

If $\mu = 55$, $\sigma_a = 4$, $\sigma_b = 10$, $\sigma_c = 15$, In this which is better

1. Introduction

In statistics, the mean (μ) represents the average value of a dataset, while the standard deviation (σ) measures the spread or variability of data around the mean. When comparing multiple datasets with the same mean but different standard deviations, the key factor that determines which is better depends on consistency and variability.

Given:

- Mean (μ) = 55
- Standard Deviation $\sigma_a = 4$
- Standard Deviation $\sigma_b = 10$
- Standard Deviation $\sigma_c = 15$

All three cases have the same average value, but they differ in how widely the data is distributed around the mean.

The main objective is to determine which distribution is better based on statistical reasoning.

2. Understanding Mean and Standard Deviation

2.1 Mean ($\mu = 55$)

- The mean represents the central value of the dataset.
- All three datasets have the same average score of 55.
- This means the central tendency is identical in all cases.

Since the mean is constant, the only difference lies in the variability of data.

2.2 Standard Deviation (σ)

Standard deviation measures how far the data values deviate from the mean.

- Small $\sigma \rightarrow$ Data tightly clustered around mean
- Large $\sigma \rightarrow$ Data widely spread from mean

Mathematically:

$$\sigma = \sqrt{(\sum(x - \mu)^2 / N)}$$

It indicates consistency and stability of performance.

3. Analysis of Each Case

3.1 Case 1: $\sigma_a = 4$

Characteristics

- Very small spread around mean
- Data points lie close to 55
- High consistency
- Low variability

Interpretation

Using the Empirical Rule:

- 68% of values lie between 51 and 59
- 95% of values lie between 47 and 63
- 99.7% of values lie between 43 and 67

This shows very stable performance with minimal fluctuation.

3.2 Case 2: $\sigma_b = 10$

Characteristics

- Moderate spread
- Values more dispersed compared to σ_a
- Medium variability
- Moderate consistency

Interpretation

- 68% of values lie between 45 and 65

- 95% of values lie between 35 and 75
- 99.7% of values lie between 25 and 85

The data shows noticeable variation around the mean.

3.3 Case 3: $\sigma = 15$

Characteristics

- Large spread
- Values widely distributed
- High variability
- Low consistency

Interpretation

- 68% of values lie between 40 and 70
- 95% of values lie between 25 and 85
- 99.7% of values lie between 10 and 100

This distribution shows large fluctuations and less stability.

4. Graphical Interpretation

If we draw three normal distribution curves with the same mean:

- $\sigma_a = 4 \rightarrow$ Tall and narrow curve
- $\sigma_b = 10 \rightarrow$ Medium height and width
- $\sigma = 15 \rightarrow$ Short and wide curve

Smaller standard deviation results in:

- Higher peak
- Narrow distribution
- More consistency

Larger standard deviation results in:

- Flatter curve
- Wider spread
- Less predictability

5. Comparison Table

Mean = 55 (Same for all)

Case 1 ($\sigma_a = 4$):

- Variability: Very Low
- Consistency: Very High
- Risk: Low
- Predictability: High

Case 2 ($\sigma_\beta = 10$):

- Variability: Moderate
- Consistency: Medium
- Risk: Medium
- Predictability: Moderate

Case 3 ($\sigma = 15$):

- Variability: High
- Consistency: Low
- Risk: High
- Predictability: Low

6. Which is Better?

6.1 If Stability and Consistency Are Important

$\sigma_a = 4$ is the best.

Reason:

- Data is tightly clustered.
- Very small deviation from mean.
- More reliable performance.
- Less uncertainty.

This is preferred in:

- Academic performance
- Manufacturing quality control
- Employee productivity evaluation
- Medical measurements

6.2 If Moderate Flexibility Is Acceptable

$\sigma_\beta = 10$ may be acceptable.

It provides some variation but still maintains reasonable stability.

6.3 If High Risk or High Variation Environment

$\sigma = 15$ is not preferable when consistency is required.

Large deviation means:

- Higher unpredictability
- Greater fluctuations
- Less reliability

7. Real-Life Example (Education Domain)

Assume three classes have average marks of 55.

Class A ($\sigma = 4$):

- Most students score between 51 and 59.
- Very consistent performance.

Class B ($\sigma = 10$):

- Students score between 45 and 65 mostly.
- Moderate variation.

Class C ($\sigma = 15$):

- Students score widely from 40 to 70.
- Large performance gap.

If we want uniform performance and fairness, Class A is better.

8. Statistical Conclusion

When the mean is constant:

Lower Standard Deviation = Better Consistency

Therefore:

$\sigma_a = 4$ is the best among the three.

Because:

- It has the smallest variability.
- It indicates stable and reliable data.
- It reduces uncertainty.
- It ensures better predictability.

9. Final Answer

Given:

$$\mu = 55$$

$$\sigma_a = 4$$

$$\sigma_\beta = 10$$

$$\sigma = 15$$

The best distribution is:

$$\sigma_a = 4$$

Reason:

- Lowest variability
- Highest consistency
- Most stable distribution
- Best for performance evaluation and quality measurement

10. Conclusion

- All three cases have the same mean (55).
- The difference lies only in variability.
- Smaller standard deviation indicates better consistency and reliability.
- Larger standard deviation indicates more fluctuation and uncertainty.