Physics 1A
Spring 2015

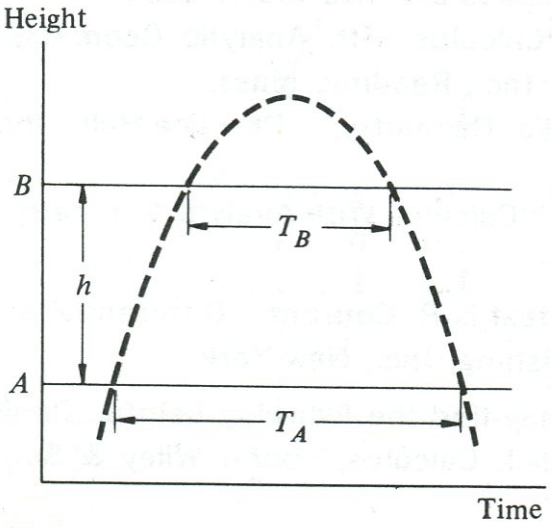
Challenge Problem 3

The acceleration due to gravity g can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A^2 - T_B^2}$$

where h is the height of line B above line A.



Solution. Suppose that v_A is the velocity of the body when it passes height A on its way up and that v_B is the velocity of the body when it passes height B on its way up, then the kinematics equation for free fall that relates

displacement to time elapsed gives

$$0 = v_A T_A - \frac{1}{2} g T_A^2 \tag{1}$$

$$0 = v_B T_B - \frac{1}{2} g T_B^2 \tag{2}$$

It follows that

$$v_A = \frac{1}{2} g T_A \tag{3}$$

$$v_B = \frac{1}{2} g T_B. \tag{4}$$

On the other hand, the kinematics equation relating displacement and velocity gives

$$v_B^2 = v_A^2 - 2gh \tag{5}$$

Combining these results gives

$$\frac{1}{4} g^2 T_B^2 = \frac{1}{4} g^2 T_A^2 - 2gh. \tag{6}$$

We now solve for g, and obtain the desired result.

$$\boxed{g = \frac{8h}{T_A^2 - T_B^2}} \tag{7}$$