



Optimal Dynamics of Vertical Jumping

Naser Mehrabi, John McPhee

Systems Design Engineering

University of Waterloo



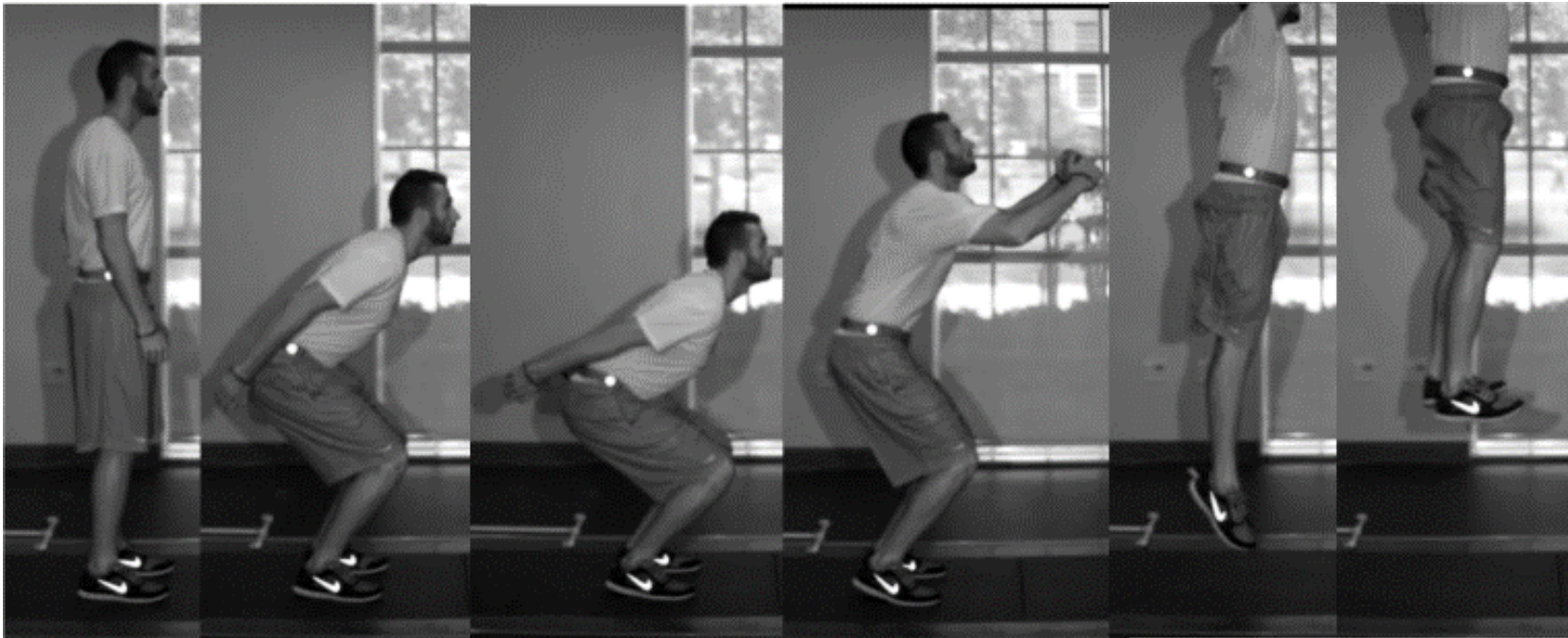
Research Motivation

- **Predictive simulation of vertical jumping**





Jumping Snapshots



Standing to toe-off

Flight



Biomechanical Jumping Model

Multibody Dynamic Model:

- 4 segment model (4-DOF):
 - HAT (Head-Arm-Trunk)
 - Thigh
 - Shank
 - Foot

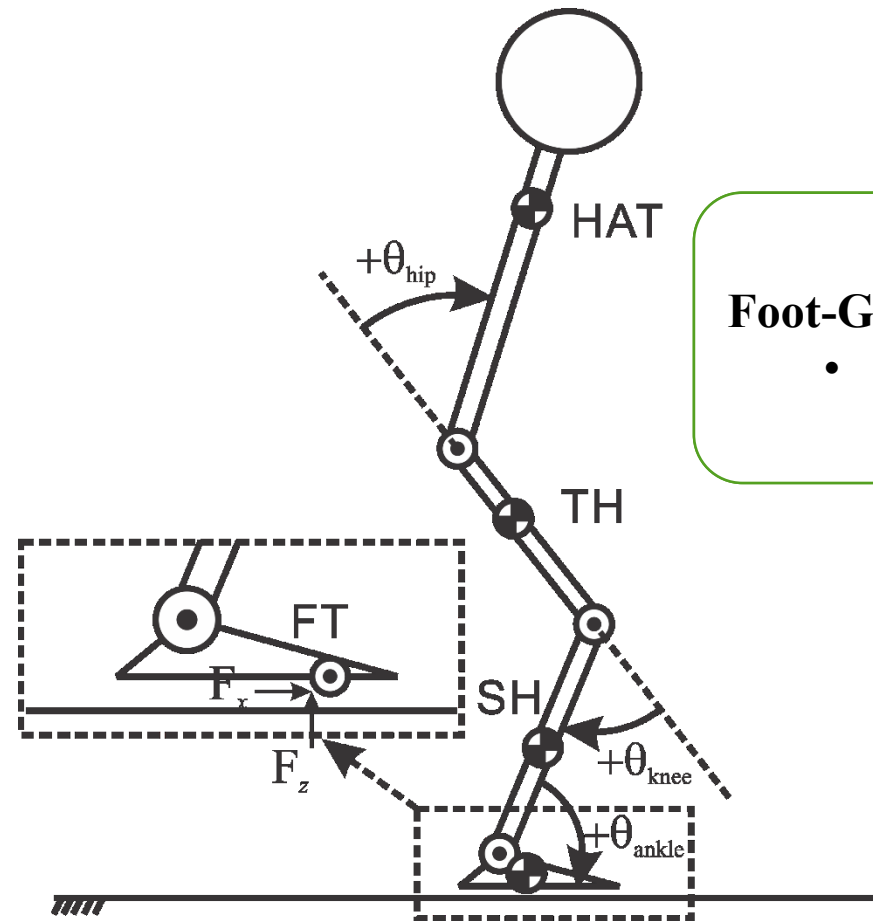
Actuation:

- 3 joint torque actuation
 $T_{\min} \leq T_i \leq T_{\max}$

Limits:

- Joint Limits
 $0 \leq \theta_{\text{hip}}, \theta_{\text{knee}} \leq 180^\circ$
 $-45^\circ \leq \theta_{\text{ankle}} \leq 45^\circ$

Pre-Flight(standing to toe-off)



Foot-Ground Contact:

- Kinematic contact joint



Biomechanical Jumping Model (Cont'd)

Multibody Dynamic Model:

- 4 segment model (6-DOF):
 - HAT (Head-Arm-Trunk)
 - Thigh
 - Shank
 - Foot

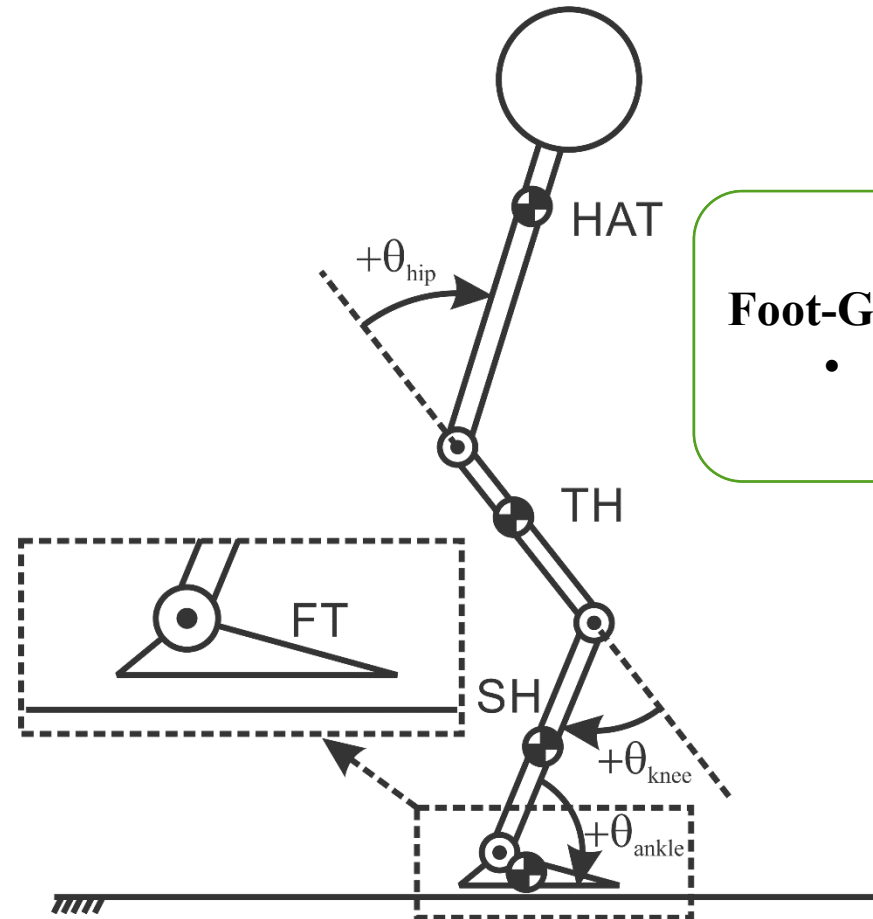
Actuation:

- 3 joint torque actuation
 $T_{\min} \leq T_i \leq T_{\max}$

Limits:

- Joint Limits
 $0 \leq \theta_{\text{hip}}, \theta_{\text{knee}} \leq 180^\circ$
 $-45^\circ \leq \theta_{\text{ankle}} \leq 45^\circ$

Flight



Foot-Ground Contact:

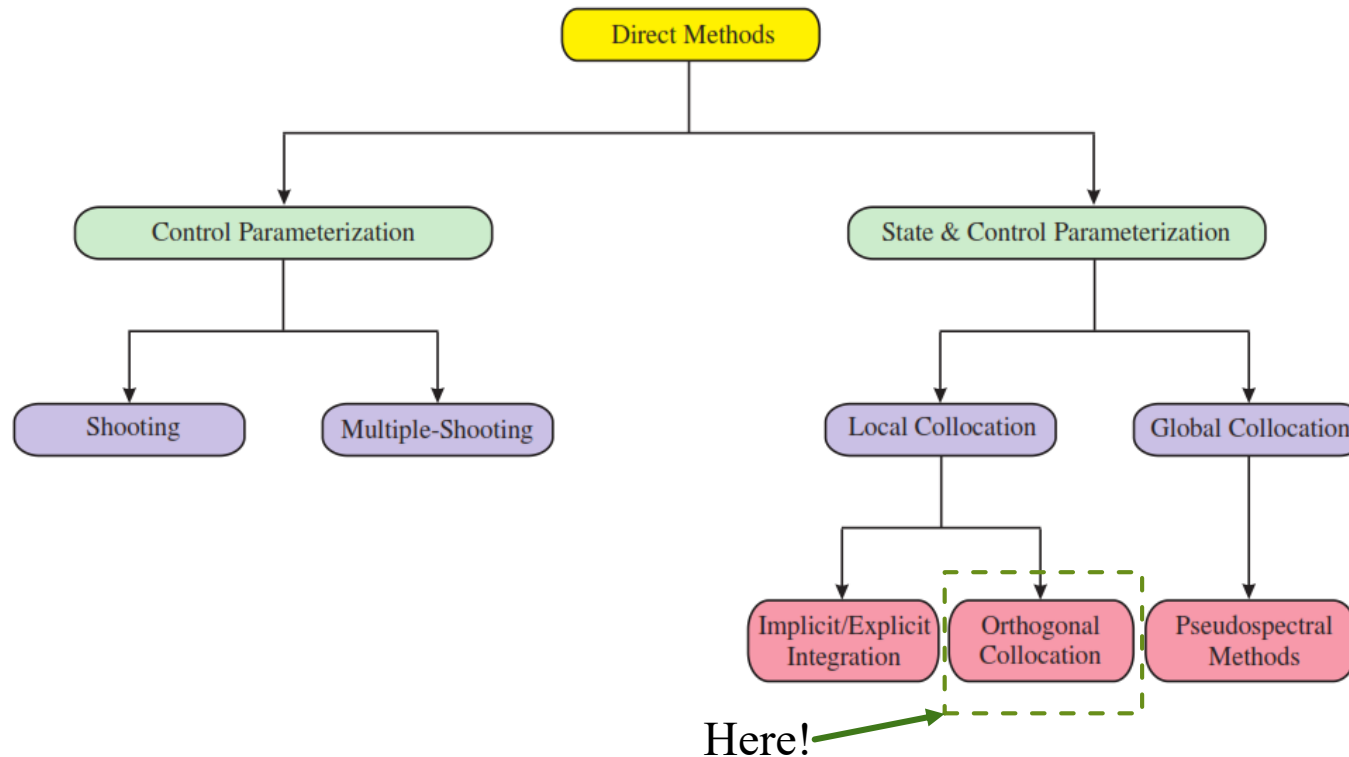
- No joint



Optimal Control Problem

Optimal Control Problem: Find the joint torque functions of time that maximize performance

Different direct methods for solving an optimal control problem¹:



Here!



Direct Collocation Method

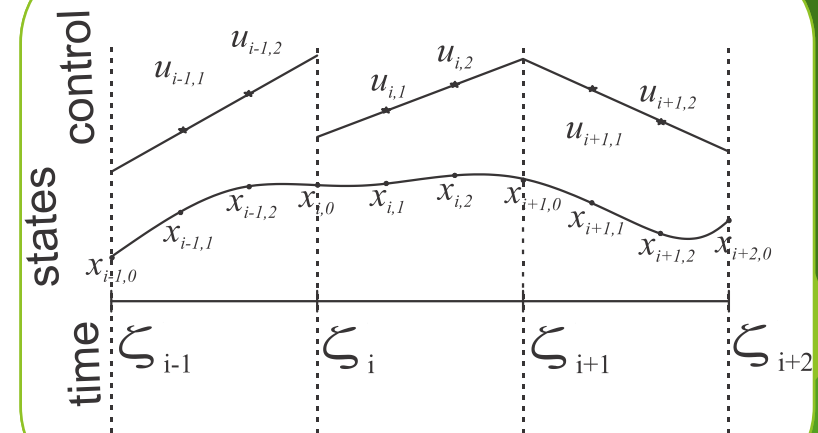
Nonlinear constraints

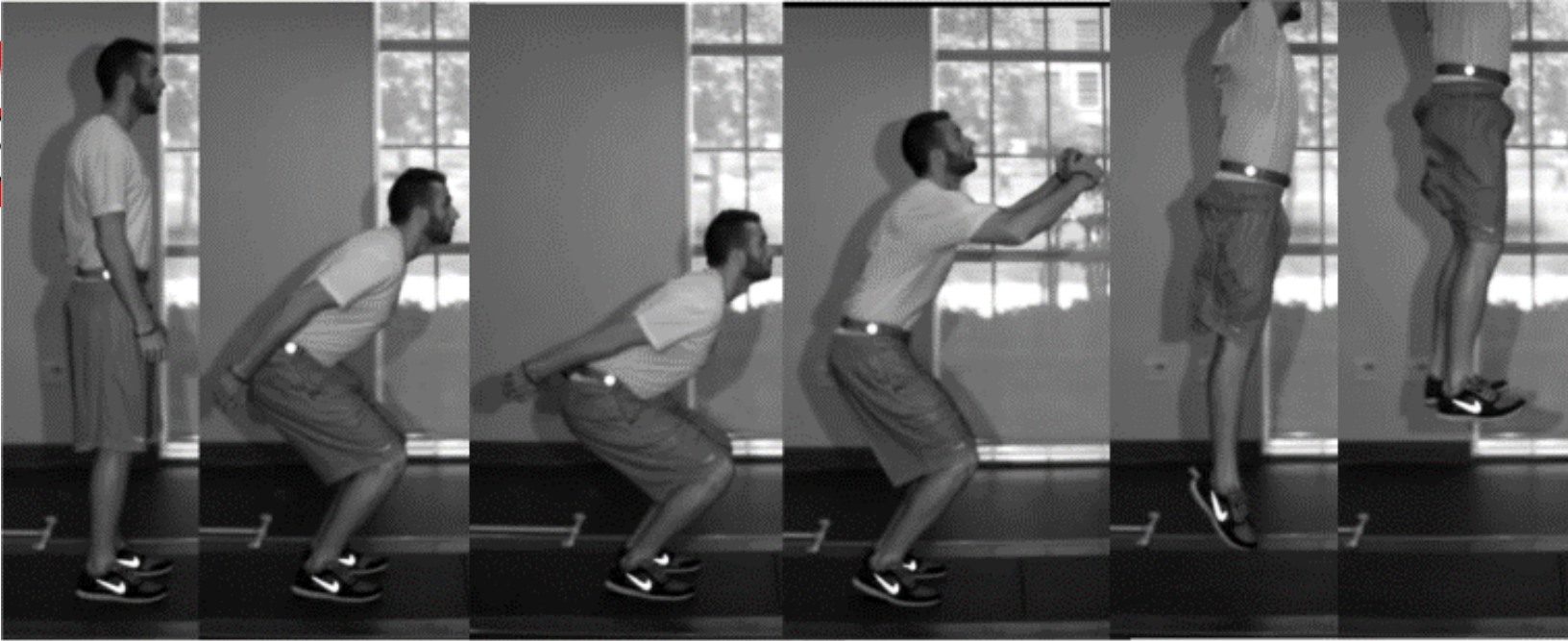
Equations of motion are satisfied at all collocation points

$$\begin{aligned}\dot{x} &= f(x, u, t), & x(t_0) &= x_0 \\ g(x, u, t) &= 0\end{aligned}$$

Objective function

$$\text{Objective function} := \Phi(x(t_f), t_f) + \int_{t_0}^{t_f} \Psi(x(t), u(t), t) dt$$





Standing to toe-off

Initial Position: Standing

Final Position: Unknown

Simulation Final Time:

$$0.2 \leq t_{f1} \leq 1.2$$

Objective Function:

$$\text{Minimize } \frac{1}{\text{Mass Center Vertical Speed}} + \int_0^{t_{f1}} T_i^2 dt$$

Constraints:

$$F_{rf} @ (t = t_{f1}) = 0$$

Flight

Initial Position: Previous final position

Final Position: Unknown

Simulation Final Time:

$$0.5 \leq t_{f2} \leq 1.8$$

Objective Function:

$$\text{Minimize } \frac{1}{\text{head height}} + \int_0^{t_f} T_i^2 dt$$

Constraints:

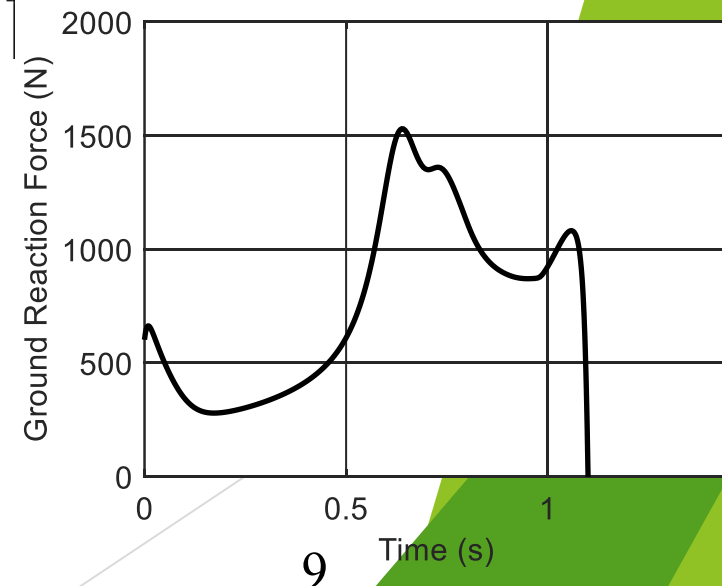
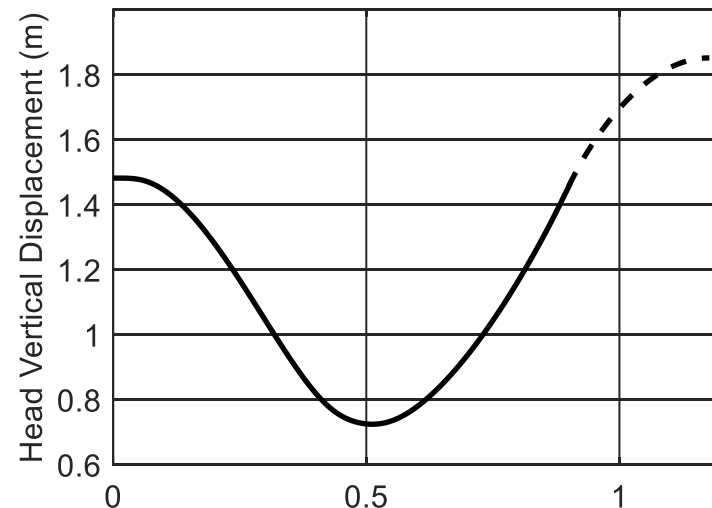
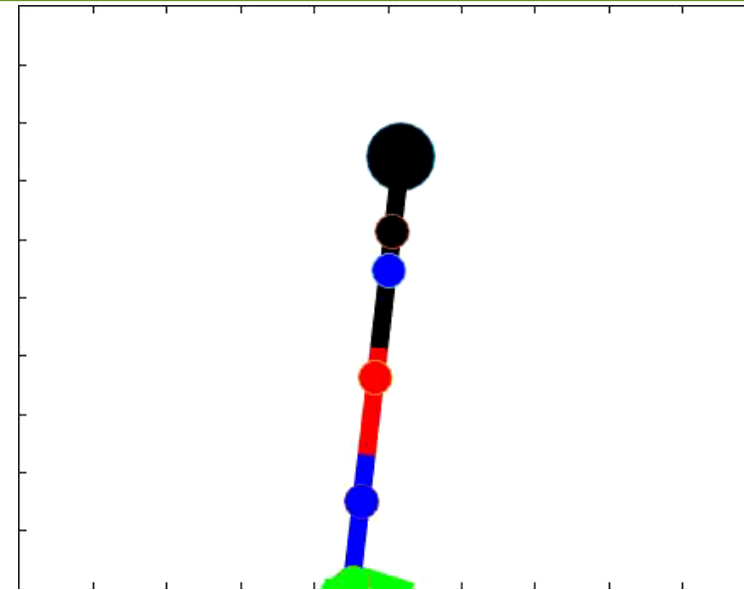
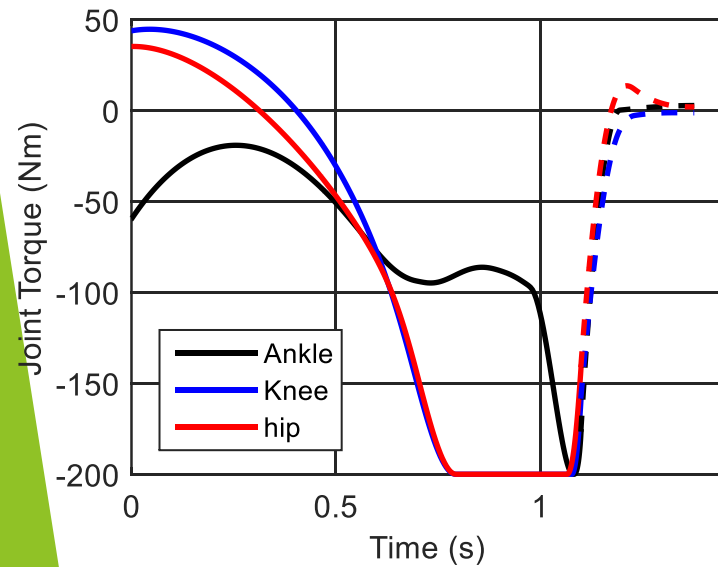
None



Simulation Results

Optimization:

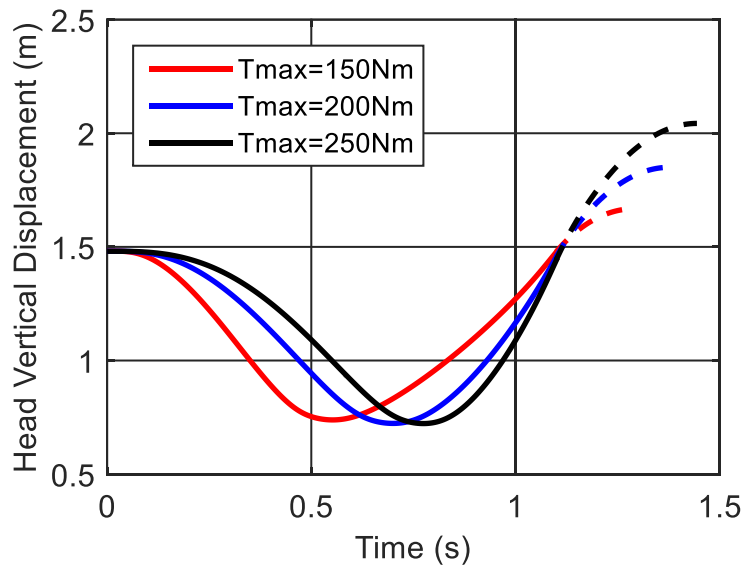
- GPOPS-II Optimal Control Toolbox
- IPOPT sparse optimizer



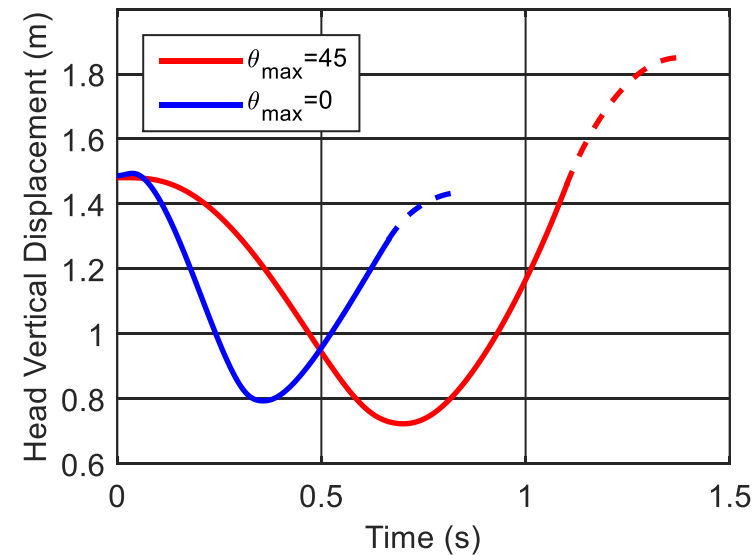


Simulation Results (Cont'd)

Maximum Joint Torque



Ankle Dorsiflexion Range





Conclusion

- ▶ Two biomechanical models of vertical jumping for the pre-flight and flight phases were developed.
- ▶ A multi-phase predictive simulation of vertical jumping was performed.
- ▶ This purely mathematical/predictive simulation seems to capture reality.

Future Work

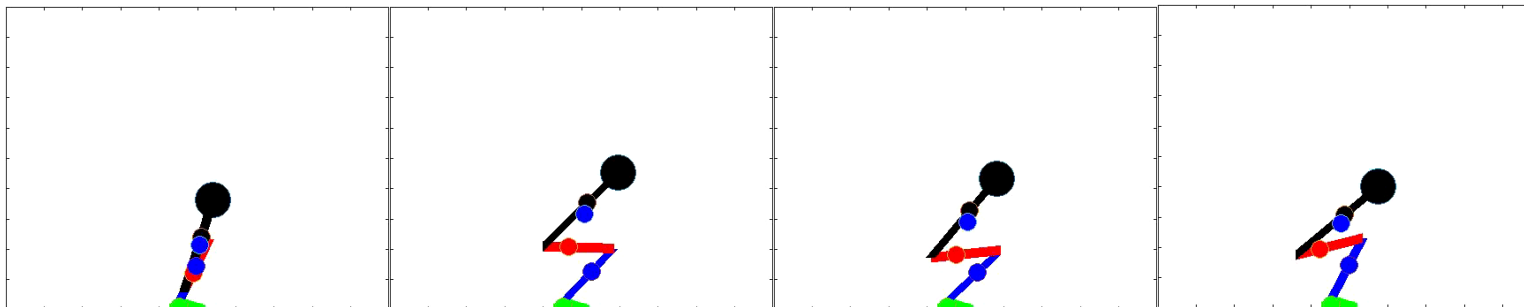
- ▶ Adding joint torque relations as a function of joint angle and angular velocity
- ▶ Adding skeletal muscle contraction dynamics
- ▶ Adding volumetric foot-ground contact model



Sponsors:

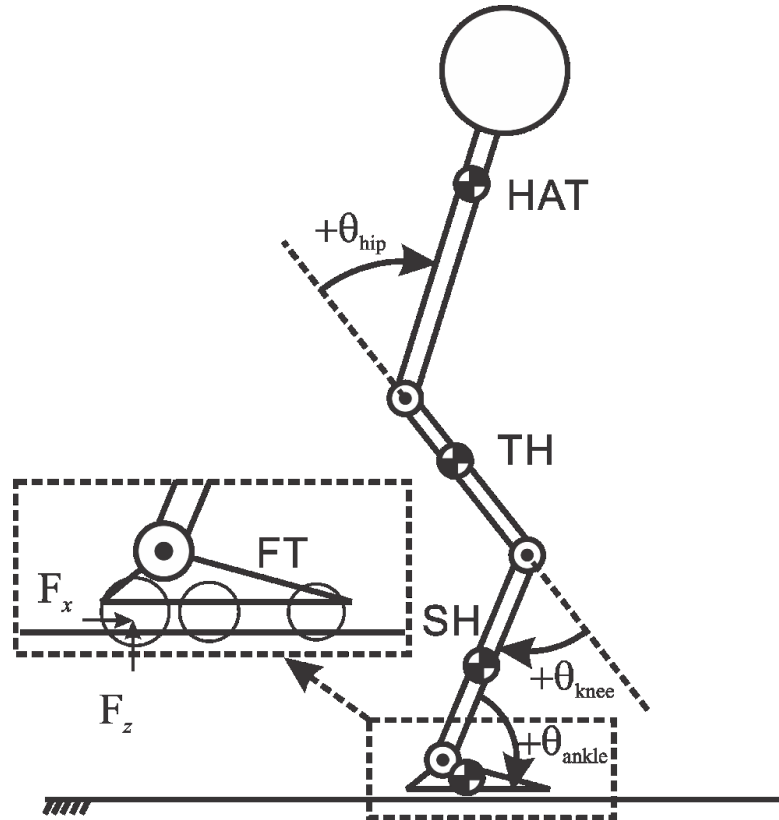


Thank you for your attention





Predictive Simulation of Jumping



- Optimization:
 - GPOPS-II Toolbox
 - IPOPT sparse optimizer

- Model:
 - 4 segment model, 6 DoF
 - 3 joint torque actuation
 - 3 volumetric sphere-plane contacts

- Optimization Cost Function:

$$\text{Minimize } \frac{1}{\text{head height}} + \int_0^{t_f} T_i^2 dt$$

Subjected to

- Equations of motion
- Volumetric contact model
- $T_{min} \leq T_i \leq T_{max}$
- $\theta_{min} \leq \theta_i \leq \theta_{max}$



Direct Collocation Method

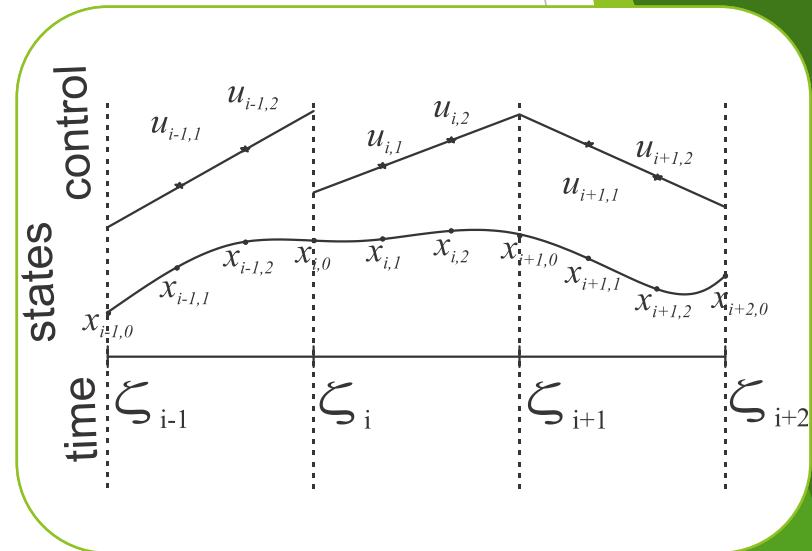
let's consider Lagrange polynomial approximation for states and control profiles.

State: K_x+1 degree polynomial with K_x collocation point

$$x^{K_x+1}(t) = \sum_{j=0}^{K_x} x_{ij} \phi_j(\tau), \quad \text{where} \quad \phi_j(\tau) = \prod_{\substack{k=0 \\ k \neq j}}^{K_x} \frac{\tau - \tau_k}{\tau_j - \tau_k}, \quad \begin{cases} t \in [\zeta_{i-1}, \zeta_i] \\ \tau \in [0, 1] \end{cases}$$

Control profile: K_u degree polynomial with K_u collocation point

$$u^{K_u}(t) = \sum_{j=1}^{K_u} u_{ij} \theta_j(\tau), \quad \text{where} \quad \theta_j(\tau) = \prod_{\substack{k=1 \\ k \neq j}}^{K_u} \frac{\tau - \tau_k}{\tau_j - \tau_k}, \quad \begin{cases} t \in [\zeta_{i-1}, \zeta_i] \\ \tau \in [0, 1] \end{cases}$$





Direct Collocation Method (Cont'd)

Nonlinear constraints

Equations of motion are satisfied at all collocation points

$$\begin{aligned} M \dot{x}(t) &= f(x(t), u(t), t), & x(t_0) &= x_0 \\ g(x(t), u(t), t) &= 0 \end{aligned}$$

At each collocation point:

$$\text{Res} := M \sum_{j=0}^{K_x} x_{ij} \dot{\phi}_j(t_k) - \Delta \zeta_i f(x_{ik}, u_{ik}, t_{ik}), \quad \begin{cases} i = 1, \dots, NE \\ j = 0, \dots, K \\ k = 1, \dots, K \end{cases}$$

Objective function

$$\text{Objective function} := \Phi(x(t_f), t_f) + \int_{t_0}^T \Psi(x(t), u(t), t) dt$$