Torsional Displacements in Base-Isolated Buildings

R. S. Jangid, M.EERI, and J. M. Kelly, M.EERI

An analytical study of the effects of torsional coupling on the seismic response of a base-isolated building is presented. The isolated structure is modeled as a rigid deck supported on axially inextensible bearings. The governing equations of motion for the coupled lateral-torsional response of the system are derived. The eccentricity in the system is that specified by the *Uniform Building Code (UBC)*. The displacement response of the isolated system with different combinations of building configuration, isolation damping, and the ratio of uncoupled torsional to lateral frequency of the system is investigated. The response of the isolated structure under a variety of near-fault and other earthquake ground motions is compared to that obtained by use of response spectrum analysis. In the response spectrum analysis the accuracy of several modal combination rules is evaluated. It is shown that torsional coupling can influence the response of the isolated structure, but if the layout of the isolation bearings is such that the torsional frequency is larger than the lateral frequency, the effect is reduced and the usual modal combination rules work well. It is also shown that in this case, the *UBC* static formula for the additional isolator displacements due to torsion is conservative.

INTRODUCTION

Generally, a seismically isolated building is designed using a time-history analysis, which is both a time-consuming and expensive process. This process is complicated by code requirements that mandate that when the displacements due to torsion are calculated, the location of the center of resistance with respect to the center of mass be selected in such a way as to provide the worst case displacement. This cannot be known in advance, thus, a time-history analysis with several locations must be carried out.

Seismic isolation in the United States has been applied mainly to buildings that house sensitive internal equipment and buildings that must remain functional after large rare earthquake events. In these cases, the additional design cost is usually justified. If the approach is to be applied to more general structures, however, for example, housing and commercial buildings, a simpler design procedure will be needed.

Although standard design codes, such as the *Uniform Building Code (UBC)* (ICBO 1997) and the *International Building Code (IBC)* (ICC 1998), permit the use of static design methods, they are rarely used because the static formulas are felt to be very conservative. Dynamic design using response spectrum methods are also permitted and could provide a very significant reduction in the design costs, but again the question of the location of the eccentricity in the torsional computation must be considered.

(RSJ) I.I.T Bombay, Mumbai 400 076, India
(JMK) Pacific Earthquake Engineering Research, 1301 S. 46th St., Richmond, CA 94804-4698

Seismically isolated structures are unusual in that the frequency of motion in the two orthogonal directions of the building is the same. If the torsional frequency is identical to the two lateral frequencies, a strong coupling can occur between the three components of the response when eccentricity is taken into account.

The present study attempts to simplify the design procedure for a seismically isolated structure by treating the torsional problem in the response spectrum method in a number of ways. It also examines the influence of damping and gives a comparison with the static design rule in the *UBC*.

The most important conclusion of the study is that the designer should arrange the layout of the isolators in such a way that the torsional frequency is larger than the lateral frequency. In this case, either of the usual modal combination rules works well and the static formula is very conservative.

SYSTEM CONSIDERED

The structure on the base isolation system is idealized as a rigid deck, with masses lumped at the corresponding column positions, as shown in Figure 1. The rigid deck is supported by mass-less, axially inextensible bearings. The center of resistance (CR) of the bearing system generally does not coincide with the center of mass (CM) of the deck. As a result, the lateral motion of the system is coupled with the torsional motion under horizontal ground excitation. The most disadvantageous situation occurs when the line joining the CR and CM is perpendicular to the direction in which excitation is considered. Thus, in the present study, an eccentricity of e_y in the y-direction of the system is considered. The two degrees-of-freedom in which coupling occurs are the lateral displacement, u_x, and the torsion, θ, when the system is subjected to ground motion in x-direction. The motion of the structure in the y-direction is uncoupled with torsion and can be considered separately.

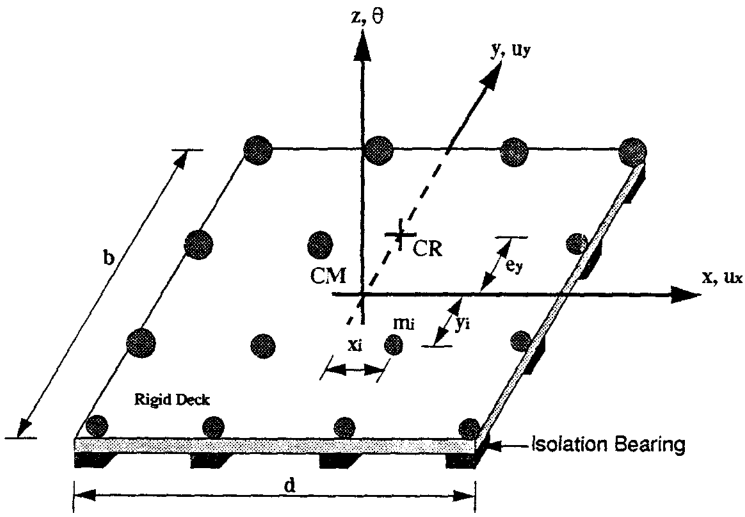

Figure 1. Simplified model of a base-isolated building.

If k_{xi} and k_{yi} represent the lateral stiffness of i[th] bearing in x- and y-directions, respectively, then

$$K_x = \sum_i k_{xi} \quad \text{and} \quad K_y = \sum_i k_{yi} \tag{1}$$

are the lateral stiffnesses of the base-isolated structure in the x- and y-directions, respectively. The torsional stiffness of the structure defined about the vertical axis passing through the CM of the deck mass is given by

$$K_\theta = \sum_i \left(k_{xi} y_i^2 + k_{yi} x_i^2\right) \tag{2}$$

where x_i and y_i denote the x- and y-direction coordinates of the i^{th} bearing, with respect to the CM of the deck mass, respectively. The torsional stiffness of each individual bearing element is negligible and ignored.

The eccentricity between the CM of deck mass and the CR of the bearings is given by

$$e_y = \frac{1}{K_x} \sum_i k_{xi} y_i \tag{3}$$

Two uncoupled frequency parameters of the system are defined as follows:

$$\omega_x = \sqrt{\frac{K_x}{m}} \quad \text{and} \quad \omega_\theta = \sqrt{\frac{K_\theta}{mr^2}} \tag{4}$$

where m is the mass of the deck, and r is the radius of gyration of deck mass about the vertical axis through the CM. The frequencies ω_x and ω_θ may be interpreted as the natural frequencies of the system if it were torsionally uncoupled, i.e., a system with $e_y = 0$, but with m, the mass of deck, K_x, K_y, and K_θ the same as in the coupled system.

GOVERNING EQUATIONS OF MOTION

The governing equations of motion of the isolated system under ground excitation in x-direction are written as follows:

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{\theta} \end{Bmatrix} + [C] \begin{Bmatrix} \dot{u}_x \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} K_x & -K_x e_y \\ -K_x e_y & K_\theta \end{bmatrix} \begin{Bmatrix} u_x \\ \theta \end{Bmatrix} = -\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \{r\}\, \ddot{u}_{gx} \tag{5}$$

where $[C]$ is the damping matrix, $\{r\} = \{1, 0\}^{\mathrm{T}}$ is the influence coefficient vector, and $\ddot{u}_{gx}$ is the ground acceleration in the x- direction.

Defining u_θ as $u_\theta = r\theta$ and carrying out the following transformation to decouple the coupled lateral-torsional motion of the system, we obtain

$$\begin{Bmatrix} u_x \\ u_\theta \end{Bmatrix} = [\Phi] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{6}$$

leading to the following set of modal equations:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = -L_i\ddot{u}_{gx} \qquad i = 1, 2 \tag{7}$$

where ξ_i is the modal damping ratio, ω_i is the natural frequency, and L_i is the modal participation factor in the i^{th} mode of the system.

The natural frequencies of the system are expressed as

$$\omega_1^2 = \frac{\omega_x^2}{2}\left[1 + \frac{\omega_\theta^2}{\omega_x^2} - \sqrt{\left(\frac{\omega_\theta^2}{\omega_x^2} - 1\right)^2 + 4\left(\frac{e_y^2}{r^2}\right)}\right] \tag{8}$$

$$\omega_2^2 = \frac{\omega_x^2}{2}\left[1 + \frac{\omega_\theta^2}{\omega_x^2} + \sqrt{\left(\frac{\omega_\theta^2}{\omega_x^2} - 1\right)^2 + 4\left(\frac{e_y^2}{r^2}\right)}\right] \tag{9}$$

The associated mode-shape matrix (normalized) is given by

$$[\Phi] = [\{\phi^1\} \quad \{\phi^2\}] \tag{10}$$

and

$$\{\phi^i\} = \begin{Bmatrix} \phi_x^i \\ \phi_\theta^i \end{Bmatrix} = \begin{Bmatrix} \dfrac{\dfrac{e_y}{r}}{\sqrt{\left(\dfrac{e_y}{r}\right)^2 + \left(1 - \dfrac{\omega_i^2}{\omega_x^2}\right)^2}} \\ \dfrac{\left(1 - \dfrac{\omega_i^2}{\omega_x^2}\right)}{\sqrt{\left(\dfrac{e_y}{r}\right)^2 + \left(1 - \dfrac{\omega_i^2}{\omega_x^2}\right)^2}} \end{Bmatrix} \qquad i = 1, 2 \tag{11}$$

Thus, the modal participation factors, L_I, simplify to

$$L_i = \frac{\dfrac{e_y}{r}}{\sqrt{\left(\dfrac{e_y}{r}\right)^2 + \left(1 - \dfrac{\omega_i^2}{\omega_x^2}\right)^2}} \qquad i = 1, 2 \tag{12}$$

The displacement response of the system is expressed as

$$u_y(t) = -\sum_{i=1}^{2} \phi_x^i L_i \int_0^t \frac{1}{\omega_{di}} e^{-\xi_i\omega_i(t-\tau)} \sin\omega_{di}(t-\tau)\,\ddot{u}_{gx}(\tau)\,d\tau \tag{13}$$

$$u_\theta(t) = -\sum_{i=1}^{2} \phi_\theta^i L_i \int_0^t \frac{1}{\omega_{di}} e^{-\xi_i \omega_i (t-\tau)} \sin \omega_{di}(t-\tau) \ddot{u}_{gx}(\tau) d\tau \tag{14}$$

in which $\omega_{di} = \omega_i \sqrt{1-\xi_i^2}$.

The displacement of the corner bearing is given by

$$u_c(t) = u_x(t) \pm \frac{b}{2r} u_\theta(t) \tag{15}$$

For design purposes it is appropriate to characterize the expected ground motion by a response spectra. The lateral and torsional displacement of the system in the i^{th} mode of the system are given by

$$u_x^i(t) = \phi_x^i L_i S_D(\omega_i, \xi_i) \tag{16}$$

$$u_\theta^i(t) = \phi_\theta^i L_i S_D(\omega_i, \xi_i) \tag{17}$$

where $S_D(\omega_i,\xi_i)$ is the displacement spectrum of the ground excitation for frequency ω_i and damping ξ_i. Three rules are used to estimate the maximum value of the response quantity from the modal quantities: the absolute sum (AS), the square root of the sum of squares (SRSS), and the complete quadratic combination (CQC).

The *1997 UBC* provides following expression for the additional displacement of the bearing due to torsion:

$$D_{TM} = D_M \left(1 + y \frac{12e}{b^2 + d^2} \right) \tag{18}$$

where D_{TM} is the total maximum displacement of the bearing, D_M is the maximum displacement at the center of rigidity of the bearing, y is the distance of the corner bearing from the center of rigidity, e is the eccentricity of the system that includes the actual plus accidental eccentricity, and b, d are the lateral dimensions of the system.

NUMERICAL EXAMPLES

The system considered in the present study is characterized by the parameters ω_x, ξ (i.e., ξ_1=ξ_2=ξ), ω_θ/ω_x, e_y/r and b/d. The building on the isolation system is assumed to have a fundamental angular frequency of $\omega_x = \pi$ rad/sec. The eccentricity in the system is considered to be 5 % of the dimension b, as specified by the *1997 UBC*. This implies that the isolated building as designed is symmetric and that torsion arises only due to the accidental eccentricity in the system.

Three types of excitation in the x-direction of the system are considered: namely (i) the unit impulse [i.e., $\ddot{u}_{gx}(t) = \dot{u}_0 \delta(t)$; $\delta(.)$ = Dirac delta function, and $\dot{u}_0$ = 1 m/sec], (ii) the N00E component of the 1940 El Centro earthquake excitation; and (iii) the N00E component of 1994 Northridge earthquake (recorded at Sylmar). Two values of the b/d ratio, i.e., 1 and 3, are considered denoting a square and rectangular configuration for the isolated building.

The variation of the peak value of u_0, u_x, u_θ and u_c against the ratio ω_θ/ω_x for different combinations of excitation and b/d ratio with system damping, $\xi = 10\%$, are shown in Figures 2 through 7. The u_0 denotes the lateral displacement of the system when it is torsionally uncoupled (i.e., $e_y/r = 0$), but the other system parameters and excitation are the same as in the torsionally coupled system. Using the modal combination rule AS provides an over-conservative response, especially for the u_θ and u_c. On the other hand, the SRSS and CQC rules under predict the u_x and over predict the u_θ when the ratio ω_θ/ω_x is in the vicinity of unity; the CQC combination rule, however, appears to be relatively better. When ω_θ/ω_x is sufficiently far from the unity (for which the two natural frequencies of the system are well separated), the CQC and SRSS methods provide the same response for the system.

Figures 8 and 9 show the displacement of the corner bearing for different excitations and b/d ratios, for systems with 5% and 15% damping, respectively. The performance of the various combination rules is similar to that observed in the earlier examples. The CQC method predicts the response with relatively good accuracy highly damped system. Both the CQC and SRSS methods under-predict the displacement of the corner bearing for ω_θ/ω_x in the range of 0.5 to 1.

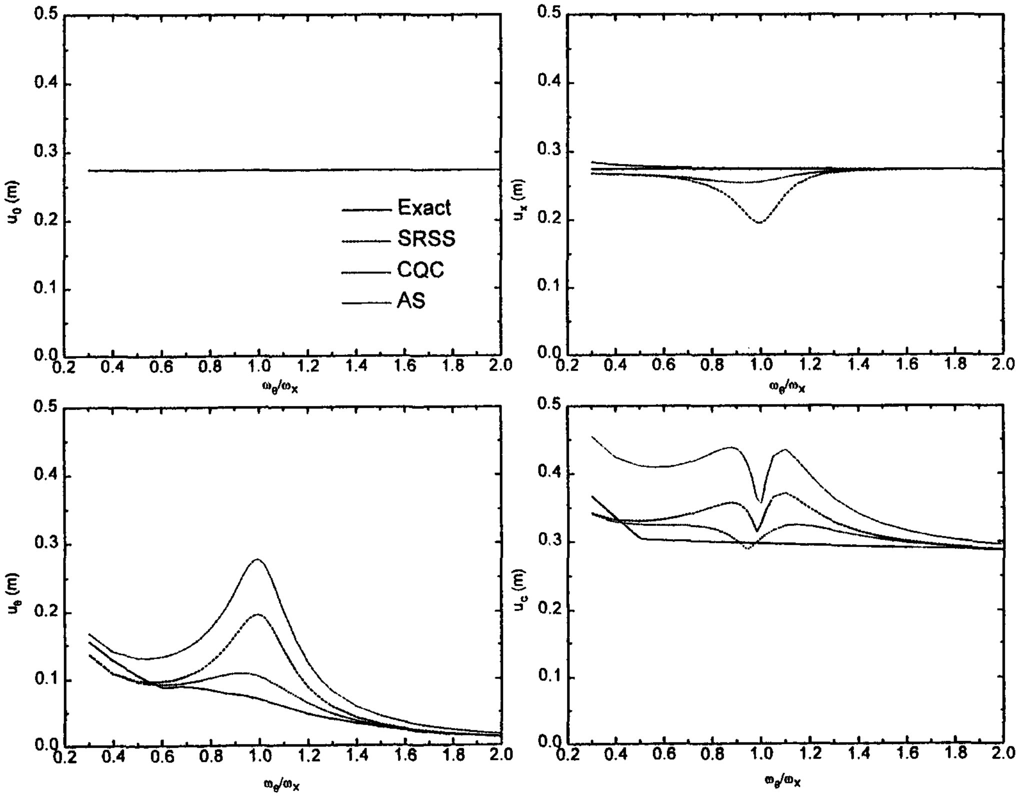

Figure 2. Displacement response of the system to unit impulse, with b/d=1 and ξ=10%.

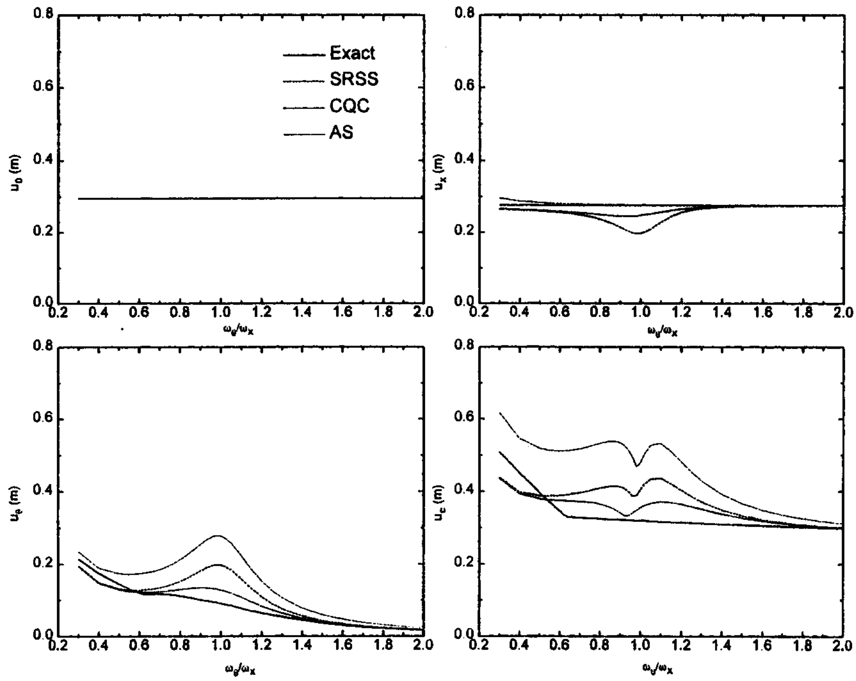

Figure 3. Displacement response of the system to unit impulse, with b/d=3 and ξ=10%.

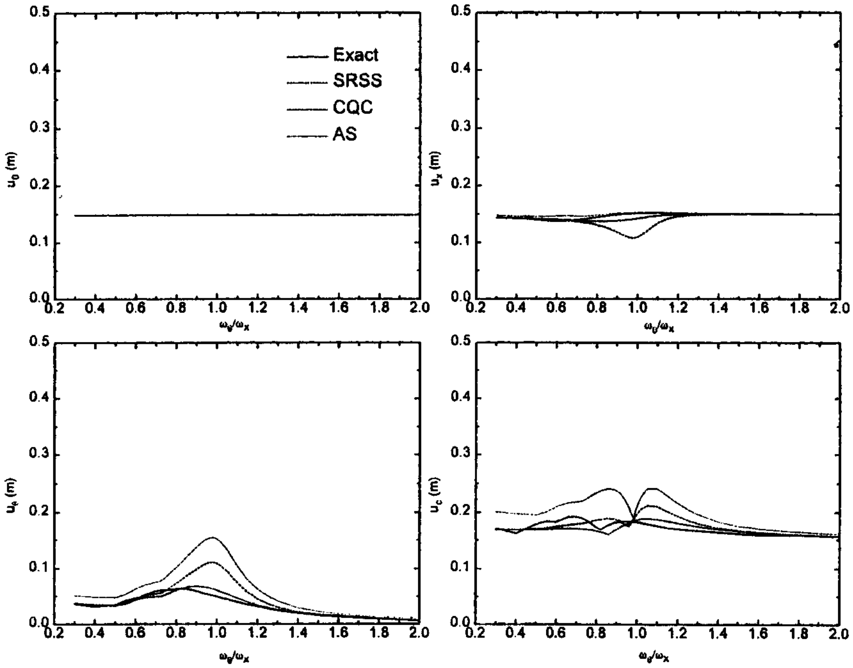

Figure 4. Displacement response of the system to El Centro earthquake, with b/d=1 and ξ=10%.

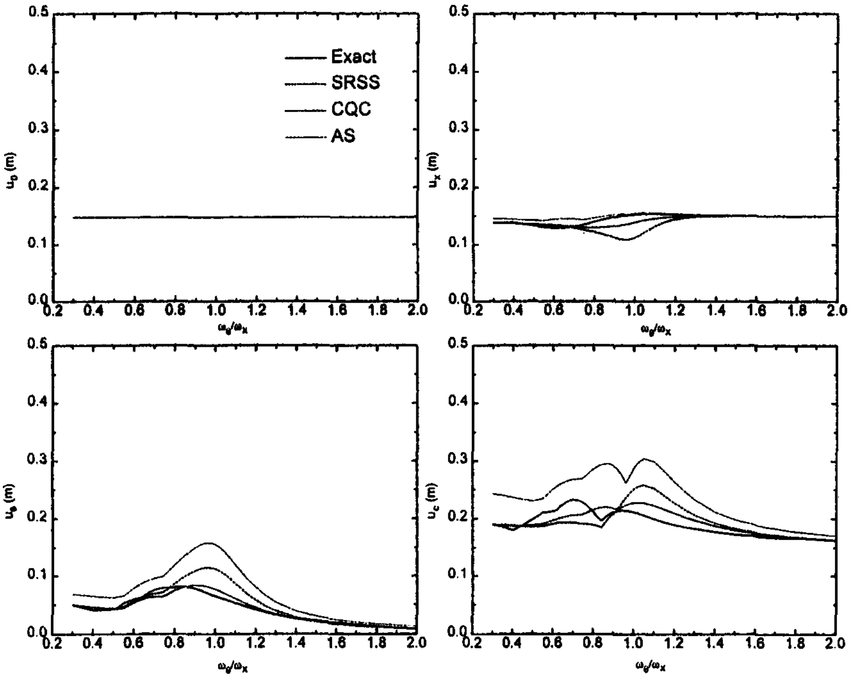

Figure 5. Displacement response of the system to El Centro earthquake, with *b*/*d*=3 and ξ=10%.

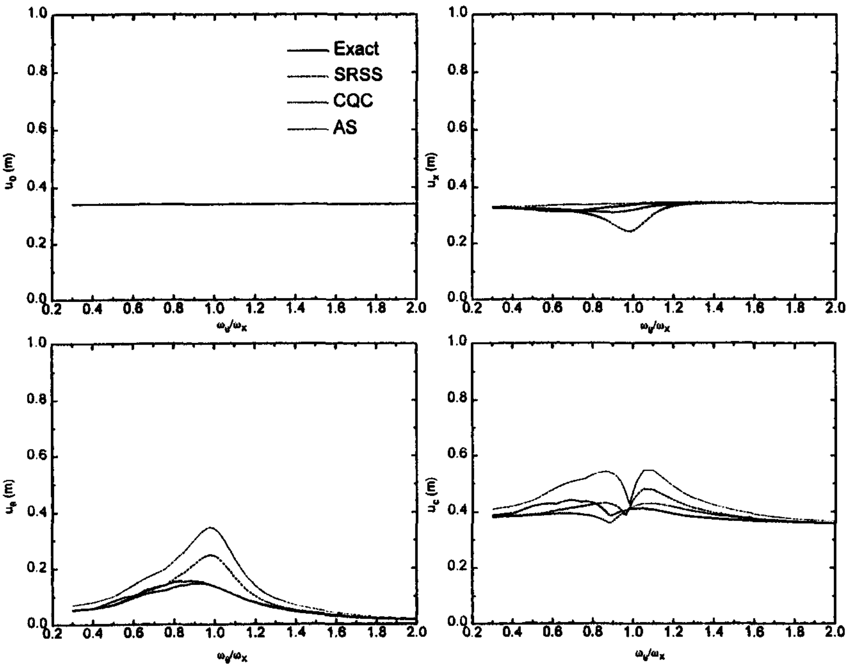

Figure 6. Displacement response of the system to the Northridge earthquake, with *b*/*d*=1 and ξ=10%.

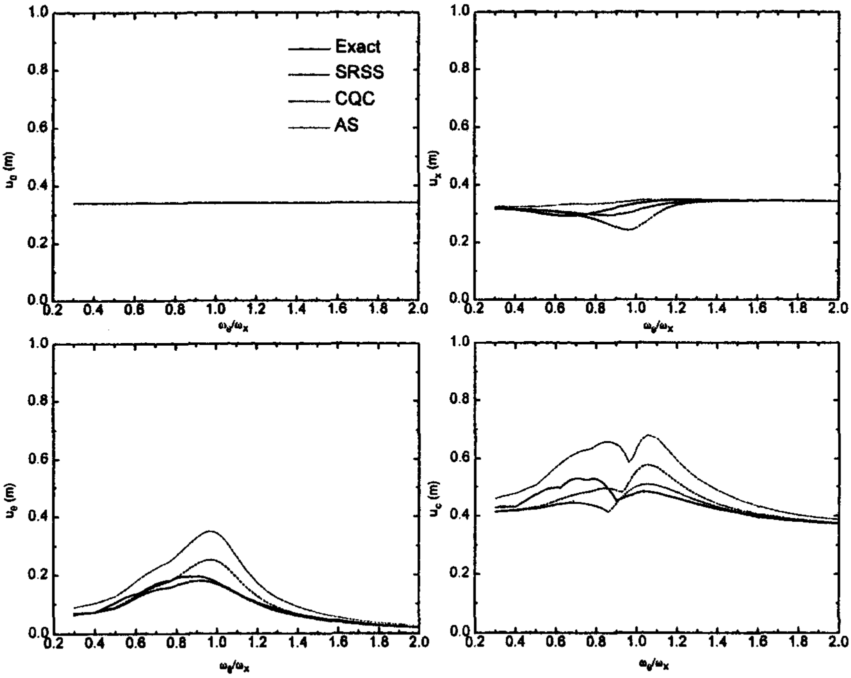

Figure 7. Displacement response of the system to Northridge earthquake, with b/d=3 and ξ=10%.

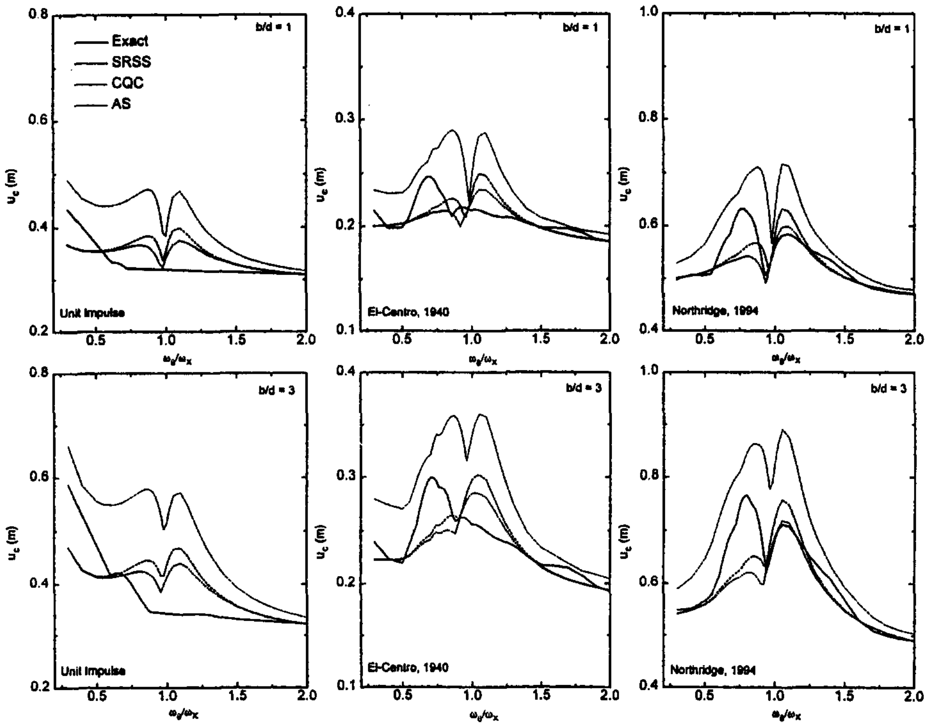

Figure 8. Displacement of corner bearing under different combinations of excitation and building configuration (ξ=5%).

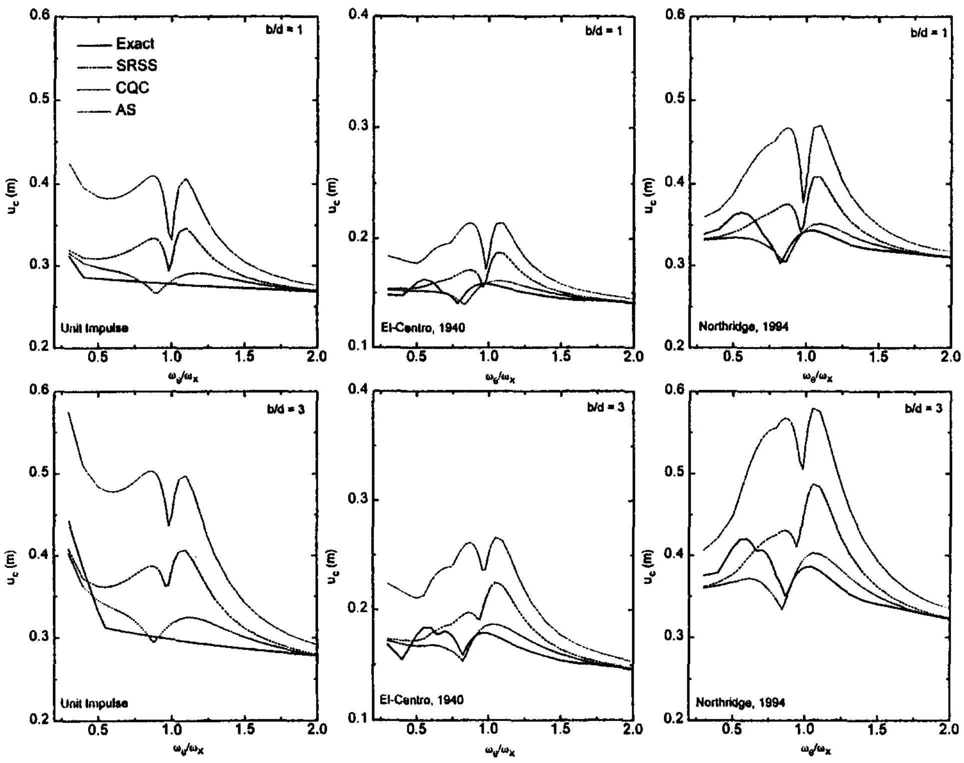

Figure 9. Displacement of corner bearing under different combinations of excitation and building configuration (ξ=15%).

A comparison of the displacement of the corner bearing under different excitations using the corresponding *1997 UBC* formula is shown in Figures 10, 11, and 12, for systems with 5, 10, and 15% damping, respectively. These figures indicate that the displacement of the corner bearing is under-predicted by the *UBC* expression for torsionally flexible systems with low damping. Further, it over-predicts the corner displacement for torsionally stiff systems, with moderate to high damping.

CONCLUSIONS

The following conclusions can be drawn from studying torsional displacement in base-isolated buildings subjected to horizontal component ground motion.

1. Using the modal combination rule CQC is more accurate than the SRSS rule, particularly for systems with high damping as the CQC method involves the damping coefficient.
2. The modal combination rule AS is always conservative and, in general, over-conservative.
3. The *1997 UBC* expression for the design displacement of the corner bearing under-predicts torsionally flexible systems with low damping.

The *1997 UBC* formula overpredicts for torsionally stiff systems with moderate to high damping.

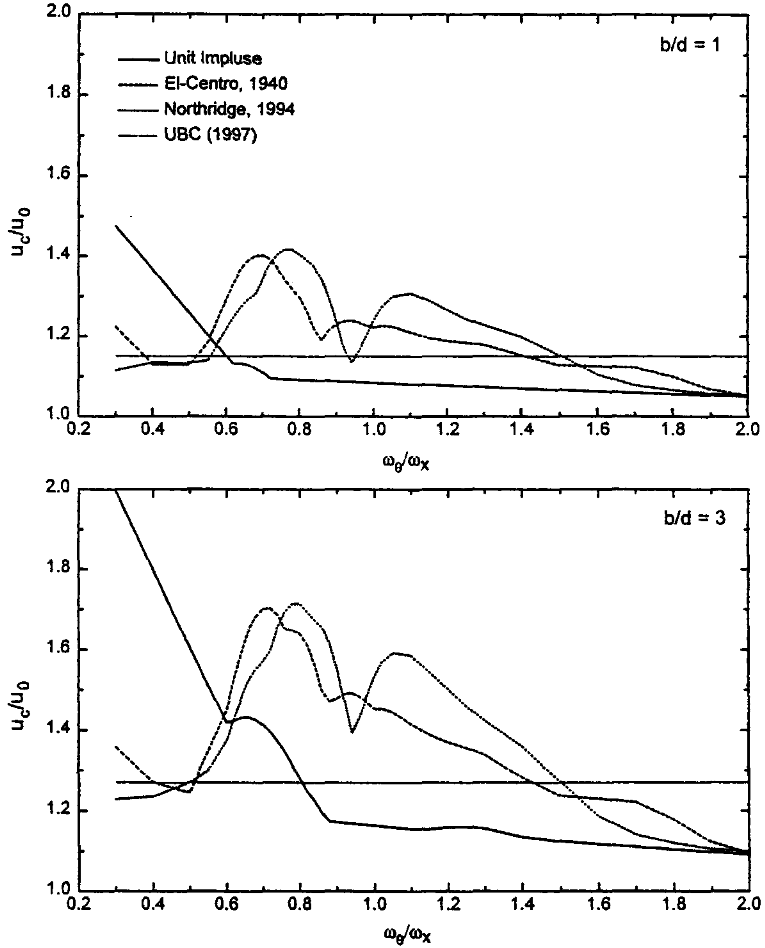

Figure 10. Comparison of normalized displacement of corner bearing with *1997 UBC* expression under different combinations of excitation and building configuration ($\xi = 5\%$).

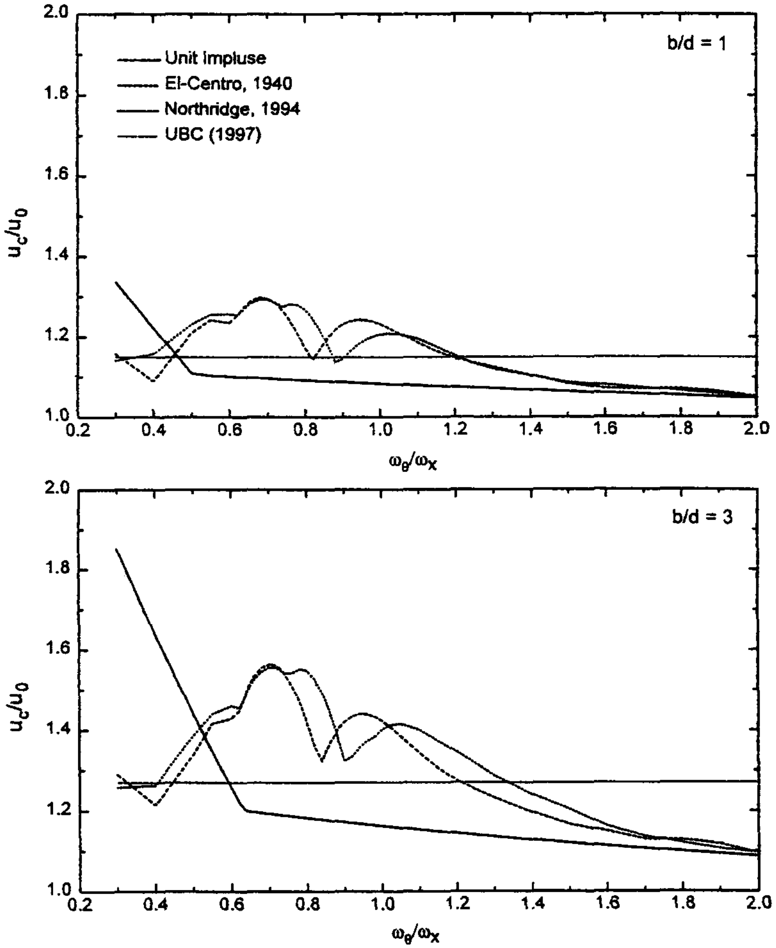

Figure 11. Comparison of normalized displacement of corner bearing with *1997 UBC* expression under different combinations of excitation and building configuration ($\xi = 10\%$).

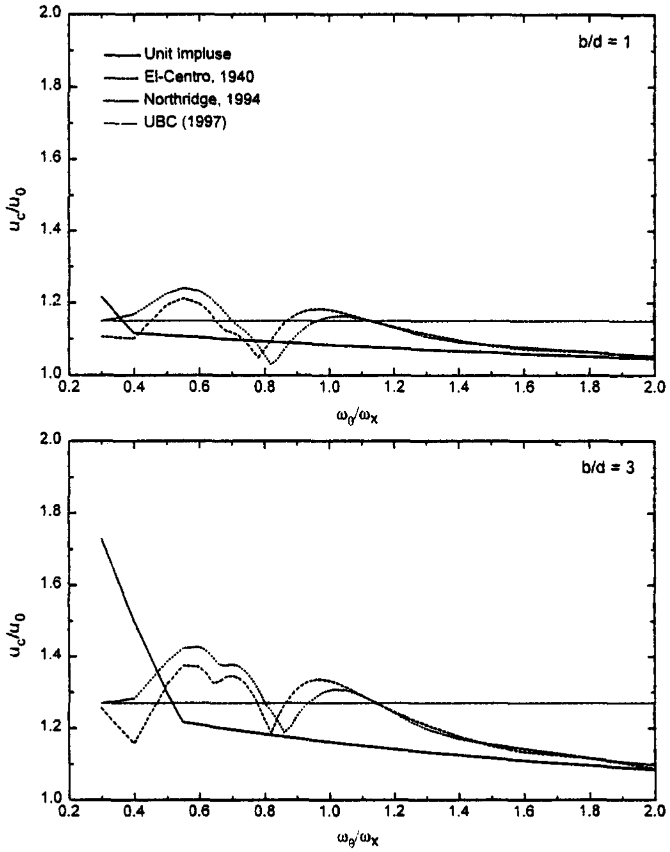

Figure 12. Comparison of normalized displacement of corner bearing with *1997 UBC* expression under different combinations of excitation and building configuration (ξ=15%).

GENERAL RECOMMENDATIONS

1. The layout of isolators should be such that the torsional frequency is greater 1.2 times the lateral frequency of the isolated system.
2. None of the combination rules work for systems that are torsionally flexible, and are not conservative for which the ratio of the uncoupled torsional frequency to the lateral frequency is in the range 0.5 to 1.

REFERENCES CITED

International Code Council (ICC), 1998, *International Building Code*, Final draft, Whittier, CA.
International Conference of Building Officials (ICBO), 1997, *Uniform Building Code*, Whittier, CA.