Definitions of static eccentricity for design of asymmetric shear buildings

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This paper assesses three definitions of eccentricity employed in the description of asymmetric multistorey buildings. Using the three approaches, formulae are developed to evaluate the asymmetry of shear-type buildings where the deformation is governed by the lateral and rotational rigid-floor displacements arising from the seismic design loadings. Simple comparisons and comments on the alternative approaches are presented with the aid of appropriate examples. This clarification of the definition of eccentricity is needed for improved understanding of the application of codified torsional design procedures. It is also a prerequisite for advanced studies of the inelastic seismic response of multistorey shear buildings.

Keywords: structures, asymmetric buildings, static eccentricity, shear buildings, seismic design

The application of the torsional design provisions of seismic building codes is based on the determination of the static eccentricity. For multistorey buildings this is a function of the distribution (horizontally and vertically) of the stiffness of structural elements, together with the vertical and horizontal distribution of the design earthquake forces. The concept of the equivalent static torsional moment is employed by all the major codes to specify the design forces arising from structural asymmetry. By this approach, the dynamic torsional effects are simulated by appropriately amplifying the static eccentricities determined for each floor or storey level of the building, prior to calculating the design torsional moments. This paper summarizes three definitions of static eccentricity employed in the description of asymmetric multistorey buildings. Despite the fundamental importance of static eccentricity as a parameter in the application of seismic design code provisions and in studies of the dynamic earthquake response of torsionally coupled structures, there is an apparent lack of consistency in its definition by codes[1] and by various research investigators[2-7].

For single storey building models the definition of static eccentricity is relatively straightforward, it being defined simply as the distance between the resultant of the earthquake-induced lateral loading at the floor level (which for the purpose of equivalent static analysis is assumed to act through the centre of mass CM), and the centre of stiffness (CS) or shear centre (SC) of the storey. The shear centre is defined as the position through which the resultant of the resisting element storey shears passes such that no rotation of the floor occurs; that is, the relative rotation between floor and base (foundation) is zero. For single storey buildings the shear centre coincides with the centre of element stiffnesses, since equivalent linear elastic response behaviour is assumed. As will be discussed below, this special feature is also present in multistorey shear-type buildings as considered in this paper.

For single storey buildings with rigid floor diaphragms, the reference position for determination of the static eccentricity may also be defined as the centre of rigidity (CRi). This is the point where the application of a lateral load will not cause any rotation of the floor. For this special case the centre of rigidity and the shear centre coincide. This position or point (the storey shear centre or centre of stiffness may be assumed to lie directly below the centre of rigidity of the floor, and

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hence the two are coincident in plan) can also be identified as the centre of twist which remains stationary when the structure is subjected to a torsional loading. Finally, it is the centre of element strengths in a corresponding inelastic analysis, for the case of building models in which the stiffness and strength are assumed proportional.

It is in the extension of these definitions to multistorey buildings, however, that essentially three definitions of static eccentricity arise. These definitions differ in terms of the reference position from which the eccentricity is measured. Wittrick and Horsington[2] defined the reference position as the shear centre at a particular storey, and this is the point where the application of the resultant of the lateral resisting element shear forces (namely the storey shear force) produces no relative rotation between the floors immediately above and below the reference storey. Cheung and Tso[2] adopted the centre of rigidity as the reference point. For multistorey buildings, this is the point located on any floor, such that when the given vertical distribution of lateral loadings passes through them no rotation of the building floors about a vertical axis occurs. These two definitions have been adopted implicitly in the application of the torsional design provisions of various building codes, as described in Reference 1. Finally, Humar[4] defined the reference point for a particular floor as the centre of resistance (CRe), and this is the point through which the lateral earthquake force at that floor should pass without the floor undergoing any rotation relative to the base (that is, absolute rotation), but other floors both above and below the reference floor may rotate. Other definitions of static eccentricity in multistorey buildings exist[5–7], but they fall essentially into one of the above categories.

The purpose of this study is to make an explicit evaluation and comparison of these definitions of static eccentricity for the case of multistorey shear-type buildings. For clarity, the definitions will be referred to in the following sections as the 'storey', 'all floors' and 'single floor' definitions, respectively.

Building model

Figure 1(a) illustrates an N-storey shear building comprised of perfectly rigid floor slabs supported by vertical resisting elements (shown as beam-column frames) which are connected rigidly to the floors. Every resisting element has two orthogonal principal axes, and those axes are oriented in the global x and y directions. The sizes, location and number of resisting elements need not be the same for each storey, and consequently the storey stiffnesses may be different. The building is subjected to a static load distribution resulting from an earthquake occurring in one or both principal axis directions, and the resulting lateral force or forces acting on

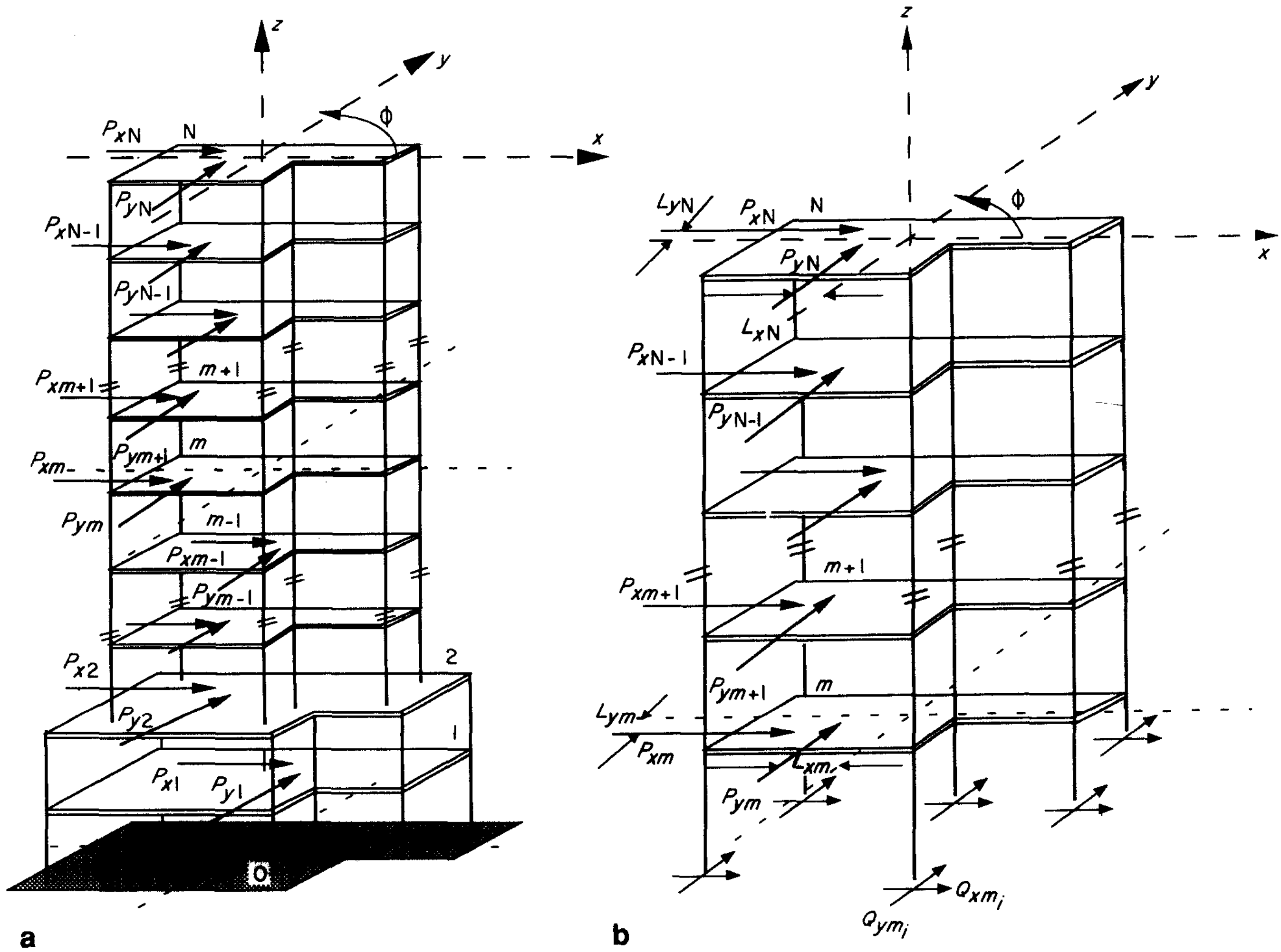

Figure 1 Multistorey building model: (a) structure and lateral load distribution; (b) example free-body diagram

each floor are expressed as P_{xm}, P_{ym} ($m = 1, 2, \ldots, N$). The vertical reference axis, z, is positioned arbitrarily, but in the case of buildings with constant floor geometry throughout their height, this axis may conveniently be located through the floor centres of mass, CM.

Displacement equations

For the building system shown in *Figure 1(a)*, a free body boundary may be drawn at any storey level, for instance at level m, resulting in a free body diagram as illustrated in *Figure 1(b)*. For clarity, only the element shear forces Q which are concerned in the following analytical evaluation are marked on the cut section of the free body. The internal forces exposed are a combination of shear forces, axial forces and bending moments which maintain the equilibrium of the free body. The torsional moments arising in the individual elements are not considered, because they are negligible compared with the resisting torque yielded by the shear forces. Translational and torsional equilibrium at the free body boundary requires that

$$\sum_{i=1}^{n_m} Q_{xm_i} = \sum_{j=m}^{N} P_{xj} \tag{1}$$

$$\sum_{i=1}^{n_m} Q_{ym_i} = \sum_{j=m}^{N} P_{yj} \tag{2}$$

$$-\sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} + \sum_{i=1}^{n_m} Q_{ym_i} X_{m_i} = -\sum_{j=m}^{N} P_{xj} L_{yj} + \sum_{j=m}^{N} P_{yj} L_{xj} \tag{3}$$

where:

Q_{xm_i}, Q_{ym_i} are the shearing forces of the ith resisting element in the mth storey in the x and y directions, respectively
X_{m_i}, Y_{m_i} are the coordinates of the ith resisting element in the mth storey in the x and y directions, respectively
n_m is the total number of resisting elements in the mth storey
P_{xj}, P_{yj} are the resultants of external lateral forces acting on the jth floor in the x and y directions, respectively
L_{xj}, L_{yj} are the coordinates of the point where the lateral forces P_{xj}, P_{yj} act
$m, j = 1, 2, \ldots, N$
$i = 1, 2, \ldots, n_m$

The shearing force and the corresponding displacements of an individual resisting element are related by

$$Q_{xm_i} = K_{xm_i}(U_{xm} - \theta_m Y_{m_i}) \tag{4}$$

$$Q_{ym_i} = K_{ym_i}(U_{ym} + \theta_m X_{m_i}) \tag{5}$$

where: θ_m and U_{xm}, U_{ym} are, respectively, the relative (anti-clockwise) rotation and relative displacements in the x and y directions, between the mth and $(m-1)$th floors; and K_{xm_i} and K_{ym_i} are the lateral stiffnesses of the ith resisting element in the mth storey.

Substituting the above equations into equations (1)−(3), the relative displacements may be written as

$$U_{xm} = \left\{ \sum_{j=m}^{N} P_{xj} + \theta_m \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i} \right\} \Big/ \sum_{i=1}^{n_m} K_{xm_i} \tag{6}$$

$$U_{ym} = \left\{ \sum_{j=m}^{N} P_{yj} - \theta_m \sum_{i=1}^{n_m} K_{ym_i} X_{m_i} \right\} \Big/ \sum_{i=1}^{n_m} K_{ym_i} \tag{7}$$

$$\theta_m = \left\{ -\sum_{j=m}^{N} P_{xj}(\epsilon_{ym} - e_{ym}) + \sum_{j=m}^{N} P_{yj}(\epsilon_{xm} - e_{xm}) \right\} \Big/ K_{\theta m} \tag{8}$$

where

$$e_{xm} = \sum_{i=1}^{n_m} K_{ym_i} X_{m_i} \Big/ \sum_{i=1}^{n_m} K_{ym_i}, \quad e_{ym} = \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i} \Big/ \sum_{i=1}^{n_m} K_{xm_i} \tag{9}$$

$$\epsilon_{xm} = \sum_{j=m}^{N} P_{yj} L_{xj} \Big/ \sum_{j=m}^{N} P_{yj}, \quad \epsilon_{ym} = \sum_{j=m}^{N} P_{xj} L_{yj} \Big/ \sum_{j=m}^{N} P_{xj} \tag{10}$$

$$K_{\theta m} = \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i}^2 - e_{ym} \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i} + \sum_{i=1}^{n_m} K_{ym_i} X_{m_i}^2 - e_{xm} \sum_{i=1}^{n_m} K_{ym_i} X_{m_i} \tag{11}$$

In equations (9)−(11): e_{xm}, e_{ym} are the distances from the storey centre of stiffness or shear centre to the origin of the global x and y axes, respectively; ϵ_{xm}, ϵ_{ym} may be interpreted as the distances measured from the resultant of external lateral forces applied at and above floor level m to the origin of the global x and y axes, respectively; and $K_{\theta m}$ is the rotational or torsional stiffness of the mth storey about the storey centre of stiffness.

It is clear from the above equations that the relative displacements U_{xm}, U_{ym} and θ_m depend only on the structural properties of the mth storey and the lateral forces applied to floors above the mth storey.

In the following sections, analytical relationships are developed with reference to the three alternative definitions of static eccentricity as outlined above. Subsequently, a series of examples are used to illustrate their application in the determination of design torsional moments in asymmetric shear buildings subjected to realistic lateral force distributions.

'Storey' eccentricity definition

Because the orthogonal lateral earthquake forces P_{xm} and P_{ym} ($m = 1, 2, \ldots, N$) are independent of each other, the rotation θ_m can be divided into two parts, θ_{xm} and θ_{ym} corresponding to the two sets of lateral forces, viz

$$\theta_{xm} = \sum_{j=m}^{N} P_{xj}(\epsilon_{ym} - e_{ym})/K_{\theta m} \tag{12a}$$

$$\theta_{ym} = \sum_{j=m}^{N} P_{yj}(\epsilon_{xm} - e_{xm})/K_{\theta m} \tag{12b}$$

The rotations θ_{xm} and θ_{ym} have similar properties and therefore discussion and further analysis is limited to one of them. It is clear that for θ_{xm} to be zero the distances ϵ_{ym} and e_{ym} must be equal. This implies that the resultant of the lateral forces applied at and above the mth floor must pass through the centre of stiffness of the mth storey. For shear-type buildings having no relative rotation between adjacent floors, the shear centre coincides with the storey centre of stiffness. This is evident from the following analysis. Based on equations (1)–(5), if θ_{xm} is zero, the following relationships exist:

$$\sum_{i=1}^{n_m} Q_{xm_i} = U_{xm} \sum_{i=1}^{n_m} K_{xm_i},$$

$$\sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} = U_{xm} \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i}$$

so

$$\sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} \Big/ \sum_{i=1}^{n_m} Q_{xm_i} = \sum_{i=1}^{n_m} K_{xm_i} K_{m_i} \Big/ \sum_{i=1}^{n_m} K_{xm_i}$$
$$= e_{ym} \tag{13}$$

Hence it is a necessary and sufficient condition for the relative rotation of adjacent floors to be zero if the line of action of the resultants of the external lateral forces (in the x and y directions) acting above the storey pass through the shear centre of the storey under consideration. In other words, the shear centre can be used as a reference point for the measurement of the relative torsional movement of a shear building subjected to specified sets of lateral forces. This is the basis of the definition of storey eccentricity, as employed in the seismic design codes of Canada (NBCC 1990[8]), the United States (UBC 1988[9]), the draft European seismic code EC8 1988[10], and others[11] which include Chile and Yugoslavia. Defining the design torsional moment for the mth storey as $M_m = M_{xm} + M_{ym}$, equations (12a, b) can be written as

$$\theta_{xm} = M_{xm}/K_{\theta m}, \quad M_{xm} = -\sum_{j=m}^{N} P_{xj}(\epsilon_{ym} - e_{ym}) \tag{14a}$$

$$\theta_{ym} = M_{ym}/K_{\theta m}, \quad M_{ym} = \sum_{j=m}^{N} P_{yj}(\epsilon_{xm} - e_{xm}) \tag{14b}$$

Hence the storey eccentricities are evaluated as

$$\eta_{xm} = \epsilon_{xm} - e_{xm}, \quad \eta_{ym} = \epsilon_{ym} - e_{ym} \tag{15}$$

The storey eccentricities defined by equation (15) represent the distances measured in the x and y directions from the storey shear centres to the line of action of the resultants of the lateral seismic forces applied to floors above the reference storey level.

Floor eccentricity ('single floor') definition

The basis of both the 'single floor' and 'all floors' definitions of static eccentricity is a consideration of the absolute rotation Φ_m ($m = 1, 2, \ldots, N$) of the mth floor, caused by the application of the lateral forces (see *Figure 1*). The building is assumed to be fixed rigidly at its base, and therefore the following relationships hold

$$\Phi_1 = \theta_1, \quad \Phi_2 = \Phi_1 + \theta_2 = \sum_{j=1}^{2} \theta_j,$$

$$\Phi_m = \Phi_{m-1} + \theta_m = \sum_{j=1}^{m} \theta_j$$

That is

$$\Phi_m = \Phi_{xm}\Phi_{ym} = -\sum_{j=1}^{m}\sum_{k=j}^{N} P_{xk}(\epsilon_{yj} - e_{yj})/K_{\theta j} + \sum_{j=1}^{m}\sum_{k=j}^{N} P_{yk}(\epsilon_{xj} - e_{xj})/K_{\theta j} \tag{16}$$

Setting the floor rotation $\Phi_m = 0$ according to the 'single floor' definition and considering the independence of P_{xm} and P_{ym}, the coordinates of the centre of resistance CRe for the mth floor, $\tilde{L}_{xm}$ and $\tilde{L}_{ym}$ may be obtained as follows

$$\tilde{L}_{xm} = e_{xm} + \left\{ -K_{\theta m} \sum_{j=1}^{m-1}\sum_{k=j}^{N} P_{yk}(\epsilon_{xj} - e_{xj})/K_{\theta j} + e_{xm} \sum_{k=m+1}^{N} P_{yk} - \sum_{k=m+1}^{N} P_{yk} L_{xk} \right\} \Big/ P_{ym} \tag{17a}$$

$$\tilde{L}_{ym} = e_{ym} + \left\{ -K_{\theta m} \sum_{j=1}^{m-1}\sum_{k=j}^{N} P_{xk}(\epsilon_{yj} - e_{yj})/K_{\theta j} + e_{ym} \sum_{k=m+1}^{N} P_{xk} - \sum_{k=m+1}^{N} P_{xk} L_{yk} \right\} \Big/ P_{xm}$$
$$(m = 1, 2, \ldots, N) \tag{17b}$$

From the above equations it can be deduced that the location of the centre of resistance on the mth floor is dependent on the complete loading distribution applied to the building, and on the structural properties of the part of the building below the mth floor. Hence the CRe of a single storey building is load independent and coincides with the CS.

The concept of the centre of resistance may also be expressed as follows

$$M_{p,m} = M_{s,bm} + M_{p,am} \tag{18}$$

where

$$M_{p,m} = P_{xm}(L_{ym} - e_{ym}) - P_{ym}(L_{xm} - e_{xm}) \tag{19a}$$

$$M_{s,bm} = K_{\theta m}\Phi_{m-1} \tag{19b}$$

$$M_{p,am} = -\sum_{j=m+1}^{N} P_{xj}(L_{yj} - e_{ym}) + \sum_{j=m+1}^{N} P_{yj}(L_{xj} - e_{xm}) \tag{19c}$$

$$m = 1, 2, \ldots, N$$

In equations (19$a-c$) the subscripts s and p refer to the structure and the applied lateral forces, respectively. The reference floor is m, and the subscripts bm and am refer to the parts of the building below and above the mth floor, respectively. It is interpreted from equation (18) that in order to maintain zero rotation of the mth floor, the torsional moment produced by the lateral forces P_{xm} and P_{ym} acting on the mth floor eccentric to the mth storey shear centre must be in equilibrium with the sum of the moments caused firstly by the rotation of the structure below the mth floor, and secondly by the lateral forces acting on the floors above the mth floor. This equilibrium relationship is illustrated in *Figure 2*.

To determine the eccentricities associated with the 'single floor' approach, the dynamic torque for the reference floor must first be defined. The rotation of the mth floor is given by equation (16), where the independent contributions of the orthogonal sets of lateral forces are defined. The corresponding 'single floor' torque, $\tilde{T}_m$ is then defined as

$$\tilde{T}_m = \tilde{T}_{xm} + \tilde{T}_{ym} = K_{\theta m}\Phi_{xm} + K_{\theta m}\Phi_{ym}$$

Taking the torque component $\tilde{T}_{xm}$ as the example, the following relationships are obtained

$$\begin{aligned}\tilde{T}_{xm} &= K_{\theta m}\Phi_{xm} = K_{\theta m}\Phi_{xm-1} + K_{\theta m}\theta_{xm} \\ &= K_{\theta m}\Phi_{xm-1} - \sum_{k=m+1}^{N} P_{xk}(\epsilon_{ym+1} - e_{ym}) \\ &\quad - P_{xm}(L_{ym} - e_{ym})\end{aligned} \tag{20a}$$

Equation (20a) can be rewritten as follows

$$\begin{aligned}\tilde{T}_{xm} &= K_{\theta m}\Phi_{xm-1} - \sum_{k=m+1}^{N} P_{xk}(\epsilon_{ym+1} - e_{ym}) \\ &\quad - P_{xm}(\tilde{L}_{ym} - e_{ym}) - P_{xm}(L_{ym} - \tilde{L}_{ym})\end{aligned} \tag{20b}$$

The first three terms on the right-hand side of equation

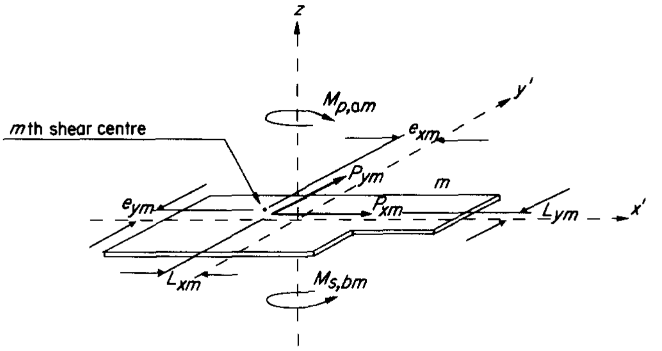

Figure 2 Moment equilibrium of mth floor ('single floor' approach). See equations (18) and (19) for moment definitions

(20b) sum to zero, a property which can be demonstrated by appropriately re-arranging equation (17b). Hence equation (20b) reduces to

$$\tilde{T}_{xm} = -P_{xm}(L_{ym} - \tilde{L}_{ym}) \tag{21a}$$

and similarly

$$\tilde{T}_{ym} = P_{ym}(L_{xm} - \tilde{L}_{xm}) \tag{21b}$$

Therefore, the eccentricities associated with the 'single floor' definition are defined as

$$\xi_{xm} = L_{xm} - \tilde{L}_{xm}, \quad \xi_{ym} = L_{ym} - \tilde{L}_{ym} \tag{22}$$

From equation (22), it can be deduced that the floor eccentricity by this approach is simply the distance measured perpendicular to the lateral force developed at a particular floor (applied in the x or y direction) to the floor centre of resistance as determined from equation (17). Furthermore, the 'single floor' torque referred to above can be written for the mth floor as follows:

$$\begin{aligned}\tilde{T}_{xm} &= K_{\theta m}\Phi_{xm} = K_{\theta m}\Phi_{xm-1} + K_{\theta m}\theta_{xm} \\ &= \left(\frac{K_{\theta m}}{K_{\theta m-1}}\right)\tilde{T}_{xm-1} + M_{xm}\end{aligned} \tag{23}$$

where M_{xm} is the storey torsional moment required in design, as defined for the 'storey' approach in equation (14a). By re-arranging equation (23), the design storey torsional moment for loads applied in the x-direction is given by

$$M_{xm} = \tilde{T}_{xm} - \left(\frac{K_{\theta m}}{K_{\theta m-1}}\right)\tilde{T}_{xm-1} \tag{24a}$$

and similarly for loads applied in the y-direction

$$M_{ym} = \tilde{T}_{ym} - \left(\frac{K_{\theta m}}{K_{\theta m-1}}\right)\tilde{T}_{ym-1} \tag{24b}$$

Floor eccentricity ('all floors') approach

The coordinates L^*_{xm} and L^*_{ym} of the centre of rigidity CRi for the mth floor can be obtained from equations (17a, b). This requires that the terms corresponding to Φ_{m-1}, $m = 1, 2, \ldots, N$ (refer also to equation (16)) be zero, according to the condition that the rotation of all floors is zero. Note that by the 'single floor' definition, $\Phi_m = 0$ for the reference (mth) floor. The centres of rigidity are therefore obtained by applying equations (17a, b) successively for $m = 1, 2, \ldots, N$. Hence:

$$L^*_{xm} = e_{xm} + \left\{ e_{xm} \sum_{k=m+1}^{N} P_{yk} - \sum_{k=m+1}^{N} P_{yk} L^*_{xk} \right\} \Big/ P_{ym} \tag{25a}$$

$$L^*_{ym} = e_{ym} + \left\{ e_{ym} \sum_{k=m+1}^{N} P_{xk} - \sum_{k=m+1}^{N} P_{xk} L^*_{yk} \right\} \Big/ P_{xm} \tag{25b}$$

$$m = 1, 2, \ldots, N$$

Although CRi at floor level m may be located by considering the lateral forces acting on and above the mth floor, together with the structural properties of the building above the mth floor, the definition requires that the set of $2N$ equations (25) should hold simultaneously. In this sense, therefore, the 'all floors' definition is stricter than the remaining two, as previously evaluated. It should be noted that the conditions relating to the 'storey' and 'single floor' definitions hold if the conditions of the 'all floors' definition are satisfied. However, in general, the sets of points corresponding to the three definitions are not coincident. This is demonstrated by the examples given in the following section.

Substituting

$$e_{ym} = \sum_{i=1}^{n_m} K_{xm_i} Y_{m_i} \Big/ \sum_{i=1}^{n_m} K_{xm_i},$$

$$U_{xm} \sum_{i=1}^{n_m} K_{xm_i} = \sum_{j=m}^{N} P_{xj}$$

and

$$U_{xm} = (V_{xm} - V_{xm-1})$$

into equation (25b), the following relationship is obtained

$$(V_{xm} - V_{xm-1}) \sum_{i=1}^{n} K_{xm_i} Y_{m_i} = \sum_{j=m}^{N} P_{xj} L^*_{yj} \tag{26}$$

where: V_{xm} is the absolute displacement of (any point on) the mth floor, which by definition undergoes pure translation; and n = maximum (n_m, $m = 1, 2, \ldots, N$).

Note that in applying equation (26) to any mth floor, if $n_m < n$ then $K_{xm_i} Y_{m_i}$ is taken to be zero when $n > i > n_m$. Substituting the $(m+1)th$ equation of (26) into the mth equation, the set of equations may be rewritten as

$$- V_{xm-1} \cdot \sum_{i=1}^{n} K_{xm_i} Y_{m_i} + V_{xm} \left\{ \sum_{i=1}^{n} K_{xm_i} Y_{m_i} + \sum_{i=1}^{n} K_{xm+1_i} Y_{m+1_i} \right\} - V_{xm+1} \sum_{i=1}^{n} K_{xm+1_i} Y_{m+1_i}$$

$$= P_{xm} L^*_{ym} \qquad m = 1, 2, \ldots, N$$

The matrix form of the above equations is

$$\{L^*_y\} = [P_x]^{-1} [K_{\theta x}] \{V_x\} \tag{27}$$

where

$$\{L^*_y\} = \{L^*_{y1} L^*_{y2} \ldots L^*_{yN}\}^T$$

$$\{V_y\} = \{V_1 V_2 \ldots V_N\}^T$$

$$[P_x] = \begin{bmatrix} P_1 & & & \\ & P_2 & & \\ & & \ddots & \\ & & & P_N \end{bmatrix}$$

$$[K_{\theta x}] = \begin{bmatrix} \Sigma K_{x1} Y_1 + \Sigma K_{x2} Y_2 & -\Sigma K_{x2} Y_2 & & & & & \\ -\Sigma K_{x2} Y_2 & \Sigma K_{x2} Y_2 + \Sigma K_{x3} Y_3 & -\Sigma K_{x3} Y_3 & & & & \\ & -\Sigma K_{x3} Y_3 & \Sigma K_{x3} Y_3 + \Sigma K_{x4} Y_4 & -\Sigma K_{x4} Y_4 & - & - & \\ & & & & - & - & -\Sigma K_{xN} Y_N \\ & & & & & -\Sigma K_{xN} Y_N & \Sigma K_{xN} Y_N \end{bmatrix}$$

and the vector $\{L_x^*\}$ may also be obtained in a similar form.

Equation (27) is identical in form to that given in Reference 3 for a frame-wall building, with differences appearing in the detail of the stiffness matrix. Alternatively, substituting;

$$e_{ym} = \sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} \Big/ \sum_{i=1}^{n_m} Q_{xm_i}$$

$$\sum_{i=1}^{n_m} Q_{xm_i} = \sum_{j=m}^{N} P_{xj}$$

$$\sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} = \sum_{j=m}^{N} P_{xj} L_{yj}^*$$

into equation (25*b*) leads to

$$L_{ym}^* = \left\{ \sum_{i=1}^{n_m} Q_{xm_i} Y_{m_i} - \sum_{i=1}^{n_{m+1}} Q_{xm+1_i} Y_{m+1_i} \right\} \Big/ P_{xm}$$

If the lateral load-resisting elements have identical locations and are the same in number for all storeys ($n_m = n_{m+1} = n$), the above equation becomes

$$L_{ym}^* = \left\{ \sum_{i=1}^{n} (Q_{xm_i} - Q_{xm+1_i}) Y_i \right\} \Big/ P_{xm} \qquad (28)$$

and L_{xm}^* has a similar form. Equation (28) is identical to that given in Reference 7, also for application to frame-wall buildings. Since

$$P_{xm} = \sum_{i=1}^{n} (Q_{xm_i} - Q_{xm+1_i}) \qquad (29)$$

the centre of rigidity CRi on the mth floor is identified as the location of the resultant of the element reaction forces acting on the floor. These reaction forces are calculated for a given element (as in the examples below) as the difference between the element shears acting in the storeys immediately above and below the reference floor.

Using equations (12*a*) and (25*b*), the relationship between the storey torsional moment m_{xm} and the position of CRi for floor level m can be found, as follows

$$\begin{aligned} M_{xm} = K_{\theta m}\theta_{xm} &= -\sum_{k=m}^{N} P_{xik}(\epsilon_{ym} - e_{ym}) \\ &= -\sum_{k=m}^{N} P_{xk}(L_{yk} - e_{ym}) \\ &= -\sum_{k=m}^{N} P_{xk}(L_{yk} - L_{yk}^* + L_{yk}^* - e_{ym}) \\ &= -\sum_{k=m}^{N} P_{xk}(L_{yk} - L_{yk}^*) = -\sum_{k=m}^{N} T_{xk} \end{aligned} \qquad (30)$$

and similarly for the component M_{ym}.

According to Reference 1, $T_{xk} = P_{xk}(L_{yk} - L_{yk}^*)$ is termed the kth floor torque, whose sum for all floors above storey level m (as in equation (30)) yields the storey torsional moment required for design. The corresponding floor eccentricities ζ_{xm}, ζ_{ym} are

$$\zeta_{xm} = L_{xm} - L_{xm}^*, \quad \zeta_{ym} = L_{ym} - L_{ym}^* \qquad (31)$$

The floor eccentricities defined in equation (31) are the basis of the equivalent static torsional design provisions of the New Zealand code NZS 4203, 1992[12], the United States reference document SEAOC 1975[13], and other codes[11] that include Iran and Portugal.

Special class of multistorey buildings

Inspection of equation (25) reveals that there is a particular or special class of buildings for which the following relationships hold

$$L_{xm}^* = e_{xm}, \quad e_{xm} = e_{xm+1}, \quad m = 1, 2, \ldots, N \qquad (32a)$$

$$L_{ym}^* = e_{ym}, \quad e_{ym} = e_{ym+1}, \quad m = 1, 2, \ldots, N \qquad (32b)$$

Equation (32) indicates firstly that the centre of rigidity on each floor is coincident with the corresponding shear centre for the storey immediately below, and that all shear centres in the building lie along a vertical line. Secondly, in view of the coincidence of the shear centres with the storey centres of stiffness in shear buildings (as studied herein), the eccentricities of this special class of buildings are load independent. Therefore the special class defines buildings in which the storey centres of stiffness lie on a single vertical line. For frame buildings the special class implies that the framing is proportional for each storey, such that the stiffness ratios of the frames are constant.

Examples and discussion

Selection of building models

Two example asymmetric buildings have been considered, as illustrated in *Figure 3*. The first building (Examples 1 and 2) is subjected to two different distributions of lateral forces representing the seismic design loading for the structure. Both buildings have four storeys and have three frames A, B and C resisting lateral forces which are applied in the y-direction only. Each two-bay by one-bay structure is assumed to be symmetric about the centre-line in the x-direction and hence eccentricities exist in the x-direction only, namely perpendicular to the seismic loading.

The floors are assumed to consist of perfectly rigid diaphragms which permit no deformation either in-plane or in flexure. Hence the floor beams of all frames are modelled as rigid elements for the purpose of developing the shear building model. Whilst this idealization is strictly applicable only to certain types of building (where the flexural stiffness of the beams is very large compared with the frame columns), it has computational advantages for determination of eccentricities and design

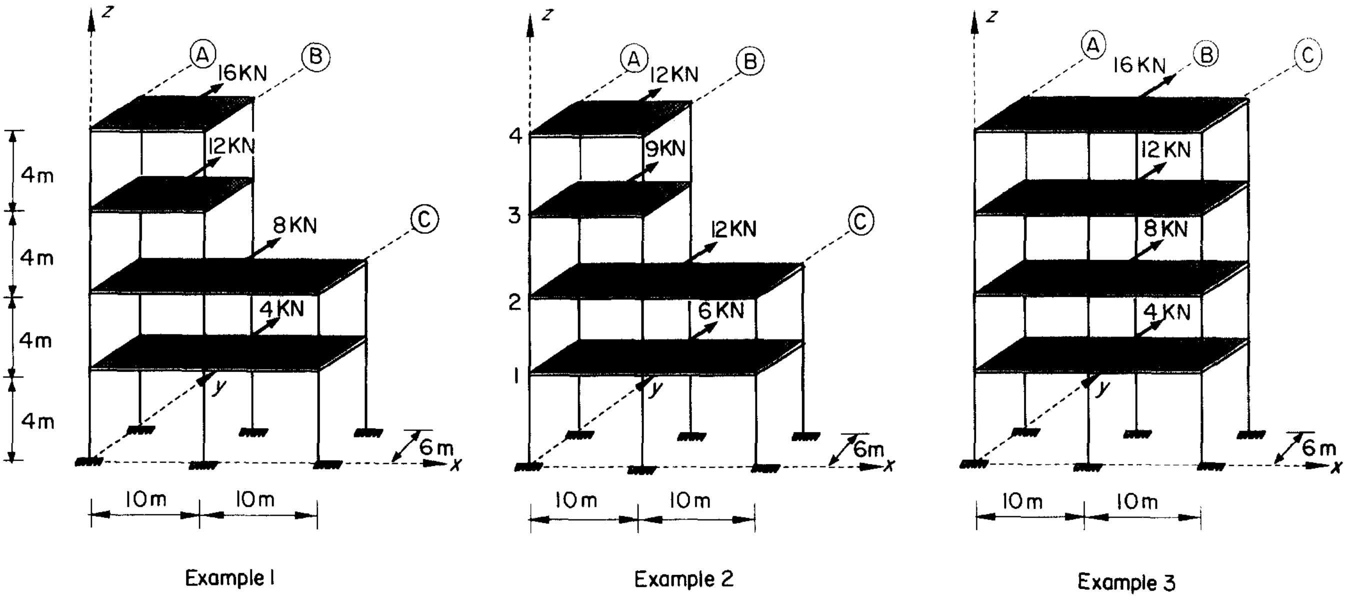

Figure 3 Example structures with lateral loading distributions

torsional moments for asymmetric structures. Furthermore, such a model may be considered to provide sufficiently accurate approximations of torsional loadings for many prototype framed buildings in seismic regions, where typically the ratio of beam-to-column stiffness may be taken to be about 3, as in Reference 1. The accuracy of the shear building approximation is examined by means of Example 1, which apart from the beam stiffness (herein assumed infinite) is identical to the building model and (second) loading distribution analysed in Reference 1.

Examples 1 and 2 refer to an identical set-back framed structure, whilst in Example 3 the structure has four identical rectangular floors. In all cases the floor centres of mass are assumed to coincide with the geometric centroid; this position is also the point of application of the lateral seismic forces. The total lateral load or base shear is taken to be 40 kN in Examples 1 and 3 (which have identical load distributions) and 39 kN in Example 2. Hence for practical purposes the total load applied in each case is identical. The load distribution applied to the structures in Examples 1 and 3 is inverted triangular in form (see *Table 1*). This is appropriate for seismically loaded, low-rise buildings with a uniform mass distribution throughout the height of the building (as in Example 3). However, in view of the reduction in floor plan area above the second floor in the asymmetric set-back structure (Examples 1 and 2), the inverted triangular load distribution is not strictly appropriate as it overestimates the seismic loading developed in the upper two floors. Hence in Example 2 the loading distribution is modified to account for the 50% reduction in plan area in floors 3 and 4 (see *Table 1*), whilst maintaining (virtually) the same total loading. Note that the storey shear forces resulting from the specified lateral loading distributions are listed in *Table 5*.

The flexural stiffness of the columns in frames A, B and C are specified in *Table 2*, in terms of the second moments of area I_c appropriate for loading in the y-direction. For the set-back structure, the columns of frames A and C have $I_c = 0.05$ m^4 and for frame B, I_c is taken as 0.1 m^4. For Example 3, the column stiffnesses in frames A, B and C are as given above for storeys 3 and 4 (i.e. the structure is symmetric above the second floor level), but in storeys 1 and 2 the column stiffnesses in frame C are increased by a factor of 3. This introduces significant stiffness eccentricity into the lower levels of the structure.

Table 1 Lateral force distributions (in kN)

Floor	Example 1	Example 2	Example 3
4	16	12	16
3	12	9	12
2	8	12	8
1	4	6	4
Total (Base shear)	(40)	(39)	(40)

Table 2 Column bending stiffnesses

		I_c (m^4)		
Example	Storey	Frame A	Frame B	Frame C
1	4	0.05	0.1	–
	3	0.05	0.1	–
	2	0.05	0.1	0.05
	1	0.05	0.1	0.05
2	4	0.05	0.1	–
	3	0.05	0.1	–
	2	0.05	0.1	0.05
	1	0.05	0.1	0.05
3	4	0.05	0.1	0.05
	3	0.05	0.1	0.05
	2	0.05	0.1	0.15
	1	0.05	0.1	0.15

Table 3 Frame shears (in kN)

Example	Storey	Frame A	Frame B	Frame C
1	4	5.33	10.67	–
	3	9.33	18.67	–
	2	9.00	18.00	9.00
	1	10.00	20.00	10.00
2	4	4.00	8.00	–
	3	7.00	14.00	–
	2	8.25	16.50	8.25
	1	9.75	19.50	9.75
3	4	4.00	8.00	4.00
	3	7.00	14.00	7.00
	2	6.00	12.00	18.00
	1	6.67	13.33	20.00

Table 3 gives the distribution of the total frame shear forces for each example (note that each frame consists of two columns in the direction of loading). The resultant of these frame shears locates the position (in the x-direction, measured from frame A) of the storey shear centres, given in *Table 5*. Due to the symmetry of both structures about the x-direction, these shear centres are located at $y = 3$ m in all cases, i.e. along the floor centre-lines (*Figure 3*). The storey shear centres are identical to the storey stiffness centres since the frame shears are simply proportional to the column stiffnesses in the shear building model. This result follows from the extrapolation of the 'single storey' eccentricity approach described earlier to the complete building, i.e. no relative and hence no absolute floor rotations occur.

The frame shears so obtained lead directly to the frame reactions listed in *Table 4*, and hence to the positions of the floor centres of rigidity given in *Table 6*. In Reference 1 the structure of Example 1 was analysed assuming the beams in all frames to be axially rigid but with flexural stiffness denoted by $I_b = 0.3$ m^4. Hence in Reference 1, the frame responses (shears and reactions) were determined by a plane frame analysis, connecting the three frames at floor levels by rigid links and subjecting the complete assembly to the specified lateral floor loadings. The shear building model offers computational advantages compared with this approach, since in the former case a plane frame analysis is not required.

Static eccentricities and torsional moments

Tables 5 and *6* (Example 1) compare the results of Reference 1 (shown in brackets) with those calculated using the 'storey' and 'all floors' approaches described above. As expected, the storey torsional moments from both studies are independent of the definition of the static eccentricities. Overall, the comparison shows that a good first approximation to the torsional moments (and eccentricities) may be obtained using the proposed simplified approach.

The static eccentricities resulting from the 'all floors' method are found, firstly, to be much more sensitive to the geometric irregularity (Example 1) than the eccentricities resulting from the 'storey' approach. This is apparent when comparing the results for floors 3 and 2 (*Table 6*), which show a very significant increase in eccentricity from -1.67 m to -11.67 m; for the same example the increase in storey eccentricity (*Table 5*) is

Table 4 Frame reactions (in kN)

Example	Floor	Frame A	Frame B	Frame C
1	4	5.33	10.67	–
	3	4.00	8.00	–
	2	–0.33	–0.67	9.00
	1	1.00	2.00	1.00
2	4	4.00	8.00	–
	3	3.00	6.00	–
	2	1.25	2.50	8.25
	1	1.50	3.00	1.50
3	4	4.00	8.00	4.00
	3	3.00	6.00	3.00
	2	–1.00	–2.00	11.00
	1	0.67	1.33	2.00

Table 5 'Storey' eccentricity approach

Example	Storey	CS/shear C. location (m)	Storey ecc. (m)	Storey* shear (kN)	Storey torsional moment (kN-m)
1	4	6.67	–1.67 (–1.00)	16	–26.7 (–16.0)
	3	6.67	–1.67 (–1.09)	28	–46.7 (–30.6)
	2	10.00	–3.89 (–4.19)	36	–140.0 (–151.0)
	1	10.00	–3.50 (–3.48)	40	–140.0 (–139.2)
2	4	6.67	–1.67	12	–20.0
	3	6.67	–1.67	21	–35.0
	2	10.00	–3.18	33	–105.0
	1	10.00	–2.69	39	–105.0
3	4	10.00	0	16	0
	3	10.00	0	28	0
	2	13.33	–3.33	36	–120.0
	1	13.33	–3.33	40	–133.3

( ): Reference 1
*Calculation based on line of action of resultant of all lateral (floor) forces above the storey under consideration

Table 6 Floor eccentricity ('all floors') approach

Example	Floor	CRi location (m)	Floor ecc. (m)	Floor torque (kN-m)	Storey torsional moment (kN-m)
1	4	6.67	−1.67 (−1.00)	−26.7 (−16.0)	−26.7 (−16.0)
	3	6.67	−1.67 (−1.22)	−20.0 (−14.6)	−46.7 (−30.6)
	2	21.67	−11.67 (−15.05)	−93.4 (−120.4)	−140.0 (−151.0)
	1	10.00	0 (+2.95)	0 (+11.8)	−140.0 (−139.2)
2	4	6.67	−1.67	−20.0	−20.0
	3	6.67	−1.67	−15.0	−35.0
	2	15.83	−5.83	−70.0	−105.0
	1	10.00	0	0	−105.0
3	4	10.00	0	0	0
	3	10.00	0	0	0
	2	25.00	−15.00	−120.0	−120.0
	1	13.33	−3.33	−13.3	−133.3

( ): Reference 1

much smaller, from -1.67 m to -3.89 m. Secondly, comparing the results for Example 1 with those obtained for the identical structure with a more realistic loading distribution (Example 2), it is observed that the floor eccentricity ('all floors') approach is more sensitive to the manner in which the lateral load is distributed over the height of the building. This is particularly noticeable with respect to the results for storey and floor level 2.

The above observations are in close agreement with those made in Reference 1 with respect to analysis of the building model as a series of rigidly connected plane frames. The basic approach and computational effort required to determine the (identical) storey torsional moments is similar, when using either the 'storey' or 'all floors' approaches for defining the static eccentricities in asymmetric shear buildings. However, in view of the above observations the former method is considered to lead to a more rational and consistent definition of torsional effects in seismically loaded asymmetric buildings.

Examples 1 and 3 lead to very similar distributions of static eccentricity and torsional moments, when comparing the results obtained using either of the above two approaches (*Tables 5* and *6*). Hence both the 'storey' and 'all floors' methods appear to be relatively insensitive to the form of eccentricity present in the example buildings considered.

The third method described above for determining the static eccentricities (namely the 'single floor' approach) is evaluated in *Table 7*, taking Example 1 for the purpose of illustration. The 'single floor' eccentricities and resultant 'single floor' torques are significantly higher than those obtained from the 'all floors' approach (*Table 6*), and the sensitivity to the geometric discontinuity above floor level 2 is clearly apparent. The location of the centres of resistance, CRe, for floors 1 and 2 is at $x = 45$m (*Table 7*, refer to *Figure 1(a)*) and hence the CRe for these two floors is located well outside the physical boundary of the building. The centres of rigidity CRi for floor 2 in Examples 1 and 3, obtained by the floor eccentricity ('all floors') approach, also lie outside the floor boundary (*Table 6*).

In *Table 7*, the storey torsional moments given in the final column are the same as those obtained by the alternative approaches (*Tables 5* and *6*), as expected. These moments, calculated using equations (24*a*, *b*), are seen to depend on the storey torsional stiffness ratio for storey levels m and $m-1$, as listed in *Table 7*. Having obtained the 'single floor' torques (equations (21*a*, *b*)) by multiplying the corresponding eccentricities with the lateral floor loadings, the required storey torsional moments are calculated, firstly for storey 1, then by working progressively upwards through the building. The 'single floor' torques for the mth and $(m-1)$th levels are required in the calculation, as well as the above-mentioned storey torsional stiffness ratio.

In view of the extremely high values of eccentricity resulting from the 'single floor' approach, together with the somewhat more complex method of calculation, this approach is not recommended for application in practical design. Indeed, the results presented herein justify the approaches adopted by current seismic codes, which are based invariably on the storey eccentricity and floor eccentricity ('all floors') approaches discussed above.

Table 7 Floor eccentricity ('single floor') approach: Example 1

Floor	$K_{\theta m}/K_{\theta m-1}$	CRe location (m)	Single floor ecc. (m)	Single floor torque (kN-m)	Storey torsional moment (kN-m)
4	1.0	16.54	−11.54	−184.64	−26.7
3	0.397	18.16	−13.16	−157.92	−47.7
2	1.0	45.00	−35.00	−280.00	−140.0
1	–	45.00	−35.00	−140.00	−140.0

Use of the equivalent static method in seismic design

The need for a simple working definition of static eccentricity arises when performing an equivalent static analysis of multistorey buildings for the purpose of seismic design, in lieu of a full dynamic analysis. Therefore, it is important to assess the application of the static method discussed in this paper in relation to the dynamic approach. It is emphasized that the various definitions of eccentricity utilized in the static approach discussed in this paper, ultimately provide identical design forces (storey torsional moments) accounting for the torsional effect in asymmetric buildings. Hence while studies comparing the results of static and dynamic analyses provide information on the validity of the static approach *per se*, their conclusions are not dependent on the definition of the static eccentricity, which has provided the focal point of the current paper. A related study[14] presents detailed comparisons of the two most widely employed analytical methods in seismic building design, namely the equivalent static procedure and the spectral modal analysis method. The emphasis in Reference 14 is on evaluating the effectiveness of such methods for controlling the inelastic seismic torsional response of asymmetric multistorey frame buildings, of the type discussed in the present study.

Conclusions

The purpose of this study was to clarify and compare three published definitions of static eccentricity in multistorey shear buildings, which are each used to provide a measure of building asymmetry. The results have also been used to comment on the suitability of each approach for determining the design torsional moments in asymmetric buildings, and to define a system parameter for the inelastic seismic analysis of such buildings. The main conclusions arising from this study in the light of the above objectives can be summarized as follows:

The shear building model may be regarded as a simple form of structural representation of multistorey buildings. It provides a valuable insight leading to improved understanding of the various definitions used to define static eccentricity. The inherent relationships between the various definitions and the methods of transforming one to another are clearly and easily understood in shear buildings. Furthermore, the simplified computational approach which can be applied to such buildings leads to a good first approximation of the torsional moments (and eccentricities), whichever of the alternative definitions of eccentricity is employed.

The static eccentricities resulting from the 'all floors' approach are found to be more sensitive to geometric or stiffness irregularities than those obtained from the 'storey' approach. Furthermore, the former approach is affected more significantly by changes in the vertical distribution of the lateral design loading. These conclusions are in close agreement with those made in Reference 1, with respect to building models analysed as a series of rigidly connected plane frames.

Both the 'storey' and 'all floors' methods appear to be relatively insensitive to the form of eccentricity present in typical asymmetric buildings. For example, similar magnitudes and distributions of static eccentricity have been obtained for an eccentric set-back structure and a building with regular floor geometry but eccentric distribution of stiffness in the lower storeys.

The static eccentricities obtained using the 'single floor' approach are the most highly sensitive, of the three methods, to changes in floor geometry in the example set-back structure. The 'single floor' eccentricities are also significantly higher than those resulting from the 'all floors' or 'storey' approaches. In view of these factors, together with the somewhat more complex method of calculation, the 'single floor' approach is not recommended for application in practical design.

For shear buildings, the storey shear centres (SCs) are considered overall to provide the most appropriate reference points for estimates of static eccentricity. The advantages of this approach are that, firstly, the resulting calculation of storey eccentricities and design torsional moments is the most straightforward of the three methods described herein. Secondly, the method is the least sensitive to changes in the lateral load distribution or the form of structural asymmetry. Thirdly, the positions of the SCs are independent of the lateral load distribution. Fourthly, the storey eccentricities can be regarded as a measure of the relative rotation between floors, which reflects directly the influence of asymmetry on the load demand for resisting elements. This latter quantity is of direct concern to structural engineers for the purpose of seismic design.

The position of the storey centres of stiffness CS is a very important structural parameter in shear buildings. This results principally from the special property that CS coincides with the storey shear centre. In addition, the storey centres of stiffness play an important role in calculations of the centres of rigidity and resistance, CRi and CRe, in multistorey asymmetric shear buildings.

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