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SIMPLE PLANAR MODELING OF ASYMMETRIC SHEAR BUILDINGS FOR LATERAL FORCES

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Abstract—A simple plane frame model is described for the lateral load analysis—static and dynamic—of frames in multistory buildings including biaxial load effects. By taking advantage of the shear force–axial force and the torque–bending moment analogies it is possible to model the spatial behavior of the structure, and to consider the combined effects of bidirectional shear due to lateral and torsional displacements on columns pertaining to two orthogonal frames. A numerical example is given.

INTRODUCTION

The structure of tall buildings is statically indeterminate to a very high degree. Yet, the increasing efficiency of the electronic computer enables the engineer to analyze more faithful mathematical models of the structure. However, it is not always advisable to take advantage of this capability because of the relatively high costs incurred when standard three-dimensional (3-D) programs are used. Moreover, the enormous bulk of output is more often than not an embarassment to the engineer, since the important information has to be sifted from the mass of practically irrelevant data. These limitations are mainly due to the fact that less significant degrees of freedom, such as frame bending normal to their plane of action and torsion of beams and columns, are considered, and their values recorded. On the other hand, in-house computers are still somewhat limited in capacity, and are quite slow in executing large-volume problems. Therefore many engineers prefer models which reduce the size of the problem by neglecting these less significant degrees of freedom, and which take advantage of the particular structural and geometric features of the building. Apart from saving the engineer's time, and time as well as storage for the computer, such an approach focuses the attention of the engineer on the more important behavioral characteristics of the structure.

Shear buildings or structures whose horizontal displacement curve is proportional to the applied shear force are natural candidates for this type of modeling. Framed structures and those laterally supported by shear walls of low aspect ratio can, under not particularly restrictive conditions, be modeled as shear buildings. Therein lies the importance of the proposed models, since the analysis of such structures, although lending themselves to story by story analysis (near coupled), is still tedious and time-consuming, particularly for the 3-D asymmetric case.

To transform the 3-D analysis of torsionally coupled shear buildings into a planar or 2-D format, two analogies have to be invoked. One analogy models the shear response of the frame—the shear force–axial force analogy [1, 2]. The other analogy models the effect of torsion. In buildings with an orthogonal framing system (Fig. 1), and only these are considered in the present paper, the torsional or rotational displacements affect the frames oriented parallel to the direction of the force, as well as those perpendicular to it. The latter effect cannot be modeled by the shear–axial force analogy and requires a separate treatment. This is the torque–bending moment analogy [2]. Finally, since columns usually pertain to two orthogonal frames, it is of some interest to obtain the maximum biaxial response of these columns. This requires that the modeling for shear and for torsion be combined in a single member. This is of particular importance in earthquake-resistant design, where biaxial interaction effects on perimeter columns have to be considered, since they may lead to larger ductility demands apart from changing the shear distribution among the resisting frames in the nonlinear range. Finally, the proposed approach can model—albeit approximately—the effect of floor flexibility on the lateral force distribution [1], which becomes important for narrow low-rise buildings with squat shear walls. As is well known, many efficient 3-D computer programs cannot consider this effect. Another advantage of the approach is that P-Delta effects can be easily incorporated into the planar formulation.

THE ANALOGIES

Shear force–axial force analogy

Consider the shear cantilever shown in Fig. 2(a), modeling a frame under a set of story forces P_i. The equivalent shear rigidity of the cantilever within the ith story is denoted by GA_{si} (G = shear modulus; A_{si} = shear area). The horizontal displacement curve along the height H is also shown in Fig. 2(a). Consider now the vertical column shown in Fig. 2(b).

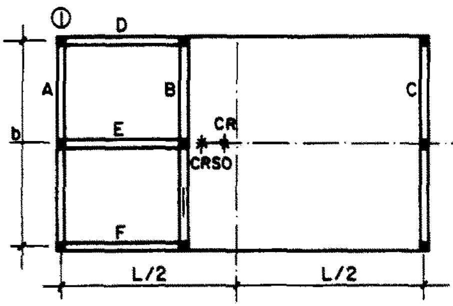

Fig. 1. Floor plan.

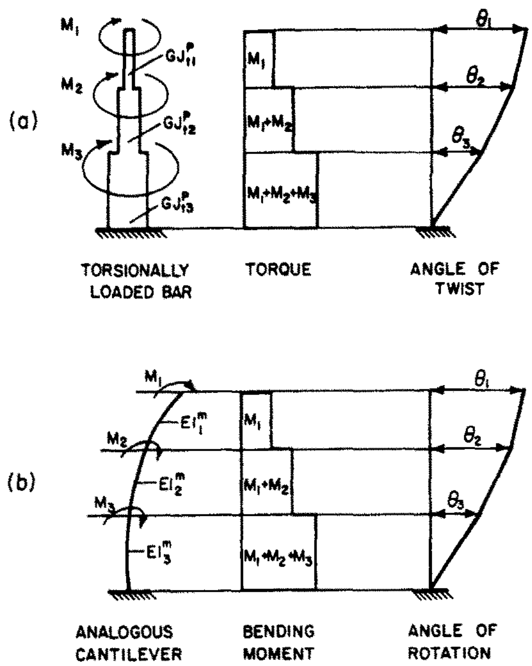

Fig. 3. Torque–bending moment analogy.

It has the same length as the shear cantilever, and its axial rigidities EA_i (E = Young's modulus; A = cross-sectional area) within the ith story are numerically equal to the corresponding shear rigidities GA_{si} of the shear cantilever. The column is acted upon by the same set of forces P_i which, however, are applied *vertically* rather than horizontally. The latter forces lead to vertical displacements of the column along the height. The deflected shape is also shown in Fig. 2(b) as a horizontal displacement curve. Since, for analogous end conditions, $\delta_a = \delta_s$, the analogy is demonstrated. Therefore, in many instances the shear rigidities of a plane frame or a shear wall can be replaced by equivalent axial rigidities of a column.

Torque–bending moment analogy

Consider now the shaft shown in Fig. 3(a). It models the torsional behavior of a shear building under a set of story torques M_i. The torsional rigidities of the shaft are denoted by GJ_{ti} (J_{ti} = torsional constant). The variation of the angle of twist θ_t along the height of the building is also shown in Fig. 3(a). Figure 3(b) shows a vertical flexural cantilever having the same length as the torsional shaft in Fig. 3(a), with flexural rigidities EI_i (I_i = moment of inertia) numerically equal to the corresponding GJ_{ti} of the shaft. Similarly, bending moments numerically equal to the torques M_i are applied at the floor levels as shown. A plot of the angle of rotation θ_b of the cantilever along the height is also shown in Fig. 3(b). Again, since $\theta_b = \theta_t$, the analogy is demonstrated.

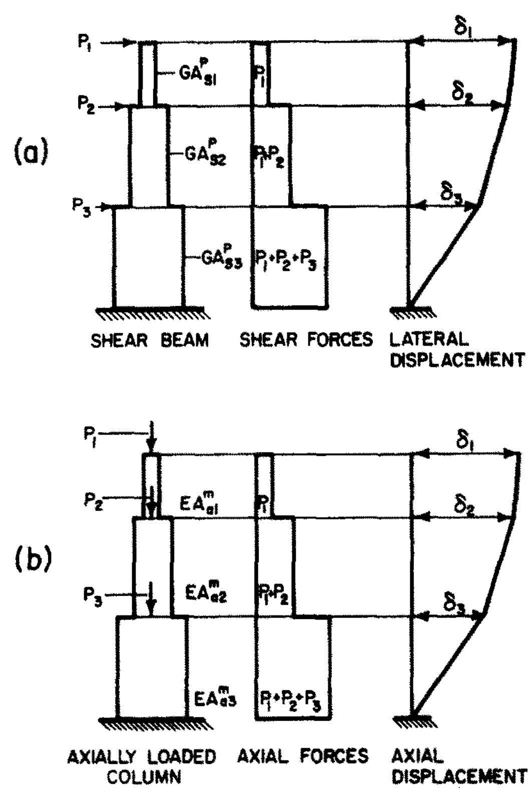

Fig. 2. Axial force–shear force analogy.

It will be observed that the model shown in Fig. 4 preserves the torque–bending moment analogy just as the one in Fig. 3(b). However, it has some advantages. In the first place, it suppresses the horizontal motion degree of freedom in the model, which are meaningless in any case, and avoids spurious axial forces in the columns of the analogous frame. Secondly, and perhaps more importantly, it permits modeling, albeit approximately, the biaxial response of any column to lateral and torsional excitation when axial and flexural properties are assigned to a given column.

It is thus seen that by assigning given axial and flexural sectional properties to the equivalent columns, shear and horizontal displacements are respectively replaced by axial compression and displacements, whereas torque and twist about the vertical axis are replaced by bending and rotation about a horizontal axis respectively. Thus, the two analogies lead to planar representation of a 3-D structural system with one axis of symmetry loaded unidirectionally.

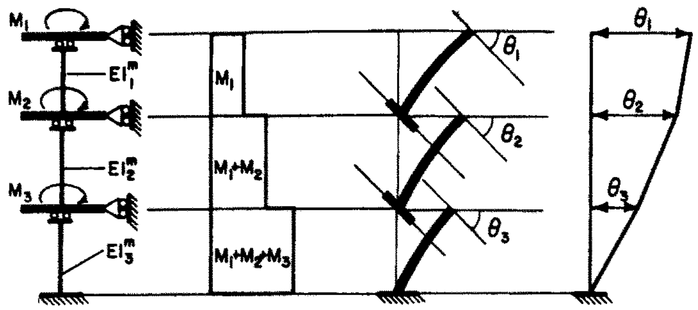

Fig. 4. Alternative modeling of analogous cantilever.

In-plane floor behavior

In the planar model described herein vertical displacements model horizontal displacements. Thus, floor slabs are modeled as continuous beams vertically supported on columns modeling the shear resistance of frames or shear walls, as shown in Fig. 1. When the floor slabs are assumed to be rigid in their own plane—the usual assumption in the lateral load analysis of tall buildings—very high values of flexural and shear rigidities should be assigned to these equivalent beams. On the other hand, assigning finite values to these rigidities permits modeling the in-plane flexibility of the floor, which becomes important for narrow low-rise buildings. However, assigning these finite values to the floor rigidities should not be done indiscriminately. This is because the span-to-depth ratio of the floor slabs acting as beams in their own plane between adjacent frames or shear walls is likely to be low. For such slabs the usual strength of materials assumptions made in standard computer programs may lead to erroneous results, particularly as to the contribution of the frames which are perpendicular to the direction of excitation (e.g. frames D and F in Fig. 1).

The single stick model

When all the floors can be assumed to be rigid in their own plane, all the shear carrying assemblages can be lumped into one equivalent column with a cross-sectional area equal to the sum of the equivalent areas of the participating assemblages, and all the torsional rigidities can be lumped into one equivalent flexural cantilever. These two can then be merged into a single member located at the center of rigidity or twist of the story CR as shown in Fig. 5. It will be observed, however, that this further reduction requires some additional work, as does the retrieval of results. Note also that by lumping the properties, the individual response of any given column—particularly the biaxial response—may not be recoverable. This aspect is important in earthquake engineering where response maxima are not concurrent. Therefore, the practical usefulness of this model appears to be limited.

SHEAR WALL AND FRAME MODELING

As noted earlier, the horizontal displacement curve of concrete walls having low aspect ratio is essentially shear-dependent, so that they can be modeled as shear beams. The deflected shape of laterally loaded multistory frames is also shear-dependent provided that the frame is not slender so that the axial deformations in the columns are relatively small, and the flexural rigidity of the girders is sufficiently high to suppress the simple cantilever bending of the col-

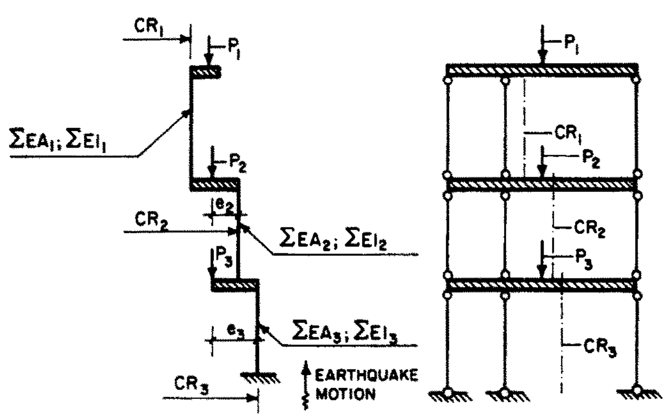

Fig. 5. Single stick model.

umns. When these conditions are met, the following procedure can be adopted for modeling these assemblages as shear cantilevers.

Equivalent column to model shear members

(a) *Squat concrete walls.* Let the shear area of the wall be A_{sw}, then the cross-sectional area A_{aw} of the equivalent column is given by:

$$A_{aw} = A_{sw}/[2(1+\nu)], \tag{1}$$

in which ν = Poisson's ratio, assuming that Young's modulus E remains unchanged in the model.

(b) *Multistory rigid frames.* The analysis is more involved since the equivalent shear stiffness of the frame in the story GA_{sf} has to be computed first. Several approximations are available which derive from Wilbur's formula [3]. The following expressions are taken from Cheong-Siat-Moy [4], for a story with m columns:

$$GA_{sf} = \frac{12E}{h^2}\sum_1^m \frac{I_c}{1+2\psi}, \tag{2}$$

in which

$$\psi = I_c/h \Big/ \left(\sum_1^n I_b/L_b\right),$$

h = story height, L = beam span, I_c = column inertia, I_b = beam inertia and n = number of beams framing into the column ($n \le 2$). Since the end restraints of the columns in the bottom and top stories are different, suitably modified expressions have to be used. For the bottom story, assuming base fixity of the columns:

$$GA_{sf} = \frac{12E}{h^2}\sum_1^m \frac{(3+\psi)I_c}{4+3\psi}, \tag{3}$$

and for the top story:

$$GA_{sf} = \frac{12E}{h^2}\sum_1^m \frac{I_c}{1+\psi}. \tag{4}$$

Once the story stiffnesses have been computed, apply the shear axial force analogy, or:

$$A_{af} = A_{sf}/[2([1+\nu)], \tag{5}$$

in which A_{af} is the axial cross-sectional area of the equivalent column.

Equivalent cantilever to model torsional resistance

(a) *Squat concrete walls.* Let the shear area of the wall perpendicular to the plane of action be A_{sw}, then the contribution of this wall to the torsional rigidity of the story is given by:

$$GJ_{tw} = GA_{sw}e^2, \tag{6}$$

in which GJ_{tw} = torsional rigidity of the wall about the center of rotation (CR) of the story and e = eccentricity of wall with respect to CR. The torque–bending moment analogy then leads to:

$$I_w = J_{tw}/[2(1+\nu)], \tag{7}$$

in which I_w = inertia of equivalent cantilever.

(b) *Multistory rigid frames.* First, evaluate the shear rigidities of the perpendicular frames using eqns (2), (3) or (4), as the case may be. Then apply the analogy

$$I_f = A_{af}e^2, \tag{8}$$

in which I_f = inertia of the equivalent cantilever. Note that zero axial rigidity should be assigned to cantilevers carrying torsion only (no shear). It will be observed that the torsional rigidity of walls and columns about their own axis GJ_{t0} can also be considered. For shear resisting members this can be done by assigning the equivalent flexural rigidity to the corresponding columns, using eqn (7), whereas for torque resisting assemblages this contribution can simply be added to GJ_{tw}:

$$GJ_{tw} = GA_{sw}e^2 + GJ_{t0}. \tag{6a}$$

BIAXIAL MODELING OF FRAME COLUMNS

Consider the structural system shown in Fig. 1. In this model columns pertain to two frames whose lines of action are at right angles. Due to the asymmetry of the system, the lateral forces, either static or dynamic, induce biaxial horizontal shear and displacement into the columns. The displacement (e.g. column No. 1 in Fig. 1) in the y direction (δ_{1y}) is obtained from the planar analysis as a vertical displacement. However, the displacement in the x direction (δ_{1x}) has to be derived from the computed rotations:

$$\delta_{1x} = e_D\theta = \frac{b}{2}\theta, \tag{9}$$

in which e_D = eccentricity of frame D from the center of rotation, and $\theta = \theta_t = \theta_b$ is the angle of twist. The shear forces acting on the column depend on its relative rigidities within the two frames. These two rigidities can be computed from the stiffness properties of the two frames (A and D). As is well known, for relatively rigid girders, the relative rigidity is proportional to the I/L^3 value of the column, and is independent of the girder properties. The share of shear force taken by column 1 in the x direction requires more computation, since the shear taken by frame D has to be computed first by dividing the torque acting on it (the bending moment in the model) by its distance e_D from the center of rotation. This additional calculation can be easily incorporated into the planar program.

When the planar model is used for seismic dynamic analysis, the simple procedure outlined above can be applied to determine the maximum forces acting on the columns. Moreover, if the planar program has nonlinear capability, then a yield surface may be fitted to model the biaxial interaction in the column (Fig. 6). Thus, the proposed model can be extended to the study of the response to unidirectional earthquake excitation of asymmetric yielding structures with nonlinear biaxial interaction. However, one should not overlook one serious limitation of the approach. Since in the planar model a frame is represented only by its columns, the relative rigidity of the column within the frame remains unchanged before any one of the other columns in the same frame is assumed to have yielded. In fact, in seismic design, columns are seldom permitted to yield, and the progressive loss of stiffness of the frame which in fact takes place is due to the yielding of the girders. This is more difficult to model within the shear force–axial force analogy.

P-DELTA EFFECTS

It has been shown that lateral and torsional P-Delta effects can respectively be modeled on standard computer programs as shear beams and torsion bars with negative stiffness properties [5, 6]. Therefore, the two analogies described herein are directly applicable.

The reduction in the shear rigidities of a column in the x and y directions ΔGA_x^* and ΔGA_y^* respectively, due to the action of the axial force ΣN, are given by:

$$\Delta GA = \Delta GA_x = \Delta GA_y = -\gamma \Sigma N, \tag{10}$$

in which γ is the correction factor to account for the deviation of the deflected shape of the column from a straight line $(1.0 \le \gamma \le 1.22)$. Therefore, the effective shear area of the column GA_{sf}^{Δ} is given by:

$$GA_{sf}^{\Delta} = GA_{sf} - \gamma \Sigma N. \tag{11}$$

Applying the shear force–axial force analogy it follows that the cross-sectional area of the fictitious column ΔA_a is given by:

$$\Delta A_a = -\gamma \Sigma N / E \tag{12}$$

and is located at the centroid of N.

Similarly, the contribution of a given column to the reduction in the torsional rigidity of the story due to gravity effects, ΔGJ_t, can be shown to be:

$$\Delta GJ_t = -\Sigma N(\gamma_y d_x^2 + \gamma_x d_y^2), \tag{13}$$

in which γ_x and γ_y are the correction factors of the column in the x and y directions respectively, and d_x and d_y are the x and y distances of the column from the center of the effective rigidities of the story, or the centroid of all the GA^{Δ}, i.e. the point CRSO in Fig. 1. Note that this point usually does not coincide with CR. Applying the torque–bending moment analogy it follows that the negative inertia of the fictitious cantilever ΔI is given by:

$$\Delta I = \Delta J_t / [2(1 + \nu)]. \tag{14}$$

For the single stick model it is necessary to combine the negative contributions of all the columns, namely:

$$\Sigma \Delta GA_y = -\Sigma \gamma_y N \tag{15}$$

$$\Sigma \Delta GJ_t = -\Sigma N(\gamma_y d_x^2 + \gamma_x d_y^2). \tag{16}$$

The latter expression can also be written as

$$\Sigma \Delta GJ_t = -r_{\mathrm{CRSO}}^2 \Sigma N, \tag{17}$$

in which the radius of inertia of the load r_{CRSO} is given by:

$$r_{\mathrm{CRSO}}^2 = \Sigma N(\gamma_y d_x^2 + \gamma_x d_y^2)/(\Sigma N), \tag{18}$$

therefore

$$\Sigma \Delta A_a = -\Sigma \gamma_y N / E \tag{19}$$

$$\Sigma \Delta I = r_{\mathrm{CRSO}}^2 \Sigma N / E. \tag{20}$$

Finally, the properties of the single stick are obtained:

$$\Sigma A_a^{\Delta} = \Sigma A_a - \Sigma \gamma_y N / E \tag{21}$$

$$\Sigma I^{\Delta} = \Sigma I - r_{\mathrm{CRSO}}^2 \Sigma N / E. \tag{22}$$

NUMERICAL EXAMPLE

The structure shown in Fig. 6 is to be analyzed for a given earthquake record. In particular, it is required to compute the maximum forces acting on the corner column I. The first 5 sec of the time history for the y direction force, $F_y(t)$, for the 1940 El Centro earthquake record is shown in Fig. 7. It is obtained by factoring the total force on frame C by the relative stiffness of the column. Similarly, the time history of the x direction force on the column is obtained by first dividing the torque time history, represented by the bending moment on the fictitions column, by the lever arm of frames D and E and then factoring the result by the relative stiffness of column I within frame E. The time history of the x direction shear on the column, $F_x(t)$ is also given in Fig. 7. In this case, for the first 5 sec of the El Centro earthquake, the maximum resultant column force is:

$$F(t)_{\max} = \sqrt{[F_x^2(t) + F_y^2(t)]}_{\max} = 7.58t,$$

compared with

$$F_y(t)_{\max} = 6.55t$$

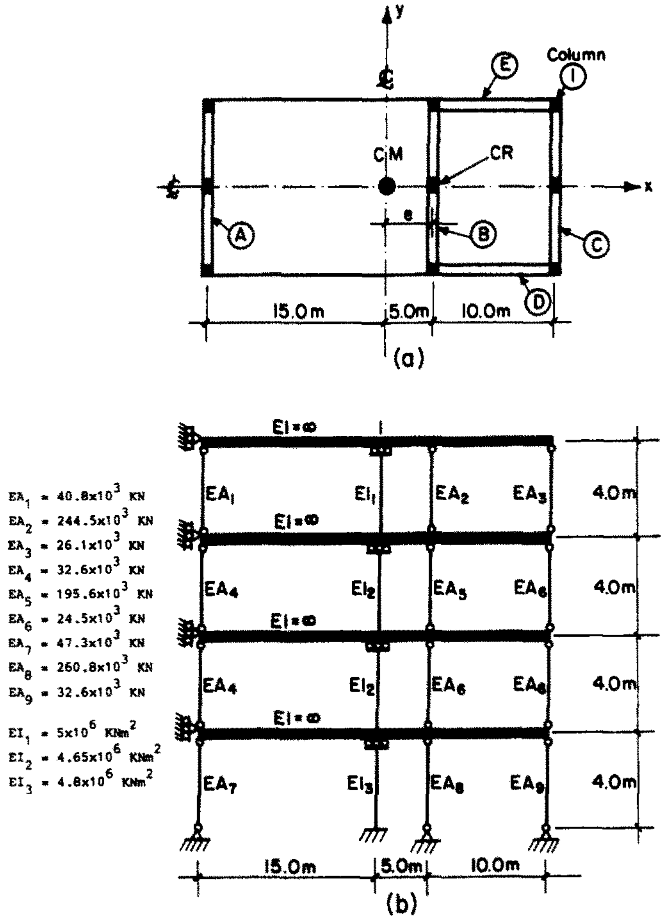

Fig. 6. Structure for example problem. (a) Floor plan. (b) Analogous frame.

and

$$F_x(t)_{\max} = 3.81t.$$

It is seen that the x–y interaction effects under undirectional excitation give a 15% increase in the shear force—for the corner column. Taking the first 16 sec of the time history (not shown), the forces are 13.8, 11.41 and 6.60t respectively. The effect of bidirectional excitation—not discussed in this paper—is believed to be even more significant. Note that a very simple subroutine is required to obtain the interaction response.

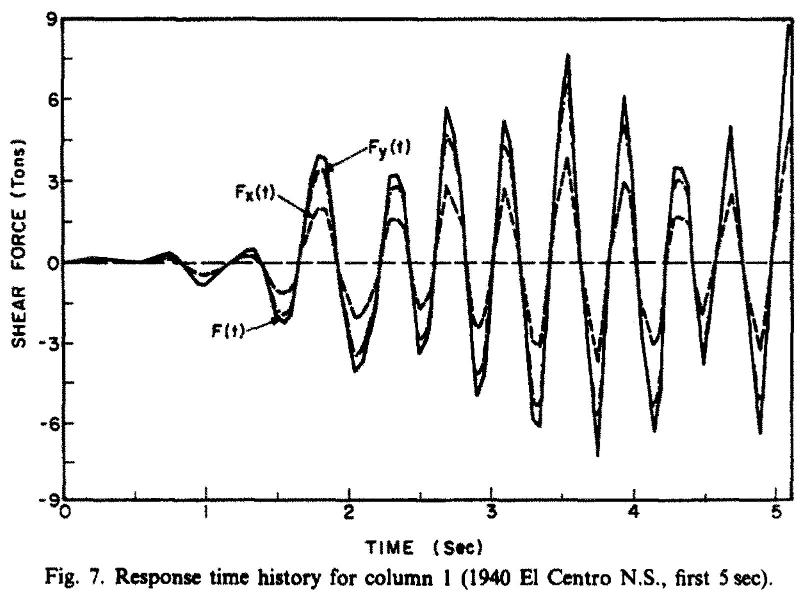

Fig. 7. Response time history for column 1 (1940 El Centro N.S., first 5 sec).

SUMMARY AND CONCLUSIONS

Simple analogies have been described that permit the lateral load analysis—static and dynamic—of a three-dimensional building structure having one axis of symmetry, to be made by means of standard first order two-dimensional plane frame computer programs. P-Delta effects may be easily incorporated in the analogous frame by adding a fictitious column with negative stiffness properties. The bidirectional interaction of columns pertaining to two perpendicular frames is achieved through a simple subroutine which combines the axial and flexural stresses in the columns of the analogous frame.

The procedure is theoretically valid only for shear buildings, therefore it is applicable to multistory buildings in which the lateral loads are carried by moment-resisting frames, and to low-rise buildings supported on walls with a low height-to-width ratio.

Large savings in computer time and storage are to be expected due to the substantial reduction in the number of degrees of fredom. Also, in view of the planar modeling, only the response of significant degrees of freedom is obtained, thereby simplifying the design process.

The procedure can easily be extended to nonlinear interaction problems. This can be done by specifying a bidirectional interaction surface and a suitable overshoot correction procedure.

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