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# CSE 546 Assignment 3 - Actor-Critic Algorithms

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## Abstract

This report presents our experiments on three environments, `CartPole-v1`, `LunarLander-v2` and `InvertedPendulum-v4`. These environments were selected from the `ClassicalControl`, `Box2D` and `MuJoCo` collections respectively in the `Gymnasium` library [2]. We applied Q Actor-Critic Algorithm to solve these environments and conducted a case study on the outcomes.

## 1 Q Actor-Critic Algorithm

As described in [3].

### 1.1 Network Architecture

The policy(actor) network takes the state as input and outputs probability distributions over the action space. In the case of discrete action space, the value function (critic) net again takes the state as input and outputs estimate the Q value for each action. In continuous action space, however, the value function (critic) net takes the state-action pair as input and outputs the Q value for that particular state-action pair.

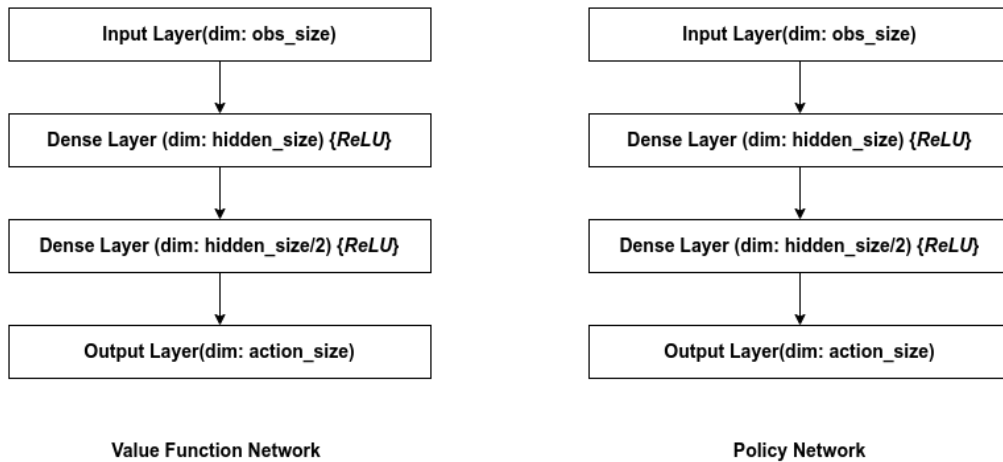


Figure 1: Networks for Discrete Action Space

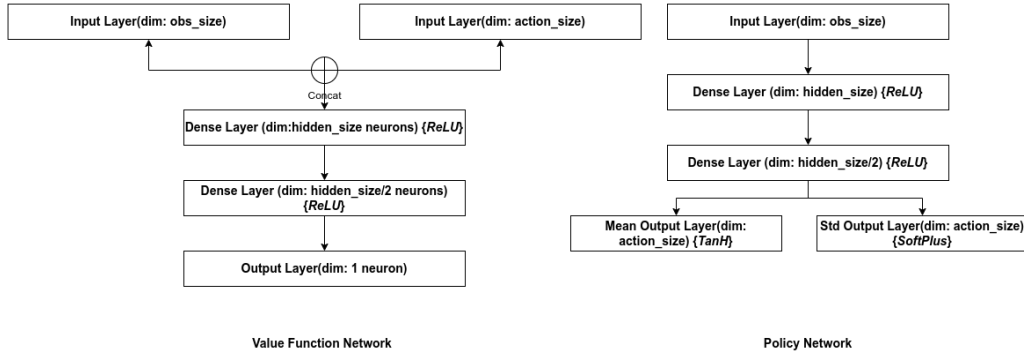


Figure 2: Networks for Continuous Action Space

## 1.2 Choosing Action

The policy network takes the current state as input and outputs a probability distribution over actions. The agent chooses an action based on this distribution.

## 1.3 Value Function update

The value function network is updated using the Q-learning algorithm, which involves computing the TD error and updating the value estimates for the current state-action pair.

## 1.4 Policy update

During each iteration of the algorithm, the actor selects an action based on the current state and the policy network. The Q-value of the chosen action is then computed using the critic network, which estimates the expected cumulative reward for taking the chosen action in the current state.

The policy network is then updated using the Q-value function to increase the probability of selecting the chosen action in the future. This is done by computing the gradient of the log probability of the chosen action with respect to the actor parameters and multiplying it by the Q-value. The resulting gradient is used to update the actor parameters using gradient ascent, which increases the probability of selecting the chosen action in the future.

By updating the policy network based on the Q-value function, the actor learns to select actions that are likely to lead to higher cumulative rewards in the future. This allows the algorithm to balance exploration and exploitation, exploring new actions to discover potentially high-reward options while also exploiting actions that have already been found to be effective.

$$\text{Actor Loss} = -1 * (\log \text{probability of action}) * (\text{TD-Error})$$

The negative value in the actor loss function is included because the goal of the optimization problem is to maximize the expected cumulative reward over time, which is equivalent to minimizing the negative of the expected cumulative reward. Therefore, the loss function is expressed as a negative quantity that needs to be minimized.

The first term in the actor loss function, which is the negative log-likelihood of the chosen action, is included to encourage the policy network to increase the probability of selecting actions that have higher expected cumulative rewards. By minimizing the negative log-likelihood, the policy network is encouraged to increase the probability of selecting actions that are more likely to lead to higher cumulative rewards.

The second term in the actor loss function, which is the TD-Error of the chosen action, is included to provide information on the value of the chosen action and encourage the policy network to select actions that are likely to lead to higher cumulative rewards. By minimizing the negative of the TD-Error, the policy network is encouraged to select actions that have higher expected cumulative

rewards. We have observed that using TD-Error instead of Q-Value here leads to faster convergence. Also, the TD-Error is detached so that it doesn't influence the learning of the value function network.

### 1.5 Other Implementation details

We have chosen to update both the actor and critic networks every timestep of the environment (similar to SGD).

## 2 Difference between Actor-Critic and Value-based approximation algorithms

The main difference between value-based and actor-critic algorithms lies in their approach to learning and updating the policy.

Value-based algorithms, such as Q-learning and SARSA, learn the optimal value function for each state-action pair in the environment. The value function estimates the expected cumulative reward that the agent can achieve by following a certain policy. Once the value function is learned, the policy can be derived by selecting the action with the highest expected cumulative reward in each state. Value-based algorithms update the value function using a temporal-difference (TD) learning approach, where the value of the current state-action pair is updated based on the value of the next state-action pair and the reward obtained.

On the other hand, actor-critic algorithms learn both a policy (actor) and an approximate estimate of the value function (critic). Since the actor-critic learns the policy function directly it is a policy gradient algorithm. The critic network in actor-critic algorithms approximates the value function, which provides an estimate of the expected cumulative reward that the agent can achieve by following the current policy. The policy network (actor) selects actions based on the output of the critic network, which is used to estimate the expected cumulative reward for each action in the current state. The critic network is updated using a TD learning approach, where the estimated value of the current state-action pair is updated based on the estimated value of the next state-action pair and the reward obtained. [1] [3]

The key difference between the two approaches is that value-based algorithms only learn the value function, while actor-critic algorithms learn both the value function and the policy. The policy in actor-critic algorithms is updated based on the value function, allowing the agent to balance exploration and exploitation and learn a more optimal policy in complex environments. By learning both the policy and the value function, actor-critic algorithms can achieve better performance and faster convergence than value-based algorithms in many reinforcement learning tasks.

Actor-Critic Algorithm (all policy gradient algorithms for that matter) have the following advantages over value-based.

1. Don't need to use exploration/exploitation trade-offs.
2. Also helps in solving perpetual aliasing issues. Where two states may have similar feature representations but require different actions. Traditional DQN and DDQN based algos will end up trapped in such scenarios.

And the following disadvantages over value-based.

1. They frequently converge to local maxima instead of a global maximum.
2. They are inefficient and generally take much longer to train.
3. They also have high variance.

## 3 CartPole-v1

In the cart-pole problem version described by Barto, Sutton, and Anderson [1] a pole is connected to a cart through a joint that cannot be moved. The cart can move on a track without any friction. The pole is positioned upright on the cart and the objective is to keep the pole balanced by applying

forces to the cart in either the left or right direction. This environment has been visualized in Figure 3.

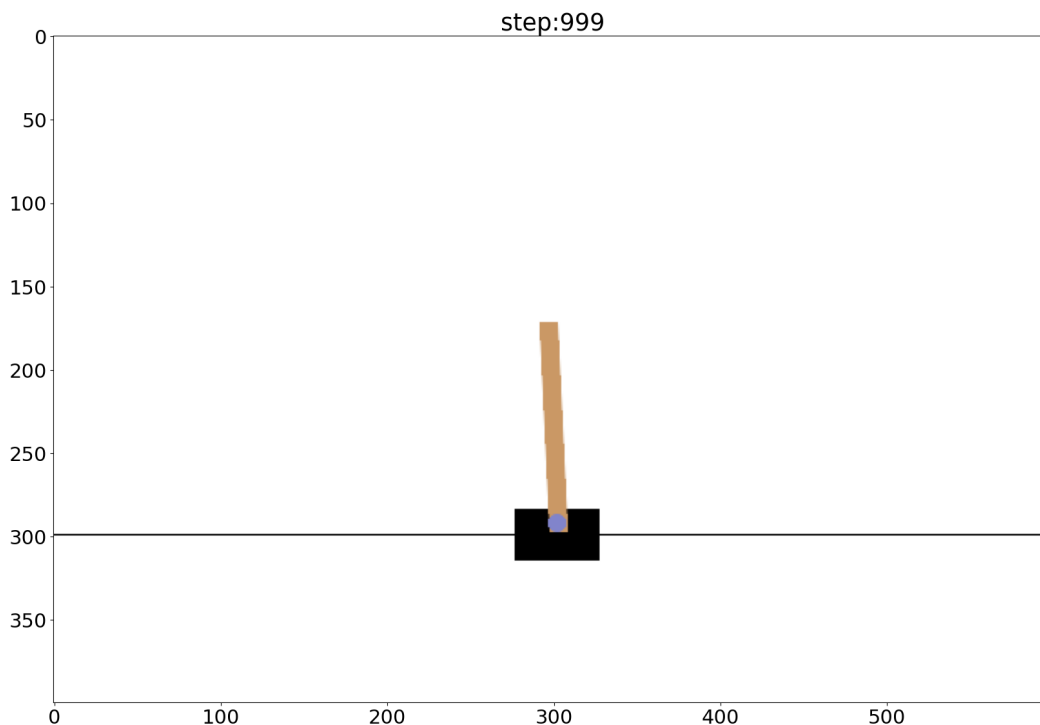


Figure 3: A frame from CartPole-v1

### 3.1 Action Space

The action is an array of shape (1,) that can take on values 0, 1 to indicate the the direction in which the cart is pushed with a fixed force.

- 0: The cart is pushed to the left
- 1: The cart is pushed to the right

The velocity that is either decreased or increased by the force applied is not constant and depends on the angle at which the pole is pointing. The center of gravity of the pole affects the amount of energy required to move the cart beneath it.

### 3.2 Observation Space

The observation is an ndarray of shape (4,) where the values represent the positions and velocities described next.

Num	Observation	Min	Max
0	Cart Position	-4.8	4.8
1	Cart Velocity	-Inf	Inf
2	Pole Angle	$\sim -0.418$ rad ( $-24^\circ$ )	$\sim 0.418$ rad ( $24^\circ$ )
3	Pole Angular Velocity	-Inf	Inf

Although the ranges above indicate the possible values for each element in the observation space, they do not reflect the allowed values of the state space in an ongoing episode. Specifically:

- The cart x-position (index 0) can take values between  $(-4.8, 4.8)$ , but the episode terminates if the cart leaves the  $(-2.4, 2.4)$  range.
- The pole angle can be observed between  $(-0.418, 0.418)$  radians or  $(\pm 24^\circ)$ , but the episode terminates if the pole angle is not in the range  $(-0.2095, 0.2095)$  or  $(\pm 12^\circ)$

### 3.3 Rewards

The objective of the task is to keep the pole upright for as long as possible. To encourage this behavior, a reward of +1 is given for every step taken, including the final step when the episode terminates. In version 1 of the task, the threshold for achieving a successful outcome is set at 475.

### 3.4 Start State

All observations are assigned a uniformly random value in  $(-0.05, 0.05)$

### 3.5 Episode End

The episode terminates under these conditions:

1. Termination: Pole Angle is greater than  $\pm 12^\circ$
2. Termination: Cart Position is greater than  $\pm 2.4$  (center of the cart reaches the edge of the display)
3. Truncation: Episode length is greater than 500

## 4 LunarLander-v2

This environment represents a classic problem of optimizing rocket trajectory. Based on Pontryagin's maximum principle, the optimal approach is to either fire the engine at full throttle or turn it off completely. As a result, this environment has discrete actions: the engine is either on or off.

Two versions of the environment are available: discrete and continuous. In our work we have used the discrete version. The landing pad is always located at coordinates  $(0, 0)$ , which are represented by the first two numbers in the state vector. It is possible to land outside of the landing pad. Since fuel is unlimited, an agent can learn to fly and land successfully on its first attempt.

### 4.1 Observation Space

The state of the environment is represented by an 8-dimensional vector that includes the  $x$  and  $y$  coordinates of the lander, its linear velocities in  $x$  and  $y$ , its angle and angular velocity, and two boolean values indicating whether each leg is in contact with the ground.

### 4.2 Action Space

Four discrete actions are available in this environment: remain idle, activate the left orientation engine, activate the main engine, or activate the right orientation engine.

### 4.3 Reward

A reward is given after each step in the environment. The total reward for an episode is calculated by summing the rewards for all steps within that episode. The reward for each step is determined by the following factors:

- The reward increases/decreases as the lander gets closer/further from the landing pad.
- The reward increases/decreases as the lander moves slower/faster.
- The reward decreases as the lander tilts more (angle not horizontal).
- The reward increases by 10 points for each leg in contact with the ground.

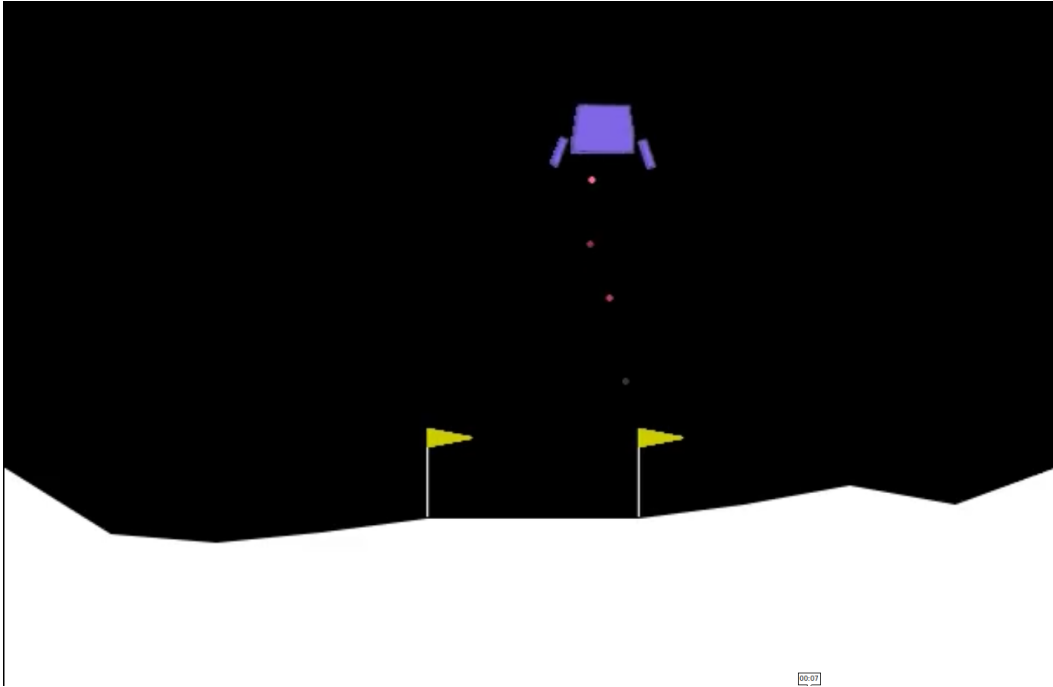


Figure 4: A frame from LunarLander-v2

- The reward decreases by 0.03 points for each frame a side engine is firing.
- The reward decreases by 0.3 points for each frame the main engine is firing.

An additional reward of -100 or +100 points is given for crashing or landing safely, respectively. An episode is considered solved if it scores at least 200 points.

#### 4.4 Starting State

At the beginning of each episode, the lander is positioned at the top center of the viewport and a random initial force is applied to its center of mass.

#### 4.5 Episode End

An episode terminates if any of the following conditions are met:

1. The lander crashes (its body comes into contact with the moon).
2. The lander moves outside of the viewport (its  $x$  coordinate is greater than 1).
3. The lander is not awake. According to the Box2D documentation, a body that is not awake does not move or collide with any other body.

### 5 InvertedPendulum-v4

This environment is the cart pole environment based on the work done by Barto, Sutton, and Anderson [1]. This has a similar problem description to the classic environment but is powered by the Mujoco physics simulator - allowing for more complex experiments (such as varying the effects of gravity). This environment involves a cart that can move linearly, with a pole fixed on it at one end and another end free. The cart can be pushed left or right, and the goal is to balance the pole on the top by applying forces on the cart.

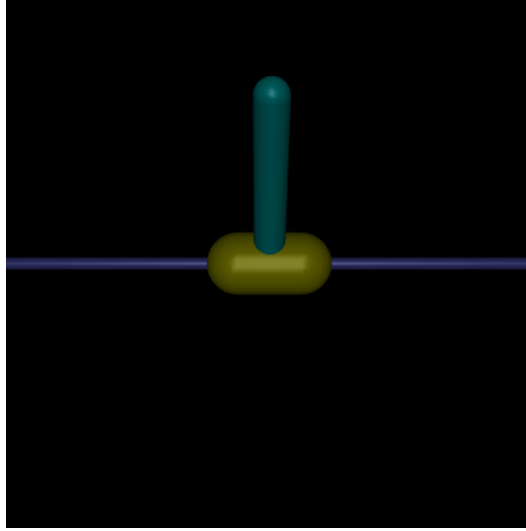


Figure 5: A frame from InvertedPendulum-v4

### 5.1 Observation Space

The state of the environment is represented by a 4-dimensional vector that includes the following.

Num	Observation	Min	Max
0	Cart Position	-Inf	Inf
1	Pole Vertical Angle	-Inf	Inf
2	Cart Linear Velocity	-Inf	Inf
3	Pole Angular Velocity	-Inf	Inf

### 5.2 Action Space

The agent takes a 1-dimensional vector for actions.

The action space is a continuous (action) in the range  $[-3, 3]$ , where action represents the numerical force applied to the cart (with magnitude representing the amount of force and sign representing the direction)

### 5.3 Reward

The goal is to make the inverted pendulum stand upright (within a certain angle limit) as long as possible - as such a reward of +1 is awarded for each timestep that the pole is upright.

### 5.4 Starting State

All observations start in the state (0.0, 0.0, 0.0, 0.0) with a uniform noise in the range of  $[-0.01, 0.01]$  added to the values for stochasticity.

### 5.5 Episode End

The episode terminates when any of the following happens:

1. The episode duration reaches 1000 timesteps.
2. Any of the state space values is no longer finite.
3. The absolute value of the vertical angle between the pole and the cart is greater than 0.2 radians.

## 6 Training and Evaluation

### 6.1 Observations

The Training on Cart-Pole was the fastest followed by Lunar-Lander and then Inverted-Pendulum. This can be attributed to the respective complexity of the three environments especially, the Cart-Pole and Lunar Lander having discrete action space while the Inverted Pendulum has a continuous action space.

### 6.2 Results

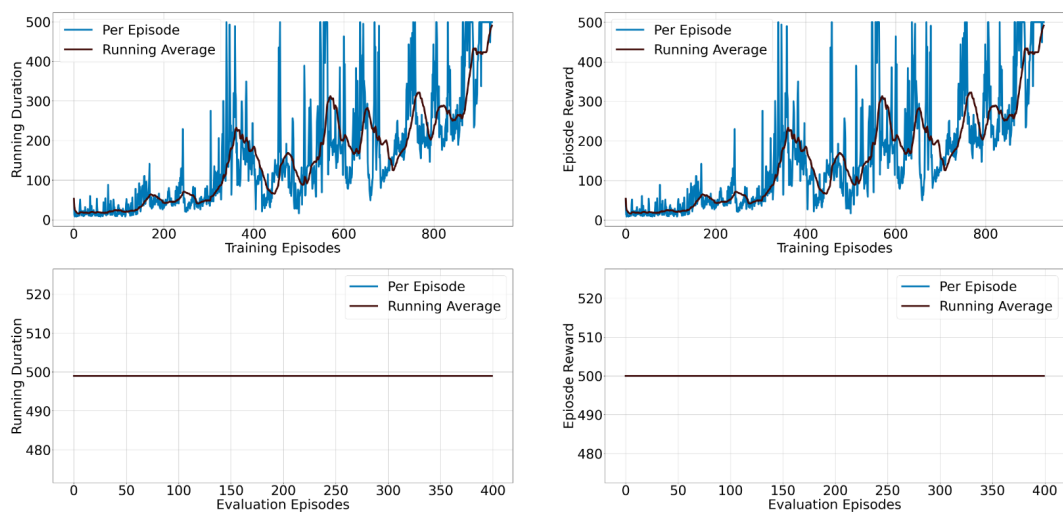


Figure 6: Cart Pole Training and Evaluation

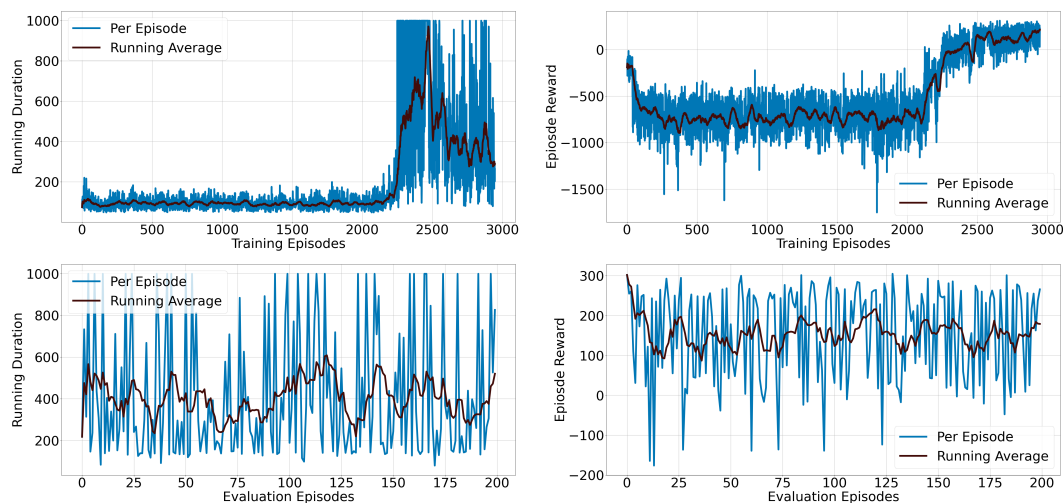


Figure 7: Lunar Lander Training and Evaluation



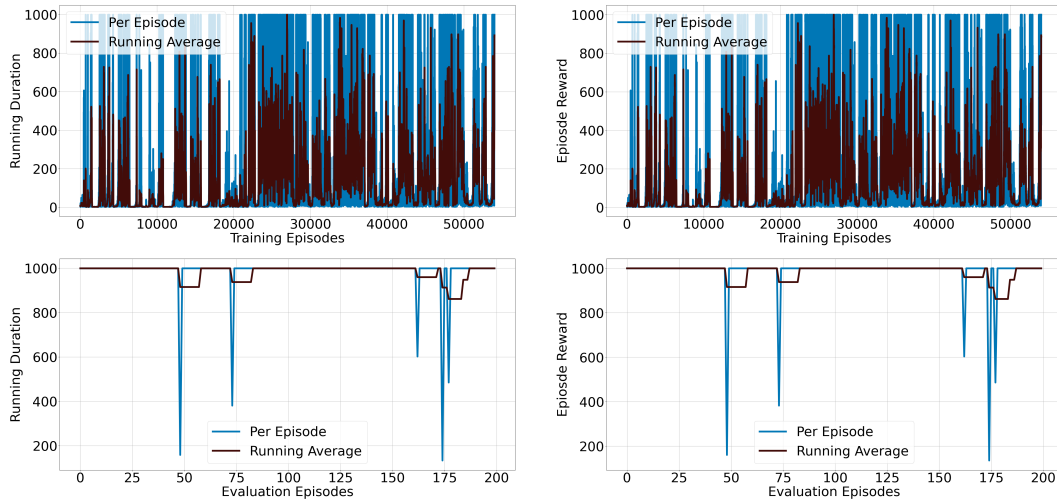


Figure 8: Inverted Pendulum Training and Evaluation

### 6.3 Training Challenges

1. The Actor's gradients can become very large, which can lead to unstable learning and slow convergence. This is because the actor's gradients are proportional to the critic's estimate of the advantage function, which can be very large or small depending on the environment and the current policy. The Solution for this is to use proximal policy optimization (PPO) instead which constrains the policy updates to be within a small range.
2. The critic's output could have a large variance, this can lead to slow training because the actor updates may be inconsistent and difficult to interpret. The Solution for this could be to use A2C instead.

## 7 Bonus - A2C

### 7.1 Results

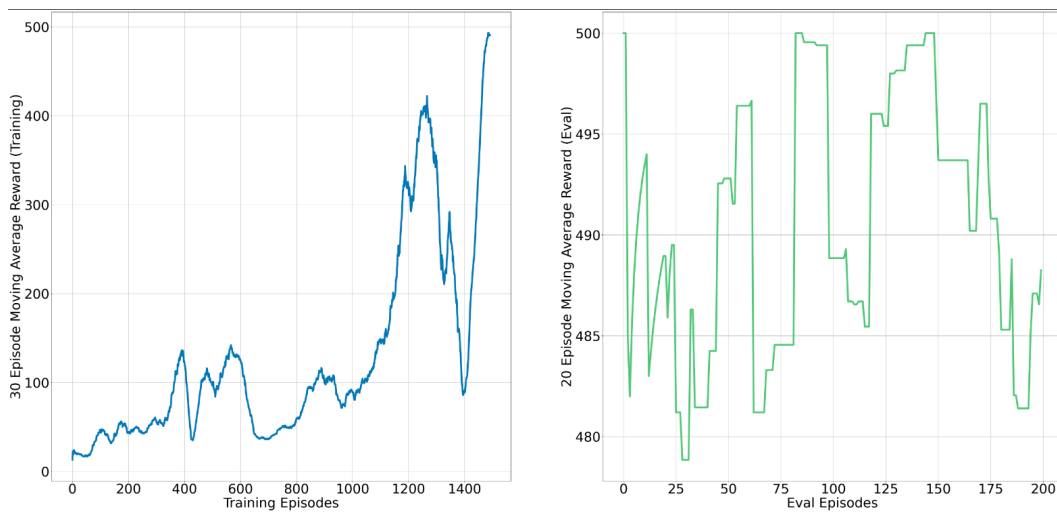


Figure 9: Cart Pole Training and Evaluation

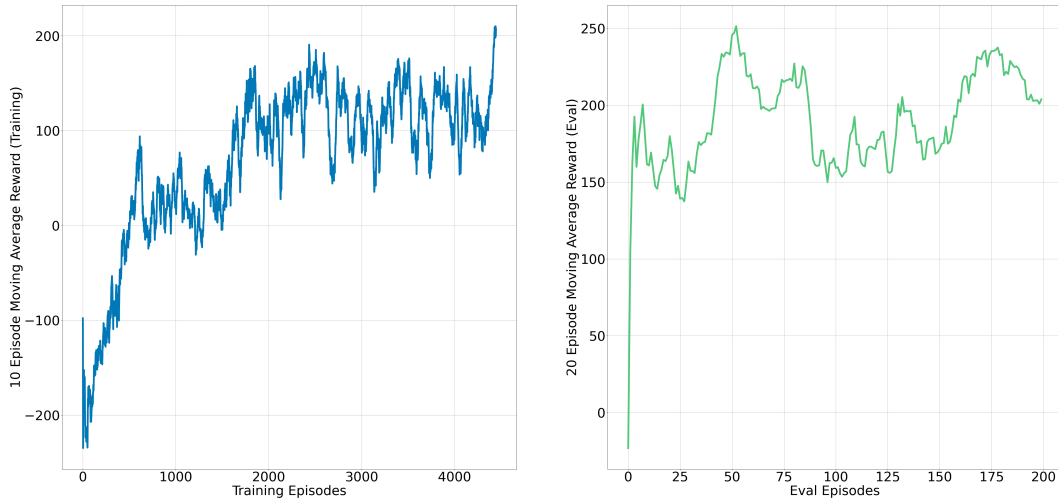


Figure 10: Lunar Lander Training and Evaluation

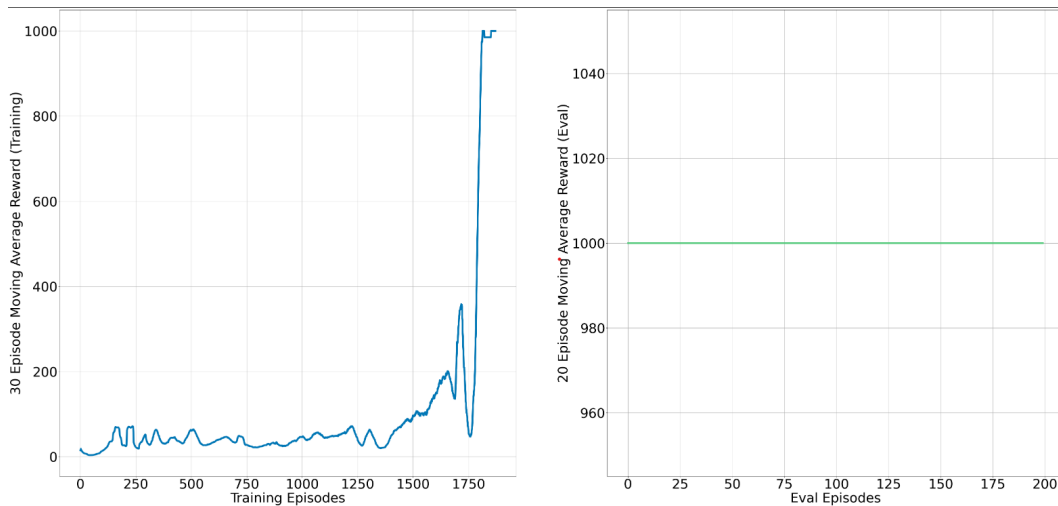


Figure 11: Inverted Pendulum Training and Evaluation

## 7.2 Comparison to QAC

We have observed that A2C converges much faster than Q Actor-Critic. This can be attributed to A2C updating the policy and the value function in parallel. This can result in faster learning, especially in environments with a small state space and a discrete action space. QAC, on the other hand, required more iterations to converge, particularly in environments with a large state space and a continuous action space such as Lunar Lander and Inverted Pendulum.

A2C has several advantages over Q Actor-Critic.

1. A2C can be more computationally efficient than QAC since it requires fewer function evaluations per update. This is because A2C uses only one network to estimate both the policy and value function, while QAC uses separate networks to estimate the value function and state-action value function.
2. A2C tends to converge faster than QAC due to its more accurate estimation of the advantage function. The advantage function in A2C is estimated using the difference between the value function and the state-action value function, which provides a better estimate of the

advantage than the difference between the state-action value function and the estimated value of the next state used in QAC.

3. A2C has a lower variance in the policy updates compared to QAC, which can lead to more stable and consistent learning. This is because A2C uses a Monte Carlo estimate of the advantage function, which can have a lower variance than the bootstrapped estimate used in QAC.

## 8 Contribution

Team Member	Part	Contribution
Sailesh Reddy Sirigireddy	Part I and II	50
Mohammed Nasheed Yasin	Part I and II	50

## References

- [1] Andrew G Barto, Richard S Sutton, and Charles W Anderson. “Neuronlike adaptive elements that can solve difficult learning control problems”. In: *IEEE transactions on systems, man, and cybernetics* 5 (1983), pp. 834–846.
- [2] Greg Brockman et al. *OpenAI Gym*. 2016. eprint: [arXiv:1606.01540](https://arxiv.org/abs/1606.01540).
- [3] Robert Crites and Andrew Barto. “An Actor/Critic Algorithm that is Equivalent to Q-Learning”. In: *Advances in Neural Information Processing Systems*. Ed. by G. Tesauero, D. Touretzky, and T. Leen. Vol. 7. MIT Press, 1994. URL: [https://proceedings.neurips.cc/paper\\_files/paper/1994/file/23ce1851341ec1fa9e0c259de10bf87c-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/1994/file/23ce1851341ec1fa9e0c259de10bf87c-Paper.pdf).