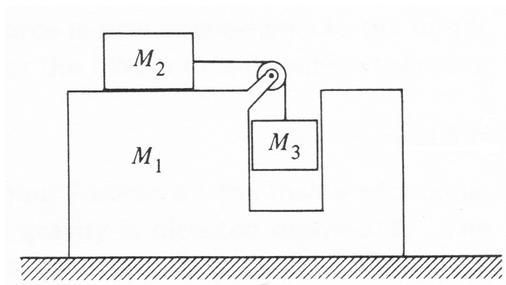
Physics 1A
Spring 2015

Challenge Problem 8

Determine the acceleration of mass M_1 relative to the ground in the following machine. All surfaces are frictionless and the pulley and rope are massless.



Solution. Applying Newton's second Law in the x-direction for all three masses and also in the y-direction for mass 3 gives the following system of equations:

$$T = M_2 a_2, \qquad -N - T = M_1 a, \qquad N = M_3 a, \qquad T - M_3 g = M_3 a_3 \quad (1)$$

Where I have already invoked the constraint that the accelerations of masses 1 and 3 in the x-direction are the same, and I called their common acceleration a. I also abbreviated $a_{2,x} = a_2$ and $a_{3,y} = a_3$. The term N represents the potential force of contact between masses 1 and 3, and I have already invoked Newton's Third Law by including N in the x-equation for mass 1 and $-N$ in the x-equation for mass 3 (we're not certain if N is positive or negative quite yet, but the equations will tell us in the end.)

At this point we have a system of 4 equations and 5 unknowns T, a_2, N, a, a_3; we need another equation. We have exhausted all dynamical information (namely information coming from the laws of motion), so there must be a constraint we're missing. Indeed, there is such a constraint, and it comes from the fact that the length of the rope is constant. This constraint can be used to determine a constraint relating the accelerations, which turns out to be

$$a = a_2 + a_3 \qquad (2)$$

Now we have five equations and five unknowns, and we can solve (which I leave to you) to give

$$\boxed{a = -\left(\frac{M_2 M_3}{M_1 M_2 + M_1 M_3 + 2M_2 M_3 + M_3^2}\right) g} \tag{3}$$