REPORT PROBLEMS ON "TIME SERIES ANALYSIS AND MATHEMATICAL STATISTIC B"

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1.Let $\{X_t : t \in \mathbf{Z}\}$ be strictly stationary. Then, show that the joint distribution function of $X_{t_1},\ldots,X_{t_n}(t_1,\ldots,t_n \in \mathbf{Z})$ depends only on $t_2 - t_1,\ldots,t_n - t_{n-1}$

2.Suppose that $\{X_t : t \in \mathbf{Z}\}$ is a Gaussian stationary process with zero mean, autocovariance function $\{R()\}$ and spectral density $f(\lambda)$. We assume that

$$\sum_{j=-\infty}^{\infty} |j||R(j)| < \infty$$

Let $I_n(\lambda) = (2\pi n)^{-1}|\sum_{t=1}^{n} X_t e^{it\lambda}|^2$. Then, show that

$$\lim_{n\to\infty} E\{I_n(\lambda)\} = f(\lambda)$$

3.Let $\{X_t : t \in \mathbf{Z}\}$ be a Gaussian stationary process with zero mean, autocovariance function $\{R()\}$. Then,show that

$$E\{X_t X_s X_u X_v\} = R(s-t)R(v-u) + R(u-t)R(v-s) + R(v-t)R(u-s)$$

4.Suppose that $\{X_t : t \in \mathbf{Z}\}$ is a Gaussian stationary process with zero mean,autocovariance function $\{R()\}$ and spectral density $f(\lambda)$. We assume that

$$\sum_{j=-\infty}^{\infty} |j||R(j)| < \infty$$

Let $I_n(\lambda) = (2\pi n)^{-1}|\sum_{t=1}^{n} X_t e^{it\lambda}|^2$. Then, show that $I_n(\lambda)$ does not converge to $f(\lambda)$ in probability.

5.Suppose that $\{X_t\}$ is generated by

$$X_t = u_t\sqrt{a_0 + \sum_{j=1}^{q} a_j X_{t-j}{}^2}, \{u_t\} \sim i.i.d.N(0,1)$$

where $a_0 > 0, a_j \geq 0, j = 1,\ldots q$. Let $\mathscr{F}_t(q)$ be the σ-algebra generated by $\{X_s : t-q \leq s \leq t-1\}$, and let

$$Q(\theta) = \sum_{t=q+1}^{n} [X_t{}^2 - E\{X_t{}^2|\mathscr{F}_t(q)\}]^2$$

where $\theta = (a_0, a_1,\ldots,a_q)'$.We define the conditional least squares estimator of θ by

$$\hat{\theta} = argmin_\theta Q(\theta).$$

Then, derive the explicit form of $\hat{\theta}$.

6. Suppose that $\{X_t\}$ is generated by

$$X_t + b_1 X_{t-1} + \ldots + b_p X_{t-p} = u_t, \{u_t\} \sim i.i.d. N(0, \sigma^2),$$

where $1 + b_1 z + \ldots + b_p z^p \neq 0 on |z| \leq 1$. Then, show that $\hat{X}_t = -(b_1 X_{t-1} + \ldots + b_p X_{t-p})$ is the best linear predictor of X_t.

7.Derive the Whittle likelihood

$$\int_{-\infty}^{\infty} \{\log f_\theta(\lambda) + I_n(\lambda) / f_\theta(\lambda)\} d\lambda$$

8.State what you know about Akaike's information criteria (AIC) and the other criteria.