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SEISMIC RESPONSE OF AN ASYMMETRIC BASE ISOLATED STRUCTURE

R. S. Jangid

Department of Civil Engineering, Indian Institute of Technology, Bombay, Powai, Bombay—400076, India

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Abstract—The response of a one-storey model of a torsionally coupled (asymmetric) building with sliding support to two component random ground motions is obtained. The base of the system consists of sliding blocks resting on sliding foundation raft. The rigid-friction force-deformation behaviour of the sliding blocks is modelled as elasto-plastic with very high initial stiffness. The response of the system is analyzed under different parametric variations to investigate the effectiveness of the sliding support. The parameter includes: eccentricity of superstructure, uncoupled torsional to lateral frequency ratio, mass ratio and coefficient of friction of sliding support. The effectiveness is studied by obtaining the response with and without consideration of interaction between the two lateral frictional resistances of the sliding support on the criterion of sliding. It was shown that the effect of bi-direction interaction is significant and should be included for the effective design of a torsionally coupled system with sliding support. Copyright © 1996 Elsevier Science Ltd.

NOTATION

b	lateral dimension of the system in y-direction
$[C]$	damping matrix of superstructure
d	lateral dimension of the system in x-direction
e_x	eccentricity between CM of deck mass and the static centre of resistance of the columns
E_i	parameter defined in eqn (10b)
F_i	parameter defined in eqn (10c)
F_{iL}	limiting value of lateral force for ith sliding block
F_{ix}	lateral resistance of ith sliding block in the x-direction
F_{iy}	lateral resistance of ith sliding block in the y-direction
g	gravitational constant
G_i	parameter defined in eqn (10a)
k_{ixb}	initial high stiffness of the sliding block in x-direction
k_{iyb}	initial high stiffness of the sliding block in y-direction
K_x	lateral stiffness of the superstructure in x-direction
K_y	lateral stiffness of the superstructure in y-direction
K_θ	torsional stiffness of the superstructure
$[k_{it}]$	tangent stiffness matrix of ith sliding block
$[k_{ie}]$	stiffness matrix of the sliding block during non-sliding phase
$[k_{is}]$	modification to the stiffness due to sliding of ith block
$[K]$	stiffness matrix of the superstructure
$[K_b]$	stiffness matrix of the sliding support
m_i	normal mass lumped on ith sliding block
m	mass of the superstructure
m_b	base mass
$[M]$	mass matrix of the superstructure
$[M_b]$	lumped mass matrix corresponding to the DOF at the base mass
r	radius of gyration of deck mass about the vertical axis through CM
$\{r\}$	matrix of earthquake influence coefficient
R	response ratio
S_0	spectral density of input white-noise
u_x	displacement of the superstructure in the x-direction relative to base mass
u_y	displacement of the superstructure in the y-direction relative to base mass
u_θ	rotation of the superstructure relative to base mass
u_{bx}	displacement of base mass in the x-direction relative to the ground
u_{by}	displacement of base mass in the y-direction relative to the ground
$u_{b\theta}$	rotation of base mass relative to the ground
$\ddot{u}_{gx}$	ground acceleration in the x-direction
$\ddot{u}_{gy}$	ground acceleration in the y-direction
$\{u\}$	vector of displacements at the deck relative to the base mass
$\{u_b\}$	vector of the displacements at base mass
$\{\ddot{u}_g\}$	ground acceleration vector
w_x	uncoupled lateral frequency of the superstructure in the x-direction
w_y	uncoupled lateral frequency of the superstructure in the x-direction
w_g	predominant ground frequency
w_θ	uncoupled torsional frequency of the superstructure
μ	coefficient of friction
ϕ	interaction curve between lateral resistances of the sliding block
δt	time interval
ξ_g	damping constant of ground filter

INTRODUCTION

Base isolation is today an accepted design philosophy for earthquake resistant design of structural system and sensitive instruments. The main concept is to isolate the structure from the ground instead of the conventional techniques of strengthening the structural member. This new design methodology appears to have considerable potential in preventing earthquake damage of structures and their internal equipment. The devices which isolate the structure at its base have two important characteristics: horizontal flexibility and energy absorbing capacity. Flexibility as such reduces the natural frequency of the structure lower than the energy containing the frequencies of earthquake excitation. Energy absorbing capacity reduces both base displacement and seismic energy being transmitted to the building. In designing a base isolated structure the aims are: (1) to provide a relatively stiff superstructure that will behave like a rigid body with little inter-storey drift and (2) to concentrate most of the horizontal deformation to the flexible mounting that supports the structure. One of the most attractive devices is the laminated rubber bearing which offers various advantages, such as lower cost than other devices, and simplification of layout and high damping at a small level of amplitude. Among the different types of bearings, the laminated rubber bearing (LRB system) or the lead rubber bearing system has been used widely in New Zealand. Base isolation of buildings by adding frictional elements in the isolation device has been a topic of considerable interest in the recent past.

A significant amount of the recent research in base isolation has focused on the use of frictional elements to concentrate flexibility of the structural system and to add damping to the isolated structure. The advantages of a frictional type system over conventional rubber bearings are (i) the friction forces developed at the base are proportional to the mass supported by that bearing, there is no eccentricity between the centre of mass of the superstructure and the centre of stiffness. Therefore, if the mass distribution is different from that which is assumed in the original design, the effect of torsion at the base is diminished, (ii) the frictional isolator has no unique natural frequency and therefore, dissipates the seismic energy over a wide range of frequency input without the risk of resonance with the ground motion and (iii) a frictional type system ensures a maximum acceleration transmissibility equal to maximum frictional force. A variety of frictional type base isolation systems have been proposed, namely earthquake barrier system (Caspe and Reinhorn [1]), Alexisismon (Ikonomou [2]), the friction pendulum system (Zayas *et al.* [3]), Tasei (Nagashima *et al.* [4]), resilient-friction base isolator (R-FBI) (Mostaghel and Khodaverdian [5], Jangid and Datta [6]) and Wabo–Fyfe earthquake protection system [7]. These systems utilize a Teflon-steel interface to create sliding bearings with some amount of restoring force or recentring capability. The simplest sliding isolation system is, however, a pure-friction base without any recentring capacity. The most attractive feature of a frictional base isolation system is its effectiveness for a wide range of frequency input. Response of a sliding structure subjected to harmonic and real ground motion has been studied by Westermo and Udwadia [8], Mostaghel and Tanbakuchi [9] and Yang *et al.* [10]. Some applications of a sliding bearing for seismic isolations are reported by Mokha *et al.* [11].

Very few studies (Jangid and Datta [12, 13]; Nagarajaiah *et al.* [14]) dealt with the effect of torsional coupling for asymmetric building models on a sliding support. In this paper, response of a simple torsionally coupled system resting on a sliding support (with pure-friction) is obtained under random ground motion acting in two horizontal directions and analyzed for a parametric study. The response analysis considers the effect of biaxial interaction on the criterion of sliding. Also, the behaviour of the system is analyzed to study the effectiveness of the base isolation of a torsionally coupled system resting on the sliding support. Although, pure-friction bearings without having any force are nearly impossible to find any practical implementation due to lack of recentring capability, this study is expected to provide considerable information on the effectiveness of a frictional base isolated torsionally coupled system.

STRUCTURAL MODEL AND EXCITATION

Figure 1 shows the structural system considered, which is an idealized one-storey building model mounted on a sliding support which consists of an array of discrete blocks resting on foundation raft as shown in Fig. 1b. The stiffness distribution of the columns and bearings are symmetric about the x-axis but not about the y-axis, as a result, the system displays a torsional effect when excited in the lateral y-direction. The degrees-of-freedom for the system are shown in Fig. 1. The force–deformation behaviour of the sliding block is assumed as rigid–plastic which is implemented in the analysis by assuming elasto-plastic behaviour with very high initial stiffness. The limiting value of lateral force (in any direction) for the ith sliding block is given by

$$F_{iL} = \mu m_i g, \tag{1}$$

where, F_{iL} is the limiting value of sliding force; μ is coefficient of friction which is assumed constant irrespective of sliding direction and velocity, g is the gravitational constant; and m_i is the normal mass lumped on the ith sliding block. It is assumed that the sliding resistance of each block remains constant irrespective of the overturning-induced axial force in the columns.

Under the combined action of loading in the two direction, the sliding is assumed to start following an interaction curve of the form

$$\phi = \left(\frac{F_{ix}}{F_{iL}}\right)^2 + \left(\frac{F_{iy}}{F_{iL}}\right)^2 \tag{2}$$

in which, F_{ix} and F_{iy} are the lateral resistance in the x- and y-direction, respectively. Sliding will take place when $\phi \geqslant 1$. ϕ is the interaction curve (shown in Fig. 2) between lateral resistances of the sliding block. For the case of no interaction, the sliding in a particular direction is independent to the sliding in the orthogonal direction.

A corresponding fixed base system or unisolated system is the same system without sliding support

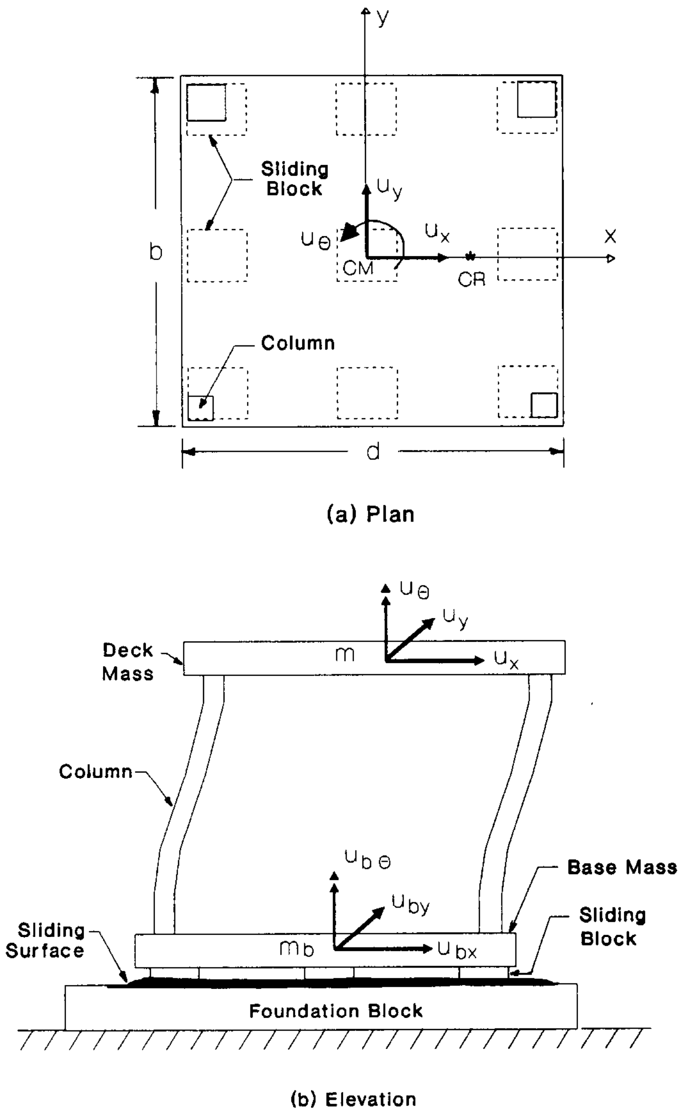

Fig. 1. Structural model.

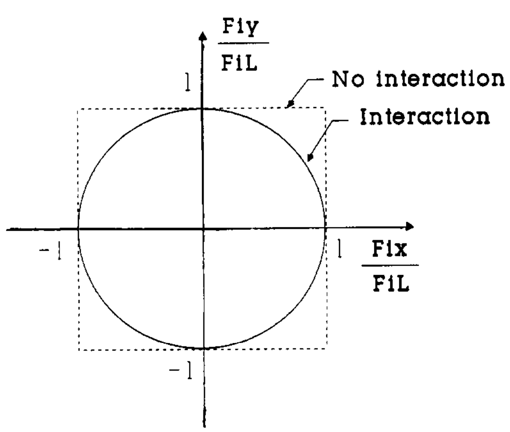

Fig. 2. Interaction curve between the two lateral resistance of the sliding block.

and base mass restrained against all movements. The eccentricity between CM of deck mass and the static centre of resistance of the columns (CR) is denoted by e_x as shown in Fig. 1. The three uncoupled frequency parameters for the fixed base building are defined as follows:

$$w_x = \sqrt{\frac{K_x}{m}}; \quad w_y = \sqrt{\frac{K_y}{m}}; \quad \text{and} \quad w_\theta = \sqrt{\frac{K_\theta}{mr^2}} \qquad (3)$$

in which m is the mass of the superstructure (including additional lumped masses); r is the radius of gyration of deck mass about the vertical axis through CM; K_x, K_y and the K_θ are the uncoupled lateral (in the x and y directions) and torsional stiffnesses of the superstructure, respectively.

The horizontal ground accelerations are simulated from the specified power spectral density function (PSDF) by Monte-Carlo simulation. The PSDF is modelled as a filtered shot noise such as the Kanai–Tajimi spectrum

$$S(w) = S_0 \frac{1 + 4\xi_g^2 w_g^2}{[1 - (w/w_g)^2]^2 + 4\xi_g^2 w_g^2}, \tag{4}$$

in which w_g and ξ_g are filter parameters and S_0 is spectral density of input white-noise.

Equations of motion

The equations of motion for a coupled lateral-torsional response of the system to the ground acceleration, acting along the x- and y-directions are written as

$$[M]\{\ddot{u} + \ddot{u}_b\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{r\}\{\ddot{u}_g\} \tag{5}$$

$$[M_b]\{\ddot{u}_b\} + [K_b]\{u_b\} - [C]\{\dot{u}\} - [K]\{u\} = -[M_b]\{r\}\{\ddot{u}_g\} \tag{6}$$

in which $[M]$, $[K]$ and $[C]$ are the lumped mass, stiffness and damping matrices corresponding to the DOF at the deck; $\{u\} = [u_x, u_y, u_\theta]^T$, vector of displacements at the deck relative to the base mass; $[M_b]$ is lumped mass matrix corresponding to the DOF at the base mass; $[K_b]$ is stiffness matrix of sliding support; $\{u_b\} = [u_{bx}, u_{by}, u_{b\theta}]^T$; $\{\ddot{u}_g\} = [\ddot{u}_{gx}, \ddot{u}_{gy}]^T$, $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ are the random ground accelerations (in the principal directions of earthquake); and $\{r\}$ is the matrix of earthquake influence coefficient. The stiffness matrices, $[K]$ and $[K_b]$ are obtained by assembling the elemental stiffness matrices of the columns and sliding blocks, respectively, after usual transformation. Let $[k_{it}]$ be the tangent stiffness matrix of ith sliding block. At any instant of time

$$[k_{it}] = [k_{ie}] - [k_{is}] \tag{7}$$

$$[k_{ie}] = \mathrm{diag}[k_{ixb}, k_{iyb}], \tag{8}$$

in which $[k_{ie}]$ is the initial stiffness matrix of the sliding block during the non-sliding phase. k_{ixb} and k_{iyb} assume a large value to ensure that the system behaves like a fixed base during the non-sliding phase. $[k_{is}]$ is modification to the stiffness due to the plastic state, i.e. sliding of ith block otherwise, it is a null matrix. The matrix $[k_{is}]$ is given by (Kan and Chopra [15])

$$[k_{is}] = \frac{1}{G_i} \begin{bmatrix} E_i^2 & E_i F_i \\ E_i F_i & F_i^2 \end{bmatrix}, \tag{9}$$

where

$$G_i = k_{ixb} H_{ix}^2 + k_{iyb} H_{iy}^2 \tag{10a}$$

$$E_i = k_{ixb} H_{ix} \tag{10b}$$

$$F_i = k_{iyb} H_{iy} \tag{10c}$$

$$H_{ix} = \frac{\partial \phi}{\partial F_{ix}} \tag{10d}$$

$$H_{iy} = \frac{\partial \phi}{\partial F_{iy}}. \tag{10e}$$

Equation (9) is essentially based on the convexity rule of the theory of plasticity.

The damping matrix $[C]$ is not explicitly known. It is constructed from the assumed modal damping for the fixed base structure using its mode shapes and frequencies. The coupled differential equations of motion [eqns (5) and (6)] are solved in incremental form by employing Newmark's method assuming linear variation of acceleration over the short time interval, δt. For the time intervals during which transition from sliding to non-sliding state or from one sliding to another sliding state occurs, the tangent stiffness, $[k_{it}]$ is re-evaluated and an average stiffness predictor–corrector iteration procedure with pull back was used to reduce the unbalanced forces created by the numerical approximation to an acceptably small value. The treatment of such procedure and state transitions used are described in Ref. [15].

NUMERICAL STUDY

The effectiveness of the sliding support of a torsionally coupled system is investigated with respect to the following parameters: e_x/d, w_θ/w_x, m_b/m and μ. These ratios may be varied suitably by varying the mass (including additional lumped masses on the deck) and stiffness properties of the superstructure and sliding system. With the assumed set of values for the parameters chosen, the inputs for the numerical study are selected as: $w_x = w_y = 2\pi$ rad s^{-1}; lateral dimension, $b = d = 6$ m; and number of sliding blocks is 9. The initial high stiffness of sliding block is taken $k_{ixb} = k_{iyb} = 10{,}000K_x$; modal damping for the superstructure is taken as 5% for all modes; the filter parameters for ground acceleration are: $w_g = 15$ rad s^{-1}; $\xi_g = 0.5$ and $S_0 = 0.05$ m^2 s^{-3}.

The average acceleration response spectrum of the simulated ground motion for the above parameters is shown in Fig. 3. Responses are obtained for both asymmetric and corresponding symmetric systems. The symmetric and asymmetric systems have the same values of uncoupled frequency parameters given by eqn (3).

In order to study the effectiveness of the sliding support and the effect of torsional coupling, response ratio R is defined as the ratio of mean peak response of the system with sliding support to mean peak response of a corresponding fixed base system. The response ratio R is an index of the performance of the

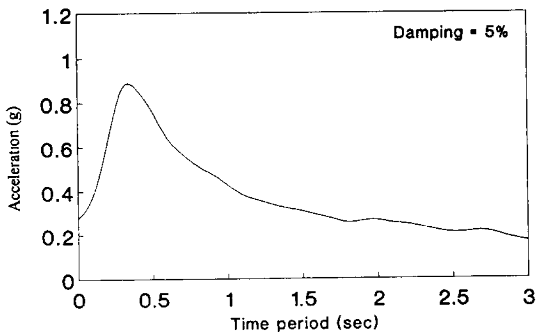

Fig. 3. Variation of the average acceleration spectrum of simulated ground motion.

sliding support and a value less than one indicates that the sliding support is effective.

Effect of eccentricity

Figure 4a shows the variation of response ratio R against eccentricity ratio, e_x/d. The ratio R for u_x remains insensitive to the variation of e_x/d, while the ratio R for responses u_y and u_θ are significantly influenced by e_x/d and increases with an increase in eccentricity. This indicates that the effectiveness of the sliding support for a torsionally coupled system decreases with an increase of the system eccentricity. Further, the effectiveness of the sliding support is underestimated (i.e. R values become greater) if the

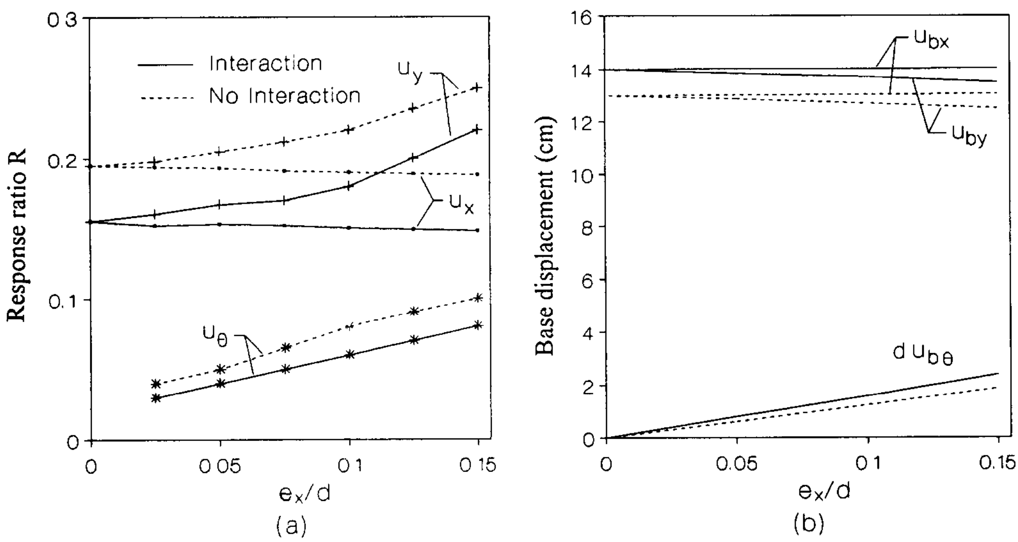

Fig. 4. Effect of e_x/d on the response ratio R and base displacements; $w_\theta/w_x = 1$, $\mu = 0.1$ and $m_b/m = 0.25$.

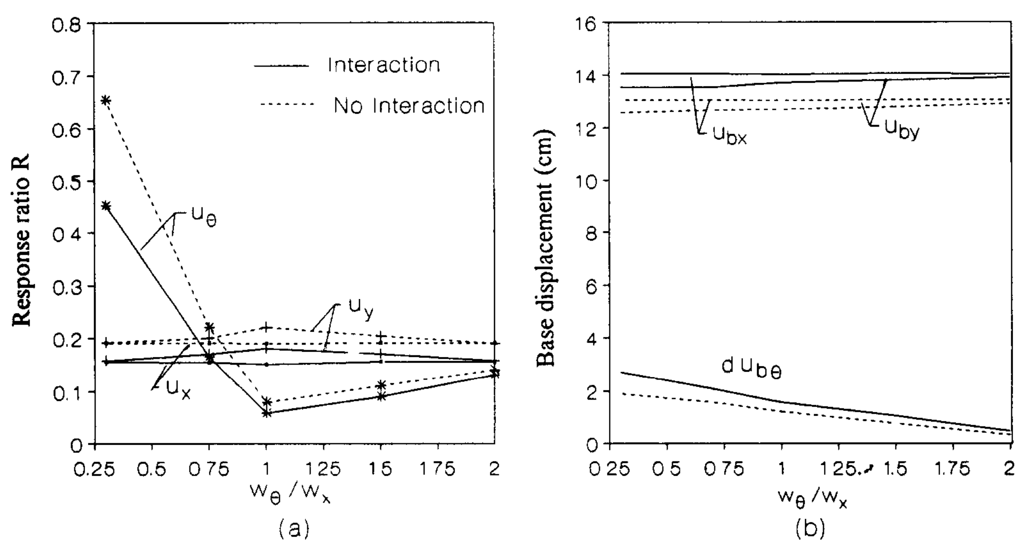

Fig. 5. Variation of response ratio R and base displacements with w_θ/w_x; $e_x/d = 0.1$, $\mu = 0.1$ and $m_b/m = 0.25$.

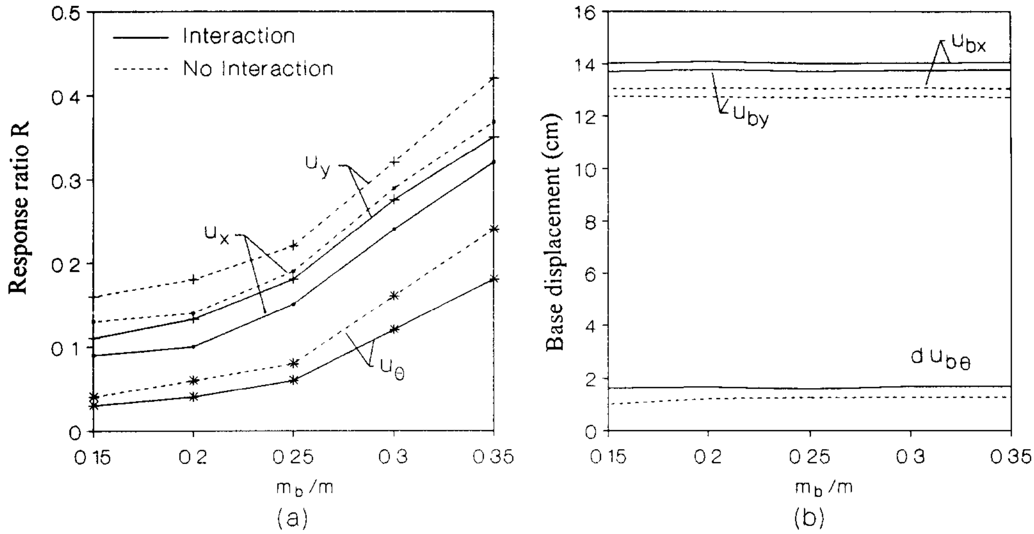

Fig. 6. Variation of response ratio R and base displacements with m_b/m; $e_x/d = 0.1$, $w_\theta/w_x = 1$ and $\mu = 0.1$.

interaction effect between the resistance of the support in the two directions is ignored. However, the trend of the variation of R with e_x/d remains the same for both cases, i.e. with and without the interaction effect considered in the analysis. From Fig. 4b, it is seen that the base displacements are significantly influenced by the e_x/d ratio. The base responses are greater for the case when interaction is considered.

Note that the value of R for the case of zero eccentricity (without interaction effect) shows the effectiveness of base isolation if the model shown in Fig. 1 were idealized as a two-dimensional system independently in the two directions (x and y). The figures clearly indicate that the effectiveness of the sliding support is overestimated if the effect of torsional coupling is ignored and the system is idealized as a two-dimensional model.

Effect of w_θ/w_x *ratio*

In Fig. 5, the variation of response ratio R against w_θ/w_x is shown. The ratio R for responses u_x is not very sensitive to the variation of the w_θ/w_x ratio. As w_θ/w_x increases (in the range of $0 < w_\theta/w_x \leqslant 1$), the ratio R for response u_θ decreases significantly, showing the effectiveness of the sliding support in reducing the torsional response. Also, because of greater torsional coupling, the ratio R for response u_y increases with increase in w_θ/w_x (for $0 < w_\theta/w_x \leqslant 1$). When w_θ/w_x becomes greater than one, u_y decreases (in the range $1 < w_\theta/w_x < 2$) and after that it remains

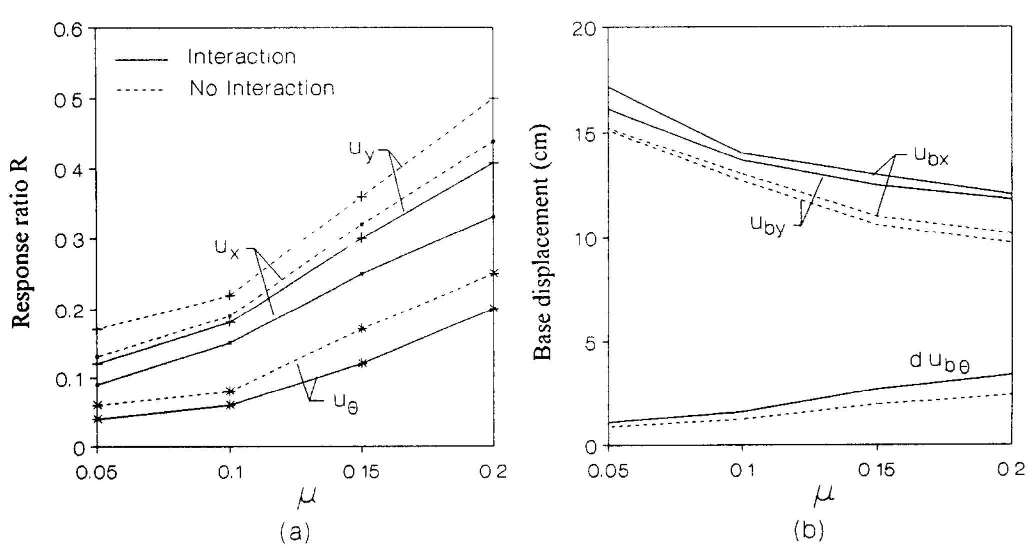

Fig. 7. Variation of response ratio R and base displacements with μ; $e_x/d = 0.1$, $w_\theta/w_x = 1$ and $m_b/m = 0.25$.

almost constant for further increase in w_θ/w_x. On the other hand, the ratio R for response u_θ increases with increase in w_θ/w_x beyond unity and u_y tends to be close to that for u_x. This is expected, since for a torsionally stiff system ($w_\theta/w_x \gg 1$) the effect of torsional coupling is considerably reduced. Further, the base responses are not greatly influenced by the variation of w_θ/w_x.

Effect of mass ratio (m_b/m)

The variation of response ratio R with m_b/m is shown in Fig. 6. The ratio R increases with increase in m_b/m. As the base mass increases, the limiting value of sliding force in the sliding block increases [see eqn (1)]. As a result, the system enters into fewer sliding phases resulting in less dissipation of seismic energy. Thus, the effectiveness of the sliding support decreases with increase in m_b/m. Further, the base responses are not much influenced by the variation of m_b/m.

Effect of coefficient of friction

As the coefficient of friction increases, the limiting value of sliding force increases. As a result, more force will be transmitted by the sliding support into the structure. It is therefore expected that the response ratio R will increase with increase in μ. This is well depicted in Fig. 7 where the ratio R is plotted against μ. Thus, the effectiveness of the sliding support decreases with increase of the value of μ. The translational base displacement decreases with increase of μ.

CONCLUSIONS

The non-linear response of a torsionally coupled system with sliding support to two component random ground motions is obtained by considering the sliding, with and without, interaction between lateral resistances. The response behaviour of the torsionally coupled system is studied for a set of important parametric variations. The parametric study is not exhaustive, but from the trend of the results, the following conclusions may be drawn:

(1) the effect of force interaction on sliding has considerable influence on the response of the system; it increases the effectiveness of the sliding support;

(2) if the eccentricity of the building is ignored and modeled as a two-dimensional system, the effectiveness of a sliding support is overestimated;

(3) the effectiveness of a sliding support is generally reduced for higher eccentricity;

(4) the effectiveness of a sliding support in reducing torsional response significantly increases with an increase in w_θ/w_x (in the range $0 < w_\theta/w_x \leqslant 1$);

(5) the effectiveness of sliding support decreases with increase in mass ratio (m_b/m) and the coefficient of friction (μ);

(6) base displacements are not significantly influenced by superstructure eccentricity e_x/d, the ratio w_θ/w_x and m_b/m, and they are underestimated if the interaction effect is ignored.

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