Physics 22: Homework 7

The following problems deal with magnetic forces on moving point charges, current-carrying wires, and wire loops, as well as magnetic torques on wire loops and magnets.

1. A uniform magnetic field, $\vec{B} = B_0\hat{x}$, has strength $B_0 = 0.10$ T. A charged particle, of charge $q = 1.0$ μC, enters the field region moving at a speed $v = 20$ m/s. Determine the magnetic force (magnetic and direction) on the charged particle at the instants described in the following scenarios.

   (a) The charged particle enters the field region along the $+x$-axis.

   (b) The charged particle enters the field region along the $+y$-axis.

   (c) The charged particle enters the field region along the $+z$-axis.

   (d) The charged particle enters the field in the xy-plane, traveling along a 45°-line relative to the x-axis in the positive sense along both the x- and y-axes.

2. Prove that there is no magnetic work done on a moving charged particle by a magnetic field.

3. Separately consider an electron and a proton moving in a plane that is at right angles to a uniform magnetic field, such that both particles have identical kinetic energies.

   (a) How do their orbital radii compare?

   (b) Consider an arbitrary charged particle, of mass m and charge q, moving with kinetic energy, K, in a plane that is at right angles to a uniform magnetic field, of strength B. Obtain an expression for the orbital radius of this moving charged particle as a function of the given quantities.

4. A particle, of charge $q = 7.80$ μC is moving with velocity $\vec{v} = -\left(3.50 \times 10^3 \text{ m/s}\right)\hat{y}$ as it enters a region of uniform magnetic field. At the instant it enters this region, the magnetic force on the charged particle is measured to be $\vec{F}_B = +(7.60 \text{ mN})\,\hat{x} - (5.20 \text{ mN})\,\hat{z}$.

   (a) Calculate the components of the magnetic field that you can calculate from the provided information.

   (b) Are there components of the magnetic field that cannot be determined from the information provided? Explain.

   (c) Calculate the dot product $\vec{B} \cdot \vec{F}_B$.

5. The general motion of a charged particle in a uniform magnetic field is helical, as shown in Figure 1. The basic features of the helix are its radius, r, and its "pitch," p. The pitch is defined as the distance between successive turns in the helix (i.e., the distance between identical points on adjacent coils representing the helix).

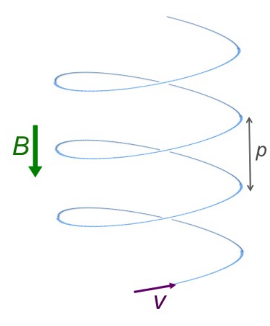

Figure 1: The helical trajectory of a positively charged particle with an initial velocity that is neither perpendicular nor parallel to the uniform external magnetic field.

   (a) Provide a detailed explanation as to why the most general motion of such a moving charged particle in a uniform magnetic field is helical.

   (b) Consider an electron moving at a speed $v = 3.8 \times 10^6$ m/s that enters a region containing a uniform magnetic field of strength $B = 18$ mT. As the electron enters this region, its velocity makes an angle of $\phi = 70°$ relative to the field direction when the field and velocity vectors are placed with their tails at a common vertex.

i. Draw the sense in which the helical trajectory will be traversed by this electron using a well-constructed diagram.

ii. Determine the radius and pitch of the helix generated by the movement of this electron.

6. One commonly used device is a so-called "velocity selector." As in Figure 2, it consists of a set of charged parallel plates that produce a uniform electric field, $\vec{E}$, in a region where an additional uniform magnetic field, $\vec{B}$, is oriented orthogonally to the plane of the page and is, therefore, at right angles to the electric field between the plates. When a beam of charged particles enters this region, a subset of the particles within the beam will emerge undeflected if that subset has the correct velocity. Suppose, in cross-section, the upper plate of the parallel-plate configuration is charged positively, and, therefore, the lower plate is charged negatively.

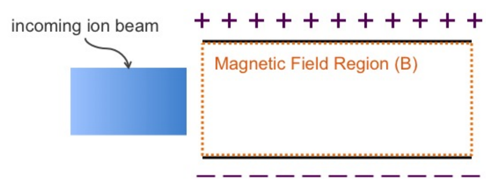

Figure 2: This is a schematic of a simplified velocity selector, showcasing its essential operating principles. A beam of charged particles with a distribution of velocities (both in direction and in magnitude) arrives at parallel-plate assembly with a equally but oppositely charged plates. A uniform magnetic field also exists within the region defined by parallel-plate assembly that is oriented perpendicular to the page.

(a) In what direction must the external magnetic field, $\vec{B}$, be in order to allow for the possibility that emerging particles from the beam can potentially pass through undeflected? Explain.

(b) Given the orientation of $\vec{B}$ determined in Part (a), determine—relative to the diagram in Figure 2—the velocity, $\vec{v}$, a particle within the beam entering the crossed-field region on the left of the plates must have in order to emerge undeflected when exiting the plates on the right side.

(c) What happens to particles that have a velocity with its speed higher or lower than the speed from Part (b)? Thus, explain why this device is called a velocity selector.

7. Another commonly used device is the mass spectrometer, of which a simple rendition is shown in Figure 3. This apparatus is designed to analyze and separate atomic and/or molecular ions with different charge-to-mass ratios. In the design shown in Figure 3, ions are accelerated through a potential difference V in a cathode-anode assembly (with the ions presumed to be positively charged, or cations), after which these ions enter a well-defined region containing a uniform magnetic field, of strength B, that is oriented perpendicular to the velocity of the cations as they enter the region through a collimator. The ions move in the circular trajectory shown in Figure 3, after which they hit a plate.

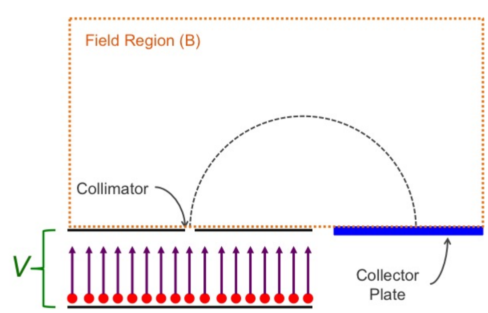

Figure 3: This is a schematic of a simplified mass spectrometer, showcasing its essential operating principles. Positive ions are boiled off an anode and made to accelerate toward a cathode with a small hole to collimate the beam. Upon exiting the anode-cathode assembly, the charged ions immediately enter a uniform magnetic field region that is directed perpendicular to the plane of the page. The ions deflect and eventually hit a collector plate, which is able to count the number of bombarding ions as a function of the target position on the plate.

(a) In what direction must the perpendicularly oriented magnetic field in Figure 3 be in order to ensure that the particles move along the trajectory shown in this figure?

(b) Suppose that the cations entering the field region have some mass m and charge q. Determine the distance between the collimator and the point of bombardment of these cations.

(c) Consider the fact that the target plate into which the cations collide is calibrated to essentially count the number of ions accumulated as a function of where the cations hit the plate. Given this fact, why is this device called a mass spectrometer?

8. The Hall effect exploits the fact that moving charges in a plane that is perpendicular to a uniform field get deflected via the magnetic force; in particular, it applies this result to the movement of a current in a conductor, showcasing that there is a clear contradiction in how the charges get deflected based on which charges are the ones actually moving. Based on Figure 4, a conventional current is running through some conducting slab of thickness, t, and depth, w, so that the cross-sectional area is the area of the plane that is perpendicular to the line defined by the drift velocity and is given by $A = wt$. There is a uniform magnetic field oriented upward, of strength B. Since we have previously discussed the invariance in analyzing circuits as far as whether we choose to look at the conventional or the electron current, notice that this no longer holds true under these circumstances. In particular, Figure 4 shows that if the charge carriers are positive, then these charges will deflect to the left side of the plate, making the left side positively charged and, thus, making the right side negatively charged; however, if the charge carriers are negative, then the charges will also deflect to the left side of the plate, making the left side negatively charged and, thus, making the right side positively charged. Thus, there is an apparent asymmetry depending on which current we use to analyze the circuit. Nonetheless, the correct answer—first discovered by Edwin Hall in 1879—is that the charge carriers are negative. Later, it is discovered that these charge carriers are, in fact, electrons. (Indeed, the prediction of the existence of the electron did not happen until the cathode-ray experiments of J. J. Thompson in 1897, and the charge and mass of the electron were not separately measured until the oil-drop experiment by Robert Millikan and Harvey Fletcher in 1909.)

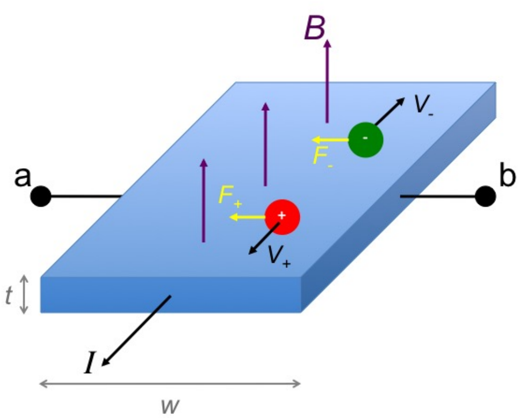

Figure 4: This is a schematic showcasing the Hall effect. The conducting slab has a conventional current flowing towards the front of the slab and is immersed in a uniform magnetic field directed upward. Based on which current one chooses to analyze (conventional vs electron current) the magnetic force on these charge carriers aims to make these drifting carriers of opposite charge drift in the same direction. A transverse potential difference is maintained with a steady current being supplied to the slab by a power supply (not pictured). The transverse potential is known as the Hall potential, and the specific potential difference between the terminals a and b—which are used to measure this Hall potential—determines the sign of the charge carriers once and for all. In particular, it is found that the charge carriers are, indeed, negatively charged.

(a) Given what actually happens, what sign is the potential difference $V_{\text{ab}} \equiv V_{\text{a}} - V_b$ between the leads a and b shown in Figure 4?

(b) One can imagine that if the current running through the conductor is steady, then the size of the potential difference $\Delta V_{\text{Hall}} \equiv |V_{\text{ab}}|$ will also be time independent. Assume that all parts of the conductor are uniform, and that the edge effects of the conductor may be ignored. Assuming the electron concentration in the conducting slab is n and that the magnitude of the charge of an electron is taken as e, determine this Hall voltage in terms of the given quantities.

9. A rectangular loop, of width a and length b, has mass m and is in the presence of the Earth's gravitational field, $\vec{g} = g\hat{x}$. As shown in Figure 5, the loop is pivoted from one of its ends about the z-axis in a region with a uniform magnetic field that is known to be along the vertical x-axis. When a current, I, runs through this loop (in the orientation shown in Figure 5), it rises to make an angle ϕ relative to the x-axis.

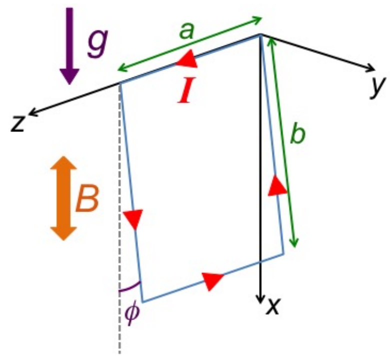

Figure 5: A rectangular loop is pivoted about the z-axis of a three-dimensional coordinate system. The current-carrying loop is seen to move through an angle ϕ in the presence of a magnetic field along the x-axis. The loop is currently in equilibrium.

(a) Determine the direction of the magnetic field.

(b) Determine the strength of the field in terms of the given quantities.

(c) It is also possible to make the loop rise in the same way with a field directed, instead, along the y-axis. Determine how this field must be directed to achieve this outcome.

(d) Determine if the state of the loop is one of stable or unstable equilibrium in each of the cases above (i.e., for the case when the field is directed along the x-axis, as well as for the case when the field is directed along the y-axis).

10. A bar magnet experiences a torque of $\tau = 12 \times 10^{-3}$ N $\cdot$ m when it is oriented with an angle of $\phi = 55°$ relative to a uniform magnetic field of strength $B = 100$ mT.

(a) Determine the magnitude of the bar magnet's dipole moment.

(b) Determine the amount of work it took to rotate this bar magnet from its initial equilibrium position to this particular angle.

(c) Determine the amount of potential energy stored in the system in this state relative to zero.

(d) Is zero potential energy the lowest energy state of this bar magnet? Explain.

11. A wire carrying current $I = 1.5$ A passes through a region containing a uniform external magnetic field of strength $B = 45$ mT. As shown in Figure 6, the wire is perpendicular to the field (which is pointing into the page) and makes a quarter-circle turn, of radius $R = 21$ cm, as it passes through the field region. Find the magnetic force (magnitude and direction) on this section of wire due to this field.

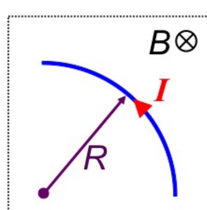

Figure 6: A circular segment of current-carrying wire—subtending an angle $\pi/4$—is in the plane of the page and immersed in a uniform magnetic field pointing into the page. The net magnetic force on the segment of wire.

12. Discuss the physical significance of Gauss's Law for magnetic fields:

$$\oint_{\partial \mathrm{V_G}} \vec{B} \cdot d\vec{A} = 0.$$