

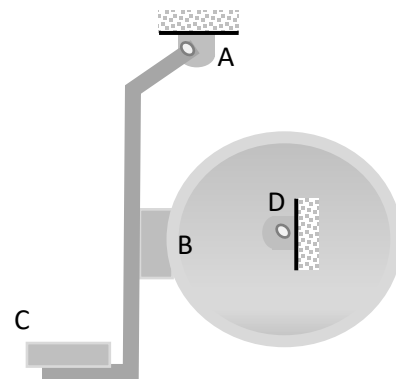
1)

The brake drum is attached to a larger flywheel that is not shown.

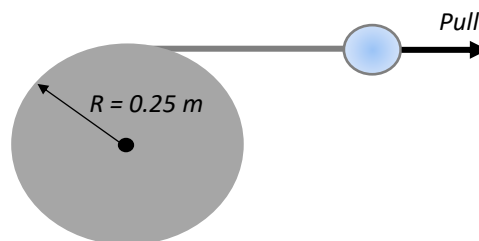
The motion of the brake drum is defined by the relation  $\theta = 36t - 1.6t^2$ , where  $\theta$  is expressed in radians and  $t$  in seconds. Determine

- the angular velocity at  $t = 2$  s.
- the number of revolutions executed by the brake drum before coming to rest.
- What is the angular acceleration of the drum at it coasts to stop.

Answer: a)  $\omega = 29.6 \text{ rad/s}$   
 b)  $\theta = 32.2 \text{ rev.}$   
 c)  $\alpha = -3.2 \text{ rad/s}^2$



to it through its center. It has very thin but strong string wrapped around its rim, and the string is attached to a ball that is pulled tangentially away from the rim of the disk. The pull increases in magnitude and produces an acceleration of the ball that obeys the equation  $a(t) = At$ , where  $t$  is in s and  $A$  is a constant. The cylinder starts from rest, and at the end of the third second, ball's acceleration is  $1.80 \text{ m/s}^2$ .



2)

- Find  $A$ .
- Express the angular velocity of the disk as a function of time.
- How long after the disk has begun to turn it reach an angular speed of  $15.0 \text{ rad/s}$ ?
- Through what angle has the disk turned as it reaches  $15.0 \text{ rad/s}$ ?

Answer: a)  $A = 0.60 \text{ m/s}^3$   
 b)  $\omega = 1.2t^2$   
 c)  $t = 3.54 \text{ s}$   
 d)  $\theta - \theta_0 = 17.7 \text{ rad/s}$  ?

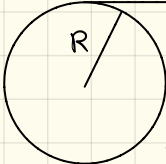
3)

The design of a 60.0 cm industrial turntable requires that it has a kinetic energy of 0.250 J when turning at 45.0 rpm.

- What must be the moment of inertia of the turntable about the rotation axis?
- If the turntable is in the shape of a uniform solid disk, what must be its mass?

Answer: a)  $I = 0.023 \text{ kg.m}^2$   
 b)  $M = 0.51 \text{ kg}$

- 1)  $\omega = 36t - 1.6t^2$   $\Theta$  (rad)  $t$  (s)
- a) find  $\omega$  @  $t = 2$  seconds  $\omega = \frac{d\Theta}{dt} = 36 - 3.2t \Rightarrow \omega(2) = 29.6 \text{ rad/s}$
- b) find number of revs until  $\omega = 0$  c) find  $\alpha$  as it comes to a stop
- $$\omega_f^2 = \omega_o^2 + 2\alpha(\Theta - \Theta_o) \left\{ \alpha = \frac{d\omega}{dt} = -3.2 \text{ rad/s}^2 \right\} - \frac{\omega_o^2}{2\alpha} = \Delta\Theta = 202.5 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 32.2 \text{ rev}$$

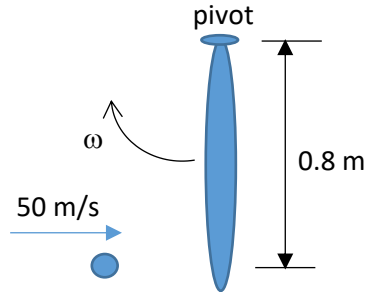
- 2)   $\alpha(t) = A + \omega_o = 0$   $\alpha(3) = 1.80 \text{ m/s}^2$   
 $R = 0.25 \text{ m}$
- a) find  $A \Rightarrow A + \omega = 1.80 \text{ m/s}^2 \Rightarrow A = \frac{1.80 \text{ m/s}^2}{3 \text{ s}} = 0.6 \text{ m/s}^3$
- b) find  $\omega(t) \Rightarrow \omega = \int \alpha dt = A \int dt = \frac{1}{2} A t^2 + C = 1.2 t^2$
- c) find  $\Delta t$  s.t.  $\omega(\Delta t) = 15.0 \text{ rad/s}$   
 $\omega(\Delta t) = 1.2 t^2 = 15.0 \text{ rad/s} \Rightarrow t = \sqrt{\frac{15.0 \text{ rad/s}}{1.2}} = 3.54 \text{ s}$
- d) find  $\Delta\Theta$  after  $\omega$  reaches  $15.0 \text{ rad/s}$   
 $\Theta = \int \omega dt = \int 1.2 t^2 dt = 0.4 t^3 + \Theta_o = 0$   
 $\Theta = 0.4 (15 \text{ s})^3 = 17.7 \text{ rad/s}$

- 3)  $r = 0.300 \text{ m}$  when  $\omega = 45.0 \text{ rpm}$ ,  $K = 0.250 \text{ J}$   $\left\{ 45 \times \frac{2\pi \text{ rad}}{60 \text{ s}} = 4.71 \text{ rad/s} \right\}$
- $I_{\text{disk}} = \frac{1}{2} m r^2$  a) find  $I$  of the disk  $\Rightarrow K = \frac{1}{2} I \omega^2 \Rightarrow I = \frac{2K}{\omega^2} = 0.023 \text{ kg} \cdot \text{m}^2$
- b) find mass of disk  $I = \frac{1}{2} m r^2 \Rightarrow m = \frac{2I}{r^2} = 0.511 \text{ kg}$

4)

A baseball of mass 0.15 kg is initially traveling horizontally at 50 m/s. It is struck by a bat, after which the baseball is still travelling horizontally but in exactly the opposite direction from its initial motion at a speed of 40 m/s. Consider the collision of the bat and the ball. Assume that before the collision, the bat is moving in a horizontal circle at an angular velocity of  $\omega$  rad/s. Assume that the player holding the bat exerts no torque. The bat has a moment of inertia of 0.3 kg.m<sup>2</sup> about the pivot and the ball hits at a point that is 80 cm away from the pivot. After the collision the bat is still swinging in the same direction around the same pivot but with a reduced angular velocity of  $0.35\omega$ . Find the numerical value of  $\omega$ .

Answer:  $\omega = 55.4 \text{ rad/s}$

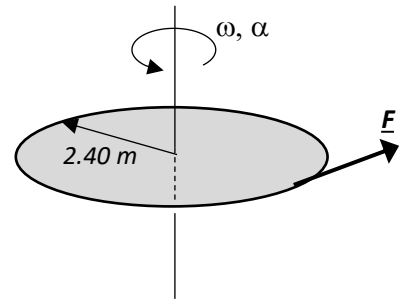


5)

A playground merry-go-round (MGR) has radius 2.40 m and moment of Inertia 2100 kg.m<sup>2</sup> about a vertical axis through its center, and it turns with negligible friction.

- A child applies an 18.0 N force tangentially to the edge of the MGR for 15.0 s. If the MGR is initially at rest, what is its angular speed after this 15.0 s interval?
- How much work did the child do on the MGR?
- What is the average power supplied by the child?

Answer: a)  $\omega = 0.31 \text{ rad/s}$   
 b)  $W = 100 \text{ J}$   
 c)  $P = 6.67 \text{ W}$



6)

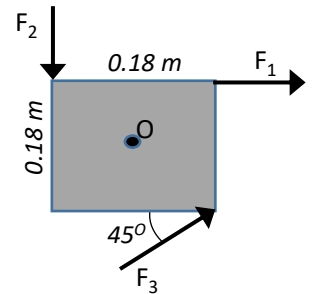
A square metal plate 0.180 m on each side is pivoted about an axis through point O at its center and perpendicular to the plate. Calculate the net torque about this axis due to the three forces shown if the magnitude of the forces are:

$$F_1 = 18.0 \text{ N}$$

$$F_2 = 26.0 \text{ N}$$

$$F_3 = 14.0 \text{ N}$$

The plate and all forces are in the plane of the page.



Answer:  $\tau_O = 2.5 \text{ N.m}$  ⤴

7)

Calculate the earth's gravity force on a 75-kg astronaut who is repairing the Hubble Space Telescope 600 km above earth's surface, and then compare this value with his weight at the earth's surface. In view of your result, explain why we say astronaut are weightless when they orbit the earth in satellite such as a space shuttle. Is it because the gravitational pull of the earth is negligibly small?

Data

$$m_E = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2$$

$$R_E = 6.38 \times 10^6 \text{ m}$$

$$r = 6.38 \times 10^6 + 6 \times 10^5 = 6.98 \times 10^6 \text{ m}$$

Answer:  $w = 735 \text{ N}$

4) horizontal, no  $u_g$

find  $W$   $\left\{ m = 0.15 \text{ kg} \right\} \left\{ v_0 = 50 \text{ m/s} \right\} \left\{ v_1 = 40 \text{ m/s} \right\} d = 0.8 \text{ m}$   
 $I = 0.3 \text{ kg} \cdot \text{m}^2$

// cons. of  $L$  gives

$$dmv_0 + Iw = -dmv_1 + I(0.35w) \Rightarrow dm(v_0 + v_1) = 0.35Iw - Iw \Rightarrow w = -\frac{dm(v_0 + v_1)}{0.65I} = 55.4 \text{ rad/s}$$

? why negative?

$\frac{m \text{ kg} \cdot \text{m/s}}{\text{kg} \cdot \text{m}^2} = \frac{1}{\text{m}} \cdot \frac{\text{m}^2}{\text{s}} = \frac{1}{\text{s}} = \text{rad/s}$  ✓

5)  $R = 2.4 \text{ m}$   $\left\{ I = 2100 \text{ kg} \cdot \text{m}^2 \right\}$  // no  $f_k$

a)  $F = 18.0 \text{ N}$ , applied for  $\Delta t = 15 \text{ s}$ ,  $w_0 = 0$  find  $w(15)$   
 $\Sigma \tau = FR = I\alpha \Rightarrow \alpha = \frac{FR}{I} = 0.02057 \text{ rad/s}^2$   
 $w(t) = w_0 + \alpha t \Rightarrow w(15) = 0.02057 \text{ rad/s}^2 \times 15 \text{ s} = 0.309 \text{ rad/s} = 0.31 \text{ rad/s}$

b) how much work did child do on MGR

**REVIEW POWER**  
 instantaneous = ?  
 $W = \tau \theta = I \alpha \theta = \Delta K$

c) what is the average power supplied by the child

$$P_{Av} = \frac{W}{\Delta t} = \frac{100 \text{ J}}{15 \text{ s}} = 6.67 \text{ W}$$

6)  $F_2$   $\left\{ F_1 = 18.0 \text{ N} \right\} \left\{ F_2 = 26.0 \text{ N} \right\} \left\{ F_3 = 14.0 \text{ N} \right\}$   
 $s = 0.18 \text{ m}$  calculate  $\Sigma \tau_o$

$$\Sigma \tau = r(F_3 + F_2 \cos 45^\circ - F_1 \cos 45^\circ) = 2.50 \text{ N} \cdot \text{m}$$

$$r = \frac{1}{2} \sqrt{2s^2} = 0.127 \text{ m}$$

7)  $F_g = G \frac{M_e m}{(r_e + r)^2}$

$M_e = 5.97 \times 10^{24} \text{ kg}$   $\left\{ r_e = 6.38 \times 10^6 \text{ m} \right\} \left\{ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right\}$   
 $m = 75 \text{ kg}$   $r = 600 \text{ km} = 6 \times 10^5 \text{ m} = 0.6 \times 10^6 \text{ m}$

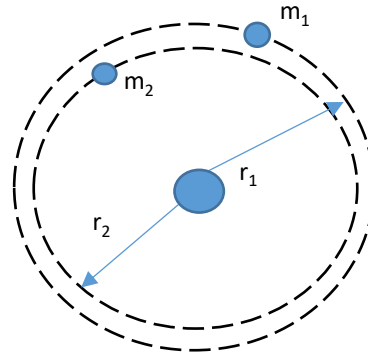
$$6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \left( \frac{5.97 \times 10^{24} \text{ kg} \times 75 \text{ kg}}{(6.98 \times 10^6 \text{ m})^2} \right) = 61.299 \text{ kg}$$

**FINISH**

8)

Two satellites are in circular orbits around a planet that has radius  $9.00 \times 10^6$  m. One satellite has mass 68.0 kg, orbital radius  $5.00 \times 10^7$  m, and initial speed 4800 m/s. The second satellite has mass 84.0 kg and orbital radius  $3.00 \times 10^7$  m. What is the orbital speed of this second satellite?

Answer:  $v_2 = 6,200 \text{ m/s} = 6.2 \text{ km/s}$



9)

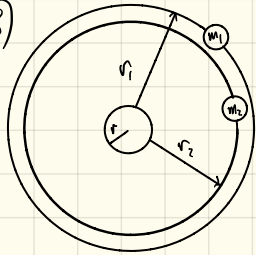
A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.09 m, it takes the block 2.7 s to travel from  $x = 0.09$  m to  $x = -0.09$  m. If the amplitude is doubled, to 0.18 m, how long does it take the block to travel

- a) from  $x = 0.18$  m to  $x = -0.18$  m
- b) from  $x = 0.09$  m to  $x = -0.09$  m

Answer: a)  
b)



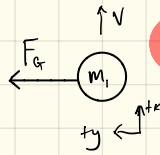
8)



$$r = 900 \times 10^6 \text{ m} \quad \left\{ \begin{array}{l} m_1 = 68.0 \text{ kg} \\ m_2 = 84.0 \text{ kg} \end{array} \right. \quad \left\{ \begin{array}{l} r_1 = 500 \times 10^7 \text{ m} \\ r_2 = 3.00 \times 10^7 \text{ m} \end{array} \right. \quad \left\{ \begin{array}{l} v_1 = 4900 \text{ m/s} \\ v_2 = 3.00 \times 10^7 \text{ m} \end{array} \right.$$

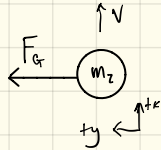
find  $v_2$ 

$$R_1 = r + r_1 = 59 \times 10^6 \text{ m} \quad \left\{ \begin{array}{l} R_2 = r + r_2 = 59 \times 10^6 \text{ m} \end{array} \right.$$



$$\Sigma F = F_g = m_1 \frac{v_1^2}{R_1} = G \frac{M m_1}{R_1^2} \Rightarrow M = \frac{v_1^2 R_1}{G} = 2.038 \times 10^{25} \text{ kg}$$

wrong



$$\Sigma F = F_g = m_2 \frac{v_2^2}{R_2} = G \frac{M m_2}{R_2^2} \Rightarrow v_2 = \sqrt{\frac{G M}{R_2}} = 5064 \text{ m/s}$$

Check:  $\frac{v_1}{v_2} = \frac{\sqrt{\frac{G M}{r_1}}}{\sqrt{\frac{G M}{r_2}}} \Rightarrow v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = 6196.7 \text{ m/s}$

