Inelastic seismic response of code designed single storey asymmetric structures

A. Rutenberg*, M. Eisenberger and G. Shohet

Division of Structures, Faculty of Civil Engineering, Technion - Israel Institute of Technology, Technion City, Haifa 32000, Israel

(Received April 1991, revised June 1991)

A parametric study of the inelastic earthquake response of single storey asymmetric structures is presented. The aim is to compare the behaviour of structures designed by the static seismic provisions of the ATC3 Code with those of the 1985 National Building Code of Canada (NBCC). Three different configurations of the resisting system were analysed for four earthquake time histories. A bilinear force-displacement relation was assumed. The study shows that the maximum ductility demand of structures designed by the NBCC is lower than those designed following ATC3. This is due to the higher total strength and the more efficient strength distribution among the resisting elements in NBCC designed structures.

Keywords: structures, earthquake engineering, inelastic response, asymmetric structures, seismic codes

Many asymmetric buildings are likely to be excited beyond the elastic limit under strong seismic action. Indeed, the damage statistics from the Mexico earthquake of September 1985 shows that close to 50% of the collapses can be directly or indirectly attributed to asymmetry[1]. Yet, most of the static code provisions governing the design and analysis of asymmetric buildings for earthquake loading were formulated mainly through calibration with results of linear dynamic elastic response analyses, usually by means of the response spectrum technique (e.g. Ref. 2). The rationale behind this approach is the belief that the peak ductility demand (*PDD*), i.e. the required maximum displacement ductility of all the resisting frames of code designed asymmetric structures will not be larger than that of similar but symmetric ones. Evidently, it is also assumed that the *PDD* expected in code designed symmetric structures can be accommodated, but this matter is beyond the scope of this paper. On the other hand, the tentative provisions of the Applied Technology Council (ATC3)[3] as well as the more recent NEHRP recommended provisions[4], ignore the linear dynamic effects of eccentricity – i.e. dynamic amplification of eccentricity due to modal coupling – on the premise that since post yield rather than elastic behaviour is to be controlled, there is no a priori reason to assume that better linear response will necessarily lead to lower levels of damage. However, accidental eccentricity effects are incorporated in the ATC and NEHRP formulation by retaining the 5% of width addition to the actual eccentricity.

There is now some numerical evidence to suggest that the peak ductility demand and maximum lateral displacements of asymmetric structures are larger than those of symmetric but otherwise similar ones[5–9]. The main difficulty with many of the available results is that they are based on asymmetric structures in which the distribution of the strength to stiffness ratio in the lateral load resisting members usually does not conform to designs based on seismic code provisions. In particular, some studies assume that all members have the same displacement at yield, i.e. strength is proportional to elastic stiffness, so that the plastic centroid (*PC*) of the system (the centroid of all member forces at yield) coincides with the centre of rigidity (*CR* in *Figure 1*). In many instances this distribution may not be realistic in the sense that code provisions allocate strength in a different manner. In these structures the plastic centroid is likely to be located closer to the mass centre (*CM*) due to the amplification of static eccentricity required by the codes. The shift in the *PC* location, coupled with the strengthening of members located at or near the flexible edge of the structure (*Figure 1*) can affect the nonlinear response appreciably. Indeed, earlier studies by the present writers have shown that structures which are not rotationally flexible have lower *PDD* when *PC* lies between *CM* and *CR*[9]. These structures, however, were not code designed.

The purpose of this paper is to investigate the inelastic response of some asymmetric single storey buildings designed using the ATC model seismic code[3] and the

*Present address: Department of Civil Engineering, McMaster University, 1280 Main Street W. Hamilton, Ontario LS8 4K1, Canada

0141-0296/92/02091-12

National Building Code of Canada[2] (NBCC) by comparing their response: *PDD* and displacements. These two codes are believed to represent the two different approaches to seismic design of asymmetric structures described earlier.

Several problems are encountered in the type of comparison that is attempted here. In the first place, the number of parameters involved in the nonlinear analysis of even the simplest single storey structures, such as those shown in *Figure 1*, is large. In other words, the elastic properties (eccentricity e, lateral and torsional rigidities) together with the plastic centroidal location are insufficient to describe the post yield response of systems comprising a small number of discrete elements arbitrarily located in the floor plan. Therefore, the models to be studied should have identical linear behaviour but different element locations and strength distributions.

Secondly, even for the same asymmetric structure, designs by different codes lead to different strength levels in the members, and so to different strength to stiffness ratios, as can easily be verified[9,17]. Generally, overall higher yield levels should result in lower *PDD*, so that designs by codes which consider dynamic (i.e. amplified) eccentricity will lead to lower *PDD* than those which do not, merely due to their increased total strength. Therefore, in order to separate the effects of total strength level from its distribution among the members, the base shears should be suitably adjusted (normalized).

Thirdly, the response is very much dependent on the type of excitation. Therefore, several earthquake time histories need to be considered; yet it is difficult to suggest a representative set. Nevertheless two categories of ground motion are often used: (i) those whose smoothed acceleration spectral shape have the standard code form (ii) those having high energy content at one or two narrow bands of low frequencies.

Fourthly, the shape of the force-displacement diagram should have some effect on the results. It is known that the bilinear relationship shown in *Figure 2* is not particularly realistic, mainly because stiffness and strength degradation and pinching due to crack closing are not modelled. More realistic modelling can be achieved by means of more sophisticated models such as those proposed by Clough[10] and Takeda[11]. Yet, studies[7] have shown that the ductility demands predicted by the

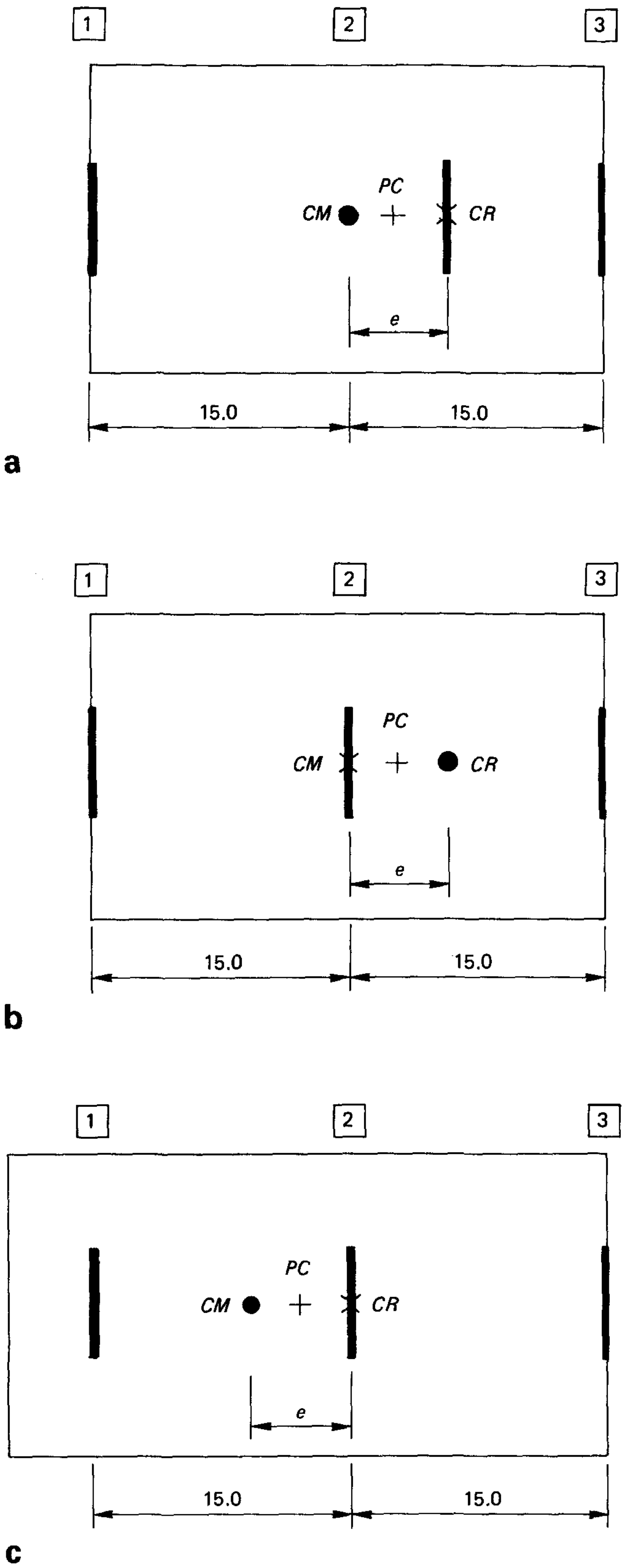

Figure 1 Plans of single storey structural models

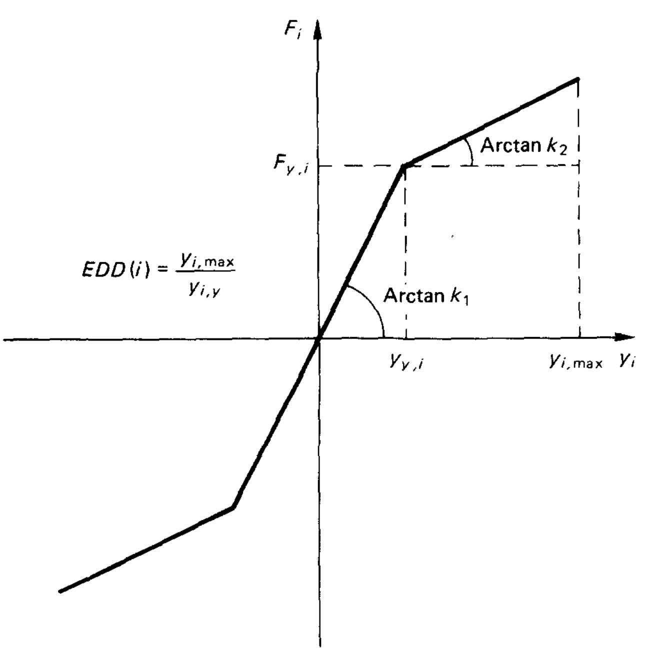

Figure 2 Force – displacement relationship

Clough degrading model are similar to those obtained from the bilinear model. Therefore, in view of its simplicity, the latter model has been used in the present study.

The previous discussion tacitly assumed that the strength to stiffness ratio within a given building frame is controlled by the designer. Although strength and stiffness are closely related, they are certainly not identical. This is more so in reinforced concrete structures where the reinforcement ratio affects the strength directly, but has only a minor effect on the initial stiffness.

Since the original report[12] that served as a basis for the present paper was written, several relevant studies have come to the attention of the writers. Bruneau and Mahin[13] studied the nonlinear response of asymmetric systems with and without initial (i.e. elastic) symmetry. However, because they used different key parameters it is difficult to compare their results with other studies. Gomez *et al.*[14] concluded, on the basis of a 3-element model (designated subsequently as the shifting model), that small *PC* eccentricity leads to large *PDD*. This appears to be inconsistent with the findings of Rutenberg *et al.*[9] but may be due to the different models used. The investigations of Diaz–Molina[15] on code designed 3-element models (to be designated the *CM* model) had similar objectives and his findings are on the whole consistent with those reached herein. Sadek and Tso[16] advocated the use of *PC* eccentricity e_{pl} as a useful parameter. A very recent study by Tso and Ying[17] using 3-element models of the CM type designed by several code provisions also concluded that code designed systems have lower ductility demand than those having strength distribution proportional to stiffness. Finally, most recently, Chandler and Duan[18] discussed several contradictory conclusions in past studies regarding the effect of eccentricity on the inelastic response, and proposed that clearer model definitions be made to ease comparison of results.

The model and equations of motion

The floor plans of the monosymmetric single storey structures chosen for this study are shown in *Figure 1*. Several simplifying assumptions were made: (a) the mass is assumed to be uniformly distributed, with the centroid at *CM*; (b) the floor slab is rigid in its own plane; (c) the lateral load resisting system consists of three elements. In the basic design designated the *CR* model (*Figure 1a*) these elements are arranged so that the centre of rigidity of the system *CR* lies in the line of action of element 2. The distance from *CM* to *CR* is the eccentricity *e*. Changes in the torsional to lateral frequency ratio were effected in that model by redistributing the stiffness among the elements. Two other models are also considered. The second, shown in *Figure 1b* denoted the *CM* model differs from the *CR* model, in that the middle element is located at *CM* rather than at *CR*. For given lateral and torsional rigidities this leads to a different stiffness distribution, which in turn leads to a different strength distribution. These two models are sometimes denoted stiffness eccentric. The model in *Figure 1c* designated as the shifting model (also denoted mass eccentric) is perhaps the simplest. In this model elements 1 and 3, which have the same stiffness, are located at equal distances from and on opposite sides of element 2. The eccentricity *e* is obtained by shifting the three elements in unison away from the mass centre, and changes in the frequency ratio are effected by changing the relative stiffnesses of the centre and edge elements.

The force-displacement response model is bilinear as shown in *Figure 2*, with a secondary response ratio of 2% of the primary. The yield levels of the elements were computed using the relevant formulae in the two codes namely: ATC3 and NBCC. As noted, code designs lead to different strength to stiffness ratios for the three elements, and bring the plastic centroid close to *CM*. The code formulae used to compute the design eccentricity e_d are as follows (*Figure 1*)

$$e_d = e \pm 0.05b \tag{1}$$

for the ATC3 code, and

$$e_d = 1.5e + 0.1b \tag{2}$$

$$e_d = 0.5e - 0.1b \tag{3}$$

for the NBCC. The plus signs in equations (1) and (2) are to be used for elements on the flexible side of *CR* (left hand side, in *Figure 1*) and the minus sign in equations (1) and (3) are applicable to elements on the rigid side of *CR* (right). The relative strength of an element (with strength = F_{yi} and lateral stiffness = K_i) was computed from the expression

$$F_{yi}^* = K_k^*\left[1 \pm e_d \frac{a_i}{r_k^2}\right] \tag{4}$$

in which

$$F_{yi}^* = \frac{F_y}{\Sigma F_y}$$

$$K_i^* = \frac{K_i}{\Sigma K}$$

$$r_k^2 = \frac{\Sigma k_i a_i^2}{\Sigma K} \tag{5}$$

and with a_i = distance of element from *CR*, ΣF_y = total yield strength of system and ΣK = total lateral stiffness of system. The resulting stiffness and strength distributions as well as e_{pl} (e_{pl} = eccentricity of *PC* from *CM*) for the three models with $e = 0.5\,\rho$ and floor width $b = 3\,\rho$ (ρ = mass radius of gyration with respect to *CM*) are given in Table 1.

As can be seen from Table 1 the three models present a wide range of strength and stiffness distributions. The differences among the models are quite apparent: the *CM* model allocates more strength to members on the rigid side of the deck (element 3) than the other two models; the shifting model allocates the least and locates element 3 at a larger distance from *CR*. It is also apparent that equation (4) allocates more strength to element 1 relative to its stiffness and less to the other elements. It will be recalled that in some earlier studies[5] equal displacements at yield were assumed for

Table 1 Stiffness and strength properties of models studied

		CR model				
		Element				
Ω		Flexible side	Central	Rigid side	ΣF_y	e_{pl}/ρ
0.8	K_i^*	0.1067	0.6800	0.2133	–	
	F_{yi}^*	0.2939	0.6182	0.0879	1.100	0.0
	F_{yi}^*	0.3341	0.4976	0.1683	1.367	0.0
1.0	k_i^*	0.1667	0.5000	0.3333	–	
	F_{yi}^* ATC	0.3485	0.4545	0.1970	1.100	0.0
	F_{yi}^*	0.3780	0.3659	0.2561	1.367	0.0
1.25	k_i^*	0.2604	0.2188	0.5208	–	
	F_{yi}^* ATC	0.4337	0.1989	0.3674	1.100	0.0
	F_{yi}^* NBCC	0.4466	0.1601	0.3933	1.367	0.0

		CM model				
		Element				
Ω		Flexible side	Central	Rigid side	ΣF_y	e_{pl}/ρ
0.8	K_i^*	0.0311	0.6044	0.3644	–	
	F_{yi}^*	0.0806	0.7784	0.1410	1.171	0.09
	F_{yi}^*	0.0819	0.6765	0.2416	1.626	0.24
1.0	k_i^*	0.1111	0.4444	0.4444	–	
	F_{yi}^* ATC	0.2255	0.5196	0.2549	1.133	0.04
	F_{yi}^*	0.2313	0.4552	0.3134	1.489	0.12
1.25	k_i^*	0.2361	0.1944	0.5694	–	
	F_{yi}^* ATC	0.3899	0.2117	0.3984	1.109	0.00
	F_{yi}^* NBCC	0.3951	0.1854	0.4195	1.401	0.03

		Shifting model				
		Element				
Ω		Flexible side	Central	Rigid side	ΣF_y	e_{pl}/ρ
0.8	K_i^*	0.1422	0.7156	0.1422	–	
	F_{yi}^*	0.3263	0.6506	0.0232	1.100	0.045
	F_{yi}^*	0.3602	0.5236	0.1163	1.367	0.134
1.0	k_i^*	0.2222	0.5556	0.2222	–	
	F_{yi}^* ATC	0.3990	0.5051	0.0960	1.100	0.045
	F_{yi}^*	0.4187	0.4065	0.1748	1.367	0.134
1.25	k_i^*	0.3472	0.3056	0.3472	–	
	F_{yi}^* ATC	0.5126	0.2778	0.2096	1.100	0.045
	F_{yi}^* NBCC	0.5102	0.2236	0.2663	1.367	0.134

the three elements (or equal strength to stiffness ratios), resulting in the plastic centroid being coincident with *CR*, an assumption which is unlikely to be realized in practice. A similar difficulty arises with the alternative models described by Tso and Bozorgnia[7,8]. On the other hand, in the *CR* model *PC* coincides with *CM*, and in the other two models the eccentricity of *PC* is quite small (see Table 1). It is believed, however, that the particular features of each model are less important that the fact that albeit being elastically equivalent the three models represent a wide range of realistic strength and stiffness distributions.

Considering the differences between the two codes, the ATC designs lead to lower total yield strength (ΣF_y) than the NBCC for all the models and for all frequency ratios (particularly the *CM* model for $\Omega = 0.8$). Also, for low and moderate Ω, the ATC code allocates appreciably less strength to element 3 than the NBCC does.

It is important to realize that the total lateral strength of a symmetric system is usually equal to the seismic factor C_s (or seismic design coefficient, assuming a load multiplier of 1.0), which is given in seismic codes as a percentage of the total weight, and is a compilation of several effects. The numerical values of C_s are different in the two codes. However, to make the comparison meaningful they are assumed herein to be equal, namely

$$\left(\frac{\Sigma F_{y0}}{G}\right)_{ATC} = \left(\frac{\Sigma F_{y0}}{G}\right)_{NBCC} = C_s \tag{6}$$

in which ΣF_{y0} = total yield strength for $e = 0$, G = the total weight of the building. As noted earlier, for asymmetric systems the amplification of static eccentricity and the added accidental eccentricity lead to a total strength (ΣF_y) that is higher than $C_s \times G$, so that the NBCC leads to a higher total strength than the ATC3. Since higher strength evidently leads to lower ductility demands, it should be expected that use of the NBCC will lead to lower *PDD* compared with ATC3 based designs. To neutralize this effect the strengths computed by the two codes were factored to the strength level of the associated symmetric system (i.e. to ΣF_{y0}). This normalization ensures that for a given eccentricity and stiffness distribution the differences between the two designs manifest themselves only in the different allocation of strength among the three elements, leaving the total strength intact. These normalized strengths are given in Table 1 for the models and frequency ratios considered.

The equations of motion for the system in *Figure 1* with respect to *CR* are given by

$$\begin{bmatrix} m & me \\ me & m(e^2+\rho^2) \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta} \end{Bmatrix} + [C] \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix} + \begin{Bmatrix} S \\ T \end{Bmatrix} = \begin{Bmatrix} m \\ me \end{Bmatrix} \ddot{u}_g = 0 \tag{7}$$

in which m = total mass of deck, C = proportional damping matrix (assumed to produce 2% damping in each of the two coupled modes), and S and T are the stiffness related resisting force and torque, respectively. The problems arising from the nonlinear nature of S and T stemming from the form of the response curves in *Figure 2* were analysed by means of the program DRAIN-2D, which solved the equations of motion. A time step of 0.005 s was chosen, which is less than 1/40 of the smallest uncoupled natural period of the elastic systems. Note that applying a plane frame program to the analysis of asymmetric multistorey structures of the type shown in *Figure 1* requires that the response be shear dependent[19].

Parametric study

The different models described in the preceding section, as well as the associated symmetric systems were subjected to the following acceleration time histories: El Centro 1940, Taft 1952, Bucharest 1977 (first 15 s), and Mexico SCT 1985 (from 20th s to 70th s). The El Centro and Taft records are very irregular and their acceleration spectra have large ordinates at the lower period range, i.e. they represent the standard code response. The Bucharest record has a single strong pulse and only two pronounced peaks in its acceleration spectrum, both at high natural periods. The Mexico record has 30 s of strong shaking with nine cycle reversals exceeding 100 gals, resulting in very large spectral accelerations at a very high natural period (circa 2.5 s). It is believed that these four records represent a sufficiently wide range of time histories. An eccentricity ratio $e^* = e/\rho = 0.5$, was assumed. This ratio appears to be typical, considering that for rectangular buildings loaded perpendicular to the longer dimension b (*Figure 1*), $b = 2.5\ \rho$ to $3.5\ \rho$ i.e. $e = 0.5\ \rho$ corresponds to eccentricities between 14 and 20% of b. The other parameters considered, were; natural lateral period: $T_0 = 0.25, 0.50, 0.75, 1.00$ s and torsional to lateral frequency ration $\Omega = 0.8, 1.0, 1.25$ ($\Omega^2 = r_k^2/\rho^2$). The main response parameters were the peak ductility demand *PDD*, i.e. the largest of the maximum ductility ratios of the three frames, and the peak lateral displacements. These two responses, which are of especial interest in design, were computed for several levels of the seismic factor of the building, namely the design base shear to total weight ratio.

Results

Peak ductility demand (PDD)

The ductility demand of the system is discussed first. This represents the maximum inelastic horizontal displacements sustained by the element during the earthquake relative to its displacement at yield *EDD* ($EDD = y_{i,max}/y_{i,y}$ in *Figure 2*). The need to define the peak ductility demand *PDD* arises from the fact that there are three elements in the system, and *PDD* is the largest *EDD* of the three. Because element 1 usually has the largest displacements in the linear and the nonlinear ranges, one expects that it would also have the largest ductility demand. However, in code designed structures this usually is not the case. This fact has important implications, and will be referred to subsequently.

Typical plots of *PDD* vs total yield strength of the system are shown in *Figure 3*. Note that in these figures the abscissa gives the lateral force leading to first yield as a fraction of the total weight of the building G ($= mg$, g = acceleration of gravity). As noted, this ratio is the seismic factor C_s. To make the plot even more meaningful, the response modification (or strength reduction) factor R, in the sense of the ATC3 and NEHRP codes is also plotted against C_s. This factor can be interpreted as the assumed ductility supply of the system, as based on the ductility factor approach[20] and hence its importance. It is possible to define R as follows

$$R = \frac{S_a}{g}\frac{G}{\Sigma F_y} \tag{8}$$

in which S_a = spectral acceleration, i.e. R = ratio of seismic factor that would result if the structure behaved elastically (S_a/g) and the design seismic factor ($\Sigma F_y/G$).

The *PDD* plots are given for the El Centro, Taft, Bucharest and Mexico records. These are presented for the three structural models in *Figure 1*, with designs based on each of the two codes (ATC and NBCC), and also for the related symmetric structure. The uncoupled natural period for all these structures was $T_0 = 0.5$ s, the eccentricity ratio $e^* = 0.5$, and the frequency ratio $\Omega = 1.0$. Note that the response values for both the non-normalized and normalized strength distributions are given. Apart from the expected hyperbolic shape of the plots, it is evident that the NBCC designs (non-normalized) always lead to lower *PDD* than the ATC designs, irrespective of record and structural model. It is also seen that whereas the *PDD* for the NBCC designs is either lower (i.e. more conservative) or similar to that of the associated symmetric system, the ATC *PDD* is higher, and in some instances, appreciably higher, than for the symmetric case, a situation that code designs are apparently intended to prevent. The inadequacy of code designs to supply the ductility demands for the Bucharest and Mexico records at low $\Sigma F_y/G$ values is evident.

As already noted, the better performance of the NBCC designs is mainly due to the higher strength built into the stiff edge member (element 3), resulting from the reduction in design eccentricity through equations (3) and (4). Thus, the differences between the predictions diminish to a certain extent when the code designs are both normalized to the level of the related symmetric systems ΣF_{y0}. The usefulness of this normalization is in that it separates the effects of strength distribution among the element from those of variations in the total strength. Comparisons with the normalized ATC *PDD* show that the resulting total strength leads to *PDD*s that are higher than in symmetric systems, a phenomenon which has already been noted. However, for the El Centro and Taft records the NBCC designs usually still lead to lower *PDD*, although for the Bucharest and Mexico records the ATC normalized *PDD*s are in some cases somewhat lower than the NBCC ones. In the present context the picture does not change significantly when the variation of *PDD* with period T_0 of the two codes designs are compared in *Figure 4* for a constant value of the seismic factor ($C_s = 0.15$ or 0.20). Note, though, the usual fall in *PDD* with natural period for the two California records, which is also true for other values of C_s, and the more complex response to the Bucharest and Mexico records. Some of these special features have been documented[8,21]. In several cases (in *Figure 4*) the values of *PDD* for ATC designed shifting models with $\Omega = 0.8$, are higher than 15, and are not shown.

For the present discussion of the relative merits of the ATC code and the NBCC, an interesting difference between the two code responses was noted: at low frequency ratios, i.e. when the torsional rigidity becomes lower than the translational ($\Omega = 0.8$ in *Figure 4*) the normalized ATC design leads to higher *PDD* than the normalized NBCC, for El Centro and Taft as well as for the Bucharest record (which for systems with higher Ω gave similar values for the two codes). Regarding the Mexico record, the ATC *PDD*s are slightly lower than those of the NBCC when the excitation level is relatively low (higher C_s) but becomes higher with increasing excitation (low C_s). It appears that in these structures the

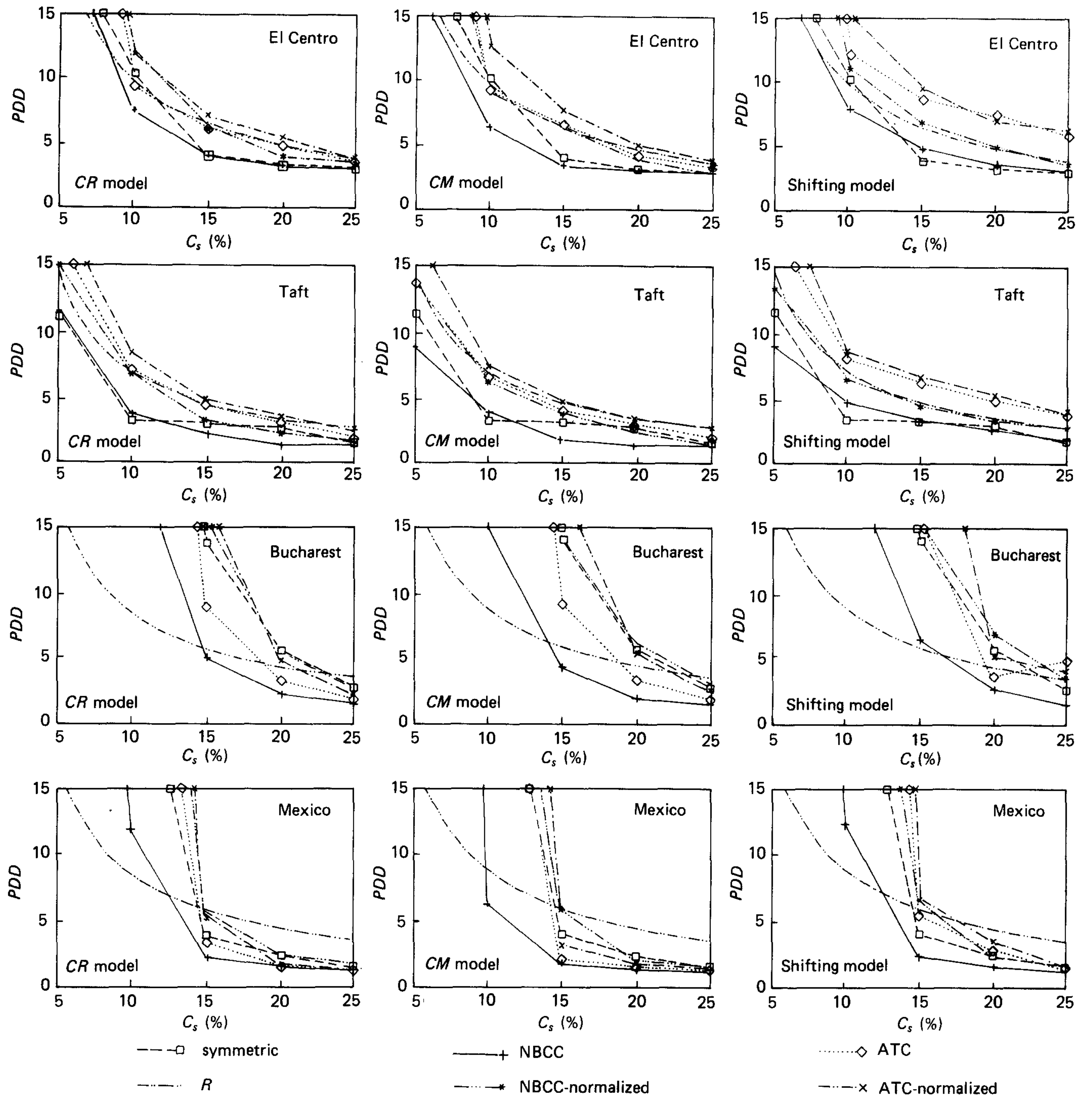

Figure 3 Peak ductility demand versus C_s, $e^* = 0.5$, $T_0 = 0.5$, $\Omega = 1.0$

higher strength built into the edge members (in this case element 3) by the NBCC is utilized. Recall however, that the non-normalized NBCC designs lead to lower *PDD* than the ATC ones for all the cases analysed.

On the whole, *Figure 4* shows that *PDD* is not insensitive to the frequency ratio Ω. This observation appears to be at variance with Bozorgnia and Tso[8] who found that the effect of Ω was marginal. The discrepancy may be explained when it is recalled that a different definition of Ω was used so that their range of $\Omega = 0.8 - 1.2$ translates into $\Omega = 0.89 - 1.34$ herein, which is to some extent torsionally stiffer, and that in their paper changes in Ω are effected through changes in the distance of the edge elements from the centre, so that the increased rotation due to lower torsional rigidity is offset by the reduced lateral displacement due to rotation of the element itself, which is now located closer to the centre of rotation. In the models used here these two effects are separated, i.e. the locations of the elements are not affected by changes in Ω, as is the case with the models used by Bozorgnia and Tso[8], and the beneficial effect of designing for higher torsional rigidity becomes apparent. Note also that larger Ω leads to larger edge member strength, as can be seen from equation (4). The correlation of *PDD* with *PC* eccentricity ratio e_{pl}/ρ did not appear to be strong for the cases studied, and since the results with

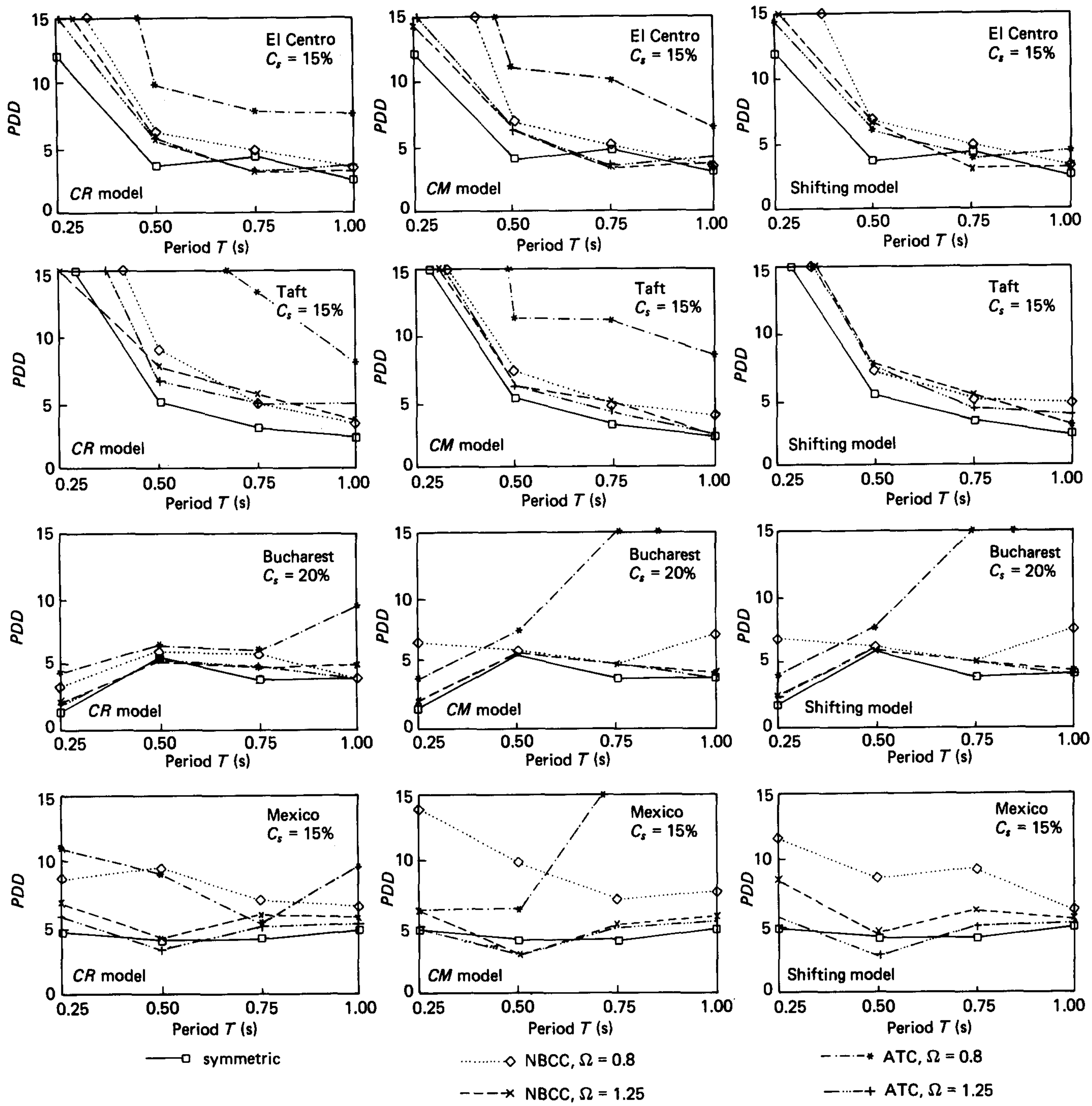

Figure 4 Peak ductility demand versus period, 'Normalized', $e^* = 0.5$, $\Omega = 0.8$ and 1.25

respect to this parameter were limited in number and range, this matter was not persued further in this investigation.

Regarding the R factor (assumed ductility supply) it is seen that R may be lower than *PDD* already at low values of *PDD* (3 to 4). This relation appears to depend strongly on the type of excitation, but it also appears that R should to a certain extent depend on eccentricity. However, full consideration of this important problem is beyond the scope of this paper.

Displacements

Upper bounds on the seismic lateral displacement and interstory drift are usually imposed by the codes in order to limit nonstructural damage, and to guard against excessive $P - \delta$ effects. Typical (non-normalized) maximum displacements (y_{max}) versus C_s of systems designed by each of the two codes for the El Centro record are plotted in *Figure 5*. Several features of the response become immediately apparent

(1) y_{max} is usually higher in asymmetric buildings than in symmetric ones – an expected result
(2) y_{max} of high period systems ($T_0 = 1.0$ s) is not appreciably affected by the yield strength level or the seismic factor (with some exceptions when the

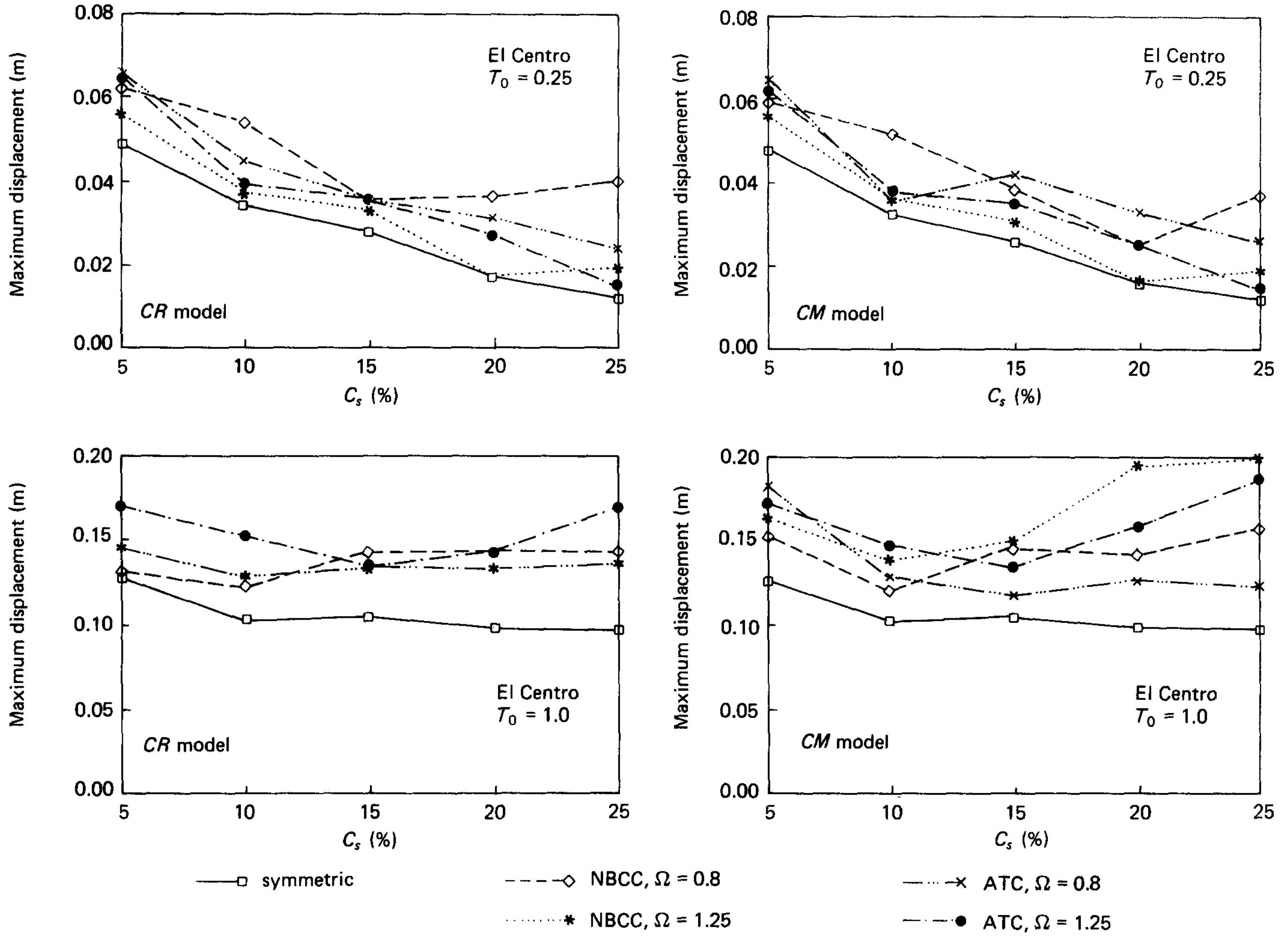

Figure 5 Maximum displacement versus C_s, El Centro, $e^* = 0.5$, $\Omega = 0.8$ and 1.25

strength level is very low), but y_{max} usually falls with increasing strength for low period systems ($T_0 = 0.25$ s), a phenomenon which is known in symmetric systems

(3) The differences in the response of the two code designs are not significant. This observation also holds for the *CM* and shifting models (not shown). The frequency ratio does not appear to affect y_{max} of the higher period systems systematically, although the effect on the $T_0 = 0.25$ s systems appears to be in the expected direction: falling displacements with increasing frequency ratio

The displacement response to the Bucharest record is given in *Figure 6*. It differs from that to the El Centro record in several respects. First, as expected, the absolute level of the displacements is much higher, and second, the response of the low period structures falls sharply with increasing strength. For $T_0 = 1.0$ s the trend is less definite, particularly for the NBCC designs with $\Omega = 0.8$. This is expected since the high energy of the input is around $T = 1.50$ s, so that systems with $T_0 = 1.0$ s can still be considered as low period ones provided Ω is not small. It appears that the sharp fall in displacements with period occurs when T_0 is below the energetic period range of the earthquake. For the El Centro record yielding of the low period structures ($T_0 = 0.25$ s) takes them away from this range. For Bucharest, yielding may take them into this range, particularly with the $\Omega = 0.8$ systems. Indeed, the maximum coupled periods of structures with $\Omega = 0.8$ and $\Omega = 1.0$ are no longer small compared with $T = 1.5$ s. Regarding the difference between the two codes, it appears that the ATC predictions are somewhat higher than those of the NBCC for $T_0 = 0.25$, but are lower for $T_0 = 1.0$ when $\Omega = 0.8$, but on the whole the two are similar.

The effect of the period T_0 on the maximum displacements is shown in *Figure 7* for the El Centro and Bucharest records using the *CR* model. It is seen that the displacements vary approximately linearly with period. However, since the graphs are based on four points only, some local peaks may have been overlooked.

Again, these graphs show perhaps a slight advantage for the NBCC over the ATC3 in limiting y_{max}, but one should not exaggerate its design implications.

Element behaviour

It has already been noted that the ductility demands of

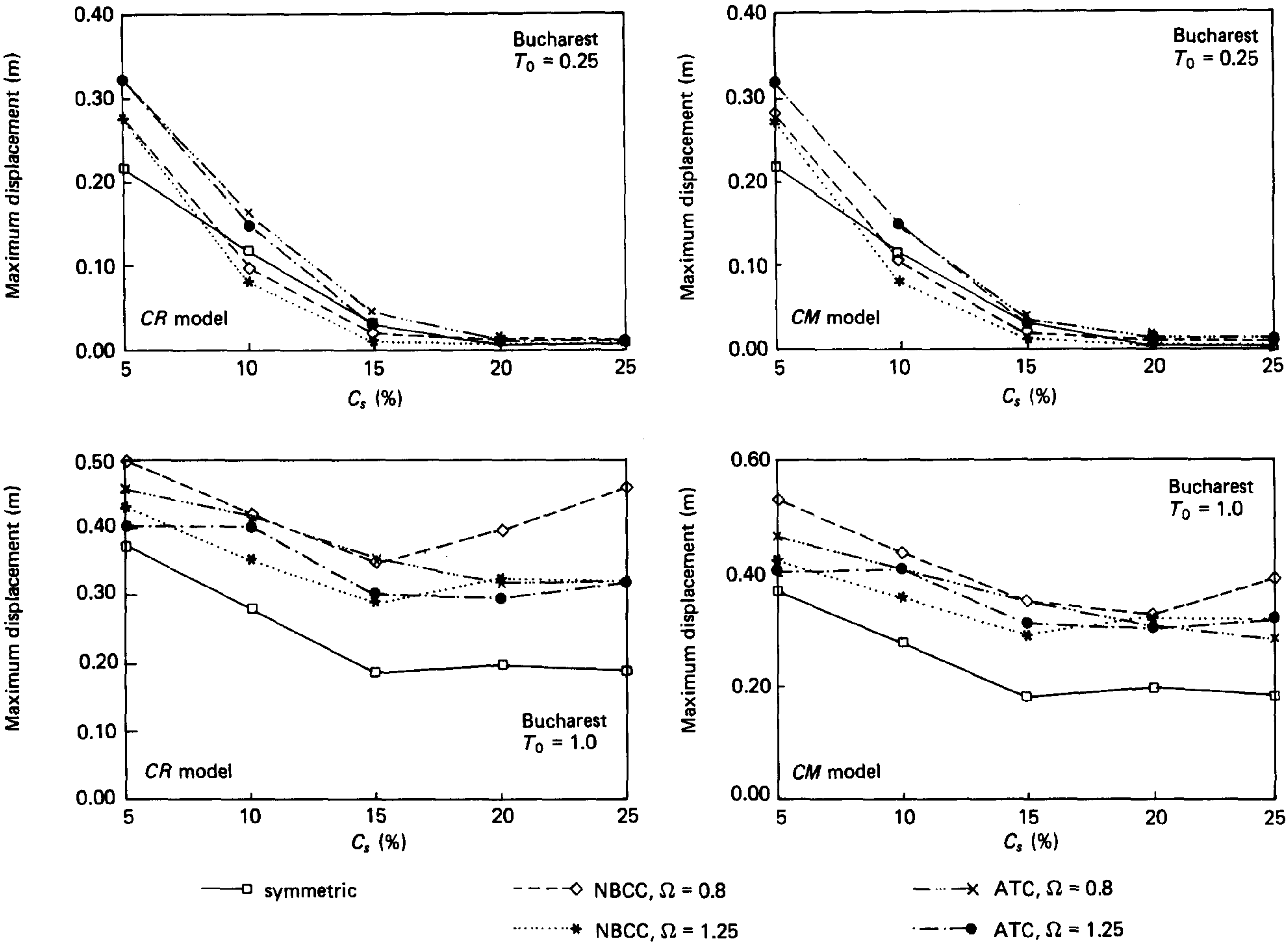

Figure 6 Maximum displacement versus C_s, Bucharest, e^* = 0.5, Ω = 0.8 and 1.25

the three elements are usually different, and so are their maximum displacements. Typical element ductility demand (*EDD*) against C_s curves are shown in *Figure 8*. Similar relationships for maximum element displacements are shown in *Figure 9*. Both are given for the El Centro record using the *CR* model. It is seen from these two figures that the ductility demand of element 1, which always has the largest displacements, is generally

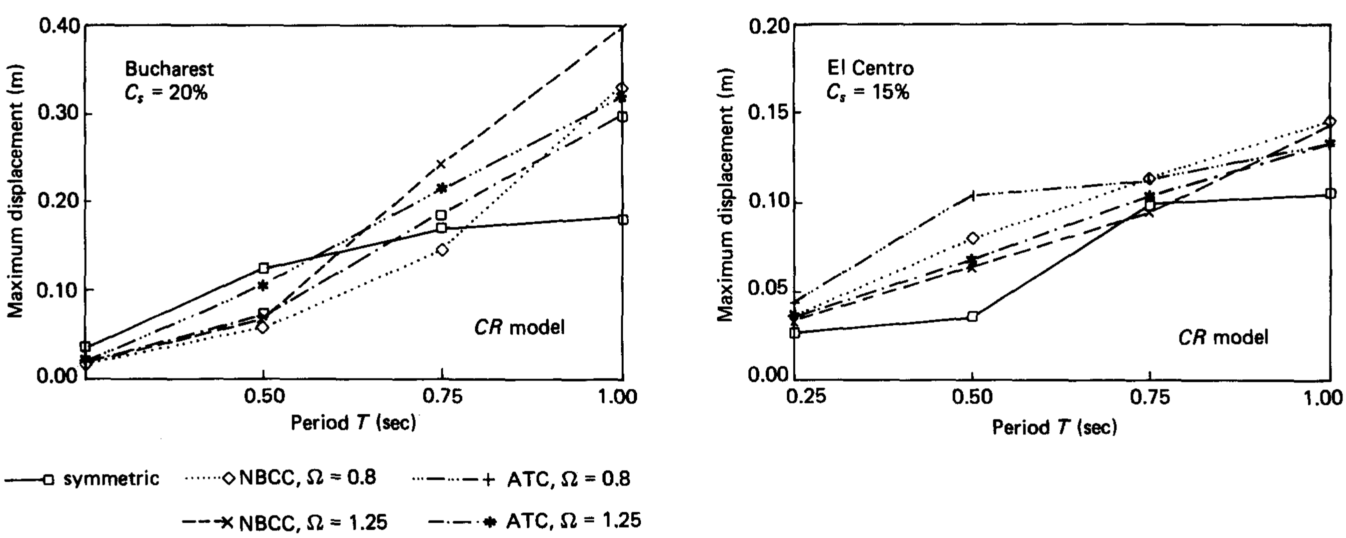

Figure 7 Maximum displacement versus period, e^* = 0.5, Ω = 0.8 and 1.25

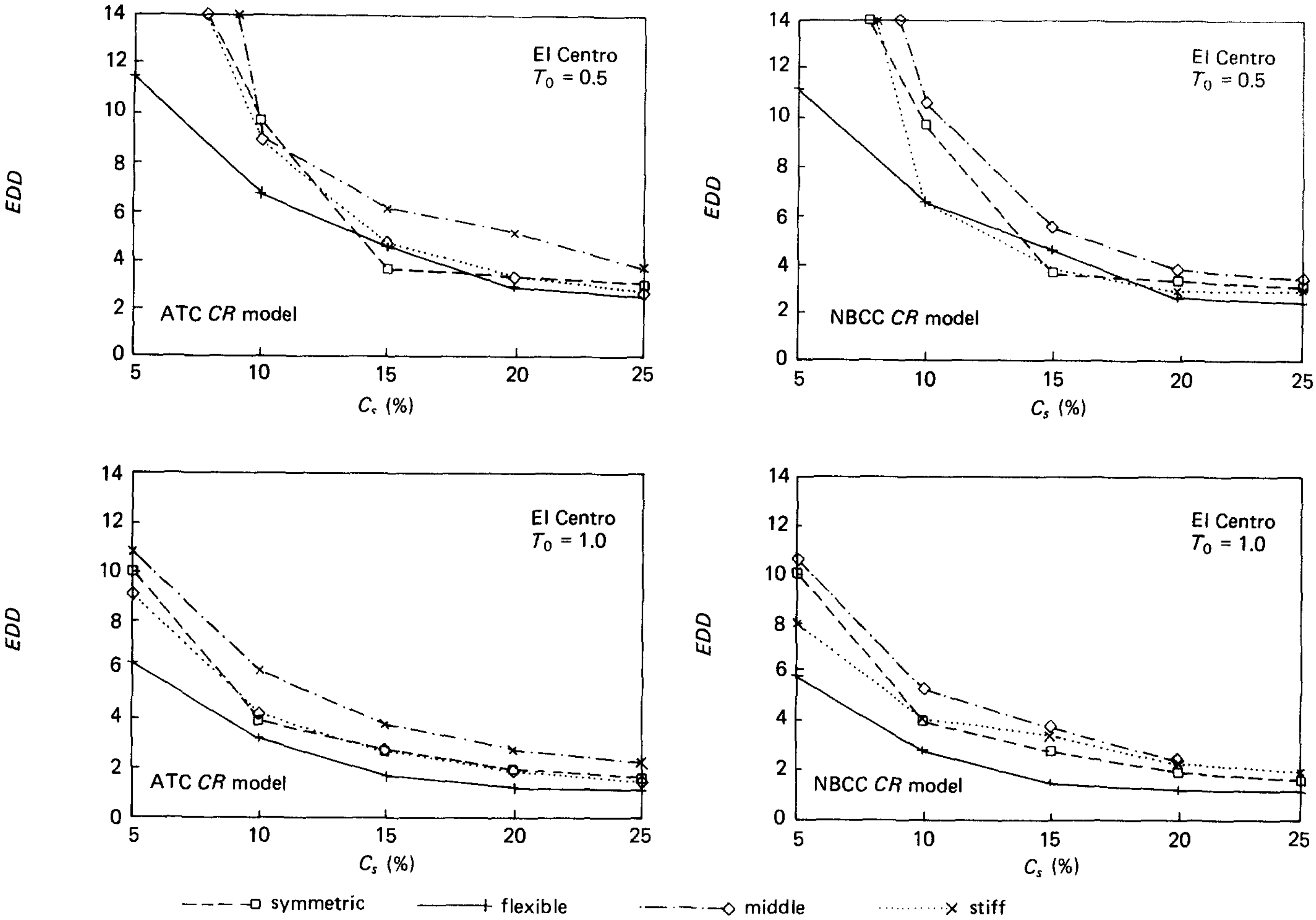

Figure 8 Element ductility demand versus C_s, El Centro, $e^* = 0.5$, $\Omega = 1.0$

lower than the *EDD* of elements 2 and 3. It is also seen that in most cases the ATC designs result in element 3 having the largest ductility demand. The NBCC designs result either in element 2 or element 3 having the *PDD*. These results appear to be consistent with those of Ref. 15. The difference in the results is mainly due to the different allocation by the two codes of relative strength among the three elements, as can be seen in Table 1.

The fact that the maximum displacement and *PDD* do not take place in the same element has some design implications. It appears that a lower *PDD* can be achieved through lowering the relative strength requirements of the elements on the flexible side. Evidently, a different strength distribution among the elements can be obtained by modifying the design eccentricities given in equations (1–3). Studies leading to alternative formulae that are better correlated with the results of the present study are now under way.

Conclusions

A parameter study was carried out on the seismic ductility demand and the maximum horizontal displacements of several monosymmetric (one axis of symmetry) three element systems excited by four ground motion time histories. The effects of yield strength level, and its distribution among the three elements, torsional to lateral frequency ratio and lateral period for a given stiffness eccentricity were investigated. On the basis of this study the following conclusions can be drawn

(1) The peak ductility demand (*PDD*) of simple asymmetric systems designed by the static code provisions of the type given in equations (2) and (3), i.e. the NBCC (1985), which allocate more strength to edge members than predicted by static analysis, is on the whole lower than that of systems designed by the ATC 3-06 (1978) rules (equation (1)). However, the resulting larger total strength of the NBCC designs (Table 1) is only partly responsible for their lower ductility demand.

(2) When the total yield strengths of the two alternative designs are made equal (normalized), the NBCC designs still lead to lower or equal *PDD* in most of the cases examined, although differences between the two are somewhat narrowed. This suggests that the strength distribution resulting from the use of equations (2) and (3) is likely to lead to a more efficient strength distribution among the elements. The same conclusion was reached by Diaz–Molina[15].

(3) The *PDD* of asymmetric systems designed by the NBCC type provisions is usually lower than that of their symmetric counterparts. This is due to the total higher total strength built into the edge

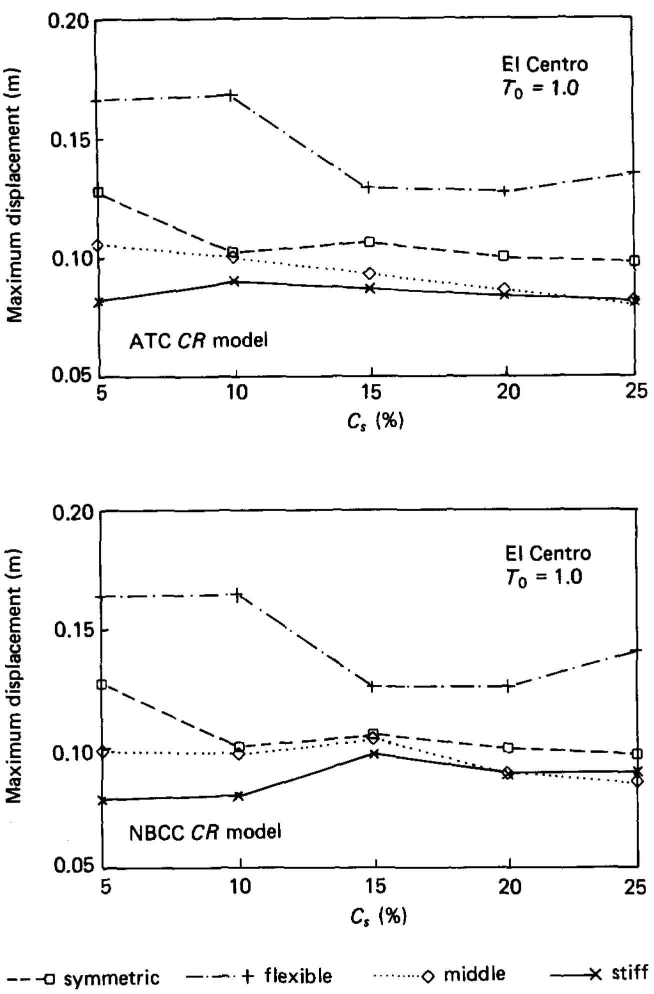

Figure 9 Element maximum displacement versus C_s, El Centro, $e^* = 0.5$, $\Omega = 1.0$

member through equations (2) and (3). However, this is not the case when total strength is normalized.

(4) The torsional to lateral frequency ratio Ω has some effect on the peak ductility demand. Larger Ω are likely to lead to lower *PDD*. The response to the California records for the periods studied of some rotationally flexible systems suggests that NBCC designs are likely to result in a lower *PDD* than ATC ones, even when the strength levels are made equal.

(5) For the two California records *PDD* usually falls with increasing natural period. For the Bucharest and Mexico records *PDD* may increase, particularly for systems with low frequency ratio designed by the ATC code.

(6) Maximum displacements (y_{max}) of the higher period asymmetric systems are not particularly sensitive to their yield strength level; but for low period systems ($T_0 = 0.25$ s) y_{max} usually falls with increasing strength, i.e. the behaviour is similar to that of symmetric structures.

(7) The maximum displacements predicted by the two code designs are on the whole similar, but for the Bucharest and Mexico records the NBCC response of the low period systems is lower than that of the ATC when y_{max} is large.

(8) The effect of frequency ratio Ω on displacements is not large, yet reduction with increasing Ω is to be expected.

(9) *PDD* and y_{max} do not necessarily occur in the same element. In fact, in the models chosen for this study, *PDD* was usually found in the element located at the rigid edge of the floor deck or in the central element (*Figure 1*), whereas the element located at the flexible edge displaced the most. This suggests that the use of current code formulae for strength distribution among elements does not lead to an optimal design for minimizing *PDD*. Since comparisons of code provisions with time history analyses in the linear range show under-estimation of the response of members located near the rigid edge of the floor, conformity with linear behaviour might prove to be beneficial also for the nonlinear range, as suggested[22]. Also, it is evident that code designs result in small plastic centroidal distance e_{pl}. Therefore, while it is recognized that *PC* location affects *PDD*, some additional parameter, or parameters, should be sought to provide guidance as to the more efficient strength allocation.

(10) The standard numerical values of the response modification factor *R* may be unconservative when applied to asymmetric systems.

The conclusions reached here are based on three different physical models of the lateral load resisting system in terms of strength and of stiffness distributions (*Figure 1*, and Table 1). It has been shown that although the results are model dependent, the behaviour patterns are quite similar. For this reason it is believed that the conclusions reached here are of some generality. Yet, similar studies of asymmetric systems in which force-displacement relations that consider strength and stiffness degradation are used should also be made. Finally, the applicability of response modification factor values given in modern codes to asymmetric structures should be explored further.

Acknowledgement

The authors wish to thank A. Benbenishti for his assistance with the computations.

References

1 Rosenblueth, E. and Meli, R. 'The 1985 earthquake: causes and effects in Mexico City', *Concrete International*, May 1986, 23–35
2 Associate Committee on the National Building Code, 'National Building Code of Canada 1985 and Supplement', National Research Council of Canada, Ottawa, 1985
3 Applied Technology Council, 'Tentative provisions for the development of seismic regulations for buildings', ATC 3-06, National Bureau of Standards, Washington D.C., 1978
4 Building Seismic Safety Council, 'NEHRP recommended provisions for the development of seismic regulations for new buildings', Earthquake Hazard Reduction Series 17, Federal Emergency Management Agency, FEMA 96, October 1988
5 Yamazaki, Y. 'Inelastic torsional response of structures subjected to earthquake ground motion', *Rep.* UBC/EERC 80/07, University of California, Berkeley, 1980
6 Tso, W. K. and Sadek, A. W. 'Inelastic seismic response of simple eccentric structures', *Earthquake Engng and Struct. Dyn.*, 1985, **13**, 255–269

7 Tso, W. K. and Bozorgnia, Y. 'Effective eccentricity for inelastic seismic response of buildings', *Earthquake Eng. Struct. Dyn.*, 1986, **14**, 413–427

8 Bozorgnia, Y. and Tso, W. K. 'Inelastic earthquake response of asymmetric structures', *J. Struct. Eng. ASCE*, 1986, **112**, (2), 383–400

9 Rutenberg, A., Eisenberger, M. and Shohet, G. 'Reducing ductility demand in asymmetric structures', *Proc. 8th European Conf. on Earthquake Eng.*, Lisbon, September 1986, Vol. 3, pp. 6.7/57–6.7/64

10 Clough, R. W. and Johnston, S. B. 'Effect of stiffness degradation on earthquake ductility requirements', *Proc. Japan Earthquake Eng Symp.*, Tokyo, 1966, 277–232

11 Takeda, T., Sozen, M. A. and Nielsen, 'Reinforced concrete response to simulated earthquakes', *J. Struct. Div. ASCE*, 1970, **96**(ST12), 2557–2573

12 Rutenberg, A., Shohet, G. and Eisenberger, M. 'Inelastic seismic response of code designed asymmetric structures', Publ. 303, Faculty of Civil Eng., Technion – Israel Inst. of Tech., Haifa, December 1989, (revised version of summer 1987 manuscript)

13 Bruneau, M. and Mahin, S. A. 'Inelastic seismic response of structures with mass or stiffness eccentricities in plan', *Rep.* UBC/EERC 87/12, University of California, Berkeley, 1987

14 Gomez, R., Ayala, G. and Jaramillo, J. D. 'Respuesta Sesmica de Edificios Asimetricos', Institute of Engineering UNAM, Mexico City, May 1987

15 Diaz–Molina I. 'Dynamic torsional behavior of inelastic systems', *MSc Diss.*, Department of Civil Engineering, Carnegie Mellon University, Pittsburgh, 1988

16 Sadek, A. W. and Tso, W. K. 'Strength eccentricity concept for inelastic analysis of asymmetrical structures', *Eng. Struct.*, 1989, **11**(3), 189–194

17 Tso, W. K. and Ying, H. 'Additional seismic inelastic deformation caused by structural asymmetry', *Earthquake Eng and Struct. Dyn.*, 1990, **19**, 243–258

18 Chandler, A. M. and Duan, X. N. 'Inelastic torsional behaviour of asymmetric buildings under severe earthquake shaking', *Struct. Eng. Rev.*, 1990, **2**, (3) 141–159

19 Rutenberg, A. and Eisenberger, M. 'Laterally loaded asymmetric shear buildings: planar analysis', in *Advances in Tall Buildings*, Van Nostrand Reinhold, New York, 1986

20 Clough, R. and Penzien, J. *Dynamics of structures*, McGraw-Hill, New York, 1975

21 Eisenberger, M. and Rutenberg, A. 'Seismic base isolation of asymmetric shear buildings', *Eng Struct.*, 1986, **8**, (1), 2–8

22 Rutenberg, A. and Pekau, O. A. 'Seismic code provisions for asymmetric structures: a reevaluation', *Eng. Struct.*, 1987, **9**, (4), 255–264