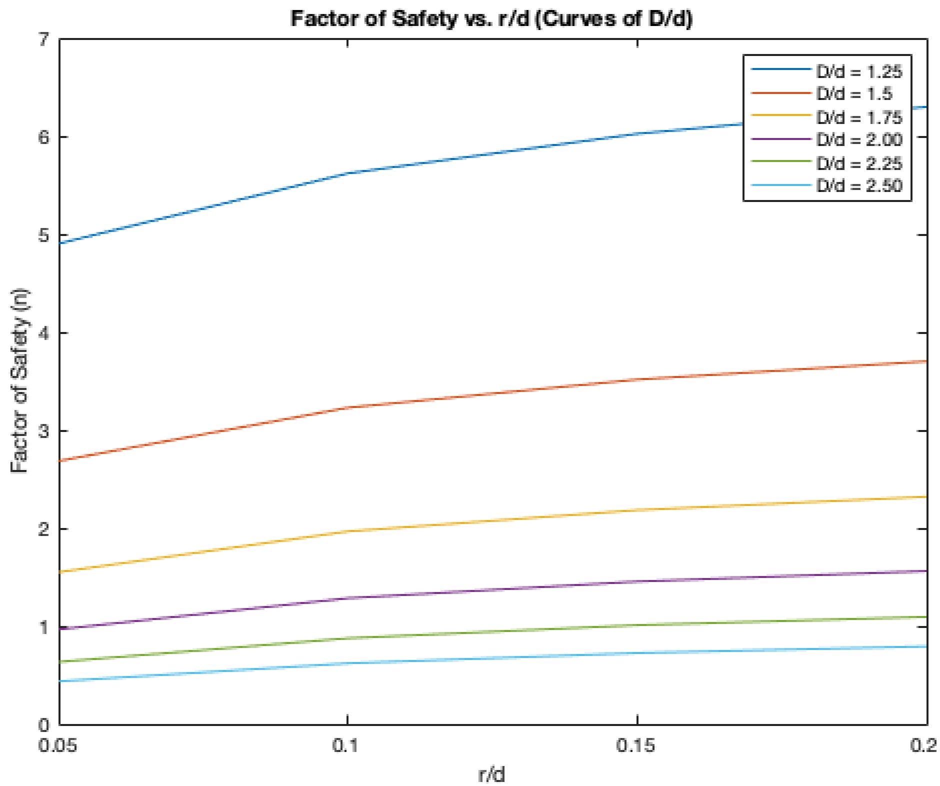
```
% NASH ELDER %
% ME 328 DESIGN I, DR. DAV0L %

% FATIGUE %
% Equations and recommended values from Shigley's 9th ed. %

% MATERIAL = STEEL %

clear;

Sy = 60.2; % ksi %
Sut = 95; % ksi %

Pa = 0.2; % kip %
Pm = 0.8; % kip %
P = 1;
Dbig = 3.5; % in. %
L = 15; % in. %
a = 8; % in. %

aSurf = 2.70; % Finish Factor a (machined finish) %
bSurf = -0.265; % Finish Factor b (machined finish) %

SePrime = 0.5*Sut; % ksi -- For Sut <= 200 %

for i = 1:6
    Doverd = 1.25 + 0.25*(i-1);
    d = (Dbig/Doverd);

    for j = 1:4
        roverd = 0.05 + 0.05*(j-1);
        r = (roverd * d);

        Ka = aSurf*Sut^bSurf;

        if d <= 2.0

            Kb = 0.879*(0.370*d)^(-0.107); % Size Factor -- small
 radius %
        else
            Kb = 0.91*(0.370*d)^(-0.157); % Size Factor -- large
 radius %
        end

        Kc = 1; % Load Factor -- Bending Load %

        Kd = 1; % Temp Factor -- Temp = 20 deg. C %

        Za = 1.288; % Transformation Variate for 90% (TBL 6-5) %

        Ke = 1 - 0.08*Za; % Reliability Factor %
```

```
        Se = SePrime * Ka * Kb * Kc * Kd * Ke;

        % Kt due to bending and torsion (ref: AMESWEB.INFO) %

        h = (Dbig-d)/2;

        if 0.1 <= h/r <= 2.0
            c1b = 0.947+1.206*(sqrt(h/r))-0.131*h/r;
            c2b = 0.022-3.405*(sqrt(h/r))+0.915*h/r;
            c3b = 0.869+1.777*(sqrt(h/r))-0.555*h/r;
            c4b = -0.810+0.422*(sqrt(h/r))-0.260*h/r;
        elseif 2.0 < h/r <= 20.0
            c1b = 1.232+0.832*(sqrt(h/r))-0.008*h/r;
            c2b = -3.813+0.968*(sqrt(h/r))-0.260*h/r;
            c3b = 7.423-4.868*(sqrt(h/r))+0.869*h/r;
            c4b = -3.839+3.070*(sqrt(h/r))-0.600*h/r;
        else
            c1b = NaN; c2b = NaN; c3b = NaN; c4b = NaN;
        end

        Kt = c1b + c2b*(2*h/Dbig) + c3b*(2*h/Dbig)^2 + c4b*(2*h/
Dbig)^3;

        if 0.25 <= h/r <= 4.0
            c1t = 0.905+0.783*(sqrt(h/r))-0.075*h/r;
            c2t = -0.437-1.969*(sqrt(h/r))+0.553*h/r;
            c3t = 1.557+1.073*(sqrt(h/r))-0.578*h/r;
            c4t = -1.061+0.171*(sqrt(h/r))+0.086*h/r;
        else
            c1t = NaN; c2t = NaN; c3t = NaN; c4t = NaN;
        end

        Kts = c1t + c2t*(2*h/Dbig) + c3t*(2*h/Dbig)^2 + c4t*(2*h/
Dbig)^3;

        % q calculation to find Kf %

        sqrtA = 0.246-3.08*10^(-3)*Sut
+1.51*10^(-5)*Sut^2-2.67*10^(-8)*Sut^3;
        sqrtA_tor = 0.190-2.51*10^(-3)*Sut
+1.35*10^(-5)*Sut^2-2.678*10^(-8)*Sut^3;


        q = 1/(1+sqrtA/sqrt(r));
        qs = 1/(1+sqrtA_tor/sqrt(r));

        Kf = q * (Kt-1) + 1;
        Kfs = qs * (Kts-1) + 1;

        % Sigma calculation %

        Ma = Pa * L;
        Mm = Pm * L;
        Ta = Pa * a;
```

```
        Tm = Pm * a;
        I = pi/4*(d/2)^4;
        J = pi/2*(d/2)^4;

        sig_a_bend = Ma*(d/2)/I;
        sig_m_bend = Mm*(d/2)/I;
        tau_a = Ta*(d/2)/J;
        tau_m = Tm*(d/2)/J;

        sig_prime_a = ((Kf*sig_a_bend)^2 + 3*(Kfs*tau_a)^2)^(0.5);
        sig_prime_m = ((Kf*sig_m_bend)^2 + 3*(Kfs*tau_m)^2)^(0.5);

        sig_prime_max = sig_m_bend + sig_a_bend;
        tau_prime_max = tau_a + tau_m;

        % Safety factors %

        n_fatigue = 1/((sig_prime_a/Se) + (sig_prime_m/Sut));% Safety
 factor for fatigue %

        n_SF = Sy / sig_prime_max; % Safety factor Yield / Allowable %

        n = min(n_fatigue, n_SF);

        SF(j,i) = n; % Allocation of safety factors on each iteration
 (for plot) %

    end
end
disp('4x6 Vector from 6 different D/d values and 4 different r/d
 values:');
disp(SF);

plot(SF);
xlabel('r/d');
ylabel('Factor of Safety (n)');
legend('D/d = 1.25', 'D/d = 1.5', 'D/d = 1.75', 'D/d = 2.00', 'D/d =
 2.25', 'D/d = 2.50');
set(gca, 'XTick', 0:1:4);
set(gca, 'XTickLabel', [0.20, 0.05, 0.10, 0.15]);
title('Factor of Safety vs. r/d (Curves of D/d)');
```

```
4x6 Vector from 6 different D/d values and 4 different r/d values:
    4.9016    2.6835    1.5490    0.9650    0.6333    0.4350
    5.6184    3.2275    1.9651    1.2834    0.8750    0.6191
    6.0222    3.5146    2.1822    1.4534    1.0085    0.7242
    6.2969    3.7004    2.3191    1.5602    1.0931    0.7918
```



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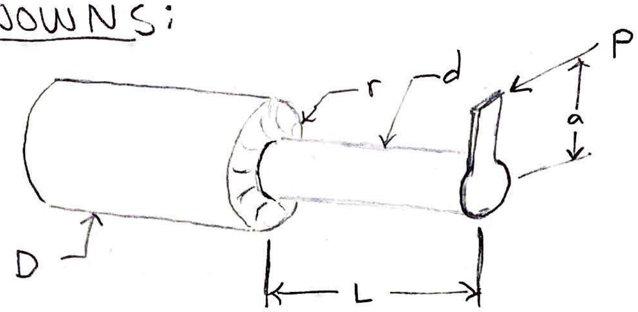
KNOWNS:



CRITICAL FAILURE AT THE NOTCH RADIUS r

OBJECTIVE:

AID SHAFT DESIGN BY GENERATING CURVES SHOWING EFFECT ON SAFETY FACTOR WITH VARYING NOTCH RADIUS, r, AND SMALLER DIAMETER, d.

INPUTS:

- D –
- L –
- a –
- P_a – ALTERNATING LOAD
- P_m – MEAN LOAD

MATERIAL: STEEL

S_y = 180 ksi

S_{UT} = 210 ksi

VARYING DIAMETERS:

D/d = 1.25, 1.5, 1.75, 2.0, 2.25, 2.5

VARYING RADII:

r/d = 0.05, 0.10, 0.15, 0.20

MODIFYING FACTORS:

$S_e \equiv$ ENDURANCE LIMIT AT CRITICAL LOCATION.

$S_e = S_e' k_a k_b k_c k_d k_e k_f$

$k_a \equiv$ SURFACE CONDITION M.F.

$\boxed{K_a = a S_{UT}^{b}}$ WHERE a AND b FROM TBL 6-2

FOR TEST CASE, ASSUME GROUND SURFACE,
$a = 1.34$, $b = -0.085$

$K_a = (1.34)(210 \text{ksi})^{-0.085}$

$K_a = 0.8506$

$K_b \equiv$ SIZE M.F.

$$K_b = \begin{cases} 0.879 d_e^{-0.107}, & 0.11 \le d \le 2 \text{ in} \\ 0.91 d_e^{-0.157}, & 2 < d \le 10 \text{ in} \end{cases}$$

$d_e = 0.370 d$ (FROM TBL 6-3)

$$\boxed{\begin{aligned} K_{b\,[\text{SMALL RADIUS}]} &= 0.879(0.370d)^{-0.107} \\ K_{b\,[\text{LARGE RADIUS}]} &= 0.91(0.370d)^{-0.157} \end{aligned}}$$

FOR TEST CASE, $d = 2$ in

$K_{bs} = 0.879(0.370(2))^{-0.107}$

$K_{bs} = 0.9078$

MODIFYING FACTORS

$K_c \equiv$ LOADING M.F.

$$K_c = \begin{cases} 1 & \text{BENDING} \\ 0.85 & \text{AXIAL} \\ 0.59 & \text{TORSION} \end{cases}$$

FOR THE BENDING $\boxed{K_c = 1}$

TORSION WILL BE TAKEN CARE OF ON THE DEMAND SIDE.

$K_d \equiv$ TEMPERATURE M.F.

$$K_d = \frac{S_T}{S_{RT}}$$

$\boxed{K_d = 1}$ (TBL 6-4, ASSUMING T = 20°C)

$K_e \equiv$ RELIABILITY M.F.

$\boxed{K_e = 1 - 0.08 Z_a}$

$Z_a = 3.091$ FOR $Re = 99.9\%$ (TBL 6-5)

FOR TEST CASE:

$K_e = 1 - 0.08(3.091)$

$K_e = 0.75272$

≈ 0.753 (AGREES WITH TBL 6-5)

NOTCH SENSITIVITY, FACTOR K_f

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{OR} \quad q = \frac{K_{fs} - 1}{K_{ts} - 1}$$

$$K_f = q(K_t - 1) + 1 \quad \text{OR} \quad K_{fs} = q(K_{ts} - 1) + 1$$

K_t EQUATION FROM AMESWEB.INFO

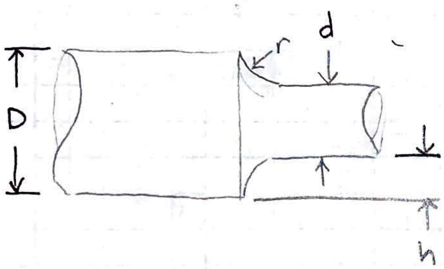
BENDING:

$$K_t = C_1 + C_2(2h/D) + C_3(2h/D)^2 + C_4(2h/D)^3$$

TORSION:

$$K_{t_s} = C_1 + C_2(2h/D) + C_3(2h/D)^2 + C_4(2h/D)^3$$

CONSTANTS DIFFER (SEE REFERENCE SHEET)



$$h = \frac{D-d}{2}$$

FOR $D = 2\text{ in}$, $\frac{D}{d} = 1.25$:

$$h = \frac{(2 - 1.6)\text{ in}}{2}$$

$$h = 0.2\text{ in}$$

$$\frac{r}{d} = 0.05$$

$$r = 0.05(1.6\text{ in})$$

$$r = 0.08\text{ in}$$

SAMPLE CALCS FOR NOTCH SENSITIVITY

$D = 2\,in$, $D/d = 1.25$, $d = 1.6\,in$, $h = 0.2\,in$, $\frac{r}{d} = 0.05$

BENDING CASE $\qquad r = 0.08\,in$

$$h/r = \frac{0.2\,in}{0.08\,in}$$

$$h/r = 2.5 \Rightarrow 2.0 \leq 2.5 \leq 20.0$$

$$C_1 = 1.232 + 0.832\sqrt{h/r} - 0.008\,h/r$$
$$= 1.232 + 0.832\sqrt{2.5} - 0.008(2.5)$$
$$= 2.528$$

$$C_2 = -3.813 + 0.968\sqrt{h/r} - 0.260\,h/r$$
$$= -3.813 + 0.968\sqrt{2.5} - 0.260(2.5)$$
$$= -2.932$$

$$C_3 = 7.423 - 4.868\sqrt{h/r} + 0.869\,h/r$$
$$= 7.423 - 4.868\sqrt{2.5} + 0.869(2.5)$$
$$= 1.899$$

$$C_4 = -3.839 + 3.070\sqrt{h/r} - 0.600\,h/r$$
$$= -3.839 + 3.070\sqrt{2.5} - 0.600(2.5)$$
$$= -0.485$$

$$K_t = C_1 + C_2(2h/D) + C_3(2h/D)^2 + C_4(2h/D)^3$$

$$\boxed{K_t = 2.014}$$

WHERE $2h/D = 2(0.2\,in/2\,in)$
$$= 0.2$$

SAMPLE CALCS FOR NOTCH SENSITIVITY

TORSION CASE

$0.25 \leq 2.5 \leq 4.0$

$C_1 = 0.905 + 0.783\sqrt{h/r} - 0.075\,h/r$

$= 0.905 + 0.783\sqrt{2.5} - 0.075(2.5)$

$= 2.042$

$C_2 = -0.437 - 1.969\sqrt{h/r} + 0.553\,h/r$

$= -0.437 - 1.969\sqrt{2.5} + 0.553(2.5)$

$= -2.168$

$C_3 = 1.557 + 1.073\sqrt{h/r} - 0.578\,h/r$

$= 1.557 + 1.073\sqrt{2.5} - 0.578(2.5)$

$= 1.809$

$C_4 = -1.061 + 0.171\sqrt{h/r} + 0.086\,h/r$

$= -1.061 + 0.17\sqrt{2.5} + 0.086(2.5)$

$= -0.577$

$K_{ts} = C_1 + C_2(2h/D) + C_3(2h/D)^2 + C_4(2h/D)^3$

$\boxed{K_{ts} = 1.676}$

VALUES FOR K_t AND K_{ts} ARE CONSISTENT WITH MATLAB CODE

SAMPLE CALCS FOR NOTCH SENSITIVITY

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$\sqrt{a}_{\text{BENDING}} = 0.246 - 3.08(10^{-3})S_{UT} + 1.51(10^{-5})S_{UT}^2 - 2.67(10^{-8})S_{UT}^3$$

WHERE $S_{UT} = 210$ ksi

$$\boxed{\sqrt{a}_{\text{BENDING}} = 0.0178}$$

$$\sqrt{a}_{\text{TORSION}} = 0.190 - 2.51(10^{-3})S_{UT} + 1.35(10^{-5})S_{UT}^2 - 2.67(10^{-8})S_{UT}^3$$

$$\boxed{\sqrt{a}_{\text{TORSION}} = 0.0110}$$

$$q = \frac{1}{1 + \frac{\sqrt{a}_{\text{BENDING}}}{\sqrt{r}}}$$

$$= \frac{1}{1 + \frac{0.0178}{\sqrt{0.08}}}$$

$$= 0.9408$$

$$K_f = q(K_t - 1) + 1$$

$$= 0.9408(2.014 - 1) + 1$$

$$\boxed{K_f = 1.954}$$

$$q_s = \frac{1}{1 + \frac{\sqrt{a}_{\text{TORSION}}}{\sqrt{r}}}$$

$$= \frac{1}{1 + \frac{0.0110}{\sqrt{0.08}}}$$

$$= 0.9626$$

$$K_{fs} = q_s(K_{ts} - 1) + 1$$

$$= 0.9626(1.676 - 1) + 1$$

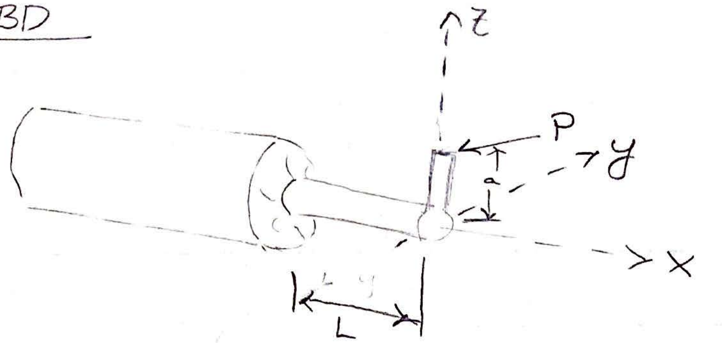
$$\boxed{K_{fs} = 1.651}$$

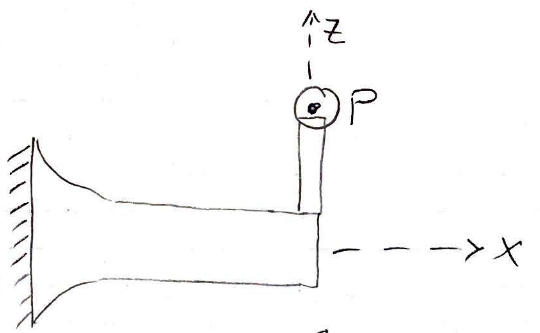
$$S_e' = \begin{cases} 0.5\, S_{UT} & S_{UT} \leq 200 \text{ ksi} \\ 100 \text{ ksi} & S_{UT} > 200 \text{ ksi} \end{cases}$$

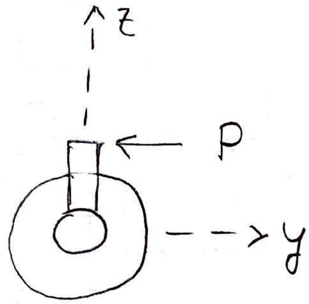
IN THIS CASE, $S_{UT} = 210$ ksi

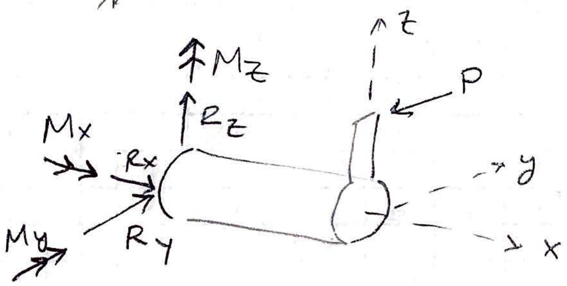
$$\boxed{S_e' = 100 \text{ ksi}}$$

FBD









$$\circlearrowleft \Sigma M_x = M_x + P(a) = 0$$

$$M_x = -Pa \quad \leftarrow \text{TORSION}$$

$$\circlearrowleft \Sigma M_z = M_z - P(L) = 0$$

$$M_z = P(L) \quad \leftarrow \text{BENDING}$$

$$\sigma = \frac{Mc}{I}$$

$$= \frac{P(L)\left(\frac{d}{2}\right)}{I}$$

$$\tau = \frac{T\left(\frac{d}{2}\right)}{J}$$

$$I_{SMALL} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4$$

$$J_{SMALL} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4$$

$$T = P(a)$$

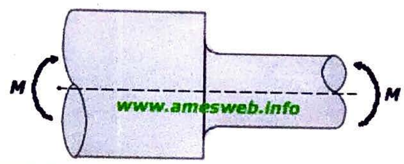
$$T_a = (200\,lb)(8\,in)$$
$$= 1600\,lb\,in$$

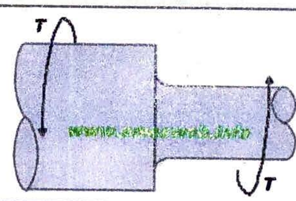
$$T_m = (800\,lb)(8\,in)$$
$$= 6400\,lb\,in$$

$$M_a = (200\,lb)(15\,in)$$
$$= 3000\,lb\,in$$

$$M_m = 800\,lb\,(15\,in)$$
$$= 12{,}000\,lb\,in$$

$$\sigma_a = M_a\,(d/2)^4 \qquad \text{FOR } d = 2\,in$$

Bending	
	
$0.1 \le h/r \le 2.0$	$2.0 \le h/r \le 20.0$
$C1 = 0.947 + 1.206\sqrt{h/r} - 0.131h/r$	$C1 = 1.232 + 0.832\sqrt{h/r} - 0.008h/r$
$C2 = 0.022 - 3.405\sqrt{h/r} + 0.915h/r$	$C2 = -3.813 + 0.968\sqrt{h/r} - 0.260h/r$
$C3 = 0.869 + 1.777\sqrt{h/r} - 0.555h/r$	$C3 = 7.423 - 4.868\sqrt{h/r} + 0.869h/r$
$C4 = -0.810 + 0.422\sqrt{h/r} - 0.260h/r$	$C4 = -3.839 + 3.070\sqrt{h/r} - 0.600h/r$
$Kt = C1 + C2(2h/D) + C3(2h/D)^2 + C4(2h/D)^3$	
$\sigma_{nom} = 32M/\pi d^3$	
$\sigma_{max} = Kt\sigma_{nom}$	

Torsion

$0.25 \le h/r \le 4.0$
$C_1 = 0.905 + 0.783\sqrt{h/r} - 0.075h/r$
$C_2 = -0.437 - 1.969\sqrt{h/r} + 0.553h/r$
$C_3 = 1.557 + 1.073\sqrt{h/r} - 0.578h/r$
$C_4 = -1.061 + 0.171\sqrt{h/r} + 0.086h/r$
$Kt = C1 + C2(2h/D) + C3(2h/D)^2 + C4(2h/D)^3$
$\tau_{nom} = 16T/\pi d^3$
$\tau_{max} = Kt\tau_{nom}$