Seismic code provisions for asymmetric structures: a re-evaluation

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Results of a parametric study on earthquake time history response of asymmetric single storey structures with one axis of symmetry modelled as two degree-of-freedom systems are compared with static seismic code provisions. A large number of such systems with a range of mass centre to rigidity centre eccentricities, a range of uncoupled lateral natural frequencies and a number of torsional to lateral frequency ratios Ω_0 were subjected to several earthquake acceleration records. Five percent damping was assumed in the two coupled modes. Maximum displacements at several locations along the roof deck were computed, normalized with respect to the symmetric case, averaged, and compared with code oriented static methods. The study shows that the static approach does not give reliable estimates for the response of frames in asymmetric buildings, even when the amplification factors provided by earthquake codes are incorporated into the formulation. In particular, code provisions often underestimate the response of frames located on the side of the rigidity centre away from the mass centre for small to moderate eccentricities when $\Omega_0^2 = 0.5$, 1.0; whereas for systems with higher torsional rigidities ($\Omega_0^2 \geqslant 2.0$), the static approach appears to yield reasonable results. For members located on the opposite side of the roof, code provisions, including recent revisions, appear to overestimate the response with decreasing frequency ratio.

Keywords: asymmetry, single storey structures, mass centre, rigidity centre, earthquakes

The lateral load provisions of earthquake codes are based on the traditional static method of seismic analysis in which the inertia forces are applied statically to the structure at the mass centre. For asymmetric structures this approach is problematic, since modal lateral-torsional coupling due to the rotatory inertia of the floor deck magnifies the effective eccentricity of the inertia forces, and affects members located on opposite sides of the rigidity centre to a different extent. This coupling effect is usually accounted for in the seismic codes by specifying a factor α multiplying the static eccentricity, e, and by assigning two values to this factor: $\alpha > 1.0$ for members located on the mass centre CM side of the rigidity centre CR (flexible side), and $\alpha < 1.0$ for members on the opposite or stiff side (*Figure 1*). Other effects, which are not dealt with in this paper, namely, torsional input, accidental eccentricity and other imponderables, are considered by means of an

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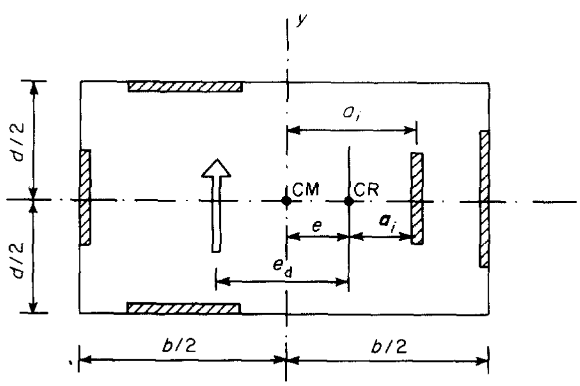

Figure 1 Plan of single storey structural model

additional eccentricity which usually is given as a fraction, β, of the width of the building, b.

Thus, the design or dynamic eccentricity e_d in most codes is given in the form:

$$e_d = \alpha e \pm \beta b \quad (1)$$

in which $\alpha e = e_{d1}$ is the dynamic eccentricity, and $\beta b = e_{d2}$ the additional or accidental eccentricity.

It will be observed that there are no special provisions for members or assemblages whose line of action is perpendicular to the direction of excitation, and the engineer is likely to design them as if they were located on the flexible side of the floor.

During the last 25 years a large number of studies have been carried out to improve the correlation of α and β with spectral and time history analyses (see reference 1 for a bibliography through 1980). Most of these analyses were carried out for asymmetric structures in which the uncoupled rotational frequency was equal to the lateral frequency. As is well-known, large dynamic amplifications of small static eccentricities are to be expected under these conditions. The main effort in many of these studies was directed towards improving the correlation of static results with dynamic analyses for edge members on the flexible side of the structure. Less attention was paid to the correlation of the static formulae for members located on the stiff edge of the floor, and for members perpendicular to the load direction which resist only torque.

In view of this, it may not be surprising that the computed response of any given frame or assemblage in the system, even when the computation is based on the theoretically correct dynamic eccentricity rather than the code values, does not correlate with those obtained from spectrum analysis when applying the RSS (square root of the sum of squares) formula to the frame's modal responses, or with the results of time history analyses. In short, the factored static methods in their present form either overestimate, or underestimate, the response in an inconsistent manner. Thus, the amplification factor for eccentricity is in fact a variable rather than a constant (or two), its numerical value depending on the location of the frame in question[2].

Therefore, if static methods are to continue in use, formulae more consistent with dynamic analysis should be adopted by earthquake codes. The purpose of the present paper is to compare the earthquake response of structural members in simple single-storey asymmetric structures dynamically analysed for several time histories, with results based on code oriented static analyses. These comparisons will permit evaluation of the adequacy of accepted static and quasi-dynamic procedures and help in formulating expressions which better correlate with reported dynamic results.

This paper considers only coupling effects due to lateral excitation in structures having one axis of symmetry. Since the response in the direction of excitation of generally asymmetric structures is usually larger when x-y coupling is neglected[3], this simplified model is conservative. The problem of accidental eccentricity is more difficult in view of the limited data available on torsional ground motion, structural imperfections and variations in mass distribution, and is not dealt with in this paper[4]. A shortened version of this paper, with an emphasis on the Canadian situation, is given in reference 5.

Two degree-of-freedom systems

The system studied is an idealized single-storey structure shown in *Figure 1*, consisting of a rigid floor deck with mass m supported laterally by several massless planar assemblages (e.g. flexural walls or frames). For simplicity, one axis of symmetry is assumed, so that only two degrees-of-freedom (DOF) are considered, namely, the lateral displacement, y, and the rotation θ about a vertical axis through the mass centre. The lateral and torsional rigidities K_y and K_θ of the 2-DOF system are obtained from the stiffnesses k_{iy} of the individual members or assemblages (if they are not simple columns or walls) in the usual way, namely

$$K_y = \sum_i k_{iy}$$

$$K_\theta = \sum_i k_{iy} x_i^2 + \sum_i k_{ix} y_i^2 + \sum k_{i\theta} = \boldsymbol{K}_\theta + K_y e^2 \quad (2)$$

in which x_i and y_i are the perpendicular distances to the mass centre, $k_{i\theta}$ the torsional rigidity of a member about its own axis (may be neglected for planar members or assemblages), and $\boldsymbol{K}_\theta$ the torsional rigidity of the system about the centre of rigidity. The eccentricity, e, of the centre of rigidity CR from the mass centre CM is given by:

$$e = \frac{1}{K_y} \sum_i k_{iy} x_i \quad (3)$$

The earthquake acceleration time history $\ddot{U}_g(t)$ is assumed to act in the y direction only. The equations of motion of the system in the linear range with respect to the mass centre are given by:[2]

$$\begin{Bmatrix} \ddot{y} \\ \rho\ddot{\theta} \end{Bmatrix} + \omega_y^2 \begin{bmatrix} 1 & e^* \\ e^* & \Omega_M^2 \end{bmatrix} \begin{Bmatrix} y \\ \rho\theta \end{Bmatrix} = -\begin{Bmatrix} \ddot{U}_g \\ 0 \end{Bmatrix} \quad (4)$$

in which m is the storey (or deck) mass, ρ the mass radius of gyration about CM, and:

$$\omega_y = (K_y/m)^{1/2} \quad \omega_\theta = (\boldsymbol{K}_\theta/m\rho^2)^{1/2} \quad \omega_\theta = (K_\theta/m\rho^2)^{1/2}$$

$$e^* = e/\rho \quad \Omega_0^2 = \omega_\theta^2/\omega_y^2 \quad \Omega_M^2 = \omega_\theta^2/\omega_y^2 = \Omega_0^2 + e^{*2}$$

Note that, in order to render equation (4) dimensionally compatible, $\rho\theta$ rather than θ has been taken as the rotational variable.

For modal analysis, it is necessary to assume proportional damping; therefore, the damping terms do not appear in the equations of motion. They are evaluated

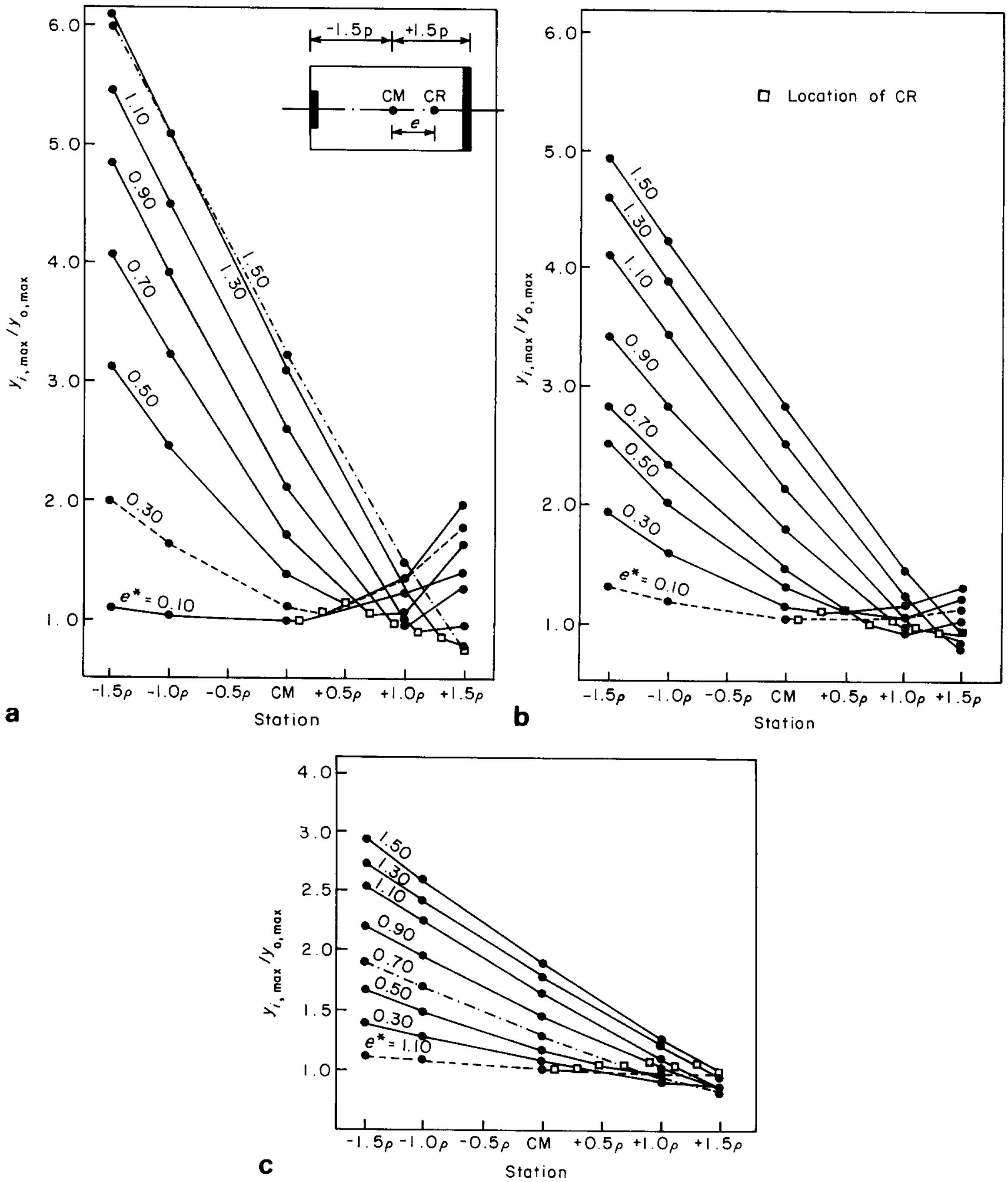

Figure 2 Maximum displacement against station location (average $+1.0\sigma$): (a) $\Omega_0^2 = 0.5$; (b) $\Omega_0^2 = 1.0$; (c) $\Omega_0^2 = 2.5$

for the uncoupled system in the usual way, namely:

$$C = c_1 M + c_2 K \tag{5}$$

where C, M and K are, respectively, the damping, mass, and stiffness matrices. For a given damping ratio η in the two modes, c_1 and c_2 are given by:

$$c_1 = \frac{4\pi}{T_1 + T_2}\eta \qquad c_2 = \frac{T_1 T_2}{\pi(T_1 + T_2)}\eta \tag{6}$$

where T_1 and T_2 are the two coupled natural vibration periods of the system.

Due to the irregular nature of $\ddot{U}_g(t)$, the maxima of the response values of interest can only be obtained by means of step-by-step (i.e. time history) solution of equation (4). Since the maximum displacement y_{max} at CR, and θ_{max} are unlikely to occur simultaneously, the maximum displacement of member i located at a distance $\boldsymbol{a}_i$ from CR (i.e. at $a_i = \boldsymbol{a}_i + e$ from CM) can be obtained from:

$$y_{i,max} = (y(t) + \boldsymbol{a}_i\,\theta(t))_{max} \tag{7}$$

The dynamic eccentricity e_{d1}, in terms of the time history analysis, can be defined either as:

$$e_{d1} = \frac{M_{max}}{V_{max}} = \frac{K_\theta \theta_{max}}{K_y y_{max}} \tag{8}$$

or as:

$$e_{d1} = \frac{M_{max}}{V_{0,max}} = \frac{K_\theta \theta_{max}}{K_y y_{0,max}} \tag{9}$$

in which V_{max} and M_{max} are, respectively, the maximum base shear and torque, and $V_{0,max}$ and $y_{0,max}$ are, respectively, the maximum base shear and displacement in a single DOF system with a circular frequency equal to ω_y and the same damping ratio. The second definition of e_{d1} is perhaps more relevant to earthquake codes, since the base shear to be taken is usually that of the associated symmetrical case, i.e. $V_{0,\max}$.

When the static approach is used, the displacement at location i is computed from the following expression:

$$y_i = \frac{V_0}{K_y} + \frac{e_d V_0}{K_\theta} a_i \tag{10}$$

in which V_0 is the base shear. It may be seen that the static code procedure is equivalent to using the expression:

$$y_{i,\max} = y_{\max} + a_i \theta_{\max} \tag{11}$$

rather than equation (7). This approach is probably responsible for some of the inconsistencies in the present static code formulae.

Parametric study

Time history analyses were performed using the computer program DRAIN 2D with an integration time step $\Delta t = 0.01$ s, for a number of 2-DOF systems of the type shown in *Figure 1*. These have lateral natural periods $T_0 = 0.25$, 0.50, 0.75, 1.00, 1.25, 1.50 and 2.0 s, and eccentricity ratios $e^* = e/\rho = 0.1$, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, and 1.5. Note that $e = 1.5\rho$ is an unrealistically high eccentricity even for very narrow floor plans ($b \approx 3.5\rho$), and thus may be considered as an upper bound. These systems were excited by five earthquake time histories: El Centro 1934 NS, 1940 NS, 1940 EW; Olympia 1949 N80E; and Taft 1952 N69W. Five percent damping was assumed for the two coupled modes. Seven torsional to lateral frequency ratios were considered: $\Omega_0^2 = 0.5$, 0.75, 1.0, 1.25, 1.5, 2.0 and 2.5. It will be observed that the choice of Ω_0^2 rather than Ω_M^2 or $\Omega_R^2 = \Omega_0^2/[1 + e^{*2}]$, commonly used by other investigators as the frequency ratio parameter, is based on the observation that this frequency ratio is independent of eccentricity, i.e. by assuming it to be constant while varying the eccentricity, it is possible to isolate the effect of eccentricity from all other properties of the system. With the other definitions, however, a constant frequency ratio implies that variations in eccentricity are accompanied by changes in stiffness or mass related parameters.

Maximum lateral displacements at several stations along the x-axis of the floor deck were computed, as well as the maximum rotations θ about the vertical axis. These responses were normalized by dividing the displacements $y_{i,\max}$ through the spectral displacements $y_{0,\max}$ for the respective translational periods of the corresponding earthquake time history and damping ratio, i.e. by the lateral response of associated systems having $e = 0$, but otherwise identical.

The normalized displacements were averaged and their standard deviations σ were computed. Thus, the results represent a sample of 35 response maxima at several stations along the floor deck for realistic ranges of eccentricities and frequency ratios.

A summary of the results is given in *Figure 2*, which shows the normalized 'average + 1.0σ' responses at different stations along the floor. As expected, practically linear variation of displacement is predicted for the torsionally rigid systems ($\Omega_0 = (2.5)^{1/2}$). With falling rotational stiffness the responses on the flexible side of the floor ($a_i = -1.0\rho$ and -1.5ρ) increase appreciably already for $\Omega_0 = 1.0$, whereas the large increase in the response at the rigid edge ($a_i = +1.5\rho$) takes place at low frequency ratio ($\Omega_0 = (0.5)^{1/2}$). The response at CR is close to that of the associated symmetric structure (i.e. $y_{\max}/y_{0,\max} \approx 1.0$). As is well known[6,2], the lateral response of asymmetric structures at CR is lower than that of their symmetric counterparts, and the similarity in results is due only to the addition of 1.0σ to the average.

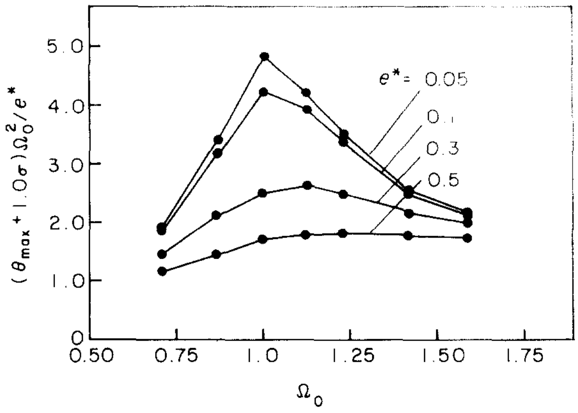

Figure 3 Dynamic eccentricity ratio against frequency ratio

It is seen that the dynamic amplification of response predicted for small e at $\Omega \approx 1.0$ is not manifested in these results. This is to be expected since a large amplification of small e may still result in a small displacement compared with the translational displacement. Also, $y_{\max}$ and $\theta_{\max}$ need not occur simultaneously. Finally, the normalization of $y_{i,\max}$ is by dividing by $y_{0,\max}$ (i.e. $e = 0$), rather than by $y_{\max}(e \neq 0)$ which is usually (but not always) lower, and this also tends to mask the effect of amplification.

The dynamic amplification of eccentricity at $\Omega = 1.0$ becomes evident in *Figure 3*. Note that the 'spiky' peak at $\Omega_0 = 1.0$ results from lack of data at adjacent values of Ω_0.

The effect of lateral vibration period T_0 on the response was also examined. As can be seen from *Figure 4*, no systematic variation of response with T_0 can be discerned with the exception of $T_0 = 0.25$ s. At this low period large displacements on the flexible side of the floor were predicted for torsionally flexible systems. This departure from the general trend tends to be confined to highly eccentric systems with decreasing frequency ratio Ω_0. As expected, the maximum rotations for $T_0 = 0.25$ s followed the same pattern (not shown). A possible explanation of the phenomenon follows. Since the coupled frequency ratio is larger than the uncoupled one, and it increases with eccentricity, the lower frequency mode tends to dominate the response provided the spectral displacement falls with frequency (or rises with period). This appears to be typical of response spectra at their low period end. With increasing period the rise in the spectral displacement is moderated (velocity bound), and so is the coupled motion relative to the uncoupled one.

A direct comparison with several static methods is given in the graphical presentations of *Figures 5–7*, which show the normalized displacements against

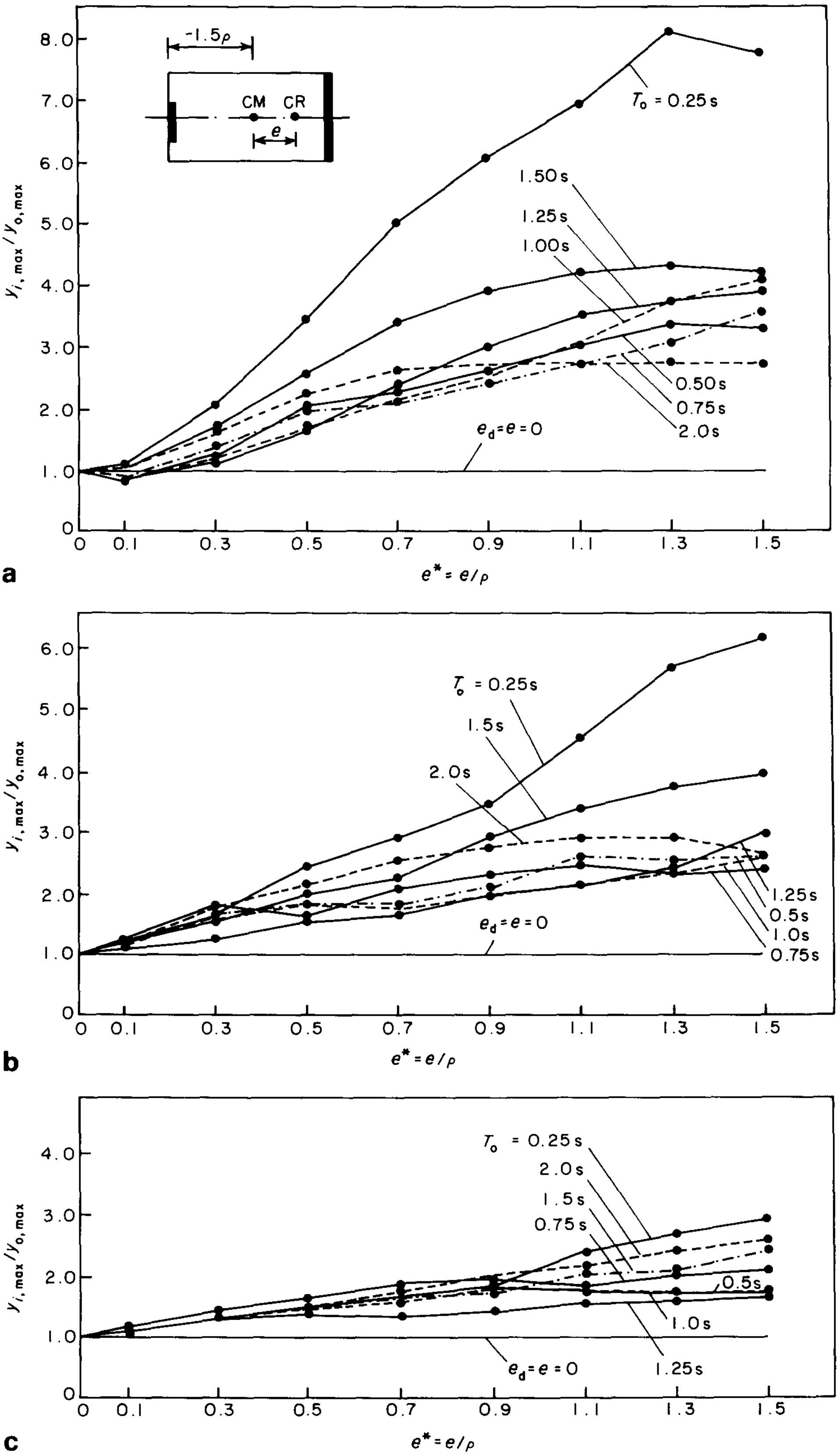

Figure 4 Average maximum displacement at flexible edge ($a_i = -1.5\rho$) against eccentricity—effect of vibration period T_0 and frequency ratio Ω_0: (a) $\Omega_0^2 = 0.5$; (b) $\Omega_0^2 = 1.0$; (c) $\Omega_0^2 = 2.5$

eccentricity ratio at edge stations $a_i = \pm 1.5\rho$. For the purpose of this comparison, it was assumed that only the first term in equation (1), namely $e_{d1} = 1.5e$ (or $0.5e$ whichever has the most unfavourable effect) was intended to account for the lateral torsional coupling, whereas the second term, βb, covers the additional eccentricity. Therefore, only the first effect is shown in these figures. Note, however, that in one of the most recent seismic codes, namely the 1985 version of the National Building Code of Canada[7], $0.05b$ out of $0.10b$ in the second term is attributed to modal coupling[8]. The effect of this contribution is shown by the curves

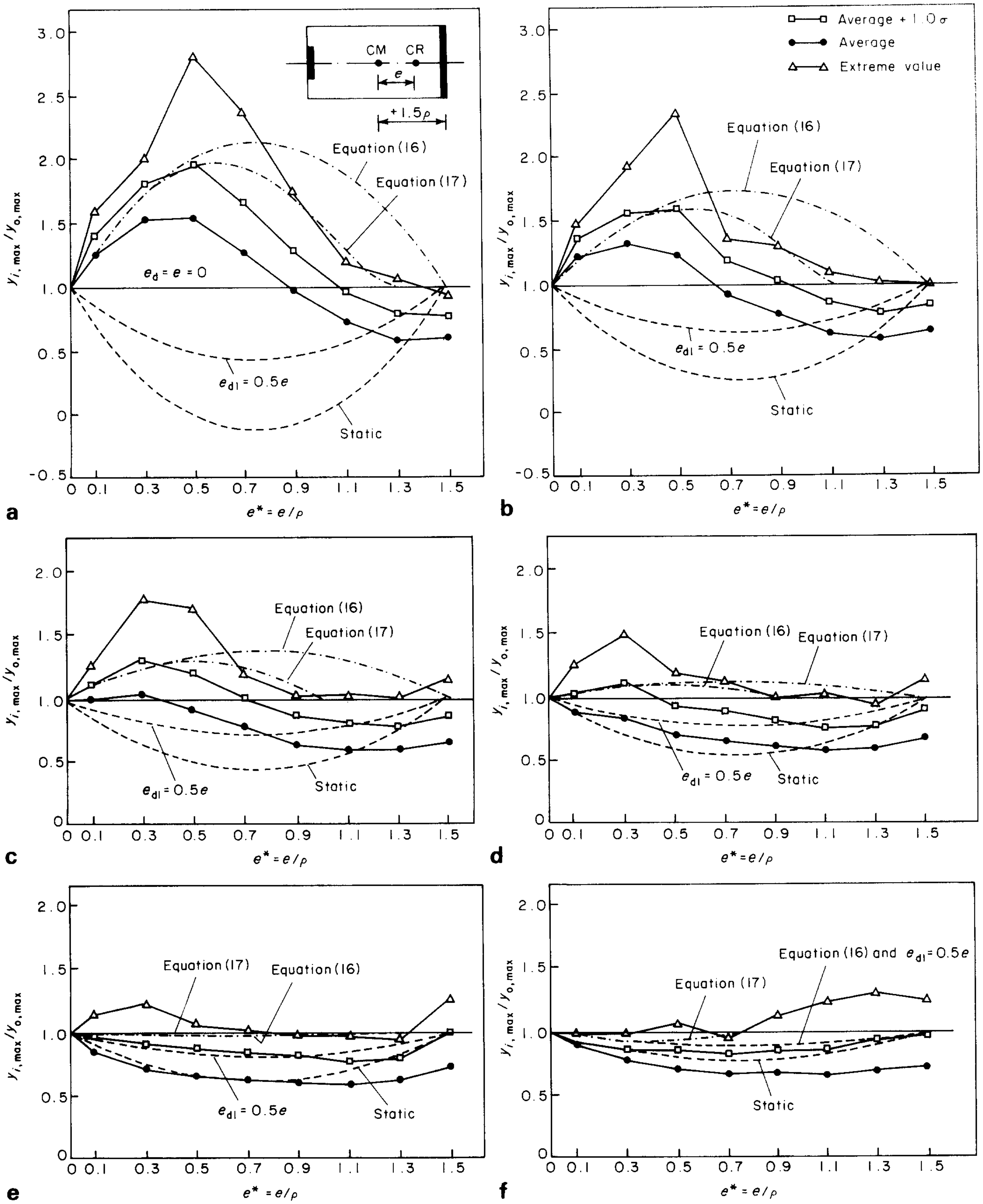

Figure 5 Maximum displacement at rigid edge ($a_i = +1.5\rho$) against eccentricity—comparison with static, code and proposed formulae: (a) $\Omega_0^2 = 0.5$; (b) $\Omega_0^2 = 0.75$; (c) $\Omega_0^2 = 1.0$; (d) $\Omega_0^2 = 1.25$; (e) $\Omega_0^2 = 1.5$; (f) $\Omega_0^2 = 2.5$

designated '$e_{d1} = 1.5e + 0.15\rho$', i.e. $b = 3.0\rho$ has been assumed. The curves designated 'static' represent $e_{d1} = 1.0e$ (i.e. inertia forces applied at CM) as advocated by the ATC recommendations[9], and those labelled $e_{d1} = 1.5e$ show the results of applying the standard amplification factor.

The different effects of increasing eccentricity on the response at the two edge stations for the different torsional to lateral frequency ratios shown are immediately apparent. The behaviour at $a_i = +1.5\rho$ (*Figure 5*) is perhaps the most interesting. Whereas, as expected, the displacements of *Figure 6* at $a_i = -1.5\rho$ (at all eccentricities) are much higher than for $e = 0$, the displacements at $a_i = +1.5\rho$ are lower than for $e = 0$ at all

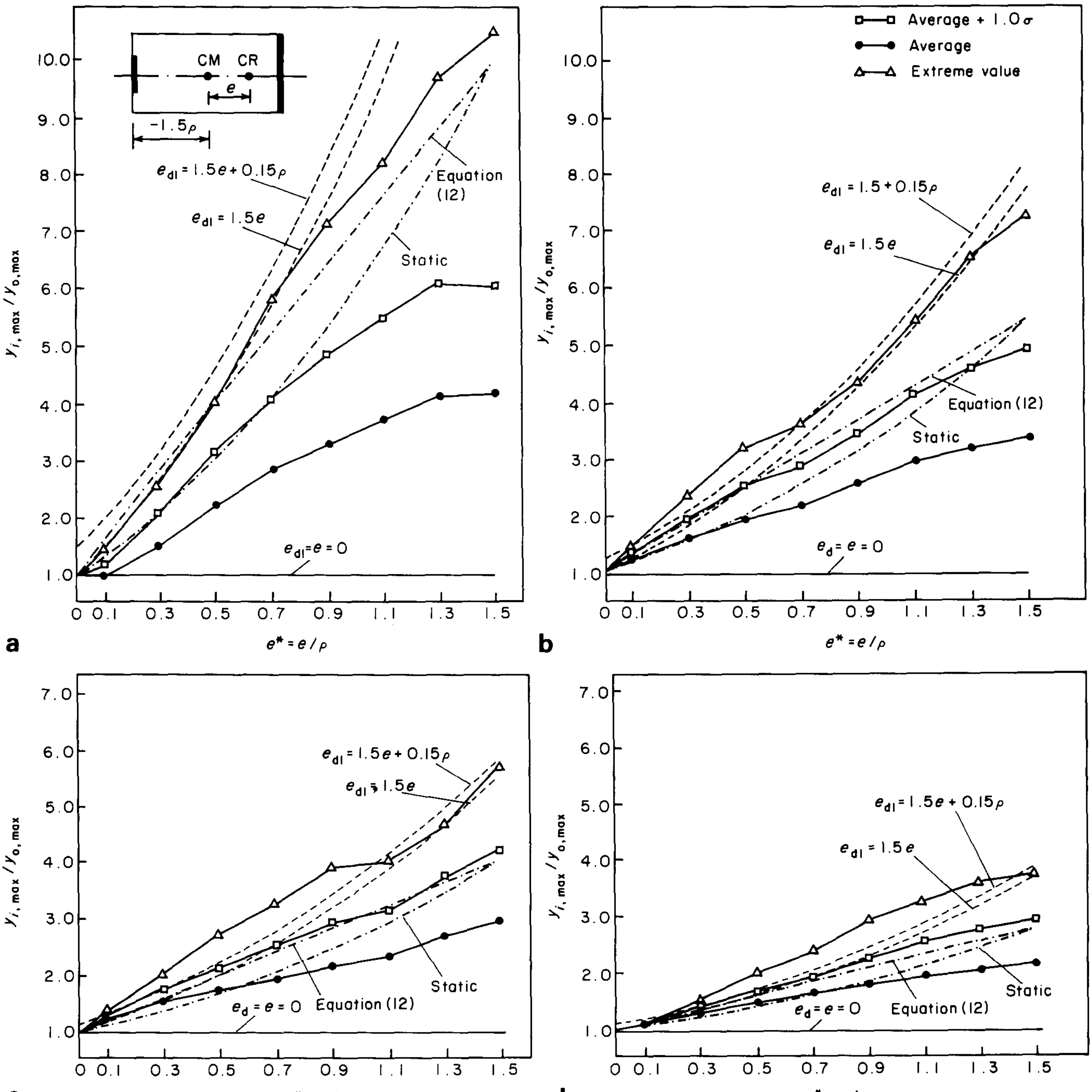

Figure 6 Maximum displacement at flexible edge ($a_i = -1.5\rho$) against eccentricity—comparison with static, code and proposed formulae: (a) $\Omega_0^2 = 0.5$; (b) $\Omega_0^2 = 1.0$; (c) $\Omega_0^2 = 1.5$; (d) $\Omega_0^2 = 2.5$

eccentricities only for the high frequency ratios ($\Omega_0^2 \geqslant 1.5$), and only at large eccentricities for intermediate and low frequency ratios. Considering now the '$e_{d1} = 1.5e$' results, it is seen that at $a_i = -1.5\rho$ (*Figure 6*) the agreement with time history analysis is satisfactory for $\Omega_0^2 \geqslant 1.0$ at low eccentricities.

Regarding the response at $a_i = +1.5\rho$ (*Figure 5*), it is evident that none of the static methods can adequately predict the dynamic behaviour unless the frequency ratio is quite high ($\Omega_0^2 \geqslant 1.5$). It will be observed that resorting to the traditional practice of ignoring the negative torsional contribution is still unconservative, although this practice narrows the gap appreciably.

The results at inner stations $a_i = \pm 1.0\rho$ (not shown) are similar to their corresponding edge stations, although the disagreement between the $e_{d1} = 0.5e$ curve and dynamic results at $a_i = +1.0\rho$ is somewhat less pronounced than at $+1.5\rho$.

The comparison between code values and dynamic results for members oriented perpendicular to the direction of excitation, thus resisting torsion only, is shown in *Figure 7*. In these graphs, the code values are plotted assuming that the torsional response is computed as for members on the flexible side of the deck. Note that the response at low eccentricities is still underestimated for $\Omega_0^2 = 1.5$, although to a lesser extent.

In the following section, possible modifications to the present code provisions are proposed which are intended to improve the agreement between the time history and static results.

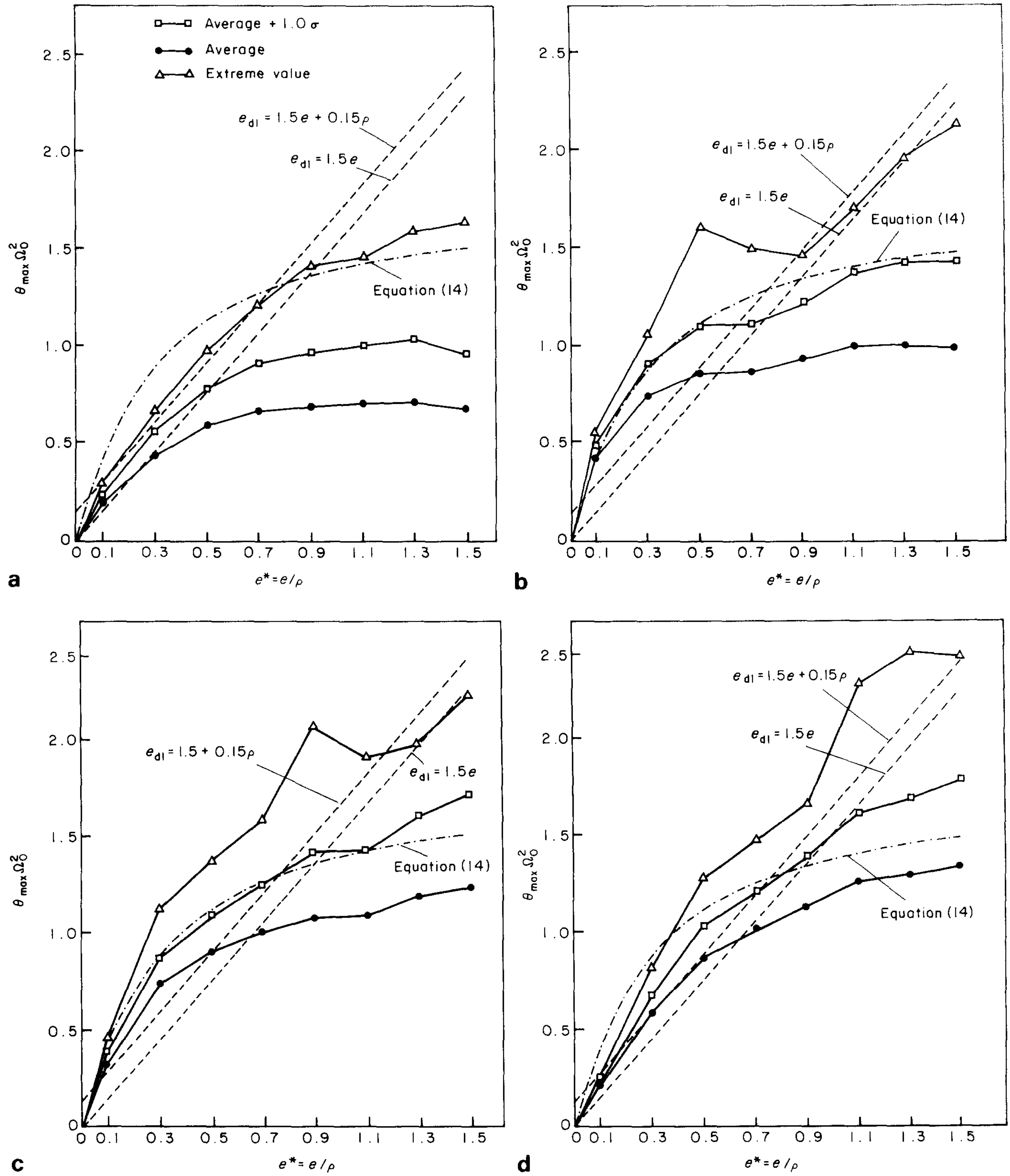

Figure 7 Maximum rotation against eccentricity—comparison with code and proposed formulae: (a) $\Omega_0^2 = 0.5$; (b) $\Omega_0^2 = 1.0$; (c) $\Omega_0^2 = 1.5$; (d) $\Omega_0^2 = 2.5$

Alternative design formulae

The parametric study described in the preceding section has shown that some modifications to the standard formulae for the design eccentricity are required if better correlation with time history results is desired. Since the extent of dynamic amplification depends on the location and orientation of the member or assemblage, it is only to be expected that different design formulae are needed for members on the flexible side, stiff side and those oriented perpendicular to the direction of excitation.

Formulae that are proposed for code purposes should be simple yet provide comparable margins of safety for different types of structures; also, they should not depart significantly from existing code provisions unless the latter are clearly inadequate. These considerations guided the authors in proposing the expressions presented in this section.

Members on the flexible side of the floor

These are members located to the left of CR in *Figure 1*. For these members the following formula is proposed:

$$e_{d1} = \frac{A\rho}{B\rho + e} e \tag{12}$$

With this formula, it is possible to reduce e_{d1} with increasing eccentricity. For example, with $A = 3.0$ and $B = 1.5$, e_{d1} varies from $2.0e$ at small e to $1.0e$ at $e = 1.5\rho$ and, when $A = 2.5$ and $B = 1.0$, $2.5e > e_{d1} > 1.0e$. Note that with the former set of coefficients, equation (12) results in linear variation of e_{d1} for $a_i = -1.5\rho$, and with the latter set for $a_i = -1.0\rho$. Also note that the second set leads to more conservative results. It can be seen from *Figure 6* ($A = 3.0$ and $B = 1.5$) that very satisfactory correlation is obtained with time history results for practically all eccentricities at medium and large frequency ratios. The only significant discrepancy between the proposed formula and 'average $+1.0\sigma$' dynamic results occurs for larger eccentricities at very low frequency ratios ($\Omega_0^2 = 0.5$). This is also the case where the static or ATC results depart appreciably from dynamic analysis. Since designs combining very low torsional rigidity with high eccentricity are not common, and in the authors' opinion should not be encouraged, this result appears to be acceptable.

It is, however, quite easy to improve the correlation at the extreme ends of the frequency range by factoring equation (12) by $(\Omega_0)^{1/2}$ to read (say for $A = 3.0$ and $B = 1.5$):

$$e_{d1} = \frac{3\rho(\Omega_0)^{1/2}}{1.5\rho + e} e \tag{13}$$

It will be recalled that the results for $T_0 = 0.25$ s were significantly different from those for the higher natural periods at moderate and large eccentricities, particularly at low frequency ratios. From the analysis of the dynamic results it appears that equation (12), with $A = 2.5$ and $B = 1.0$, is practically above the 'average $+ 1.0\sigma$' displacements of the five samples with $T_0 = 0.25$ *s*, up to $e = 0.7\rho$. This is close to the range within which equation (12) is more conservative than $e_{d1} = 1.5e$. Therefore, pending a detailed study on the behaviour of laterally rigid asymmetric systems one may use equation (12) in the following form:

$$e_{d1} = \frac{2.5\rho}{1.0\rho + e} e \geqslant 1.5e \tag{12a}$$

for $T_0 < 0.5$ s.

Torsion resisting members

Since these members are oriented perpendicular to the direction of excitation they do not participate in resisting the lateral forces. A formula similar in form to equation (12) is proposed:

$$e_{d1} = \frac{1.8\rho}{0.3\rho + e} e \tag{14}$$

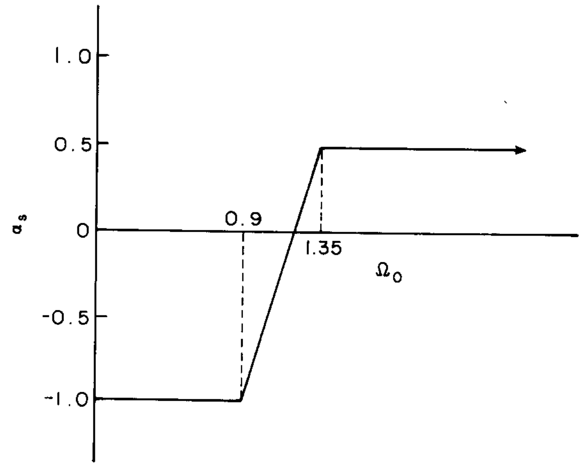

Figure 8 Variation of α_s in equation (16) with Ω_0

In this formula, e_{d1} varies from $6.0e$ at very small eccentricities to $1.0e$ at $e = 1.5\rho$. The comparison between the dynamic results, equation (14) and $e_{d1} = 1.5e$ is given in *Figure 7*. Again, one may choose to improve the agreement with dynamic results at the extreme ends of the frequency range by letting:

$$e_{d1} = \frac{1.8\rho(\Omega_0)^{1/2}}{0.3\rho + e} e \tag{15}$$

Again, pending a detailed study for $T_0 < 0.5$ *s*, it is advisable to limit the use of equation (14) as follows:

$$e_{d1} = \frac{1.8\rho}{0.3\rho + e} e \geqslant 1.5e \tag{14a}$$

for $T_0 < 0.5$ *s*.

Members on the rigid side of the floor

These are members located to the right of CR in *Figure 1*. The response of these members is the most difficult to describe in simple mathematical terms, as can be seen from the curves in *Figure 5*.

The simplest way to improve the correlation between the code formulae and the dynamic results is to reverse the sign of the eccentricity in either the static or the $0.5e$ results in *Figure 5* and factor them so that with increasing Ω_0 the negative eccentricity would diminish, namely:

$$e_{d1} = \alpha_s e \tag{16}$$

in which α_s for the static results is given in *Figure 8*. However, it can be seen from *Figure 5* that, for $\Omega_0^2 < 1.25$, this formula tends to overestimate the response at larger eccentricities. For negative eccentricities, an expression of the form:

$$\begin{aligned} \alpha_s^* &= \alpha_s(1 - \Omega_0^2 e^{*3}), \quad (\Omega_0^2 e^{*3} < 1.0) \\ \alpha_s &= 0, \quad (\Omega_0^2 e^{*3} > 1.0) \end{aligned} \tag{17}$$

lowers the response with increasing e and Ω_0, and so is better correlated with the dynamic results, but perhaps is not simple enough. Better agreement with time history results can be obtained by means of exponential functions, but these are even more complicated.

It will be observed that some engineers prefer using natural building dimensions b and d in the design eccentricity formulae rather than the 'artificial' dimension ρ. In this case, one may substitute $\rho \approx D/3$ where D is the largest diagonal building dimension.

Conclusions

A parametric study of the earthquake time history response of asymmetric single storey structures with one axis of symmetry modelled as two-degree-of-freedom systems has been reported. Five earthquake records have been applied to a number of eccentric systems with a range of eccentricities, torsional to lateral frequency ratios, and uncoupled lateral periods ranging from 0.25 to 2.0 s. The 35 maximum displacements per eccentricity obtained at several locations along the floor were normalized with respect to the corresponding symmetric cases and averaged. The 'average + 1.0σ' is the response with which standard static code provisions were compared.

It was found that static code provisions underestimate the response of members on the rigid side of the rigidity centre at low frequency ratio, as well as of torque resisting members (line of action perpendicular to direction of excitation) at inner resonance ($\Omega_0 \approx 1.0$ and $e^* \ll 1.0$). Also, the correlation of results on the flexible side of the floor is not always satisfactory, with overestimates at large eccentricities and at low frequency ratios. A difference between the maximum displacements and rotations of systems with uncoupled lateral period $T_0 = 0.25$ s and those with higher periods for members located on the flexible side of the floor was noted. The extent of this phenomenon and its effect on the design eccentricity expression deserve further study.

Simple formulae which better correlate with dynamic response have been proposed, and these could replace the existing design eccentricity expressions in seismic codes.

It has been argued[9] that refinements in the code formulae are not warranted because structures in seismic areas are designed to yield, and ductility demand is not simply related to linear response. There is some evidence[10], however, that ductility demand in asymmetric structures can better be controlled when the strength of the edge members is not appreciably lower than that predicted by linear dynamic analysis.

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Nomenclature

a_i	location of element from CM
$\boldsymbol{a}_i$	location of element from CR
b	building dimension perpendicular to excitation
CM	centre of mass
CR	centre of resistance
d	building dimension in direction of excitation
D	largest diagonal building dimension
e	static eccentricity
e_d	design eccentricity
e_{d1}	dynamic eccentricity
e_{d2}	accidental eccentricity
e^*	eccentricity ratio ($= e/\rho$)
K_y	structure translational stiffness
K_θ	structure rotational stiffness about CM
$\boldsymbol{K}_\theta$	structure rotational stiffness about CR
m	structure mass
M_{max}	maximum dynamic torque
V_{max}	maximum dynamic base shear
V_0	code base shear
$V_{0,max}$	maximum dynamic base shear of symmetric structure
T_0	translational period of symmetric structure ($= 2\pi/\omega_y$)
y	axis of excitation
$y_{i,max}$	maximum displacement of element i
$y_{0,max}$	maximum displacement of symmetric structure
y_{max}	maximum displacement at CR of asymmetric structure
α_s	dynamic eccentricity coefficient for rigid side elements (equation (16))
η	damping ratio
θ	torsional displacement of structure
ρ	mass radius of gyration about CM
σ	standard deviation
ω_y	uncoupled translational frequency ($= (K_y/m)^{1/2}$)
$\boldsymbol{\omega}_\theta$	uncoupled rotational frequency ($= (\boldsymbol{K}_\theta/m\rho^2)^{1/2}$)
Ω_0	uncoupled torsional to translational frequency ratio ($= \boldsymbol{\omega}_\theta/\omega_y$)

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