Physics 22: Homework 5 Hints

1. For Part (a), the capacitors are in series, so that the equivalent capacitance is $C_a = C/3$. For Part (b), the perfect wire makes it so that the potential in between the leftmost capacitor's right plate and the middle capacitor's left plate are at the potential of point b. Moreover, the left plate of this middle capacitor and the right plate of the right-most capacitor are at this potential as well. Thus, the wire shorts through these right two capacitors, making the equivalent capacitance $C_b = C$. For Part (c), note that the left plate of the leftmost capacitor, and the right and left plates of the middle and right-most capacitors, respectively, are at the potential of point a. However, the right and left plates of the left-most and middle capacitors, as well as the right plate of the right-most capacitor, are all at the potential of point b. As such, charges coming into point a have three different transit routes to b, all of which will be equivalent since they would each run through one capacitor. This allows one to reconfigure the capacitors into a single parallel array, resulting in an equivalent capacitance of $C_c = 3C$.

2. In these problems, a battery is to be placed across the points a and b. This helps one resolve exactly how the charge outputted by the battery is going to be deposited onto the capacitors under question.

   (a) Here, the parallel configuration of C_{48} is, as a unit, in series with C_4.

   i. The answer is:
   $$C_{\text{eq}} = \frac{C_4 C_{48}}{C_4 + C_{48}} = 3C.$$

   ii. The potential difference across the parallel configuration of C_4 and C_8 is the same (essentially by definition of the meaning of "parallel"). However, $V_4 + V_{48} = V_{\text{ab}}$.

   iii. The charge on the stand-alone C_4 is the same as the charge on C_{48} because these are in series, so that $Q_4 = Q_{48}$. However, these charges are the same as the outputted charge from the battery:
   $$Q = C_{\text{eq}} V_{\text{ab}}.$$
   Since C_4 has half the capacity to hold charge as compared to C_8 , then the charge on C_4 in the parallel configuration will be half the charge as on C_8.

   iv. The equipotentials are determined by figuring out which plates are connected via perfect wires. In particular, note that the top plates of the parallel configuration of C_4 and C_8 are at the same potential as the bottom plate of the stand-alone C_4. The bottom plates of the parallel configuration of C_4 and C_8 are at the same potential as point b, while the top plate of the stand-alone C_4 is at the same potential as point a.

   v. In solving the circuit, note that
   $$V_4' = V_8 = V_{48} = \frac{Q_{48}}{C_{48}} = \frac{1}{12}\frac{Q}{C} = \frac{1}{12}\frac{C_{\text{eq}}}{C} V_{\text{ab}} = \frac{1}{4} V_{\text{ab}},$$
   where the prime is placed to indicate the distinction between C_4 in the parallel configuration versus the stand-alone C_4. Moreover,
   $$V_4 = \frac{Q_4}{C_4} = \frac{Q}{4C} = \frac{C_{\text{eq}}}{4C} V_{\text{ab}} = \frac{3}{4} V_{\text{ab}}.$$
   As we "juice-up" the battery, then notice that when $V_{\text{ab}} = 4V$, then $V_8 = V$, while $V_4' = V$ and $V_4 = 3V$. Thus, the C_4 capacitances are still below their rating threshold, while C_8 is not any more. In this sense, the limiting capacitor is C_8, resulting in the maximum applied voltage to be $V_{\text{ab}}^{(\text{max})} \lesssim 4V$.

   (b) Note that C_{12} is a parallel configuration of C_1 and C_2, while C_{12} is in series with C_3 and C_6. The procedure for solving the problem is the same as in Parts (ai) - (aiv). As for Part (bv), since the overall equivalent capacitance will decrease if C_1 is disconnected, then the outputted charge from the battery will be less. Since the charge on all the remaining series capacitors will be the same, then the charge on C_6 will be smaller than in the presence of C_1. As such, since the energy in a capacitor is proportional to the square of the charge on the capacitor, then the energy on C_6 will have decreased. To see if the entire circuit's energy decreased, it helps to use the same capacitor formula for the energy applied to the new equivalent capacitance in terms of the battery voltage. In s
   $$U' = \frac{1}{2} C_{\text{eq}}' V_{\text{ab}}^2.$$
   Since $C_{\text{eq}}' < C_{\text{eq}}$ and the potential of the battery is the same, then the energy will have decreased. This could also be seen to follow from the fact that the charge outputted by the battery has decreased much more substantially than the decrease in the equivalent capacitance. As such, the energy has overall decreased.

(c) Note that C_{36} is a series configuration of C_3 and C_6, while C_{36} is in parallel with C_2, forming the equivalence C_{236}. Finally, note that C_{236} is in series with C_4. The procedure for solving the problem is the same as in Parts (ai) - (aiv), or equivalently for Parts (bi) - (biv). As for Part (cv), the goal would be to increase the capacitance in the branch with C_3 and C_6 as much as possible. This can only be done by maximizing the capacitance of C_6, because the capacitance in that branch adds inversely. Thus, the largest capacitance that one can harvest from that branch is by replacing C_6 with the largest possible capacitance that one can find, or, better yet, with an infinite capacitance. Indeed, as in Part (bv), increasing the capacitance of the equivalent circuit as much as possible will result in a larger energy for the same potential difference. Achieving this would ideally amount to taking $C_6 \to \infty$. Indeed,

$$C_{36} = \frac{C_3 C_6}{C_3 + C_6} = \frac{C_3}{1 + C_3/C_6}$$

is maximized when C_6 is as large as possible.

3. The point of this problem is to note that the capacitors will exchange charge until the plates on either side no longer have a potential difference between them.

(a) In this case, the capacitors are connected with the touching plates having the same polarity. There will not be a tremendous amount of charge transfer, but there will be some nonetheless to ensure that the connected plates reach the same potential. The common voltage will be 64 V.

(b) In this process, charges had to inevitably flow from one capacitor to the other until the common voltage in Part (a) was achieved across the plates of each of the parallel configuration of these capacitors. Since these wires have some resistance, the practical way to explain this is to suggest that the wires get hot as the current travels across the wires. (For the ideal case of perfectly conducting wires, the result may actually be determined by noting the loss in energy via electromagnetic radiation achieved by the accelerating charges as they move from one plate to the other unhindered. The formal process to show this is beyond the scope of this class, but it can be derived using the classical power formula for electromagnetic radiation: Larmor Radiation.)

(c) In this case, the capacitors are connected with the touching plates having opposite polarity. In this case, there will be quite a lot of charge transfer in comparison to Part (a), as not only do the potentials have to eventually reach the same value on either end, one of the capacitors has to switch polarity in the process in order to do so. In the end, the common voltage will be 7.1 V.

4. When the switch is open, the capacitors in the top branch—as well as the capacitors in the bottom branch—are in series with one another. Moreover, these top and bottom branches are in parallel with each other. However, when the switch is closed, C_2 and C_1 to the left are in parallel with each other, while C_1 and C_2 on the right are in parallel with each other. Furthermore, these two parallel branches are in series with one another. Note that since point a is grounded, determining the potential relative to ground simply means to determine the potential relative to a.

(a) The results are: $V_{\mathrm{c}} = 2V$, and $V_{\mathrm{d}} = 4V$.

(b) The results are: $V'_{\mathrm{c}} = V'_{\mathrm{d}} = 3V$.

(c) Prior to the closing of the switch, the right plate of C_2 on top has a charge of $-4CV$, while the left plate of C_1 on top has a charge $+4CV$. Also, the right plate of C_1 on the bottom has a charge of $-4CV$, while the left plate of C_1 on the bottom has a charge $+4CV$. Thus, the net charge at c (factoring in the left and right plates) is zero, and this is also the case at d. After the switch is closed, the right plate of C_2 on top has a charge of $-6CV$, while the left plate of C_1 on the top has a charge $+3CV$. Also, the right plate of C_1 on the bottom has a charge of $-3CV$, while the left plate of C_1 on the bottom has a charge $+6CV$. Thus, the net charge at c (factoring in the left and right plates) is $-3CV$, while at d it is $+3CV$. Ultimately, this implies that $3CV$ of positive charge had to be taken away from the plates adjacent to c and redistributed to the plates adjacent to d, resulting in a decrease of charge at c by $-3CV$ and a gain of charge at d by $+3CV$.