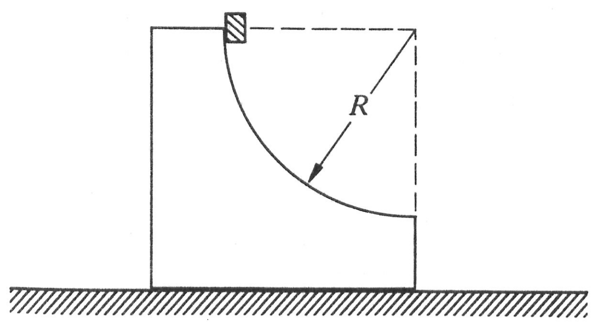
Physics 1A
Spring 2015

Challenge Problem 18

A small cube of mass m slides down a circular path of radius R cut into a large block of mass M. The block of mass M rests on a table, and both the cube and the block move without friction and are initially at rest. The cube of mass m starts from the top of the path. Find the velocity v of the cube relative to the ground as it leaves the block.



Solution. We analyze the system from the perspective of an inertial observer on the ground. The net work done by the forces of constraint (the normal forces between the block and the ramp and the ramp and the ground, is zero, so since there is also no friction, there is no non-conservative work done on the system. It follows that we can impose conservation of mechanical (kinetic plus potential) energy between any two points in time. We choose to do so between the initial point when the block is let go, and the point at which it is about to leave the ramp. Since the ramp does not move at all in the y-direction, it's potential energy remains constant and therefore can be neglected (it cancels on both sides of the conservation of energy equation). We set $y = 0$ to be at the initial position of the small block, so mechanical energy conservation gives

$$0 = -mgR + \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \tag{1}$$

where v is the velocity of the block just as it leaves the ramp as measured by an observer on the ground, and V is the velocity of the ramp at that same time. Note that we can treat these velocities as scalars at the instant the block leaves the ramp because at that moment, the velocities only have an x-compnent.

Since the net external force on the system in the x-direction is zero, we can impose momentum conservation in the x-direction.

$$0 = mv + MV \tag{2}$$

Solving for V in the momentum conservation equation, plugging the result into the energy conservation equation, and solving for v in what results yields:

$$\boxed{v = \sqrt{\frac{2gR}{1 + m/M}}} \tag{3}$$