigid square top plate supported by nine columns. The coordinate system of the structure is illustrated in Figure 3. The x-axis (along the lateral direction) crosses the y-axis (along longitudinal direction) at the center of mass of the superstructure (top plate + added weights).

The columns of the model are assumed to be inextensible and their mass negligible compared to the mass of the whole system. The top plate is quasi-rigid in and out of plane. Thus, the system has three degrees of freedom, namely u_x, u_y, and u_θ as illustrated in Figure 3. Columns are placed symmetrically in both the x and y directions so that the center of rigidity is located at the geometric center of the top plate. Different levels of eccentricity are introduced into the structure by placing additional concentrated masses at various pre-determined locations on the top plate. As a result, the center of mass of the whole system varies according to the location of the added masses. For the purpose of this experiment, the model is excited only in the y-direction and eccentricity is imposed only along the x-axis. The distance between the two centers (center of mass, c.m., and center of stiffness, c.s.) along the x- and y-direction is denoted as eccentricity along the x-direction, E_x, and eccentricity along the y-direction, E_Y, respectively.

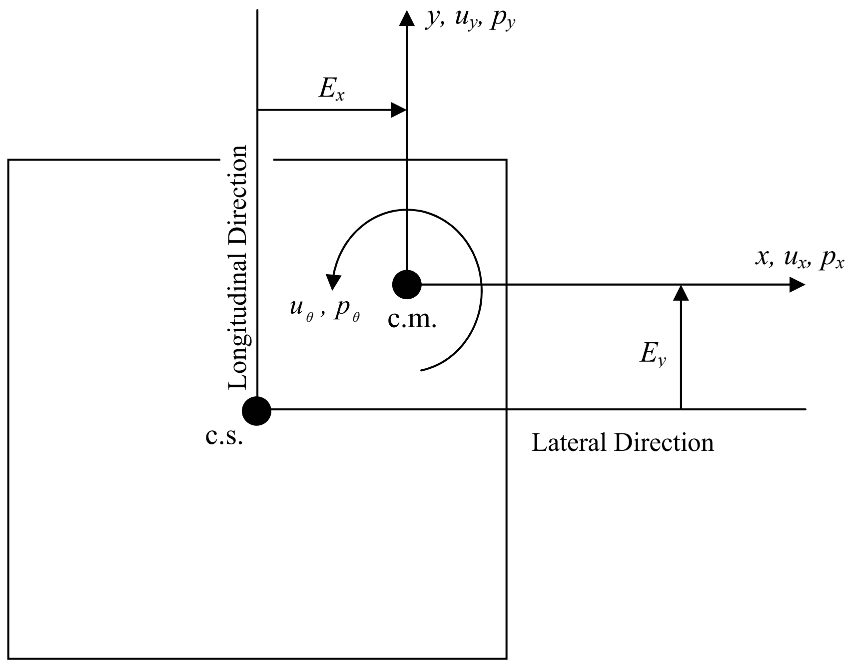

Figure 3: Coordinate System

The general equation of motion for a multiple-degree-of-freedom system is derived from Newton's second law and takes the form

$$\underset{(1)}{\boldsymbol{m}\ddot{\boldsymbol{u}}(t)} + \underset{(2)}{\boldsymbol{c}\dot{\boldsymbol{u}}(t)} + \underset{(3)}{\boldsymbol{k}\boldsymbol{u}(t)} = \underset{(4)}{\boldsymbol{p}(t)} \tag{1}$$

where $\boldsymbol{u}(t)$ is the displacement vector of the system as a function of time and a superimposed dot denotes a single differentiation with respect to time; $\boldsymbol{m}$, $\boldsymbol{c}$, and $\boldsymbol{k}$ are the mass, damping, and stiffness matrices, respectively; $\boldsymbol{p}(t)$ is the dynamic forcing vector function. Equation (1) expresses the equilibrium between the system's inertial forces (1), damping forces (2), elastic restoring forces (3), and the external dynamic forces (4).

The dynamic response of the eccentric system is derived under two hypotheses:

(1) The lateral stiffness of each base isolator (column) is uniform in all directions of deformation.

(2) The rotational response u_θ developed under dynamic excitation is small so that $u_\theta \approx \sin(u_\theta) \approx \tan(u_\theta)$.

Hence, the dynamic response is governed by the following set of coupled differential equations of motion. Note that based on the assumption stated above, $E_y = p_x = p_\theta = 0$.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & (m\cdot\rho^2) \end{bmatrix} \cdot \begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \ddot{u}_\theta(t) \end{bmatrix} + [C'] \cdot \begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \dot{u}_\theta(t) \end{bmatrix} + \begin{bmatrix} k & 0 & (k\cdot E_y) \\ 0 & k & (-k\cdot E_x) \\ (k\cdot E_y) & (-k\cdot E_x) & k_{\theta\theta} \end{bmatrix} \cdot \begin{bmatrix} u_x(t) \\ u_y(t) \\ u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ p_\theta(t) \end{bmatrix} \quad (2)$$

where

$u_x(t)$, $u_y(t)$, $u_\theta(t)$ = translation along the x- and y- directions and rotation along the z-axis, respectively, of the base-isolated system;

m = total mass of the superstructure (top plate + added weights);

I_p = polar mass moment of inertia of the superstructure with respect to the z-axis which passes through the center of mass;

$\rho = \sqrt{\dfrac{I_p}{m}}$ = mass radius of gyration of the superstructure with respect to the z-axis;

[C'] = damping matrix;

x_i, y_i = x- and y- coordinates of the i^{th} base isolator;

k_i = lateral stiffness of the i^{th} base isolator in any direction;

N = Total number of base isolators ;

$k = \sum_{i=1}^{N} k_i$ = lateral stiffness of the total base isolation system in any direction;

$E_x = \dfrac{\left(\sum_{i=1}^{N} k_i \cdot x_i\right)}{k}, E_y = \dfrac{\left(\sum_{i=1}^{N} k_i \cdot y_i\right)}{k}$ = eccentricity (positive as shown in Figure 3) in the x- and y- direction, respectively, of the center of stiffness of the total base isolation system relative to the center of mass;

$k_{\theta\theta} = \sum_{i=1}^{N} k_i \cdot (x_i^2 + y_i^2)$ = rotational stiffness (about the z-axis) of the total base isolation system;

$p_x(t)$, $p_y(t)$, $p_\theta(t)$ = external dynamic forces/moment applied in the x-, y-, and z-direction, respectively.

To identify the *dimensionless* parameters governing the coupled lateral-torsional response of the system, Equation (2) can be rewritten as (note that $E_y = p_x = p_\theta = 0$ in our application),

$$m\cdot\begin{bmatrix}\ddot{u}_x(t)\\ \ddot{u}_y(t)\\ \rho\cdot\ddot{u}_\theta(t)\end{bmatrix}+\left[C\right]\cdot\begin{bmatrix}\dot{u}_x(t)\\ \dot{u}_y(t)\\ \rho\cdot\dot{u}_\theta(t)\end{bmatrix}+m\cdot\omega_L^{\,2}\cdot\begin{bmatrix}1 & 0 & (e_y\cdot\sqrt{12})\\ 0 & 1 & (-e_x\cdot\sqrt{12})\\ (e_y\cdot\sqrt{12}) & (-e_x\cdot\sqrt{12}) & \gamma^2\end{bmatrix}\cdot\begin{bmatrix}u_x(t)\\ u_y(t)\\ \rho\cdot u_\theta(t)\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \dfrac{1}{\rho}\end{bmatrix}\cdot\begin{bmatrix}p_x(t)\\ p_y(t)\\ p_\theta(t)\end{bmatrix} \quad (3)$$

where

$$\left[C\right]=\left[C'\right]\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \dfrac{1}{\rho^2}\end{bmatrix} = \text{damping matrix;}$$

$\omega_L = \sqrt{\dfrac{k}{m}}$ = uncoupled lateral (longitudinal or transversal) natural circular frequency of vibration;

$\omega_\theta = \sqrt{\dfrac{k_{\theta\theta}}{I_p}}$ = natural circular frequency of rotational vibration of a fictitious non-eccentric structure having the same rotational stiffness and mass moment of inertia (with respect to the z-axis) as the eccentric system considered;

$\gamma = \dfrac{\omega_\theta}{\omega_L} = \sqrt{\dfrac{k_{\theta\theta}}{\rho^2\cdot k}}$ = ratio of rotational uncoupled natural frequency ω_θ to the lateral uncoupled natural frequency ω_L as defined above;

$D_e = \rho\cdot\sqrt{12}$ = "equivalent diagonal" of the eccentric system which, for rectangular shape and uniform mass distribution, coincides with the actual length of the diagonal of the system;

$e_x = \dfrac{E_x}{D_e}$ = relative eccentricity in the x-direction, recalling that in our case the eccentricity in the y -direction is zero, i.e., $E_y = e_y = 0$.

Special Case of Support Excitation

In the case of support (earthquake) excitation in the y-direction, the effective earthquake load vector is given as the product of the mass matrix, a load influence vector, and the ground acceleration [Chopra, 2000]:

$$\mathbf{p}_{\mathrm{eff}}(t) = -\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & (m \cdot \rho^2) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \ddot{\mathbf{u}}_{\mathbf{g}}(t) \tag{4}$$

Through modal analysis, it is possible to uncouple the equations of motion of a linear system by making use of the natural vibration mode shapes [Chopra, 2000]. The solution of the eigenvalue problem

$$[\mathbf{k} - \omega_n^2 \mathbf{m}]\boldsymbol{\varphi}_{\mathrm{n}} = \mathbf{0} \tag{5}$$

which governs the undamped free vibration response of a linear system provides the following frequencies for the first three natural modes of vibration of the system, ω_1, ω_2, and ω_3 (ω_n and $\boldsymbol{\varphi}_{\mathrm{n}}$ denote the circular frequency in rad/sec and mode shape, respectively, of the *n*-th natural mode of vibration):

$$\Omega_1 = \left(\frac{\omega_1}{\omega_L}\right)^2 = \frac{1}{2} \cdot \left\{1 + \gamma^2 - \sqrt{(\gamma^2 - 1)^2 + 48 \cdot e^2}\right\} = 1 + \frac{e}{2} \cdot \Theta_1 \tag{6a}$$

$$\Omega_2 = \left(\frac{\omega_2}{\omega_L}\right)^2 = 1 \tag{6b}$$

$$\Omega_3 = \left(\frac{\omega_3}{\omega_L}\right)^2 = \frac{1}{2} \cdot \left\{1 + \gamma^2 + \sqrt{(\gamma^2 - 1)^2 + 48 \cdot e^2}\right\} = 1 + \frac{e}{2} \cdot \Theta_3 \tag{6c}$$

where

$$e^2 = e_x^2 + e_y^2; \quad e_y = 0 \tag{7}$$

$$F = \frac{e}{\gamma^2 - 1} \tag{8}$$

$$\Theta_1 = \frac{1}{F} \cdot \left(1 - \sqrt{1 + 48F^2}\right) \tag{9}$$

$$\Theta_3 = \frac{1}{F} \cdot \left(1 + \sqrt{1 + 48F^2}\right) \tag{10}$$

Equations (6)a-c indicate that the modal frequencies of the subject eccentric system are functions of structural parameters γ and e only.

Another important structural parameter, the "alpha parameter", α, is defined as the product of the mass radius of gyration of the structure and the ratio of the maximum rotational to the maximum longitudinal displacement response developed by an undamped one-story eccentric system in free vibration induced by an initial displacement in the longitudinal direction (Trombetti and Conte 2001).

$$\alpha = \rho \frac{|u_\theta|_{\max}}{|u_y|_{\max}} = \frac{\sqrt{48F^2}}{\sqrt{48F+1}} \tag{11}$$

The γ, e, and α parameters reveal fundamental characteristics of the coupled lateral-torsional response of eccentric systems. They are used in designing the physical model.

3.2 Stiffness Determined From Simplified Analytical Model

Figure 4 shows a prismatic column that is fixed at both ends. It is displaced by one unit in the horizontal direction. From Bernoulli-Euler linear beam theory, the lateral stiffness, k_i, of a single column of height h, with material Young's modulus E, and moment of inertia I of the cross-section can be determined by Equation (12). All columns in the model are assumed to have the same lateral stiffness. Multiplying k_i from Equation (12) by the total number of columns N in the structure gives the total stiffness k of the system.

$$k_i = \frac{12EI}{h^3} \tag{12}$$

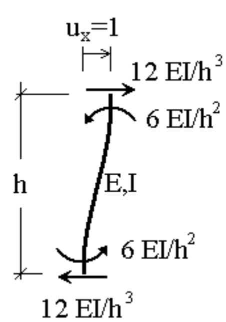

Figure 4: Stiffness Coefficients of a Prismatic Column

3.3 Damping Ratio Determined by Logarithmic Decrement of Amplitude Decay

The damping ratio characterizes the rate of decay of motion of the system. Therefore, the damping ratio can be obtained experimentally from free vibration records of a system. For lightly damped systems, the damping ratio can be determined from the following equation [Chopra, 2000]:

$$\zeta = \frac{1}{j2\pi} \ln \frac{\ddot{u}_i}{\ddot{u}_{i+j}} \tag{13}$$

where j denotes the number of cycles during which successive peaks $\ddot{u}_i$, $\ddot{u}_{i+1}$, … , $\ddot{u}_{i+j}$ are used in the calculation as shown in Figure 5.

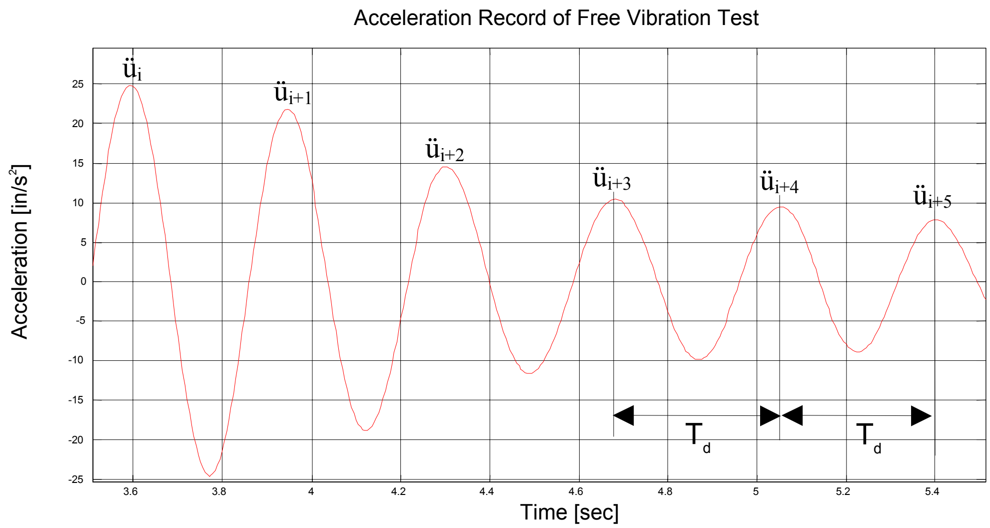

Figure 5: Damping Ratio Determined by Logarithmic Decrement

4. PHYSICAL MODEL

4.1 Target Prototype

The target prototype structure is a three-story base-isolated structure with planar dimensions $67\ \text{ft} \times 67\ \text{ft}$ and an average floor height of 10 ft. The prototype structure is assumed to have an uncoupled lateral natural period of $T_L = \dfrac{2\pi}{\omega_L} = 2$ seconds, which is very realistic. The ratio of the rotational uncoupled natural frequency to the lateral uncoupled natural frequency, γ, ranges from 1.0 to 1.8, while the relative eccentricity, $e = e_x$, ranges from 0.02 to 0.22.

Similitude Requirements

Scaling factors for the various geometrical and physical quantities are selected to produce a reasonably sized model and to accommodate the capacity of the small shaking table available. The time scale factor of prototype to model is $\lambda_T = \dfrac{T_{prototype}}{T_{model}} = 5$ and the length scale factor of prototype to model is $\lambda_L = \dfrac{L_{prototype}}{L_{model}} = 40$. Therefore, the physical model is designed to have an uncoupled lateral natural period of vibration of 0.4 second, and plan dimensions of the top plate of $20\ \text{in} \times 20\ \text{in}$.

4.2 Design of the Model

To maximize the visual effects of the model's dynamic behavior, the material for the columns must be flexible enough, yet resistant enough against buckling. Using the MTS uni-axial testing machine, tension and compression tests were performed on both Plexiglas and Lexan samples. Plexiglas was selected as the basic material for the model because it possesses more desirable properties than other materials considered, such as light weight, flexibility in the elastic range, and workability. The Young's modulus and strength of Plexiglas were measured to be approximately 422,000 psi and 11,000 psi, respectively. The density of Plexiglas was measured to be 0.0426 lb/in^3.

Nine columns of 0.25 inch in diameter are set 8 inches apart. As stated before, the model must show visual effects of torsional response when excited; in other words, the "alpha parameter", defined in Equation (11), of the system is to be maximized. From the equations in Section 3.1, the "alpha parameter" is a function of structural parameters γ and e, which in turn are functions of many physical and geometrical variables such as columns stiffness, amount of added masses, and position of added masses. A program written in MATLAB (see *masspos.m* in the *Matlab* folder on the CD-ROM) is used to calculate the "alpha parameter" from basic (geometric and material) model parameters. Several cycles of iterations produced an optimized model with a top plate thickness of 0.375 inch, 9 columns of 9.0 inches in length (measured from the top of the base plate to the middle of the top plate) placed symmetrically with 8 inches spacing, 2 added circular weights of 6.2 lbf each, and $\gamma = 1.2$.

To facilitate referencing the location of the added masses, a secondary coordinate system is defined with its origin set at the geometric center of the top plate. The coordinates of the added masses and the resulting locations of the center of mass of the superstructure and the relative eccentricities are tabulated in Table 1. Figure 6 illustrates the pre-determined positions of added masses at two inch intervals from 1 to 9 inches away from the geometric center of the plate along the x-axis.

TABLE 1: Coordinates of Added Mass and Resulting Parameters

Position reference	**x coordinate of added masses** [inches]	**y coordinate of added masses** [inches]		**E_x center of mass** [inches]	**e_x (relative eccentricity)** [%]	**α Alpha parameter**
1	1	7.46	-7.46	0.66	2.48	.36
3	3	7.52	-7.52	1.99	7.29	.75
5	5	7.65	-7.65	3.31	11.70	.88
7	7	7.84	-7.84	4.64	15.54	.93
9	9	8.09	-8.09	5.96	18.79	.95

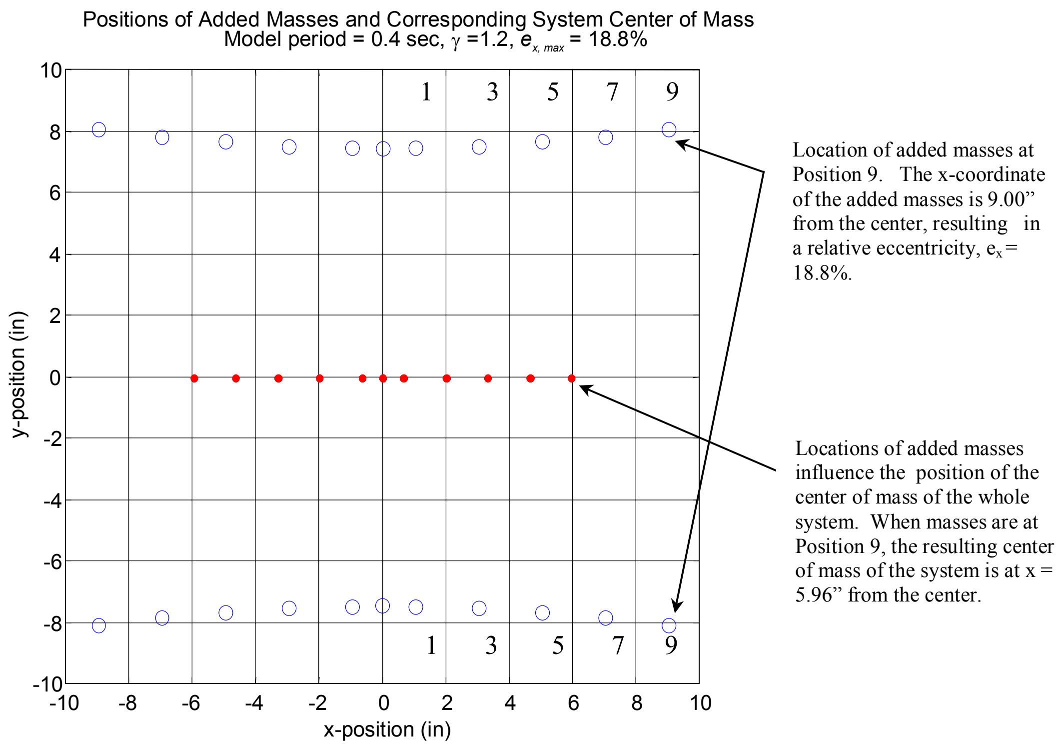

Figure 6: Position of Added Masses

4.3 Model Construction

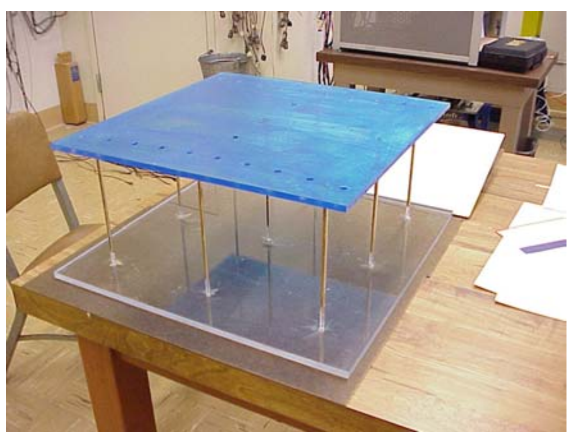

Figure 7: The Completed Model

Material and Equipment

- 2 Plexiglas plates of size 20" × 20" × 0.375" (top plate) and 22" × 22" × 0.375" (bottom plate);
- 9-pieces of 0.25" diameter Plexiglas solid rod 9.19" in length; make sure that both end surfaces form a 90 degrees angle with the axis of the rod;
- Liquid Plexiglas welding agent;
- 3 small C-clamps;

- Various sizes of washers, nuts, and fasteners;
- Circular weights with external diameter of approximately 4 inches;
- Electric scale;
- Punching bit (or nail) and hammer for marking top plate;
- Milling machine (at machine shop);
- "G" drill bit, or drill bit slightly larger than 0.25" diameter for columns to fit through plate;
- 3/8" drill bit.

Construction Procedure

The Plexiglas model must be fabricated carefully and precisely to minimize any construction discrepancy (e.g., accidental eccentricity) that could effect significantly the experimental results.

(1) Referring to Table 1, mark the exact positions of the added masses on a piece of paper of size 20" x 20".

(2) On the same paper, mark the exact positions of the columns using a different color pen.

(3) Tape the paper securely on the top Plexiglas plate.

(4) Hold the punching bit straight up and firmly at one of the marks on the paper; hammer the punching bit lightly to make an indentation on the Plexiglas under the paper. Repeat for all the columns and added mass locations.

(5) Clamp both pieces of the Plexiglas plates together, make sure that the plates are centered relative to one another and the edges are parallel.

(6) Using the milling machine and a "G" drill bit, drill the holes for the nine columns through both plates. Before you turn on the milling machine, lower the drill bit to line up the tip of the drill bit with the indentation. (You might want to practice drilling on some scrap Plexiglas plates first!).

(7) Unclamp the plates (be sure that you can re-align the holes on the top plate with the holes on the bottom plate later, you may want to make marks as a reminder). Using a 3/8" drill bit, drill the holes at the positions of the added masses through the TOP PLATE ONLY.

(8) Drill two extra holes at coordinates (0.00, 9.00) and (0.00,-9.00) for the non-eccentric case.

(9) Flip the top plate over, place the nine Plexiglas rods into the column holes. Make sure that the ends flush with the bottom side of the plate. Using the welding agent, place a few drops around the column and in the hole of the plate. Allow for 30 minutes to dry.

(10) Weigh the assembly of top plate and columns. Record this value.

(11) After the whole assembly has dried, flip it over (it should look like a table with 9 legs), insert the "legs" into the corresponding holes in the bottom plate. As before place a few drops of the welding agent around the column and in the hole of the plate.

(12) Using a combination of circular weights, washers, and fasteners, make up two weights of 6.22 lb each.

(13) Allow for 24 hours to dry before subjecting the model to the static and dynamic tests described in the next section.

5. EXPERIMENTS

5.1 Stiffness Test

The first test performed on the physical model of a one-story eccentric system is to experimentally determine the lateral stiffness of the model (which represents the lateral stiffness of the total base isolation system). According to the equation of static equilibrium, $F = ku$, the lateral stiffness of the structure k is obtained by simply measuring the applied lateral force F it takes to displace the structure through a displacement u in any horizontal direction.

Experimental Set-Up and Equipment

The equipment required to perform the static test consists of

- Load cell or potentiometer (calibrated);
- Voltmeter;
- Dial gage;
- String;
- C-clamps.

Testing Procedure

(1) Securely clamp the base plate of the model to the edge of a large table to prevent sliding.

(2) Place the two added masses (each consisting of several circular weights) on the model at the zero eccentricity position.

(3) Place the dial gage in the position indicated in Figure 8, (note the orientation of the model in the figure which has x as the vertical axis and y as the horizontal axis). Make sure that the spring of the dial gage is in contact with the top plate with a zero reading.

(4) Tie the string to the top plate through a hole at location (0., -9.) and to the hook at the end of the load cell.

(5) Slowly pull the load cell horizontally in the longitudinal direction; for each increment of 0.05 *in* measured by the dial gage, read the corresponding voltage on the voltmeter measuring the applied force.

(6) To stay away from the deformation capacity of the columns, do not displace the top plate by more than 0.5 *in*.

(7) Convert the voltage reading from the voltmeter to force in pounds (*lbf*) using the calibration constant of the load cell, i.e., number of Volts per pound. Graph the Force in pounds (*lbf*) versus the displacement in inches (*in*) as discrete data points.

(8) Find the slope of the best-fit line through the data points (using linear least-squares fit) to obtain the total lateral stiffness k in (*lbf/in*).

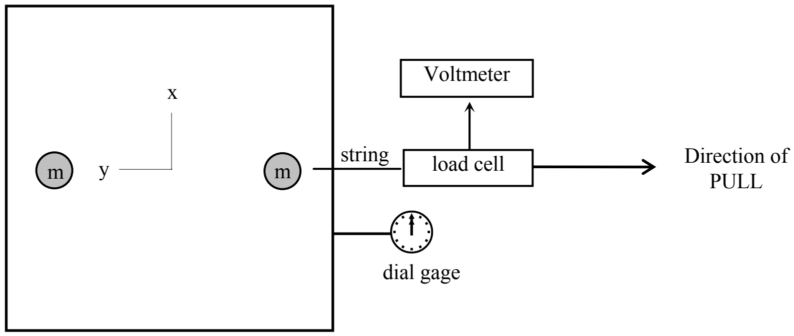

Figure 8: Stiffness Test Apparatus

Questions:

(a) What is the experimental lateral stiffness *k* of the structure?

(b) Determine the lateral stiffness *k* of the structure analytically using Equation (12) and compare with the experimental stiffness obtained under (a). Are the experimental and analytical values of *k* identical? If not, why?

5.2 Free Vibration Test

You will perform free vibration tests on the physical model under two conditions:

(1) When the system has no eccentricity ($e = e_x = 0$).

(2) When the system is at various degrees of eccentricity.

With the non-eccentric system, you can determine the damping ratio, ζ. In addition, you can calculate the structure parameter γ from the experimentally identified uncoupled lateral natural frequency, $f_L = 2\pi \cdot \omega_L$, and uncoupled torsional natural frequency, $f_\theta = 2\pi \cdot \omega_\theta$.

5.2.1 Non-Eccentric System

Experimental-Setup and Equipment

The following equipment is required to perform this free vibration test commonly referred to as a *snap-back* test:

- 3 single-channel accelerometers;
- Signal amplifier unit(s) for accelerometers;
- Terminal board;
- Multiple-channels dynamic data acquisition board;
- Data acquisition software (LabView or equivalent);

- Spectrum Analyzer.

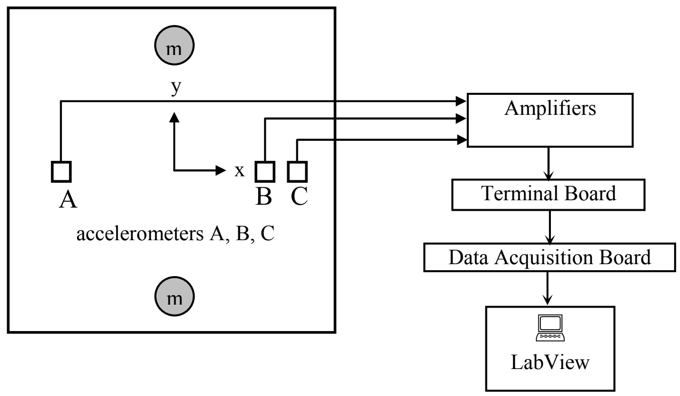

Figure 9: Free Vibration Test Apparatus

The various components of the equipment required to perform the free vibration tests are connected according to Figure 9. The added masses are fixed at the zero transversal eccentricity position ($e_x = 0$). The three accelerometers are attached at the following locations: accelerometer A at (-8.0, 0.0), accelerometer B at (8.0, 0.0), and accelerometer C at (9.0, 0.0). Make sure that all accelerometers are placed in the right orientation (see arrow engraved on each accelerometer) so that accelerometers A and B measure the voltage proportional to the system's acceleration in the y-direction, while accelerometer C measures the acceleration response in the x-direction. The voltage signals are amplified before they are recorded by a data acquisition software (here LabView) through a dynamic data acquisition board.

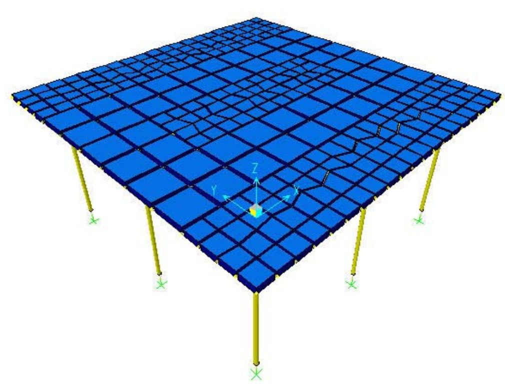

Figure 10: SAP2000 Model

Testing Procedure

Test 1:

(1) Pull the top plate slowly until you displace it approximately 0.25" in the y-direction. Pull as straight as possible.

(2) Start the data acquisition program so it is ready to record the data.

(3) Release the top plate to set the model under free longitudinal vibration. Record and save 10 to 15 sec of free vibration response data. Be aware of the sampling rate in your data acquisition program. In the present case, a sampling rate of 200 Hz is recommended, although any sampling rate above 50 Hz is acceptable.

Test 2:

(1) Place your hands at opposing corners of the top plate, pull-and-push in a twisting motion; caution: do not twist too much so as to keep the model in its safe domain!

(2) Start the data acquisition program so it is ready to record the data.

(3) Release the top plate to set the model under free rotation response. Record and save 10 to 15 sec of free vibration response data.

Task:

(1) Convert the voltage measured by the accelerometers in **Test 1 and Test 2** to acceleration in (in/s^2). Note that you will also have to reduce the three measurements to accelerations ($\ddot{u}_x$, $\ddot{u}_y$, and $\ddot{u}_\theta$) along the three degrees of freedom at the center of mass (see Figure 3). The reader is referred to Appendix 1 for instruction on how to accomplish this.

(2) Graph the results obtained from **Test1** and **Test2** in the form of acceleration versus time.

(3) Observe the acceleration record in the y-direction for **Test 1,** and determine the uncoupled natural lateral frequency, f_L, of the model.

(4) Observe the acceleration record in the y-direction for **Test 1**, and determine the damping ratio ζ of the model (for its uncoupled lateral mode of vibration) using the logarithmic decrement of amplitude decay according to Equation (13) in Section 3.3.

(5) Observe the acceleration record in the y-direction for **Test 2**, and determine the uncoupled natural torsional frequency, f_θ , of the model.

(6) Run the SAP2000 model from the *model.sdb* (or *model.$2k*) file in the *SAP2000* folder on the CD-ROM. This model and analysis simulate numerically the physical experiments performed under Test 1 and Test 2. Click on the heading Display --> Show Time History Traces. At the Time History Case pull down menu, select FREEVIB, plot the time histories of response quantities 10x, 10y, and 10z (they denote the translational acceleration in the x- and y- direction, respectively, and the rotational acceleration at Joint 10 = center of mass). Click on Display to actually display these acceleration response histories.

(7) Change the Time History Case to FREEROT, plot response quantities 10x, 10y, and 10z. (they denote the translational acceleration in the x- and y- direction, respectively, and the rotational acceleration at Joint 10 = center of mass). Click on Display to actually display these acceleration response histories.

(8) Obtain the theoretical uncoupled natural lateral frequency, f_L, and uncoupled torsional frequency, f_θ , from the above SAP2000 acceleration response histories.

Questions:

(a) What is the value of the damping ratio (in percent) obtained experimentally?

(b) Complete the following chart:

(c)

	SAP2000	Experiment
Natural lateral frequency, f_L [Hz]		
Natural torsional frequency, f_θ [Hz]		
Natural lateral period, T_L [s]		
Natural torsional period, T_θ [s]		
$\gamma = \omega_\theta / \omega_L$		

(c) Do you see a general agreement between the SAP2000 results and experimental results? What are the possible sources of discrepancy?

5.2.2. Eccentric System

In this section, you will perform free vibration tests on the physical model with various degrees of transversal eccentricity (e_x) and find the corresponding modal frequencies through the use of a spectrometer and frequency domain analysis (discrete Fourier transform) of the experimental data. The experimental results will be compared to theoretical results obtained from SAP2000 and Equations (6)a-c.

Experimental Set-up and Equipment

Use the same apparatus described for the non-eccentric case, with the addition of a Spectrum Analyzer. A spectrum analyzer performs on the fly a Fourier transform of the data and displays the resonant frequencies (in the form of very narrow spectral peaks) of the structure on a monitor.

Testing Procedure

(1) Secure the two weights on the top plate at the holes 1-inch away from the center of the plate.

(2) Displace the model to approximately 0.25” in the negative y-direction.

(3) Release the top plate to set the model under free vibration. Record and save 10 to 15 sec of free vibration response data.

(4) Using the MATLAB FFT program (*fft.m* in the *Matlab* folder on the CD-ROM), perform a Fast Fourier Transform of the three channels of saved data. (You may need to make some slight modifications in the program). A graph of transformed functions called Fourier amplitude spectra will appear on the screen.

(5) Identify the natural frequencies from the Fourier amplitude spectra of the experimental data.

(6) Disconnect accelerometer A from the terminal board and connect it to the Spectrum Analyzer. Tap the top plate using your finger. The frequencies corresponding to the obvious narrow spectral peaks appearing on the monitor of the Spectrum Analyzer are the natural frequencies determined experimentally. You may need to tap the top plate in different directions in order to obtain good readings from the Spectrum Analyzer for all three natural modes of vibration.

(7) Open the SAP2000 model (from the *model.sdb* file in the *SAP2000* folder on the CD-ROM) and assign concentrated (or nodal) translational masses to the locations defining the first

level of eccentricity. (You may find the locations easily by clicking on Select ➠ Groups ➠ Mass1; you will see two nodes being selected that correspond to coordinates (7.46, 1.00) and (-7.46,1.00). Remember to delete the masses originally assigned to the zero eccentricity position.

(8) Run the modified SAP2000 model, display the first three natural mode shapes of vibration and the corresponding modal frequencies. (click on Display-> Display Mode Shapes. Use the arrow keys at the lower right hand corner of the screen to show the next mode).

(9) Observe the first three natural modes of vibration using the dynamic animation option by clicking the *start* button at the lower right hand corner of the screen.

(10) Using Equations (6)a-c, calculate the theoretical natural frequencies for the first three natural modes of vibration.

(11) Repeat steps (1) through (10) for eccentric mass positions 3, 5, 7, and 9. Complete the following chart:

Position Name	Relative Eccentricity [%]		Theoretical Equation (6)	SAP2000	FFT	Spectrometer
1	2.48	Mode1 [Hz]				
		Mode2 [Hz]				
		Mode3 [Hz]				
3	7.29	Mode1 [Hz]				
		Mode2 [Hz]				
		Mode3 [Hz]				
5	11.7	Mode1 [Hz]				
		Mode2 [Hz]				
		Mode3 [Hz]				
7	15.54	Mode1 [Hz]				
		Mode2 [Hz]				
		Mode3 [Hz]				
9	18.79	Mode1 [Hz]				
		Mode2 [Hz]				
		Mode3 [Hz]				

Questions:

(a) Compare the results obtained from each of the method used. Do they show a general agreement?

(b) Why are we only concerned about the first three modes of vibration?

(c) Why does the frequency of the second mode of vibration remain the same at different levels of eccentricity?

5.3 Shaking Table Test

In this experiment, you will excite the physical building model on the shaking table with a scaled down version of the El Centro 1940 earthquake record. You will observe how the model responds to seismic excitation under two scenarios:

(1) When the system is non-eccentric ($e = e_x = 0$).

(2) When the system is eccentric.

Experimental Set-Up and Equipment

The equipment required to perform the shaking table test consists of

- Shaking table;
- Four (single channel) accelerometers;
- Signal amplifier unit(s) for accelerometers;
- Terminal board;
- Multiple-channels dynamic data acquisition board;
- Data acquisition software (LabView or equivalent);
- Software: SAP2000 and MATLAB.

The test apparatus is similar to that for the Free Vibration Test with the addition of a fourth accelerometer unit, accelerometer D, to measure the shaking table acceleration as shown in Figure 11. You will need to make some small modifications in the data acquisition program in order to acquire the data from accelerometer D as well. Again, be sure to place Accelerometer D in the correct orientation so as to measure the table acceleration in the positive y-direction.

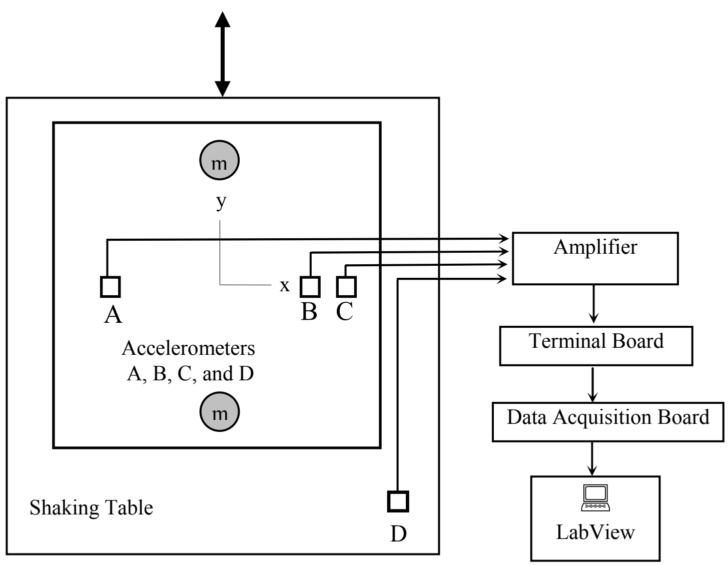

Figure 11: Shaking Table Test Apparatus

Testing Procedure

(1) Clamp the Plexiglas model securely to the shaking table.

(2) Create a non-eccentric system by placing the two added masses at the zero transversal eccentricity positions as shown in Figure 11.

(3) Copy the scaled El Centro 1940 earthquake record from the CD-ROM (file *ELC40.prn* in *SAP2000* folder). Use it as the input function for the shaking table test. The earthquake record is an acceleration record with 1501 data points and a constant time step of 0.004 second.

(4) Operate the shaking table to excite the physical model with the scaled El Centro earthquake record.

(5) Record and save all measurements from all four accelerometers.

(6) Convert the raw data (in Volts) from accelerometers A, B, C, and D to acceleration in (in/s^2) using the accelerometer calibration constant (acceleration per Volt). Create a text file (*.txt) with time (s) in the first column and the corresponding measured shaking table acceleration (in/s^2), from accelerometer D, in the second column.

(7) Reduce the measurements from accelerometers A, B, and C to the three components of acceleration, $\ddot{u}_x$, $\ddot{u}_y$, and $\ddot{u}_\theta$, at the center of mass of the system according to the procedure outlined in Appendix 1. Plot the measured acceleration response histories $\ddot{u}_x(t)$, $\ddot{u}_y(t)$, and $\ddot{u}_\theta(t)$, on three separate graphs versus time in seconds.

(8) Use the above text file as the Time History Function for the SAP2000 model of the experiment and perform the SAP2000 simulation for the same level of eccentricity as in the experiment. It is important that you do not use the scaled El Centro 1940 earthquake record provided in the CD-ROM as the SAP2000 input function, since it is unlikely that the shaking table is capable of reproducing exactly this earthquake record. Save the response histories $\ddot{u}_x(t)$, $\ddot{u}_y(t)$, and $\ddot{u}_\theta(t)$ simulated using SAP2000.

(9) Create a video of the building response simulated in SAP2000 or open a SAP2000 video file in the CD-ROM (in the *PowerPoint and Videos* folder).

(10) Repeat steps 2 – 9 for the physical model with two different levels of eccentricity (e.g., $e = e_x = 7\%$ and 19%).

(11) Obtain the ground record for a second earthquake, such as Coalinga 1989 (from the 1989 Loma Prieta earthquake). Scale the record using the time and length scaling factors mentioned in section 4.1. Repeat steps 1-10 for the scaled Coalinga 1989 earthquake record.

Questions:

(a) By comparing the response of the non-eccentric system to that of the eccentric system with two levels of eccentricity, describe the effects of eccentricity of the center of mass relative to the center of stiffness.

(b) Do you observe any response in the transversal (or x-) direction? If yes, how does the amplitude of this response compare to that in the longitudinal (or y-) direction?

(c) Is the response in the transversal (or x-) direction expected from the theory? Refer to Equations (2) and (4).

(d) How does the response simulated in SAP2000 compare with the actual response? Plot each of the response histories $\ddot{u}_x(t)$, $\ddot{u}_y(t)$, and $\ddot{u}_\theta(t)$ simulated using SAP2000 and observed experimentally on separate graphs using the same scaling of axes and same grid for better comparison.

(e) What are the possible sources of discrepancy between analytical/numerical response predictions and experimental results?

(f) Answer questions (a) through (e) for the numerical and experimental data obtained using the scaled Coalinga 1989 earthquake record.

6. SAP2000 MODEL

The numerical model, created using the finite element analysis software SAP2000, is developed with modeling assumptions and parameters representing as closely as possible the real physical model. Most of the measurable parameters used in the SAP2000 model, such as Young's modulus and mass/weight density of Plexiglas, are all determined experimentally. However, there is always some deviation between the real physical model and its numerical counterpart. The reader is referred to the SAP2000 model in the CD-ROM (file *model.sdb* in the *SAP2000* folder).

6.1 Geometry and Joint Constraints

The top plate with overall dimensions of 20" × 20" × 0.375" is modeled using quadrilateral shell elements. For the areas where the added masses are located, shell elements are meshed into a reduced size of 1" × 1" to locate exactly the added masses. Column frame sections have a height of 9.0" and a cross sectional diameter of 0.25". The bottoms of the columns are defined as fixed supports.

6.2 Material

A new material, labeled as *Plexi1*, must be defined in SAP2000. *Plexi1* is used for both the top plate and the columns. The properties of *Plexi1* were determined through material uni-axial testing and information from the manufacturer. The following steps are followed to define the *Plexi1* material:

Define Material: *Plexi1*

Material Name: *Plexi1*

Type of Material: Isotropic

Mass per unit Volume: $1.100\text{E}\times10^{-4}$ [$\text{lb}\cdot\text{sec}^2/\text{in}^4$]

Weight per unit Volume: 0.0426 [lb/in^3]

Modulus of Elasticity: 422,000 [psi]

Poisson's Ratio: 0.35

Coefficient of Thermal Expansion: 3.3

Shear Modulus: 156,296.3 [psi]

6.3 Dynamic Analysis

The model is analyzed using Ritz-Vector Analysis with 20 modes. The modal damping ratio identified experimentally in Section 5.2 is used.

Appendix 1
Reducing Accelerometer Measurements at the Center of Mass of the System

Referring to Figure 12 below, in the general case, the center of mass (c.m.) and the center of stiffness (c.s.) of the system do not coincide. When the plate is displaced, the displacement at point A, B, and C are measured. The three accelerometers at points A, B, and C measure the acceleration response of the system along degrees of freedom s_1, s_2, and s_3, respectively. Using simple linear kinematics, these three degrees of freedom can be reduced into the three degrees u_x, u_y, and u_θ at the center of mass of the system. The distance between point A and the center of mass is denoted by d_1, while d_2 represents the distance between points B and the center of mass.

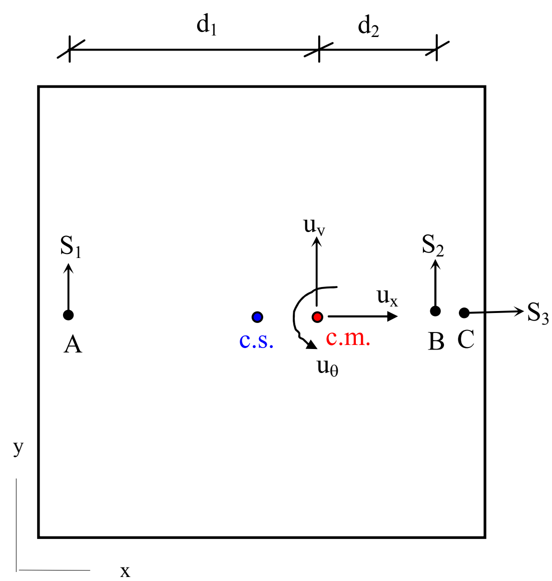

Figure 12. Reduction of Measurement DOF's s_1, s_2, and s_3 into Analysis DOF's u_x, u_y, and u_θ

By simple geometry, it follows that

$$s_1 = u_y - d_1 u_\theta \tag{14}$$

$$s_2 = u_y + d_2 u_\theta \tag{15}$$

$$s_3 = u_x \tag{16}$$

Solving the above equations for u_x, u_y, and u_θ yields

$$u_x = s_3 \tag{17}$$

$$u_\theta = (s_2 - s_1) / (d_1 + d_2) \tag{18}$$

$$u_y = s_1 + d_1 u_\theta \tag{19}$$

References:

Chopra, A. K., *Dynamics of Structures,* 2nd Ed., Prentice Hall, 2000.

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Conte, J. P. and Trombetti, T. L., "New Insight Into and Simplified Approach to Analysis of Laterally-Torsionally Coupled One-Story Systems: Part II – Numerical and Experimental Verifications," submitted for possible publication in the *Journal of Sound and Vibration*, August 2001.