Physics 1A
Spring 2015

Challenge Problem 14

This problem was designed for you to explore one of the simplest, most important, and most ubiquitous systems in all of physics: the classical **harmonic oscillator**.

Consider a block of mass m sliding on a frictionless surface in the x-direction. Suppose, additionally, that the block is connected to a Hooke's Law spring of spring constant k that is attached to a wall. Let $x = 0$ denote the position of the block when the spring is at its natural length.

(a) Determine the equation of motion for the block. In other words, determine the equation satisfied by the position as a function of time $x(t)$ of the block.

(b) Define a variable $\omega = \sqrt{k/m}$. What are the units of this variable?

(c) Write your equation of motion from part (a) in terms of ω.

(d) Determine the most general solution to your equation from part (c) as follows:

- **(i)** Try solutions to the equation of the form $\cos(\alpha t)$ and $\sin(\alpha t)$, and determine what α needs to be for these trial solutions to work.
- **(ii)** The general solution is then $x(t) = A\cos(\alpha t) + B\sin(\alpha t)$ (in math, this is called a **linear combination** of the cos and sin solutions), where A and B are constants that are determined by "initial conditions." More concretely, A and B can be solved for once one specifies the initial position $x(0)$ and the initial velocity $\dot{x}(0)$.
- **(iii)** Suppose that the following initial conditions are given:

$$x(0) = x_0, \qquad \dot{x}(0) = v_0. \tag{1}$$

  Solve for A and B in your general solution in terms of x_0 and v_0.

(e) In light of your solution, what is a physical interpretation of ω?

(f) Suppose that at time $t = 0$, the spring is at its natural length, and that the block is given a push so that its initial velocity is v_0.

- **(i)** What is the position $x(t)$ of the block as a function of time? Sketch it.
- **(ii)** What is the velocity $v(t)$ of the block as a function of time? Sketch it.
- **(iii)** What is the acceleration $a(t)$ of the block as a function of time? Sketch it.
- **(iv)** What is the magnitude of the largest displacement from its initial position that the block ever reaches? This maximum displacement is called the **amplitude** of its motion.
- **(v)** What is the maximum speed of the block during its motion? At what point in its motion does this speed occur?
- **(vi)** What is the magnitude of the maximum acceleration of the block during its motion? At what point during its motion does this maximum occur?