Physics 22: Homework 8

The following problems deal with magnetic field calculations using the Biot-Savart Law, magnetic dipoles, magnetic forces and torques, as well as the theory and applications of Ampere's Law.

1. An electron and a proton are each moving at 300 km/s in perpendicular paths at the instant shown in Figure 1. The coordinate system adopted in the figure at this instant has the electron moving along the $-x$-direction positioned at $(x_e, y_e) = (0, b) = (0, 5.00\text{ nm})$, and has the proton moving along the $-y$-direction positioned at $(x_p, y_p) = (a, 0) = (4.00\text{ nm}, 0)$. Note that the speed of each of these particles is much, much smaller than the speed of light, $c = 3.0 \times 10^5$ km/s.

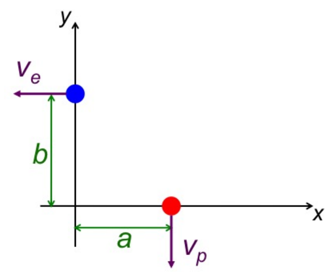

Figure 1: An electron and a proton are moving in orthogonal directions at some instant. They will not only exert a magnetic force on one another, but also an electric force.

   (a) Determine the total magnetic field (magnitude and direction) that these charges produce at this instant at the origin of the xy-plane.

   (b) Determine the magnetic field (magnitude and direction) that the electron produces at the location of the proton.

   (c) Determine the Lorentz force (magnitude and direction) that the electron exerts on the proton.

   (d) Show that Newton's 3$^{\text{rd}}$ Law is not satisfied for this problem by determining the Lorentz force (magnitude and direction) that the proton exerts on the electron, and comparing it to the result of Part (c).

2. An electron is moving in the vicinity of a very long, straight wire that is placed on the x-axis of an xyz-coordinate space. The wire runs a current $I = 9.00$ A in the $-x$-direction. At an instant when the electron is at point $(x_e, y_e, z_e) = (0, 0.200\text{ m}, 0)$ and the electron's velocity is $\vec{v}_e = (50.0\text{ km/s})\,\hat{x} - (30.0\text{ km/s})\,\hat{y}$, determine the magnetic force (magnitude and direction) on this electron due to the wire at this instant.

3. Four long, parallel wires occupy the corners of a square of side length ℓ and carry identical currents, each of magnitude I. As in Figure 2, wires 1 and 2 have the currents flowing into the page, while wires 3 and 4 have the currents flowing out of the page.

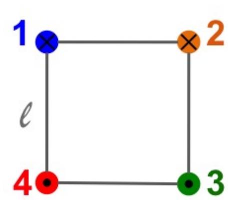

Figure 2: Long, straight wires are arranged parallel to one another at the corners of a square of length ℓ. The wires carry currents and, thus, generate magnetic fields.

   (a) Find the magnetic field at the center of the square.

   (b) Find the magnetic field at the location of wire 4 due to the other wires.

   (c) Find the force *per unit length* on wire 4.

   (d) Repeat Parts (a) - (c) if the direction of the current running through wires 2 and 3 was flipped.

4. Consider the structure in Figure 3, which consists of two conducting rods, each of length $\ell = 95$ cm. The bottom rod is rigidly attached to vertical conducting posts. The top rod has conductive sliders on either end that make

electrical contact with the vertical posts and is, thus, able to move freely (i.e., with minimal friction) along the vertical posts in such a way as to maintain its horizontal nature. A battery (not pictured) produces a current that flows in a rectangular loop, notably making it so that the current flows in opposite directions between the two horizontal portions of the assembly. Suppose that the sliding rod has a mass of $m = 22$ g, and that the current flowing in the assembly is $I = 66$ A. Assuming the top and bottom rods are in fairly close proximity throughout this process, determine the height at which the upper wire will be in equilibrium.

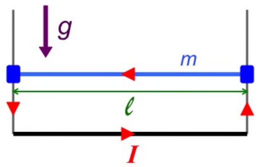

Figure 3: A closed loop of current runs through a conductive rod, of mass m and length ℓ, that can adjust its height away from the fixed rod below it via conductive sliders on vertical conductive posts. The sliding rod is seen to be in equilibrium.

5. Figure 4 shows a straight segment of wire, of length ℓ, carrying a current I from left to right that is fed by other wires that are not shown. Consider the points A and B shown, where A lies on a perpendicular bisector of the segment at a distance, s, away from the segment, and where B lies also a perpendicular distance, s, from the segment but at a point that is at the edge of the segment.

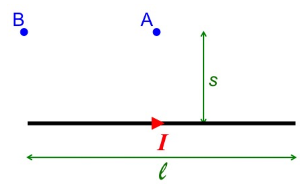

Figure 4: A current runs from left to right within a straight segment of wire. The magnetic field of this straight segment is to be determined at the points A and B.

   (a) Determine the magnetic field at these two points.
   (b) Show that if point A is brought very, very close to the wire segment (i.e., $s \ll \ell$) that the resulting field is the same as that of an infinitely long wire.

6. A ring, of radius R, carries a total charge Q and is rotating about its axis of symmetry with angular frequency ω. The rotating ring creates the effect of a current loop.

   (a) Determine the value of this current.
   (b) Determine the magnetic dipole moment of this ring.

7. A long, straight wire carries a current $I_1 = 20$ A. As shown in Figure 5, a rectangular wire loop—of length $\ell = 10$ cm and width $w = 5.0$ cm—has its length parallel to the long wire, with its nearest side a distance $a = 2.0$ cm away from the long wire. This rectangular loop also runs a current of $I_2 = 500$ mA in an Anti-Clock-Wise (ACW) fashion.

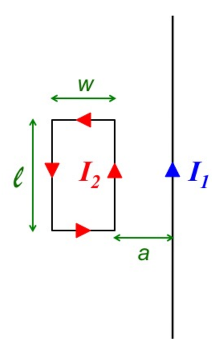

Figure 5: A long straight wire and a rectangular loop of wire both carry current. The net magnetic force between the wires is desired.

(a) Determine the net magnetic force (magnitude and direction) on the rectangular loop.

(b) Discuss what happens if one of the currents (I_1 or I_2) reverses direction.

8. Consider two very long wires that perpendicularly intersect with some insulating boundary between them at the point of intersection. Suppose that the vertically oriented wire in Figure 6 carries a current I_1 upward, while the horizontally oriented wire carries a current I_2 rightward.

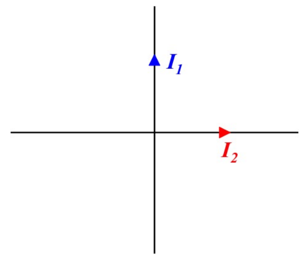

Figure 6: Perpendicular wires carry their own respective currents without any leakage of current from one wire to the other due to an insulating barrier between the wires where they intersect.

(a) If $I_1 = I_2 = I$, describe the magnetic field in each of the quadrants formed by these wires.

(b) If $I_2 = 2I_1$, determine at what points, if any, the magnetic field is zero.

9. As in Figure 7, a long, flat conducting ribbon, of width w, has its length parallel to a long, straight wire, with its nearest edge a distance a away from the wire. Suppose that the currents running through the wire and the ribbon are in the same direction and of the same magnitude ($I_1 = I_2 = I$). Moreover, suppose that the current is distributed uniformly over the ribbon.

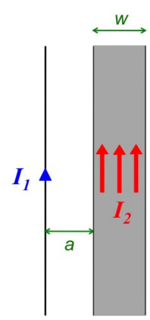

Figure 7: A very long, straight, thin ribbon of wire is parallel to a very long, straight, thin wire. Both carry currents in the same direction, and the net magnetic force per unit length between the conductors is desired.

(a) In terms of the given quantities, determine the force *per unit length* between the two conductors.

(b) Suppose the ribbon is replaced with a single wire running exactly the same current. Where would this replacement wire have to placed so that the force between these wires would be identical to the force between the original wire and the ribbon?

10. As in Figure 8, consider two identical circular loops, each of radius R and each carrying the same current I. Suppose they share the same axis of symmetry and are separated by a distance d.

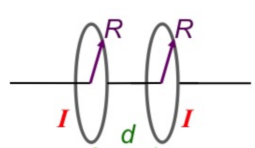

Figure 8: Two conducting rings share the same axis of symmetry and are separated by a distance d. The rings run identical currents. The net magnetic field of the two-ring system is desired at various locations of interest.

(a) If the current in each loop flows in the same direction, determine the magnetic field (magnitude and direction) at

i. the center of either loop.

ii. the midpoint between the loops.

(b) If the current in each loop flows in opposition to the other, determine the magnetic field (magnitude and direction) at

i. the center of either loop.

ii. the midpoint between the loops.

11. A very long wire has a part of it bent into a quarter circle of radius a connected to another quarter circle of radius $b > a$ via a radial segment extending from a to b, as shown in Figure 9. A current, I, flows through the entire assembly. Determine the magnetic field (magnitude and direction) at point P, which lies along the continuation of the straight wire when bypassing the semicircular region, but also serves as the common center of the circles that would be drawn for each of the quarter circles described.

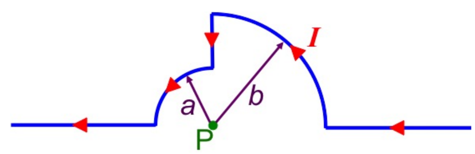

Figure 9: A very long wire, at some region, is bent into two quarter circles, which are connected by a radial line. The wire carries a current I. The magnetic field due to this assembly is to be determined at point P, which serves as the common center of these quarter circles.

12. A conducting semi-annular loop is formed from two concentric semicircles—of radii a and $b > a$—that are connected via radial lines that connect the two semicircles, as shown in Figure 10. The loop carries a current I. Determine the magnetic field (magnitude and direction) at point P, which serves as the common center of the circles that would be drawn for each of the semicircles shown.

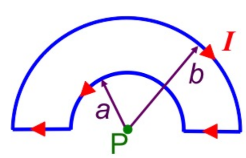

Figure 10: A wire is bent into two concentric semicircles connected via radial segments. Given that the wire carries a current I, the magnetic field at point P is to be determined, which serves as the common center to these concentric semicircles.

13. Consider a thin, hollow conducting pipe of radius R and length ℓ. This pipe carries a uniform current, I, that flows circumferentially, as shown in Figure 11. Suppose the pipe is placed with the z axis serving as its axis of symmetry, and also with the xy-plane cutting the pipe in half length-wise. Think of this pipe as a stack of infinitesimally thin hoops supporting an infinitesimal current that flows along the circumference of the hoop.

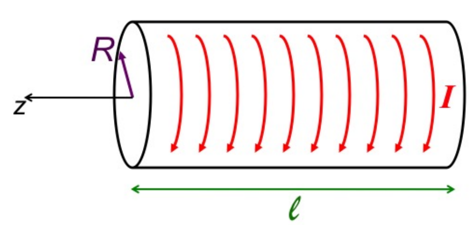

Figure 11: A hollow cylinder has current running circumferentially and uniformly across the length of the cylindrical surface.

(a) Determine the magnetic field of this assembly for $|z| > \ell/2$.

(b) Determine the magnetic field of this assembly for $|z| \leq \ell/2$.

14. A thin insulating disk, of radius R, carries a uniform surface charge density σ. This disk is spinning at constant angular speed ω about its axis of symmetry, which is taken to be the z-axis.

    (a) Determine the magnetic field at the disk's center (i.e., at $z = 0$).

    (b) Determine the magnetic field at any point, z, along the disk's axis of symmetry.

    (c) Determine the magnetic dipole moment of this disk.

15. Demonstrate the premise of Ampere's Law,

$$\oint_{\partial\mathcal{S}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}},$$

by determining the circulation of the magnetic field of a long, straight wire (with a current, I, going into the page) about the closed paths $\partial\mathcal{S}_1$, $\partial\mathcal{S}_2$, and $\partial\mathcal{S}_3$ shown in Figure 12. Path $\partial\mathcal{S}_1$ is a circular path of radius a with an Anti-Clock-Wise (ACW) orientation. Path $\partial\mathcal{S}_2$ consists of a Clock-Wise (CW) movement through an angle θ along a circle of radius a, a radially outward movement from radius a to $b > a$ an ACW movement through angle θ along a circle of radius b, and a radially inward movement from radius b to a. Path $\partial\mathcal{S}_3$ consists of a CW movement through an angle π along a circle of radius b, followed by a radial movement inward from radius b to a, leading to another CW movement through an angle π along a circle of radius a, followed by a radial movement outward from radius a to b.

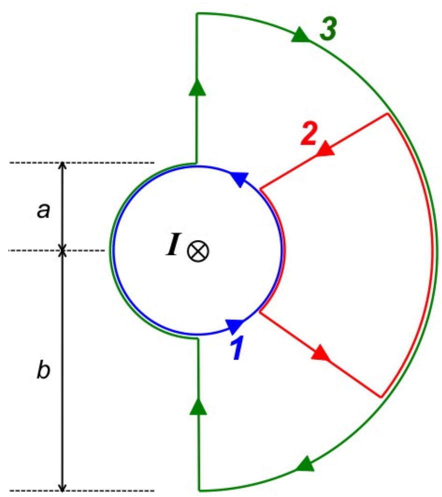

Figure 12: A very long, straight wire carries a current I into the page. The various closed loops $\partial\mathcal{S}_1$, $\partial\mathcal{S}_2$, and $\partial\mathcal{S}_3$ are drawn to help illustrate the premise of Ampere's Law in this simplest of cases.

16. Consider thin, straight conducting wires in the cross-sectional view shown in Figure 13. The current $I_1 = 2I$ is coming out of the page, current $I_2 = I$ is also coming out of the page, while current $I_3 = 2I$ is going into the page. Determine the circulation of the magnetic field (magnitude and direction) around each of the closed loops A, B, C, and D shown in Figure 13.

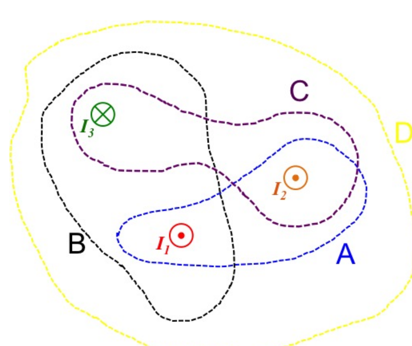

Figure 13: A variety of very long, straight wires are running a current perpendicular to the page. The magnetic-field circulation is to be determined around the loops A, B, C, and D that have been drawn.

17. Consider a magnetic field with a gradient in its strength along y but with a direction that consistently points along the $+x$ direction. In particular, the field profile has the form

$$\vec{B}(y) = (\alpha + \beta y)\,\hat{x},$$

with α and β as positive constants. Consider also an imaginary rectangular loop, which lives in Quadrant I of the xy-plane. As shown in Figure 14, the $+x$-axis serves as the boundary of the bottom side of the loop, which has length ℓ, while the $+y$-axis serves as the boundary of the left side of the loop, which has length h. Determine the total current (magnitude and direction) that must be flowing through the cross-section of the rectangular loop.

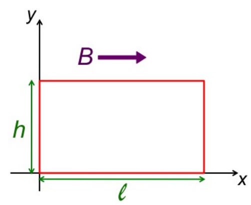

Figure 14: An imaginary rectangular loop is placed in Quadrant I of the xy-plane, with one of its corners at the origin of coordinates. There is a nonuniform magnetic field pointing consistently in the $+x$-direction, with a linear gradient along the y-axis. The total current piercing the rectangular loop is to be determined based on the field's circulation about this loop.

18. A very long solenoid consists of a circular cross-section, of radius a, has a coil density n, and carries a current I. Determine the magnetic field of this solenoid both inside and outside the solenoid.

19. A toroidal coil, of inner radius $R_1 = 15$ cm and outer radius $R_2 = 17$ cm, is wound from $N = 1200$ turns of wire.

    (a) Show that the magnetic field inside the hole of the toroid (i.e., thought as the hole of a donut) or outside the toroid is zero.

    (b) Determine the minimum and maximum strengths of the magnetic field when the toroid carries a current $I = 10$ A.

20. Figure 15 shows a coaxial cable—looking directly along the symmetry axis—that consists of a solid inner conductor, of radius a, and a concentric cylindrical conducting shell of inner radius $b > a$ and outer radius $c > b$. The solid conductor and the conducting shell carry equal but opposite currents, the total current magnitude in each being denoted as I and being presumed to be uniformly distributed in each of the conducting regions.

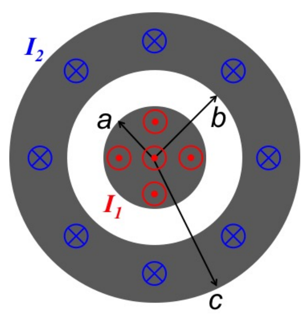

Figure 15: A coaxial cable—consisting of a solid inner conducting cylinder and a concentric outer conducting shell—runs equal but opposite currents along the inner and outer conducting regions. The magnetic field of this configuration is to be determined everywhere in space.

    (a) Determine the magnetic field everywhere (i.e., for $r < a$, $a < r < b$, $b < r < c$, and $r > c$).

    (b) Sketch a graph of the magnetic field of this configuration as a function of the radial position, r.

    (c) Repeat Parts (a) and (b) assuming that the two conductors have counter-propagating currents, but that the current in the inner conductor is twice the current in the outer conductor.

21. A long conducting rod, of radius R, carries a total current I that is non-uniform across the cross-section of the rod. In particular, the current density varies linearly with the radial distance, r, from the rod's axis of symmetry,

$$J(r) = J_0 \frac{r}{R},$$

where J_0 is a positive constant to be determined.

(a) In terms of the given quantities, obtain an expression for J_0.

(b) Determine the magnetic field everywhere (i.e., for $r < R$ and for $r \geq R$).

(c) Sketch a graph of the magnetic field of this configuration as a function of the radial position, r.

22. A conducting slab extends infinitely along the $x$and y directions of an xyz-coordinate system, and has a nonzero thickness h along the z-direction. The slab supports a uniform current density, $\vec{J}$, that flows in the $+x$ direction, as shown in Figure 16. Determine the magnetic field (magnitude and direction) everywhere (i.e., at any point inside or outside the slab).

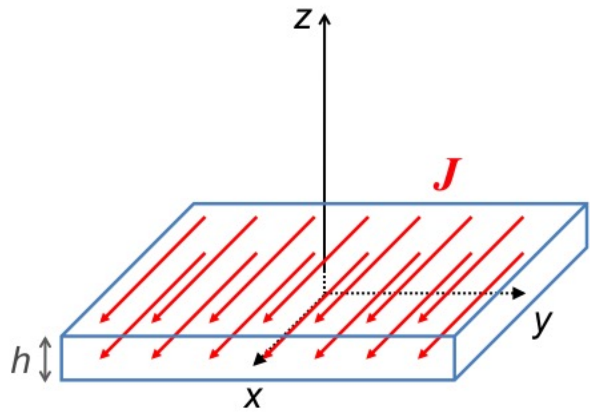

Figure 16: A slab of current has finite thickness but is otherwise infinite. Being a conducting slab, there is a uniform current density, $\vec{J}$, running through the cross section of the slab along the $+x$ direction.