

**Midterm 1**  
**Discrete Mathematics (Math 61-001)**

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. **You must show your work.**

Name: \_\_\_\_\_ U ID: \_\_\_\_\_

Section: \_\_\_\_\_

Question:	1	2	3	4	Total
Points:	5	5	5	5	20
Score:					

1. 5 points Prove by induction that  $4^{n-1} > n^2$  for all  $n \geq 3$ .

2. 5 points Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions such that  $g \circ f : X \rightarrow Z$  is surjective, i.e., onto. Then prove that  $g$  is **surjective**.

3. 5 points Let  $\mathbb{N} = \{1, 2, 3, \dots\}$  be the set of natural numbers. Let  $R$  be a relation on  $\mathbb{N}$  defined as  $(x, y) \in R$  if and only if 5 divides  $x + 4y$ . Prove that  $R$  is an **equivalence relation**.



4. 5 points Compute that the number of  $n$ -bit strings defined on the set  $S = \{0, 1\}$  which contains exactly  $k$  number of 0's such that no two 0's are sitting next to each other.  
(Warning: You must explain how you are arriving to your solution for this problem.  
Writing only the answer will earn you 0 point.)