Physics 1A
Spring 2015

Challenge Problem 13

An instrument-carrying projectile accidentally explodes at the top of its trajectory. The horizontal distance between the launch point and the point of explosion is L. The projectile breaks into two pieces which fly apart horizontally. One piece has three times the mass of the other. To the surprise of the scientist in charge, the small piece returns to earth precisely at the launching station. How far away does the larger piece land? Neglect air resistance and effects due to the earth's curvature.

Solution. When the instrument blows up at the top of its trajectory, its momenta in the x-direction before and after the explosion are the same because there is no net force in the x-direction. This gives

$$4mv_x = mv_1 + 3mv_2 \tag{1}$$

where v_1 is the velocity of the smaller piece after the explosion, and v_2 is the velocity of the larger piece, and v_x is the x-component of the original launch velocity of instrument. Since the small piece returned back to the launch location, and since the explosion occurred at the apex of the trajectory of the launched instrument, we have

$$v_1 = -v_x. \tag{2}$$

Combining these equations gives

$$v_2 = \frac{5}{3}v_x = -\frac{5}{3}v_1. \tag{3}$$

In other words, the speed of the larger piece is 5/3 the speed of the smaller piece in the other direction. Therefore, the distance traveled by the larger piece is 5/3 the distance traveled by the smaller piece, and it goes in the opposite direction, so it lands at a position $L + \frac{5}{3}L$ from the launch point;

$$\boxed{x_{\text{large-land}} = \frac{8}{3}L}. \tag{4}$$