

$$T_{pq}(a, b) = (bq + aq + ap, bp + aq) \\ = (bq + a(q + p), bp + aq)$$

$$T_{p'q'}(a, b) = T_{pq}(T_{pq}(a, b)) \\ = T_{pq}(bq + aq + ap, bp + aq) \\ = ((bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p, (bp + aq)p + (bq + aq + ap)q) \\ = (bpq + aq^2 + bq^2 + aq^2 + apq + bpq + apq + ap^2, bp^2 + apq + bq^2 + aq^2 + apq) \\ = (2bpq + 2aq^2 + bq^2 + 2apq + ap^2, bp^2 + 2apq + bq^2 + aq^2) \\ = (b(2pq + q^2) + a(2q^2 + 2pq + p^2), b(p^2 + q^2) + a(2pq + q^2))$$

$$bq' + a(q' + p') = b(2pq + q^2) + a(2q^2 + 2pq + p^2)$$

$$q' = 2pq + q^2$$

$$p' = 2q^2 + 2pq + p^2 - q' \\ = 2q^2 + 2pq + p^2 - (2pq + q^2) \\ = 2q^2 + 2pq + p^2 - 2pq - q^2 \\ = q^2 + p^2$$