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VIBRATIONS OF NON-LINEAR ASYMMETRIC BUILDINGS ON A FLEXIBLE FOUNDATION

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The dynamic response of inelastic asymmetric shear buildings supported on a linear elastic half-space is investigated for sinusoidal excitation. The resisting elements of the building are assumed to be bilinear hysteretic. The influence of non-dimensional system parameters on the lateral and torsional response spectra, as well as the peak ductility of the resisting elements, are determined. It is found that for tall linear and non-linear structures as well as short non-linear structures the response increases as the soil becomes more flexible. It is also observed that the flexible soil has a profound effect on the peak ductility of the element further away from the centre of stiffness.

1. INTRODUCTION

The dynamic response of asymmetric building–foundation system has been studied in the past by Balendra *et al.* [1, 2] and by Tsicnias and Hutchinson [3] on the assumption that the behaviour of both the structure and the soil is linear. While linear analysis provides valuable information on the dynamic response, in the event of an extreme earthquake the designer is not economically justified in restricting the behaviour of the structure to be linear elastic. Thus, recently, Kan and Chopra [4], Syamal and Pekau [5] and Tso and Sadek [6] have studied the inelastic response of asymmetric buildings, but have treated the foundation as rigid. The purpose of this investigation is to study the effects of the foundation on the inelastic response of asymmetric buildings. A similar study for a symmetric building has been carried out by Bielak [7].

A single-storey monosymmetric shear building supported by an elastic homogeneous, isotropic half-space is considered as the building–foundation system. The resisting elements are assumed to be bilinear hysteretic, for which linear and elasto-plastic are two limiting cases. The free field motion is assumed to be sinusoidal. The non-dimensional system parameters are identified, and their influence on the steady state response of the system is determined.

2. FORMULATION OF THE PROBLEM

2.1. IDEALIZATION OF THE MODEL

A monosymmetric structure with a floor plan shown in Figure 1(a) is considered. The degrees of freedom of the corresponding building–foundation system are shown in Figure 1(b). The model represents a single-storey shear building with bilinear stiffness, resting on the surface of a linear elastic half-space. The base is assumed to be a massless rigid plate of infinitesimal thickness.

When the ground motion is in the x direction, due to vertically incident shear waves, the structure–foundation system will have five degrees of freedom; namely, horizontal

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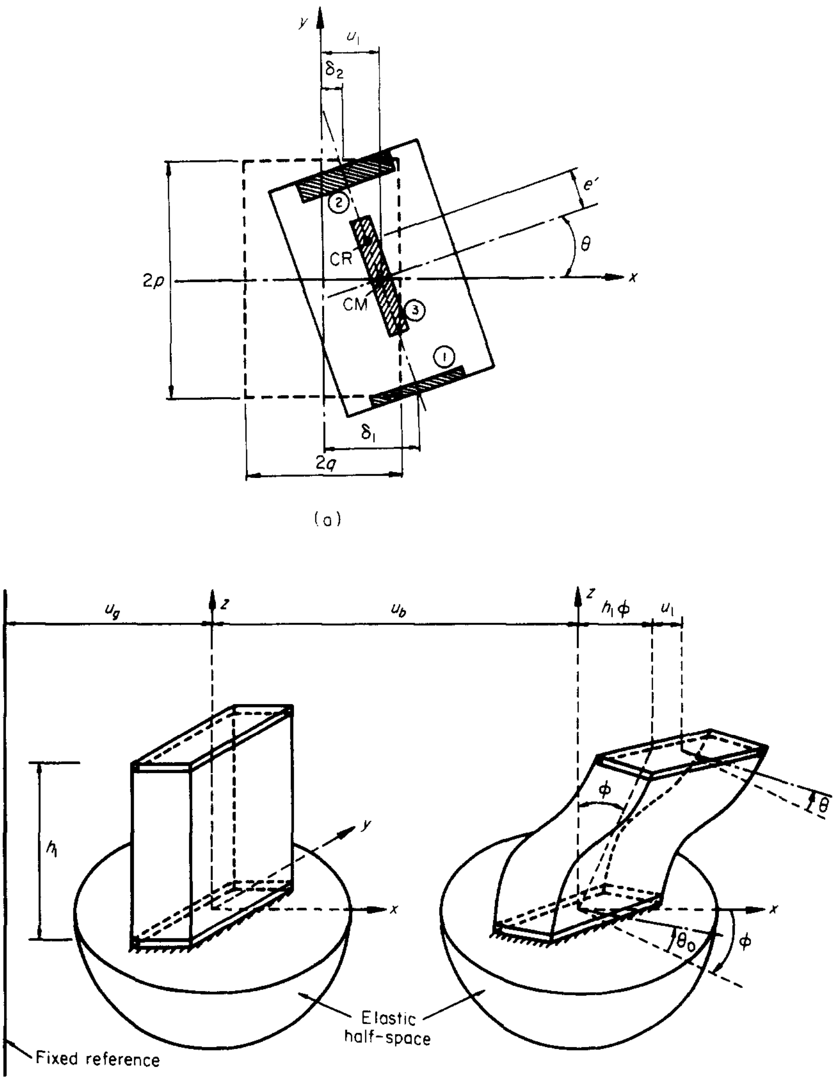

Figure 1. (a) Lateral–torsional displacement of the floor; (b) degrees of freedom of a monosymmetric building foundation model.

translation of the base, u_b, rocking of the system in the plane of excitation, ϕ, twisting of the base, θ_0, translation, u_1, and twist, θ, of the top floor mass with respect to the base mass.

2.2. EQUATION OF MOTION

For horizontal ground acceleration of $\ddot{u}_g$ along the x-axis, the equation of motion for translation of the whole system along the x-axis can be written as

$$M_1\ddot{u}_t + C_v\dot{u}_b + K_v u_b = 0, \tag{1}$$

where $\ddot{u}_t = \ddot{u}_g + \ddot{u}_b + h_1\ddot{\phi} + \ddot{u}_1$ is the total acceleration of the top floor mass, h_1 is the height of the building, M_1 is the mass of the top floor, and K_v and C_v are the stiffness and damping coefficients associated with the interaction force at the structure–foundation interface.

The equation of motion for rocking of the whole system can be written as

$$I_1\ddot{\phi}+M_1h_1\ddot{u}_t+C_\phi\dot{\phi}+K_\phi\phi=0, \tag{2}$$

where K_ϕ and C_ϕ are the stiffness and damping coefficients associated with the interaction moment at the structure–foundation interface, and I_1 is the mass moment of inertia of the top floor.

The equation of motion for twist of the whole system about the vertical axis through the centre of base mass can be written as

$$J_1\ddot{\theta}_t+C_\theta\dot{\theta}_0+K_\theta\theta_0=0, \tag{3}$$

where $\ddot{\theta}_t=\ddot{\theta}_0+\ddot{\theta}$ is the total twist of the top floor mass, J_1 is the polar mass moment of inertia of the top floor mass, and K_θ and C_θ are the stiffness and damping coefficients associated with the torsional moment at the structure–foundation interface.

For translational and torsional motions of the top floor, the equations of motion are

$$M_1\ddot{u}_t+C_1\dot{u}_1+R_{1x}+R_{2x}=0, \qquad J_1\ddot{\theta}_t+C'_\theta\dot{\theta}+(R_{1x}-R_{2x})p=0, \tag{4, 5}$$

where C_1 and C'_θ are the translational and torsional viscous damping coefficients of the first storey, and p is the distance of the resisting elements from the centre of mass. Furthermore, R_{1x} and R_{2x} are the hysteretic restoring forces of the resisting elements, and are defined as

$$R_{ix}(\delta_i)=k_{ix}F_i(\delta_i, \alpha), \qquad i=1, 2, \tag{6}$$

where $F_i(\delta_i, \alpha)$ is a general function for the geometric description of the hysteretic skeleton

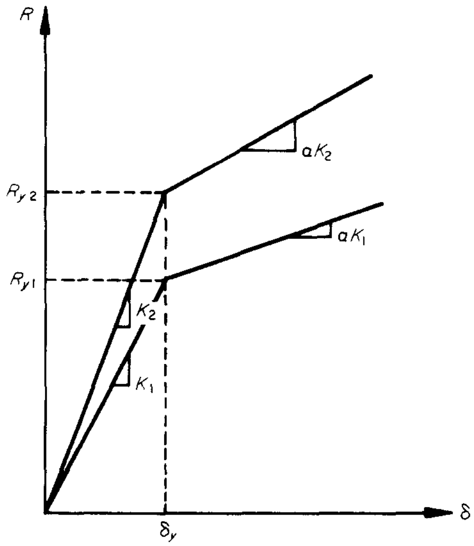

Figure 2. Restoring force-displacement relation.

curve shown in Figure 2, in which k_{ix} is the linear stiffness of the ith resisting element, α is the bilinear coefficient and δ_i ($i=1, 2$) is the displacement of the ith element. The latter can be expressed as linear functions of system displacements u_1, θ as

$$\delta_{1,2}=u_1\pm p\theta. \tag{7}$$

It is evident from the hysteretic curve that the yield displacement δ_y is the same for both elements. The linear stiffnesses of elements 1 and 2 are assumed to be related to each other by $k_{2x} = (1+\eta)k_{1x}$, where η is the unbalanced stiffness factor, as a measure of eccentricity. The normalized hysteretic force–displacement relation is shown in Figure 3.

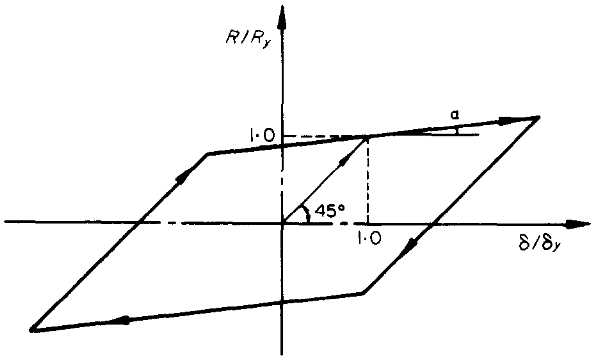

Figure 3. Normalized hysteretic force–displacement relation.

3. METHOD OF SOLUTION

The governing equations (1)–(5) are non-linear because of the terms R_{1x} and R_{2x} representing the restoring forces in the superstructure. The problem is then one of a localized non-linearity of the type studied by Iwan [8]. When the bilinear coefficient $\alpha = 1$, the response will be sinusoidal under steady state harmonic excitation. If α is close to one, the sinusoidal character of the response will be preserved [9, 10]. Iwan [11] has shown that the approximate solutions obtained for hysteretic behaviour close to the elasto-plastic state, on the assumption of essentially sinusoidal response, is within 10% of exact values. In anticipation of the same degree of accuracy, the response of the building–foundation system under study is assumed to be essentially sinusoidal when the ground acceleration is of the form

$$\ddot{u}_g = \hat{u}_g \cos wt, \tag{8}$$

where $\hat{u}_g$ is the amplitude of the ground acceleration and w is the excitation frequency. Solving equations (1)–(3) for u_b, ϕ and θ_0 in terms of u_1 and θ, and substituting the resulting expressions into equations (4) and (5) leads to

$$\ddot{u}_1 + [2\zeta_x w_x + \Omega^2 w_x \sigma_1]\dot{u}_1 + [\Omega^2 w_x^2 \sigma_2]u_1 + w_x^2[F_1 + (1+\eta)F_2]/(2+\eta)$$
$$= -\hat{u}_g[\{1-\sigma_2\}\cos(wt) + \Omega\sigma_1 \sin(wt)], \tag{9}$$

$$\ddot{\theta} + [2\zeta_\theta w_\theta + w_x\sigma_3]\dot{\theta} + w_x^2\sigma_4\theta + (k_{1x}F_1 p - k_{2x}F_2 p)/J_1 = 0, \tag{10}$$

in which $\Omega = w/w_x$, w_x is the natural frequency of uncoupled vibration in the x direction of the linear system on a rigid ground, determined as $\sqrt{(k_{1x}+k_{2x})/M_1}$, and w_θ is the corresponding uncoupled torsional frequency, determined as $\sqrt{k_{\theta_m}/J_1}$, where k_{θ_m} is the linear torsional stiffness about the centre of mass. Furthermore,

$$\zeta_x = C_1/2M_1 w_x, \qquad \zeta_\theta = C'_\theta/2J_1 w_\theta \tag{11}$$

and the expressions for σ_1, σ_2, σ_3 and σ_4 appearing in equations (9) and (10) are defined in the Appendix.

In order to express the equations of motion in a non-dimensional form, setting $\tau = w_x t$ in equations (9) and (10) leads to

$$(\mathrm{d}^2u_1/\mathrm{d}\tau^2) + (2\zeta_x + \Omega^2\sigma_1)(\mathrm{d}u_1/\mathrm{d}\tau) + \Omega^2\sigma_2 u_1 + [F_1 + (1+\eta)F_2]/(2+\eta)$$
$$= (-\hat{u}_g/w_x^2)[(1-\sigma_2)\cos(\Omega\tau) + \Omega\sigma_1 \sin(\Omega\tau)], \quad (12)$$

$$w_x^2(\mathrm{d}^2\theta/\mathrm{d}\tau^2) + w_x(2\zeta_\theta w_\theta + w_x\sigma_3)(\mathrm{d}\theta/\mathrm{d}\tau) + \sigma_4 w^2\theta + (k_{1x}F_1 p - k_{2x}F_2 p)/J_1 = 0. \quad (13)$$

Now, by setting

$$\Gamma_x(\tau) = u_1(\tau)/\delta_y, \qquad \Gamma_\theta(\tau) = \theta(\tau)/\theta_y, \quad (14)$$

where $\theta_y = \delta_y/p$ is the initial yield torsional displacement of the structure, one obtains the non-dimensional forms of the equations of motion as

$$\ddot{\Gamma}_x + [2\zeta_x + \Omega^2\sigma_1]\dot{\Gamma}_x + \Omega^2\sigma_2\Gamma_x + \{1/(2+\eta)\}\{f(\delta_1/\delta_y, \alpha) + f(\delta_2/\delta_y, \alpha)\}$$
$$+ \{\eta/(2+\eta)\}\{f(\delta_2/\delta_y, \alpha)\}$$
$$= \{-\hat{u}_g/(\delta_y w_x^2)\}\{[1-\sigma_2]\cos(\Omega\tau) + \Omega\sigma_1\sin(\Omega\tau)\}, \quad (15)$$

$$\ddot{\Gamma}_\theta + [2\zeta_\theta\Omega_\theta + \sigma_3]\dot{\Gamma}_\theta + \sigma_4\Gamma_\theta$$
$$+ \{\beta_2^2/(2+\eta)\}[f(\delta_1/\delta_y, \alpha) - f(\delta_2/\delta_y, \alpha)] - \{\eta\beta_2^2/(2+\eta)\}[f(\delta_2/\delta_y, \alpha)] = 0, \quad (16)$$

where

$$\beta_2 = p/r, \qquad \Omega_\theta = w_\theta/w_x, \qquad r^2 = J_1/M_1. \quad (17)$$

It is convenient to express the functions $f(\delta_i/\delta_y, \alpha)$, $i = 1, 2$, explicitly in terms of Γ_x and Γ_θ. It can be shown that

$$\Gamma_x + \Gamma_\theta = \delta_1/\delta_y, \qquad \Gamma_x - \Gamma_\theta = \delta_2/\delta_y. \quad (18)$$

In view of equation (18), equations (15) and (16) become:

$$\ddot{\Gamma}_x + [2\zeta_x + \Omega^2\sigma_1]\dot{\Gamma}_x + \Omega^2\sigma_2\Gamma_x + \{1/(2+\eta)\}\{f(\Gamma_x + \Gamma_\theta, \alpha) + f(\Gamma_x - \Gamma_\theta, \alpha)\}$$
$$+ \{\eta/(2+\eta)\}\{f(\Gamma_x - \Gamma_\theta, \alpha)\}$$
$$= \{-\hat{u}_g/(\delta_y w_x^2)\}\{[1-\sigma_2]\cos(\Omega\tau) + \Omega\sigma_1\sin(\Omega\tau)\}, \quad (19)$$

$$\ddot{\Gamma}_\theta + [2\zeta_\theta\Omega_\theta + \sigma_3]\dot{\Gamma}_\theta + \sigma_4\Gamma_\theta + \{\beta_2^2/(2+\eta)\}$$
$$\times [f(\Gamma_x + \Gamma_\theta, \alpha) - f(\Gamma_x - \Gamma_\theta, \alpha)] - \{\eta\beta_2^2/(2+\eta)\}[f(\Gamma_x - \Gamma_\theta, \alpha)] = 0. \quad (20)$$

For the solution of equations (19) and (20), the Kryloff and Bogoliuboff [12] method has been used. This method of solving non-linear simultaneous equations is essentially based upon the principle of slowly varying amplitude. The method has been successfully used for inelastic harmonic excitation of single- and multi-degree-of-freedom systems [5, 9, 10].

In the Kryloff and Bogoliuboff method, one assumes the solution to be of the form

$$\Gamma_x(\tau) = A_1(\tau)\cos\{\Omega\tau + \Phi_1(\tau)\}, \qquad \Gamma_\theta(\tau) = A_2(\tau)\cos\{\Omega\tau + \Phi_2(\tau)\}, \quad (21\mathrm{a, b})$$

where A_1 and A_2 are the slowly varying system amplitudes for translation and torsion, respectively, and Φ_1 and Φ_2 are the corresponding slowly varying phase angles for the respective system responses. The resisting element displacements are also functions of the system parameters,

$$\delta_1/\delta_y = \Gamma_x + \Gamma_\theta = A_3(\tau)\cos\{\Omega\tau + \Phi_3(\tau)\}, \qquad \delta_2/\delta_y = \Gamma_x - \Gamma_\theta = A_4(\tau)\cos\{\Omega\tau + \Phi_4(\tau)\}, \quad (22\mathrm{a, b})$$

where A_3 and A_4 are the slowly varying amplitudes and Φ_3 and Φ_4 are the corresponding slowly varying phase angles for the translational displacements of resisting elements 1 and 2 respectively. These four parameters are related to the system parameters as follows:

$$A_3^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Phi_1 - \Phi_2), \qquad A_4^2 = A_1^2 + A_2^2 - 2A_1A_2\cos(\Phi_1 - \Phi_2), \quad \text{(23a, b)}$$

$$\tan\Phi_3 = \{A_1\sin(\Phi_1) + A_2\sin(\Phi_2)\}/\{A_1\cos(\Phi_1) + A_2\cos(\Phi_2)\}, \quad \text{(23c)}$$

$$\tan\Phi_4 = \{A_1\sin(\Phi_1) - A_2\sin(\Phi_2)\}/\{A_1\cos(\Phi_1) - A_2\cos(\Phi_2)\}. \quad \text{(23d)}$$

Substituting equations (21)–(23) into equations (19) and (20), neglecting first order variational terms since these remain essentially constant when averaging over one cycle, and replacing A_j and Φ_j by their average values $\bar{A}_j$ and $\bar{\Phi}_j$, leads to the following four non-linear coupled algebraic equations:

$$\begin{aligned}(2\zeta_x + \Omega^2\sigma_1)\Omega\bar{A}_1 &- [S_3(\bar{A}_3)\cos(\bar{\Phi}_3 - \bar{\Phi}_1) - C_3(\bar{A}_3)\sin(\bar{\Phi}_3 - \bar{\Phi}_1)]/(2+\eta)\\ &- (1+\eta)[S_4(\bar{A}_4)\cos(\bar{\Phi}_4 - \bar{\Phi}_1) - C_4(\bar{A}_4)\sin(\bar{\Phi}_4 - \bar{\Phi}_1)]/(2+\eta)\\ &= (u_g/\delta_y w_x^2)[\{1-\sigma_2\}\sin(\bar{\Phi}_1) + \Omega\sigma_1\cos(\bar{\Phi}_1)], \end{aligned} \quad \text{(24a)}$$

$$\begin{aligned}(1-\sigma_2)\Omega^2\bar{A}_1 &- [C_3(\bar{A}_3)\cos(\bar{\Phi}_3 - \bar{\Phi}_1) + S_3(\bar{A}_3)\sin(\bar{\Phi}_3 - \bar{\Phi}_1)](2+\eta) - (1+\eta)\\ &\times [C_4(\bar{A}_4)\cos(\bar{\Phi}_4 - \bar{\Phi}_1) + S_4(\bar{A}_4)\sin(\bar{\Phi}_4 - \bar{\Phi}_1)]/(2+\eta)\\ &= (u_g/\delta_y w_x^2)[\{1-\sigma_2\}\cos(\bar{\Phi}_1) - \Omega\sigma_1\sin(\bar{\Phi}_1)], \end{aligned} \quad \text{(24b)}$$

$$\begin{aligned}(2\zeta_\theta\Omega_\theta + \sigma_3)\Omega\bar{A}_2 &- \beta_2^2[S_3(\bar{A}_3)\cos(\bar{\Phi}_3 - \bar{\Phi}_2) - C_3(\bar{A}_3)\sin(\bar{\Phi}_3 - \bar{\Phi}_2)]/(2+\eta)\\ &+ (1+\eta)\beta_2^2[S_4(\bar{A}_4)\cos(\bar{\Phi}_4 - \bar{\Phi}_2) - C_4(\bar{A}_4)\sin(\bar{\Phi}_4 - \bar{\Phi}_2)]/(2+\eta) = 0, \end{aligned} \quad \text{(24c)}$$

$$\begin{aligned}(\Omega^2 - \sigma_4)\bar{A}_2 &- \beta_2^2[C_3(\bar{A}_3)\cos(\bar{\Phi}_3 - \bar{\Phi}_2) + S_3(\bar{A}_3)\sin(\bar{\Phi}_3 - \bar{\Phi}_2)]/(2+\eta)\\ &+ (1+\eta)\beta_2^2[C_4(\bar{A}_4)\cos(\bar{\Phi}_4 - \bar{\Phi}_2) + S_4(\bar{A}_4)\sin(\bar{\Phi}_4 - \bar{\Phi}_2)]/(2+\eta) = 0. \end{aligned} \quad \text{(24d)}$$

Here

$$C_i(\bar{A}_i) = \begin{cases}(\bar{A}_i/\pi)(1-\alpha)\cos\theta_i^* + \alpha\pi - (1-\alpha)\sin(2\theta_i^*)/2, & \bar{A}_i > 1\\ \bar{A}_i, & \bar{A}_i < 1\end{cases}, \quad \text{(25a)}$$

$$S_i(\bar{A}_i) = \begin{cases}-(\bar{A}_i/\pi)(1-\alpha)\sin^2(\theta_i^*), & \bar{A}_i > 1\\ 0, & \bar{A}_i < 1\end{cases}. \quad \text{(25b)}$$

where $\theta_i^* = \cos^{-1}[(\bar{A}_i - 2)/\bar{A}_i]$, $i = 3, 4$.

The solution of the set of non-linear algebraic equations, namely equations (23) and (24), by using the IMSL subroutine ZSPOW [13], will yield the steady state average response of the system.

4. SYSTEM PARAMETERS

4.1. STRUCTURAL PARAMETERS

The eccentricity e' may be expressed in terms of the unbalanced stiffness factor η as

$$e' = p\eta/(2+\eta). \quad \text{(26)}$$

Since $p = \sqrt{k_{\theta_m}/(k_{1x} + k_{2x})}$, the non-dimensional eccentricity $\bar{e}$ may be expressed as

$$\bar{e} = e'/r = \eta\Omega_\theta/(\eta + 2), \quad \text{(27)}$$

where e' is the eccentricity. Thus, the non-dimensional structural parameters are as

TABLE 1

Numerical values of system parameters

Description	Definition	Value
Amplitude of ground acceleration	$G_a = \hat{u}_g/\delta_y w_x^2$	1·00
Poisson ratio	v	0·33
Height ratio	h_1/R	1·0 (short and stocky) 4·0 (tall and slender)
Eccentricity ratio	$\bar{e} = e'/r$	0·05 (low) 0·20 (medium) 0·30 (large)
Translational damping ratio	ζ_x	0·05
Torsional damping ratio	ζ_θ	0·05
Frequency ratio	Ω_θ	0·5 (torsionally weak) 1·0 1·5 (torsionally stiff)
Density ratio	$M_1/\pi R^2 h_1 \rho$	0·15
Bilinear coefficient	α	1·00 (linear) 0·10 (bilinear) 0·00 (elasto-plastic)
Shear velocity ratio	$\chi = 2\pi V_s/h_1 w_x$	100 (rigid soil) 10 5 3 (soft soil)

follows: (1) non-dimensional eccentricity, $\bar{e}$; (2) strain-hardening ratio, α; (3) uncoupled torsional frequency/translational frequency ratio, Ω_θ; (4) translational damping ratio, ζ_x; (5) torsional damping ratio, ζ_θ.

4.2. FOUNDATION PARAMETERS

The frequency independent stiffness and damping coefficients associated with the interaction forces corresponding to translational, rocking and torsional motions have been given by Richart and Hall [14] as

$$K_v = 32(1-\gamma)GR/(7-8v), \qquad C_v = 2D_v\sqrt{K_v M_1},$$

$$D_v = 0{\cdot}288\sqrt{B_v}, \qquad B_v = (7-8v)M_1/32(1-v)\rho R^3,$$

$$K_\phi = 8GR^3/3(1-v), \qquad C_\phi = 2D_\phi\sqrt{K_\phi I_1},$$

$$D_\phi = 0{\cdot}15/(1+B_\phi)\sqrt{B_\phi}, \qquad B_\phi = 3(1-v)I_1/8\rho R^5,$$

$$K_\theta = 16GR^3/3, \qquad C_\theta = 2D_\theta\sqrt{K_\theta J_1}, \qquad D_\theta = 0{\cdot}5/(1+2\beta_\theta), \qquad B_\theta = J_1/\rho R^5. \quad (28)$$

Here G is the shear modulus of the soil, ρ is the mass density of the soil, v is the Poisson ratio of the soil, and R is the radius of the footing.

Thus, the non-dimensional parameters of the interaction system are as follows: (1) density ratio, $M_1/\pi R^2 h_1 \rho$; (2) height ratio, h_1/R; (3) shear wave velocity ratio, $\chi = \pi V_s/h_1 w_x$, where V_s is the shear wave velocity.

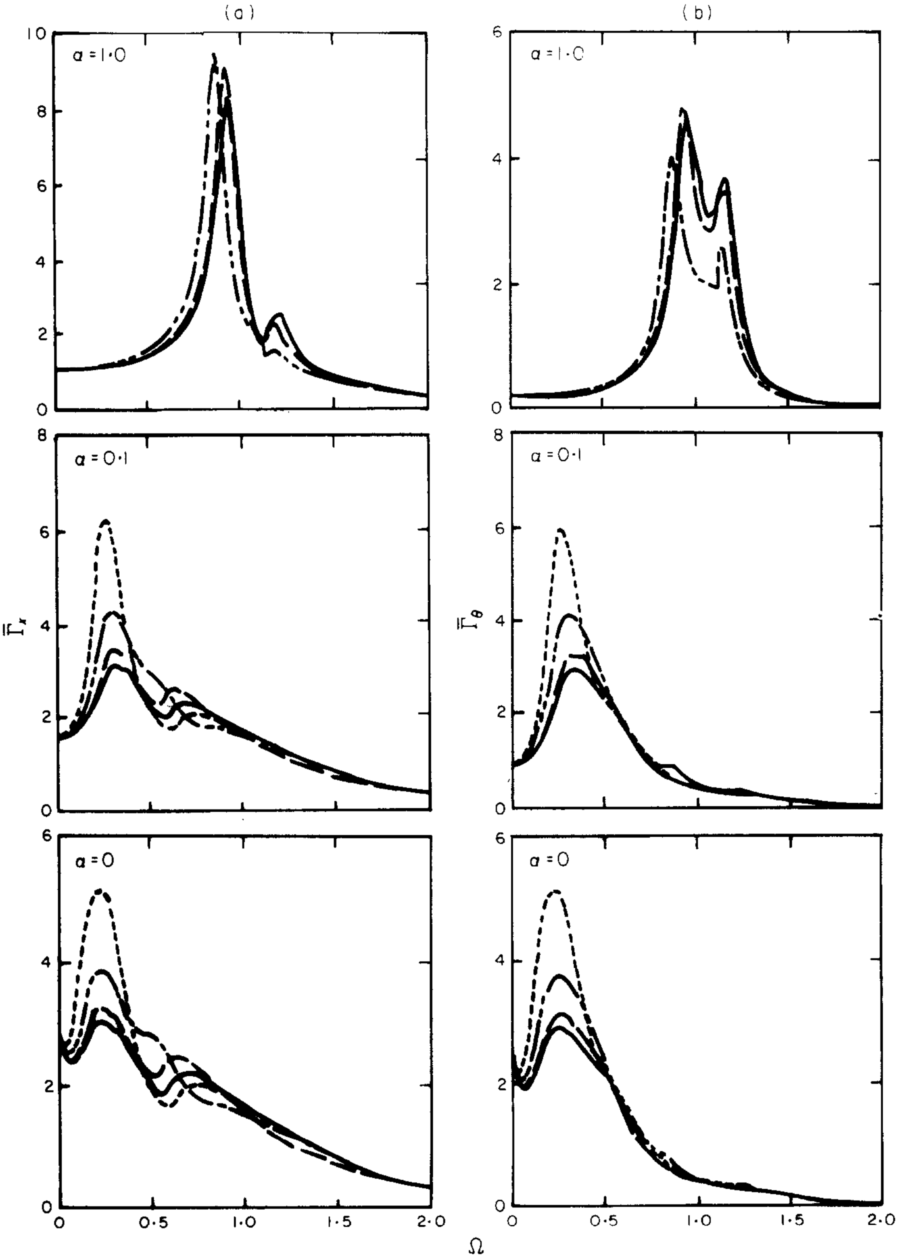

Figure 4. Influence of soil condition on response spectra of tall structures: (a) translational displacement and (b) torsional displacement of the floor. χ values: ——, 100; — —, 20; - - - -, 10; - · - ·, 3.

4.3. EXCITATION PARAMETERS

The system response will also depend on the following non-dimensional parameters of the input motion: (1) the amplitude of the ground acceleration, $G_a = \hat{u}_g/\delta_y w_x^2$; (2) the frequency ratio, $\Omega = w/w_x$.

For the numerical study, the values chosen for the non-dimensional parameters are given in Table 1. Except for parametric variation, the system variables G_a, Ω_θ and $\bar{e}$ are fixed at 1·0, 1·0 and 0·2, respectively.

5. RESULTS AND DISCUSSION

5.1. EFFECTS OF SOIL

The translational and torsional displacements spectra of tall linear, bilinear hysteretic and elastoplastic structures on different soil conditions are depicted in Figure 4. It is seen

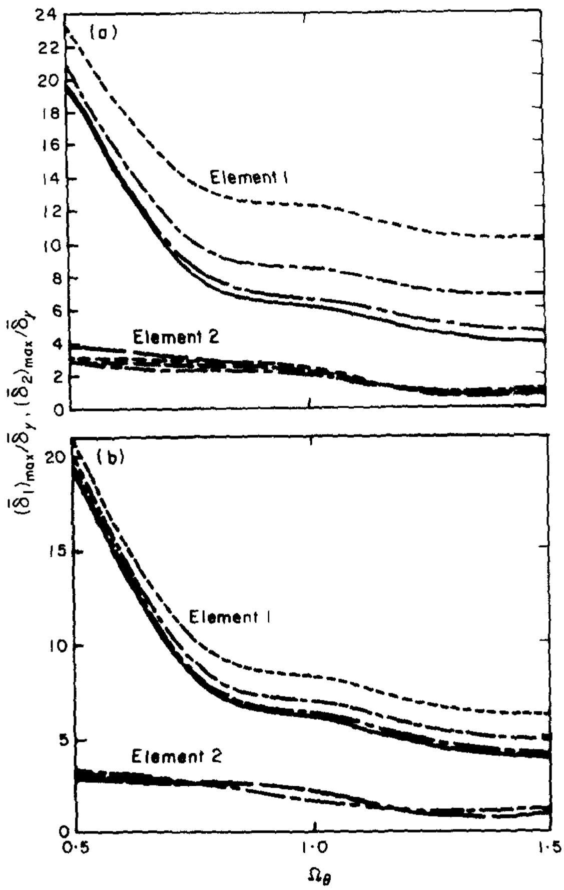

Figure 5. Influence of soil condition on the peak ductility demand of the elements in (a) tall structures and (b) short structures. Element 1 χ values: ——, 100; - — -, 10; - - -, 5; - - -, 0; element 2 χ values: — —, 100; - - -, 10; — —, 5; - — -, 3.

that as the soil becomes more flexible, the first resonant amplitude shifts towards the lower values of frequency, Ω, with a corresponding increase in magnitude.

As the soil becomes softer, the interaction system becomes more flexible, and hence it has a lower resonant frequency. This explains the reasons for shift of the resonant peaks into lower frequencies.

The response of the system is dependent on two mechanisms. First, since the energy generated in the structure is dissipated into the supporting medium by radiation of waves, a flexibly mounted structure offers more damping. This results in a decrease in the response

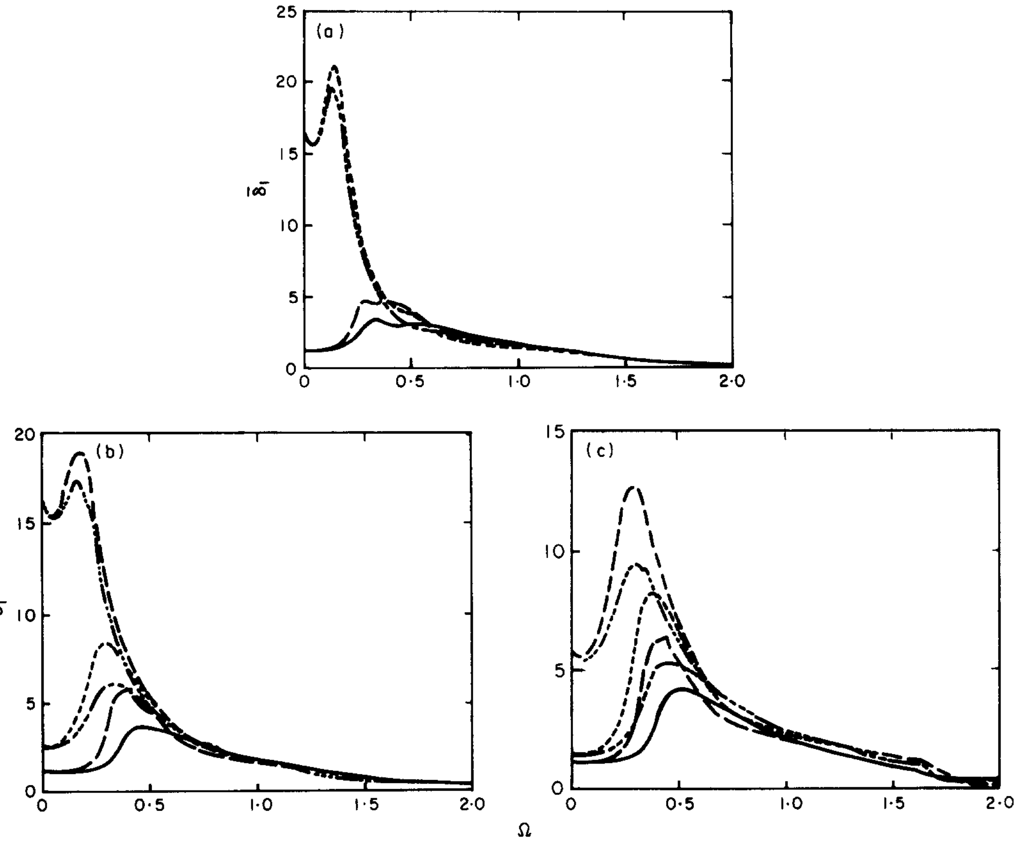

Figure 6. Influence of eccentricity on the displacement response spectra of element 1. (a) $\Omega_\theta=0{\cdot}5$; (b) $\Omega_\theta=1{\cdot}0$; (c) $\Omega_\theta=1{\cdot}5$. χ, $\bar{e}$ values: – –, 5, 0·4; - - - -, 100, 0·4; – – –, 5, 0·2; —- - -, 100, 0·2; — —, 5, 0·05; ——, 100, 0·05.

amplitudes. Second, as the soil becomes softer, the rocking and twisting effects of the building become more pronounced. This will increase the acceleration of the mass the associated inertia force. This "whipping" effect leads to a corresponding increase in the response. Whether the resonant amplitude of a building will increase or decrease as the soil becomes softer will depend upon which mechanism predominates. Thus, in the case of tall structures, the controlling factor seems to be the "whipping" effect, since the amplitude increases as the soil becomes softer. Similar results were obtained by other investigators for symmetrical linear [15] and non-linear [7] tall buildings on a viscoelastic half-space.

It is noted that the second resonant amplitude gradually disappears as the soil becomes softer. This is because as the soil becomes more flexible, it offers more damping. Hence, it is able to absorb most of the energy thereby reducing the second resonant peak.

In short elastic structures, it is observed that a softer soil will result in a decrease in the resonant amplitude. This implies that the damping effect of the soil is the controlling factor for short elastic structures. A similar observation was made for symmetrical short buildings in reference [15]. For short non-linear buildings, it is found that the resonant amplitude increases as the soil becomes softer. This could be due to tuning of the superstructure with the system, when the reduced frequency of the superstructure arising from non-linearity coincides with reduced system resonant frequency.

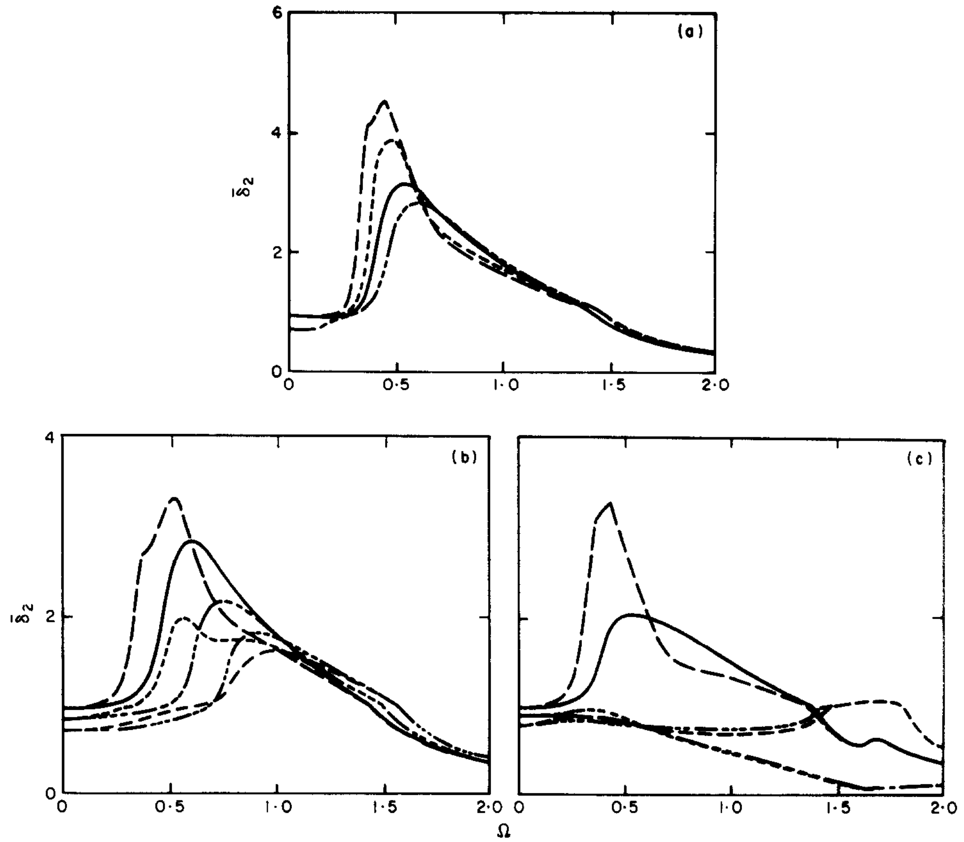

Figure 7. Influence of eccentricity on the displacement response spectra of element 2. (a) $\Omega_\theta = 0{\cdot}5$; (b) $\Omega_\theta = 1{\cdot}0$; (c) $\Omega_\theta = 1{\cdot}5$. Key as Figure 6.

5.2. PEAK DUCTILITY DEMAND

The peak ductility demand in the elements of a structure is very important information for designers. In Figure 5 is shown the influence of soil condition on the peak ductility demand in the far (element 1) and near (element 2) elements of tall and short bilinear structures with different Ω_θ. It is seen that the ductility of the far element is highly dependent on Ω_θ for both tall and short structures. The stiffness of the soil marginally affects the ductility of the near element. However, it has a profound effect on the ductility of the far element, for which the ductility demand increases as the soil becomes more flexible. This effect is greater for tall structures.

In Figures 6 and 7 is shown the effect of non-dimensional eccentricity $\bar{e}$ on the element response spectra for tall structures with different torsional stiffnesses under different soil conditions. The frequency ratios chosen are $\Omega_\theta = 0{\cdot}5$, $1{\cdot}0$ and $1{\cdot}5$. Soil conditions corresponding to stiff soils ($\chi = 100$) and soft soils ($\chi = 5$) have been considered.

From the response spectra of element 1 (Figure 6), it can be seen that, as the eccentricity increases, the response increases for both soil conditions. This is especially so near the resonant condition. The reason for this is that the response of element 1 is dominated by torsional displacement of the system, which increases with eccentricity near the resonant frequencies.

The response of element 2 (Figure 7), is exactly opposite to that of element 1. At lower values of Ω, the less eccentric structure will produce a larger response. However, at higher

Ω, the trend reverses. This is because the response of element 2 is dominated by the translational displacement of the system, which decreases with eccentricity near the resonant frequencies.

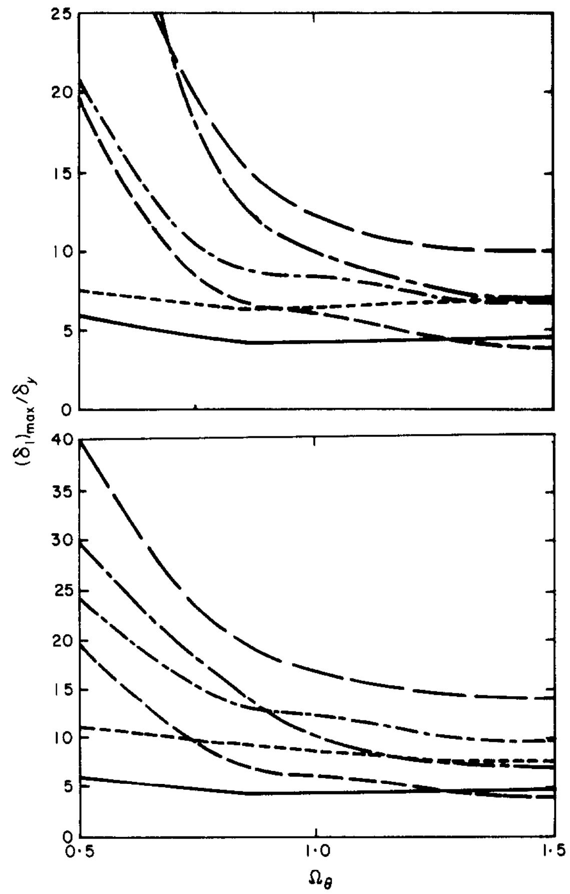

Figure 8. Influence of eccentricity on the ductility demand of element 1 in (a) tall structures and (b) short structures. χ, $\bar{e}$ values: ——, 100, 0·1; - - -, 5, 0·1; – –, 100, 0·2; — — —, 5, 0·2; - — -, 100, 0·3; — —, 5, 0·3.

In Figure 8 is depicted the effect of eccentricity on the peak ductility demand of the far element in tall and short structures. The corresponding results for the near element are not presented, as the eccentricity is found to have only a marginal effect on the peak ductility of the near element. In the case of the far element, the higher the eccentricity, the greater is the peak ductility demand. Another worthwhile observation is that, as Ω_θ becomes smaller (torsionally weak structures), the ductility demand of the far element for a more eccentric structure increases significantly. From the graphs, it is also seen that at all frequencies (Ω_θ), element 1 has completely yielded (the peak ductility demand is greater than unity) regardless of the eccentricity.

6. CONCLUSIONS

The steady state response of tall and short non-linear asymmetric structures, with soil–structure interaction taken into consideration, has been extensively investigated in this study. The system parameters have been identified and their effects on the system and element responses determined. It is found that for tall linear and non-linear structures as well as short non-linear structures, a softer soil leads to an increase in the resonant amplitude. However, in short elastic structures, a softer soil leads to a decrease in the resonant amplitude. This is due to the two opposing mechanisms that control the response of the system; namely, the damping of the soil and the "whipping" effect. The resonant peaks are found to occur at lower frequencies (Ω) as the soil softens since a softer soil corresponds to a lower resonant frequency.

The effects of the ground acceleration are found to be similar to that of soil: i.e., the resonant amplitude increases as the ground acceleration increases because a larger acceleration can impart greater energy, and the resonant peaks shift into lower frequencies as the acceleration increases since the structure becomes more flexible due to an increase in the degree of non-linearity.

The bilinear coefficient also has an important effect on the system and element responses. Generally, it is observed that the resonant peaks shift into lower frequencies, with a corresponding decrease in amplitudes when the system becomes more inelastic. The latter effect is due to the loss of energy through hysteresis. At higher frequencies, the non-linear systems are found to behave elastically.

The stiffness of the soil as well as Ω_θ are found to have a profound effect on the peak ductility demand of the far element, but only marginally affect the near element. The results also show that the eccentricity of the structure has only a marginal effect on the peak ductility of the near element. Its effect on the far element is significant.

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APPENDIX: EXPRESSIONS FOR σ_1, σ_2, σ_3 AND σ_4

Let

$$a=\Omega^2(J_1K_\phi+C_\phi C_\theta)-K_\phi K_\theta/w_x^2, \qquad b=J_1w_xC_\phi\Omega^2-(K_\phi C_\theta+K_\theta C_\phi)/w_x,$$

$$c=\Omega^2(J_1K_v+C_vC_\theta)-K_vK_\theta/w_x^2, \qquad d=J_1C_vw_x\Omega^2-(K_vC_\theta+C_vK_\theta)/w_x,$$

$$\Delta=\alpha_1^2+\Omega^2\alpha_2^2, \tag{A1}$$

$$\begin{aligned}\alpha_1=[1&-(\Omega^2K_1h_1^2/K_\phi)(1+K_\phi/K_vh_1^2+J_1K_\phi w_x^2/K_\theta K_1h_1^2+C_\phi C_\theta w_x^2/K_\theta K_1h_1^2\\&+C_\phi C_vw_x^2/K_vK_1h_1^2+K_\phi C_\theta C_vw_x^2/K_\theta K_vK_1h_1^2)\\&+(\Omega^4K_1h_1^2/K_\phi)(J_1w_x^2/K_\theta+K_\phi J_1w_x^2/K_\theta K_vh_1^2\\&+J_1w_x^4C_vC_\phi/K_\theta K_vK_1h_1^2+w_x^2C_\phi C_\theta/K_\theta K_vh_1^2+w_x^2C_vC_\theta/K_\theta K_v)],\end{aligned} \tag{A2a}$$

$$\begin{aligned}\alpha_2=[&(\Omega^4K_1h_1^2/K_\phi)(J_1C_\phi w_x^3/K_\theta K_vh_1^2+J_1C_vw_x^3/K_\theta K_v)-(\Omega^2K_1h_1^2/K_\phi)\\&\times(K_\phi C_\theta w_x/K_\theta K_vh_1^2+C_\theta w_x/K_\theta+C_\phi w_x/K_vh_1^2+J_1C_\phi w_x^3/K_\theta K_1h_1^2+C_vw_x/K_v\\&+J_1K_\phi C_vw_x^3/K_\theta K_vK_1h_1^2-C_vC_\phi C_\theta w_x^3/K_1K_\theta K_vh_1^2)\\&+(K_1h_1^2/K_\phi)(K_\phi C_\theta w_x/K_\theta K_1h_1^2+C_\phi w_x/K_1h_1^2+K_\phi C_vw_x/K_vK_1h_1^2)].\end{aligned} \tag{A2b}$$

Then,

$$\sigma_1=\frac{\Omega^2K_1h_1^2}{K_\phi\Delta}\left[\frac{\alpha_1w_x^2(b/h_1^2+d)}{K_\theta K_v}-\frac{\alpha_2w_x^2(a/h_1^2+c)}{K_\theta K_v}\right], \tag{A3a}$$

$$\sigma_2=\frac{\Omega^2K_1h_1^2}{K_\phi\Delta}\left[\frac{\alpha_1w_x^2(a/h_1^2+c)}{K_\theta K_v}+\frac{\Omega^2\alpha_2w_x^2(a/h_1^2+d)}{K_\theta K_v}\right], \tag{A3b}$$

$$\begin{aligned}\sigma_3=(J_1\Omega^4w_x^2/K_\phi\Delta)\{&(\alpha_2/K_\phi K_v)[K_\phi K_v-\Omega^2(K_1K_\phi+K_1K_vh_1^2+w_x^2C_\phi C_v)]\\&-(\alpha_1w_x/K_\theta K_v)[(K_vC_\phi+K_\phi C_v)-\Omega^2(K_1C_\phi+K_1h_1^2C_v)]\},\end{aligned} \tag{A3c}$$

$$\begin{aligned}\sigma_4=(J_1\Omega^4w_x^2/K_\phi\Delta)\{&(\alpha_1/K_\theta K_v)[K_\phi K_v-\Omega^2(K_1K_\phi+K_1K_vh_1^2+w_x^2C_\phi C_v)]\\&+(\alpha_2\Omega^2w_x/K_\theta K_v)[(K_vC_\phi+K_\phi C_v)-\Omega^2(K_1C_\phi+K_1h_1^2C_v)]\}.\end{aligned} \tag{A3d}$$