

The influence of stiffness and strength eccentricities on the inelastic earthquake response of asymmetric structures

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Abstract

This paper presents a study into the effects of stiffness and strength eccentricities on inelastic earthquake response of asymmetric structures. Previous studies in this area have often drawn contradictory conclusions regarding the effects of eccentricity—both stiffness and strength. It was concluded that these differences arose from the use of different analytical models. An analytical model was developed for this study that contains the key properties and dynamic characteristics of actual buildings, and as such provides accurate information on structural response. In most of the previous studies conducted, the strength eccentricity was not considered as an independent structural parameter. It was found that for a structure experiencing significant inelastic action during earthquake loading the strength eccentricity has greater influence on the structural response than the stiffness eccentricity. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

An earthquake induces horizontal inertia forces associated with the mass distribution in a structure. At any particular floor level, the resultant force may be considered to act through the centre of mass at that floor level. These resultant forces are resisted by the vertical elements of the structural system (resisting elements), and the resultant acts through the shear centre or centre of stiffness of the storey under consideration. In many structures, for any particular storey, these opposing forces are not coincident. This gives rise to eccentricities and the corresponding structure is referred to as asymmetric.

When asymmetric buildings are subjected to lateral loads (e.g. earthquakes), an interaction between the lateral and torsional responses occurs. This coupling of responses can lead to an increase in the damage that an asymmetric building would sustain during an earthquake compared with an equivalent symmetric one. Statistics

showed that 15% of the buildings that suffered severe damage or collapse in Mexico City during the 1985 Michoacan earthquake could be classified as possessing pronounced asymmetry [1].

The objective of this study is to assess the influence of stiffness and strength eccentricities on the inelastic earthquake response of asymmetric structures. Usually, the stiffness eccentricity of a building is determined from geometric considerations, and the strength eccentricity is determined from the distribution of lateral strength in the resisting elements that is specified by the relevant earthquake design code. The strength eccentricity, therefore, is not an independent structural parameter in code designed buildings. In the present study the strength eccentricity was treated as an independent structural parameter, and the purpose of this study is to undertake a parametric study on the effects of stiffness and strength eccentricities on structural response, and not to assess the performance of various earthquake design codes.

2. Structural model

The numerical study presented in this paper was conducted using a model that consists of a single-storey

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structure with a perfectly rigid floor diaphragm, supported by moment-resisting frames or other forms of lateral load-resisting elements. These elements are assumed to be massless and axially inextensible. The torsional inertia of individual elements is assumed to be negligible and thus ignored in the analysis. These models facilitate the introduction of eccentricity by varying the distributions of element stiffness.

Although this study was limited to single-storey structures, its results are also applicable to multi-storey ones. This is due to the fact that although idealized in form, the single-storey model used in this study incorporates the key properties and dynamic characteristics of actual buildings: fundamental lateral and torsional frequencies, and stiffness and strength distributions. Higher-mode effects can influence the response of multi-storey structures to earthquake excitation; however, these effects are usually small compared to the response of the fundamental modes [2]. The area where single-storey models are no longer an accurate representation of reality is when the stiffness and/or strength distribution of the structure varies drastically from floor to floor, although for many multi-storey structures this is not the case.

The analytical model consists of a uniform rigid slab of mass m , and radius of gyration r taken about the centre of mass. The slab is supported by three massless, axially inextensible load-resisting elements: 1, 2 and 3. Elements 1 and 3 are located at equal distance $d/2$ from, but on opposite sides of, element 2. The centre of mass of the slab (CM) is taken to be coincident with element 2, as shown in Fig. 1. The system is symmetrical about the x -axis and is subjected to ground motion \ddot{u}_{gy} along the y -axis. The two degrees-of-freedom of the system are the horizontal displacement u_y of the centre of mass of the deck—relative to the ground—along the y -axis, and the rotation of the deck u_θ about the vertical z -axis. The elements have in-plane stiffness k_i ($i = 1, 2, 3$) and yield strengths f_i ($i = 1, 2, 3$). The elements have zero out-of-plane stiffnesses and strengths.

The force–displacement relationship for each resisting element—in the y direction—is taken to be bilinear hys-

teretic, with the slope of the post-yield branch set to 1% of the initial elastic slope.

The total translational stiffness K_y and the torsional stiffness K_θ —about the CM—of the system are given by

$$K_y = k_1 + k_2 + k_3, \quad (1)$$

$$K_\theta = \frac{d^2}{4} (k_1 + k_3), \quad (2)$$

assuming that the torsional stiffness of each member about its own axis is negligible. The total translational strength F_y is given by

$$F_y = f_1 + f_2 + f_3. \quad (3)$$

For this system, the centre of stiffness (CS) is located at distance e_s , the static eccentricity, with

$$e_s = \frac{d}{2K_y} (-k_1 + k_3), \quad (4)$$

and the centre of strength (CR) is located at distance e_r , the strength eccentricity, with

$$e_r = \frac{d}{2F_y} (-f_1 + f_3), \quad (5)$$

measured from CM along the x axis as shown in Fig. 1.

The equations of motion for the system are given by

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{u}_y \\ \dot{u}_\theta \end{Bmatrix} + \begin{bmatrix} K_y & K_y e_s \\ K_y e_s & K_\theta \end{bmatrix} \begin{Bmatrix} u_y \\ u_\theta \end{Bmatrix} = - \begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_{gy} \\ 0 \end{Bmatrix}, \quad (6)$$

or

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\ddot{\mathbf{u}}_g. \quad (7)$$

The elastic slopes of the individual resisting elements k_i may be different, and their relative values will be adjusted to give rise to different stiffness eccentricity e_s (Eq. (4)) values for the system. Similarly, the yield strengths f_i of the resisting elements may be different and their relative values will be adjusted to give rise to different strength eccentricity e_r values (Eq. (5)).

The damping matrix \mathbf{C} in Eq. (7) was made proportional to the stiffness matrix \mathbf{K} , thus

$$\mathbf{C} = a\mathbf{K}, \quad (8)$$

where the proportionality constant a has units of sec.

The proportionality constant a was chosen such that

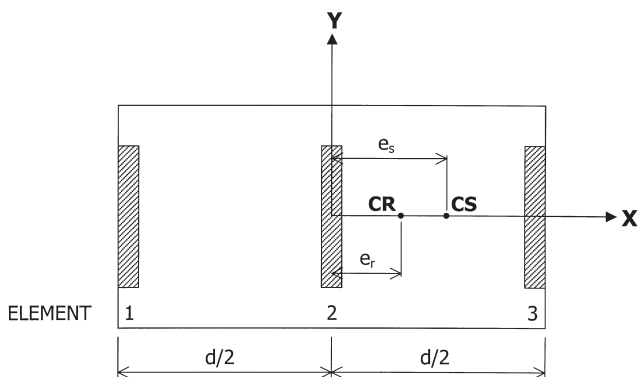


Fig. 1. Plan view of structural model.

the uncoupled lateral mode of vibration has damping equal to 2% of critical damping. This was to account for the nominal elastic energy dissipation that occurs in any real structure. The critical damping coefficient c_c for a single degree-of-freedom system is given by

$$c_c = 2m\omega, \quad (9)$$

where m is the mass and ω the natural circular frequency. In this case m is the mass of the deck and ω the natural frequency of the uncoupled lateral mode, given by

$$\omega = \sqrt{\frac{K_y}{m}}. \quad (10)$$

Thus, using Eqs. (9) and (10) gives a as

$$a = \frac{0.02 \times 2m \sqrt{\frac{K_y}{m}}}{K_y}, \quad (11)$$

which is then used in Eq. (8) to calculate the damping matrix \mathbf{C} .

3. Previous studies

In studies where the resisting elements of the structure remained within the elastic limit, results of all researchers agree that as the stiffness eccentricity increases the effects of torsional coupling increase.

In inelastic studies, however, researchers' results are more uncertain. Initial studies did not include the strength eccentricity as an independent parameter, instead the strength eccentricity was set equal to the stiffness eccentricity. Kan and Chopra [3] found that the effects of torsional coupling on deformations depend on the stiffness eccentricity in a complicated manner with no apparent trends except for systems with high torsional stiffness, where they found that the effects of torsional coupling increase with increasing eccentricity. Tso and Sadek [4], Bozorgnia and Tso [5], and Syamal and Pekau [6] all found that the ductility demand is very sensitive to the stiffness eccentricity—in some cases they found a large increase in ductility demand caused by large stiffness eccentricity. Bruneau and Mahin [7], however, found that the peak ductility demand is not affected by the stiffness eccentricity at all.

Chandler and Duan [8] point out that most of the inconsistencies in results can be attributed to the fact that researchers have used different structural models when conducting their studies.

Differences exist in the number of resisting elements used in the models. Kan and Chopra [3] conducted the

studies using a simplified model having only one resisting element. Bruneau and Mahin [7] used a two-element model; Tso and Sadek [4], and Bozorgnia and Tso [5] used a three-element model; whereas Syamal and Pekau's [6] model has four resisting elements—two parallel and two perpendicular to the direction of ground motion.

One major difference in the structural models used is the presence or absence of resisting elements perpendicular to the direction of ground motion. Goel and Chopra [9] undertook a study where they compared models with and without elements perpendicular to the direction of ground motion for a system subjected to a uni-directional earthquake loading. They concluded that element deformations and maximum ductility demands are affected significantly by the contribution to the torsional stiffness from the resisting elements perpendicular to the direction of ground motion. They also state that as most actual buildings invariably include resisting elements in both lateral directions to provide resistance to both horizontal components of ground motion, the findings of studies that did not include elements perpendicular to the direction of ground motion are unrealistic and therefore not valid.

Tso and Zhu [10] refute the logic behind these statements. They point out that the preceding studies were conducted with a uni-directional ground excitation, which resulted in the transverse elements remaining mostly in the elastic state. In reality, a structure will be exposed to two horizontal components of ground motion and the transverse elements are therefore likely to be excited well beyond the elastic state during strong shaking. The contribution of these elements to the torsional stiffness of the system is, therefore, questionable. This effect would account for the findings of Kan and Chopra [3], who found that after initial yielding, their system has a tendency to yield further—primarily in translation—and behave more and more like an inelastic single-degree-of-freedom system. This was due to the fact that Kan and Chopra's [3] model had high torsional stiffness after yield because of the transverse elements remaining within the elastic limit. Tso and Zhu [10] state that only a systematic study of structural models subjected to bi-direction ground motions will conclude whether these elements should or should not be included in uni-directional ground excitation studies.

Such a study has been carried out by Correnza, Hutchinson and Chandler [11], who concluded that for most cases accurate results are obtained for systems where the transverse elements are neglected. Errors may occur for systems with short lateral period; however, these errors are over-conservative, whereas models incorporating transverse elements may underestimate system response by as much as 100%.

The other major difference in the structural models used is the number of resisting elements parallel to the

direction of ground motion. Goel and Chopra [9] compared models using two, three and sixteen elements parallel to the direction of ground motion, and concluded that the number of resisting elements oriented along the direction of ground motion has little influence on the overall response. Models containing only two such elements are, therefore, sufficient.

The results of Goel and Chopra's [9] study are, however, in doubt owing to the fact that they included resisting elements perpendicular to the direction of ground motion, which as stated above can lead to errors in the results. Bozorgnia and Tso [5] state that because two element models are statistically determinate, the element forces are independent of the distribution of element stiffnesses, and therefore independent of the stiffness eccentricity. As practically all building structures are statically indeterminate, models with three or more elements in the direction of ground motion should be used.

This discussion explains the form of the model used in this study (Fig. 1). As shown, it contains three resisting elements parallel to, and no elements perpendicular to, the direction of ground motion.

In all of the studies discussed previously, the stiffness eccentricity, e_s and the strength eccentricity e_r were coincident. Thus when the eccentricity was varied, it was both e_s and e_r that were varied. Some studies have been conducted where the strength eccentricity was considered as an independent structural parameter. Sadek and Tso [12], Goel and Chopra [13], and De Stefano et al. [14] all treated the strength eccentricity as an independent structural parameter, and they all found that increasing the strength eccentricity would lead to greater torsional response. Unfortunately, these studies were conducted using models containing resisting elements perpendicular to the direction of ground motion that—as stated above—can lead to under-conservative results. Tso and Ying [15] undertook a study using a model that contained three resisting elements perpendicular to, and no resisting elements parallel to, the direction of ground motion. They considered two values of the strength eccentricity: zero, and equal to the stiffness eccentricity. They reported higher torsional response when the strength eccentricity was equal to the stiffness eccentricity compared with when it was zero. Mittal and Jain [16] used the same form of model as Tso and Ying [15] and considered a range of strength eccentricity values. They found that the ductility demand of the flexible side element (element 1 in Fig. 1) increased, and that of the stiff side (element 3) decreased, with increasing strength eccentricity. It appears clear, therefore, that the strength eccentricity is an important parameter in classifying structural response.

Many other studies (for example, see References [17–19]) have been conducted where the strength distribution of the resisting-elements was defined by one of the vari-

ous earthquake design codes in existence. Different design codes often used different methods for calculating the design strengths of the resisting elements, resulting in different strength eccentricities. Significant differences in structural response were observed between models designed by different codes, again highlighting the importance of strength distribution—and thus strength eccentricity—on structural response.

4. Ground motion

The ground motion that was used in this study was the north–south component of the El Centro record obtained during the Imperial Valley earthquake of 18 May 1940. The El Centro record has been widely used in earthquake response analysis, and amongst existing records it represents one of the most severe combinations of strong-motion ground acceleration over a long duration [20]. For these reasons it was chosen for use in this study.

5. Input and response parameters

The different input parameters used in this study were:

5.1. Lateral period

The uncoupled lateral period of the structure T_y is given by

$$T_y = \frac{2\pi}{\sqrt{\frac{K_y}{m}}} \quad (12)$$

5.2. Normalized stiffness and strength eccentricities

The stiffness eccentricity e_s and strength eccentricity e_r were defined earlier (Eqs. (4) and (5) respectively). These eccentricities were then normalized to give non-dimensional parameters by dividing by the radius of gyration of the deck r .

5.3. Torsional to lateral frequency ratio

The uncoupled frequencies of the structure ω_y and ω_θ are given by

$$\omega_y = \sqrt{\frac{K_y}{m}}; \text{ and } \omega_\theta = \sqrt{\frac{K_\theta}{mr^2}} \quad (13)$$

These uncoupled frequencies are the true natural frequencies of the structure only if the stiffness eccentricity

e_s is zero. The torsional to lateral frequency ratio Ω is then defined as

$$\Omega = \frac{\omega_\theta}{\omega_y}. \quad (14)$$

5.4. Strength reduction factor

The final parameter that was taken into account in this study was the degree of inelastic response experienced by the structure during earthquake loading. This was measured by a non-dimensional strength reduction factor C . An equivalent symmetric structure is defined as a structure with the same total lateral stiffness K_y and total lateral strength F_y as the asymmetric structure under consideration, but with the stiffness eccentricity e_s and strength eccentricity e_r set to zero. The equivalent symmetric structure responds in a purely translational manner during earthquake loading—experiencing no torsional rotation. The equivalent symmetric structure undergoes maximum lateral displacement u_{sym} during earthquake loading. The minimum total lateral strength F_{min} of the equivalent symmetric structure that is required for it to remain elastic throughout the earthquake loading is given by

$$F_{\text{min}} = K_y u_{\text{sym}}. \quad (15)$$

The strength reduction factor C is then defined as

$$C = \frac{F_{\text{min}}}{F_y} = \frac{K_y u_{\text{sym}}}{F_y}, \quad (16)$$

where F_y is the total lateral strength of the structure. The higher the value of C the more inelastic deformation the structure will undergo during earthquake loading. It is important to note that although a symmetric structure will remain perfectly elastic for $C = 1.0$, an asymmetric structure will not, due to torsional coupling. This method of defining the total lateral strength of the model is similar to those used by other researchers and is equivalent to that used by Bozorgnia and Tso [5].

The response parameters employed in this study were:

5.5. Normalized ductility demand

The ductility demand is a measure of amount of yield an element undergoes during earthquake loading. For an element i with stiffness k_i and strength f_i the displacement at which yield occurs u_{yield} is given by

$$u_{\text{yield}} = \frac{f_i}{k_i}. \quad (17)$$

If the maximum displacement this element undergoes

during earthquake loading is u_{max} then the ductility demand is given by

$$\text{ductility demand} = \frac{u_{\text{max}}}{u_{\text{yield}}}. \quad (18)$$

If the ductility demand is less than 1.0 this indicates that the element remained within the elastic limit throughout earthquake loading.

The normalized ductility demand is a measure of the extra ductility demand caused by structural asymmetry. The normalized ductility demand for an element is then defined as the ductility demand of the asymmetric structure's element divided by the ductility demand of the same element in the equivalent symmetric structure, and is given by

$$\text{normalized ductility demand} = \quad (19)$$

$$\frac{\text{ductility demand}}{\frac{u_{\text{sym}}}{\left(\frac{F_y}{K_y}\right)}} = \frac{\left(\frac{u_{\text{max}}}{u_{\text{yield}}}\right)}{\frac{u_{\text{sym}}}{\left(\frac{F_y}{K_y}\right)}}.$$

5.6. Normalized element displacement

This is a measure of the extra element displacement caused by structural asymmetry. If the maximum element displacement experienced by the equivalent symmetric structure is u_{sym} then the normalized edge displacement is given by

$$\text{normalized edge displacement} = \frac{u_{\text{max}}}{u_{\text{sym}}}. \quad (20)$$

6. Results of numerical investigation

In this section, the results of the investigation into the effects of the normalized stiffness eccentricity e_s/r and the normalized strength eccentricity e_r/r on the response of the structure are presented. The normalized stiffness eccentricity e_s/r and the normalized strength eccentricity e_r/r were varied between 0.0 and 1.0. The uncoupled lateral period T_y was kept constant at 1.0 s; the torsional to lateral frequency ratio Ω was kept constant at 1.0; and the strength reduction factor C was kept constant at 5.0—indicating relatively high inelastic action. The d/r ratio for the model used was 3.0.

Firstly the effects of e_s/r and e_r/r on the normalized ductility demands of the resisting elements are examined (Figs. 2–4).

As shown in Fig. 2, increasing e_r/r caused an increase in the normalized ductility demand for element 1 (flexible side element). This increase is much more pronounced for a structure with small e_s/r . For a structure with $e_r/r = 1.0$ and $e_s/r = 0.0$ element 1 experienced 25 times more ductility demand than the equivalent symmetric structure. It is important to note that this increase is somewhat artificial. A structure with high strength eccentricity and low stiffness eccentricity results in element 1 having low strength but high stiffness—giving this element a very low yield displacement. This means that even though this element may only undergo a relatively small displacement, the ductility demand value will be large due to the low yield displacement. This kind of structural configuration is highly unlikely in practice.

The normalized ductility demand of element 2 (centre element) was not affected by e_s/r and e_r/r (it is important to note the different scales on the vertical axis for Figs. 2–4).

The normalized ductility demand of element 3 (stiff side element) showed the opposite trend to that of element 1—the normalized ductility demand decreased as e_r/r was increased (Fig. 4). The numerical values of the normalized ductility demands for element 3 are, however, significantly lower than those of element 1.

Finally, the effects of e_s/r and e_r/r on the normalized element displacements are examined (Figs. 5–7).

Increasing the normalized strength eccentricity e_r/r caused an increase in the normalized element displacement of element 1 (flexible side element) and a decrease for that of element 3 (stiff side element), with element 2 (centre element) showing no real trends. The normalized element displacements show no clear trends for increasing normalized stiffness eccentricity e_s/r .

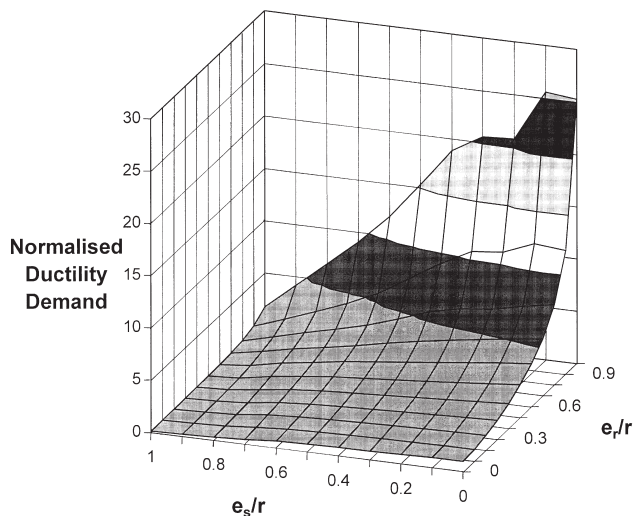


Fig. 2. The effects of e_s/r and e_r/r on the normalized ductility demand of element 1.

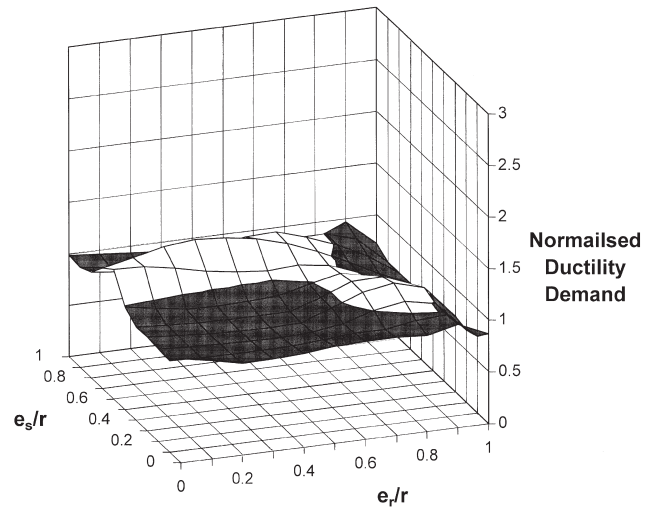


Fig. 3. The effects of e_s/r and e_r/r on the normalized ductility demand of element 2.

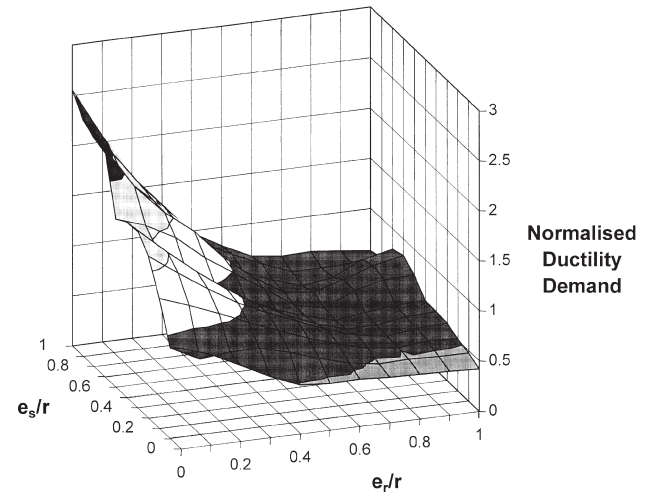


Fig. 4. The effects of e_s/r and e_r/r on the normalized ductility demand of element 3.

7. Discussion

The results presented here highlighted some important points. The trends in response of element 1 (normalized ductility demand and element displacement) were almost identically opposite to those of element 3, but with the magnitude of the response of element 3 being less than that of element 1. Syamal and Pekau [6] found that the normalized ductility demand of the element at the flexible edge of the structure (element 1) grows rapidly with increasing eccentricity, whereas the normalized ductility demand of the stiff side element (element 3) decreases only slowly with an increase in eccentricity. These results agree with those of the study presented here, where the magnitudes of the responses for element 3 were smaller than those of element 1. Bruneau and Mahin [7] concluded from the results of their study that

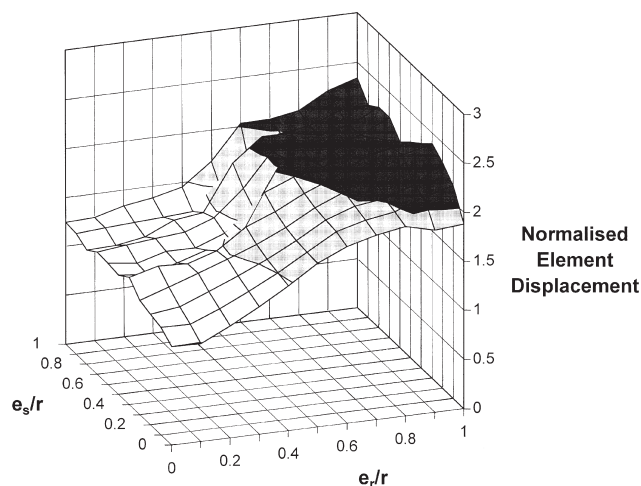


Fig. 5. The effects of e_s/r and e_r/r on the normalized element displacement of element 1.

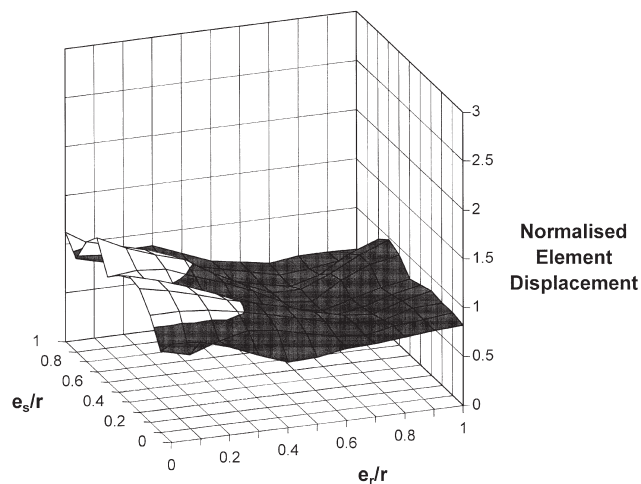


Fig. 7. The effects of e_s/r and e_r/r on the normalized element displacement of element 3.

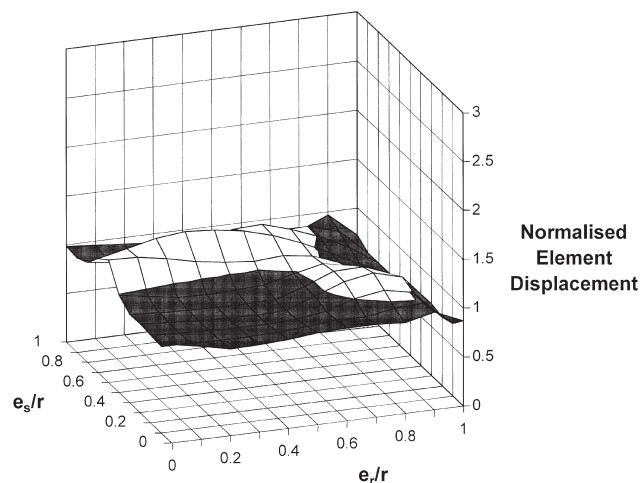


Fig. 6. The effects of e_s/r and e_r/r on the normalized displacement of element 2.

the normalized ductility demands were not affected by eccentricity. Their analytical model had only two resisting elements, making it statically determinate. As discussed earlier, this resulted in the element forces being independent of the stiffnesses, which accounts for their finding that the ductility demands are independent to the eccentricity.

Mittal and Jain [16] found that the ductility demand of the flexible side element (element 1) increased with, and that of the stiff side element decreased, with increasing strength eccentricity e_r . These findings agree with the results of the current study.

The normalized stiffness eccentricity e_s/r had no discernible effect on the normalized element displacements during earthquake loading (Figs. 5–7); however, e_s/r did have a significant effect on the normalized ductility demands (Figs. 2–4). As e_r/r was increased the stiffness of element 1 was decreased, while there was no real

change in the maximum displacement experienced by this element. This reduction in stiffness of element 1 caused the ductility demand to increase, even though the maximum displacement was not affected.

8. Conclusions

From the above it can be concluded that the normalized strength eccentricity e_r/r has a greater effect on structural response than the normalized stiffness eccentricity e_s/r for structures experiencing high inelastic action. This result is not surprising because when one or several of the resisting elements undergo yield, the stiffness distribution is drastically modified from the original elastic condition, and it is the stiffness distribution that determines the normalized stiffness eccentricity e_s/r .

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