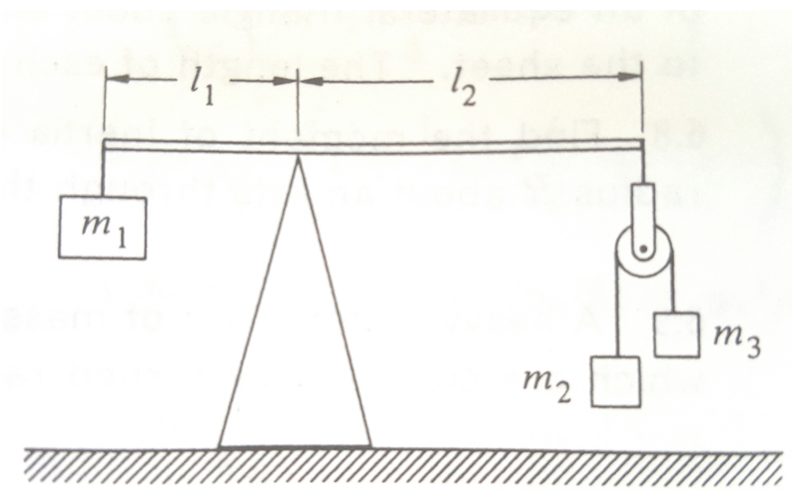
Physics 1A
Spring 2015

Challenge Problem 24

A pivoted beam has mass m_1 suspended from one end and an Atwood's machine suspended from the other with masses m_2 and m_3 suspended on either side. The frictionless pulley has negligible mass and size. Find the relation between.

(a) Find the relation between m_1, m_2, m_3, ℓ_1, and ℓ_2 which will ensure that the beam has no tendency to rotate just after the masses are released.

(b) What would you predict the relation would be in the case that all three masses are equal? Does your answer from part (a) agree with that prediction?



Solution.

(a) We take the location of the vertex of the fulcrum to be the origin and let the positive x-direction point to the right and the positive y-direction point vertically upward. The tension in the string holding mass m_1 is $m_1 g$, so that tension exerts a torque on the left hand side equal to

$$\boldsymbol{\tau}_L = m_1 g l_1 \mathbf{k} \tag{1}$$

On the right hand side, denoting the tension connecting the masses T, noting that the accelerations of the two masses are equal and opposite, and applying Newton's Second Law in the y-direction to each mass gives

$$T - m_2 g = m_2 a, \qquad T - m_3 g = -m_3 a \tag{2}$$

On the other hand, applying Newton's second Law to the pulley, and using the fact that it's massless, we find that the tension $'$ in the rope holding the pulley satisfies

$$T' - 2T = 0. \tag{3}$$

These equations can be solved for the tension T' to give

$$T' = 4g\frac{m_2 m_3}{m_2 + m_3} \tag{4}$$

The torque due to the string on the right hand side is therefore

$$\boldsymbol{\tau}_R = -T' l_2 \mathbf{k} = -4g\frac{m_2 m_3}{m_2 + m_3} l_2 \mathbf{k} \tag{5}$$

Since the angular momentum of this system doesn't change while it's in equilibrium, the net external torque is zero;

$$\boldsymbol{\tau}_L + \boldsymbol{\tau}_R = 0 \tag{6}$$

which gives the desired relationship:

$$\boxed{m_1 l_1 = 4\frac{m_2 m_3}{m_2 + m_3} l_2} \tag{7}$$

(b) When all three masses are equal (say to m), the tension in the left string will be mg while the tension on the right string will be $2mg$ (there will be no accelerations of the right-hand masses in this case). For the torques to balance, the tension that is twice as small needs to be acting twice as far away on the rod to compensate, so I would expect the relation to reduce to $l_1 = 2l_2$. In this special case, the relationship we just derived gives

$$ml_1 = 4\frac{m^2}{2m}l_2 \tag{8}$$

and this implies $l_1 = 2l_2$ as expected! Can you think of any more limiting cases to check?