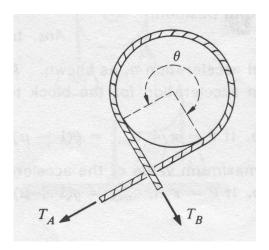
Challenge Problem 9

A device called a capstan is used aboard ships to control a rope which is under great tension. The rope is rapped around a fixed drum, usually for several turns (the drawing shows about three-fourths of a turn). The load on the rope pulls it with a force T_A , and the sailor holds it with a much smaller force T_B .

(a) Show that

$$T_B = T_A e^{-\mu\theta}$$

where μ us the coefficient of static friction of the drum's surface, and θ is the total angle subtended by the rope on the drum.



(b) Alice can pull a rope with a force of magnitude $500 \,\mathrm{N}$. If the drum has a diameter of $1/2 \,\mathrm{m}$, the surface has coefficient of static friction 1/2, and Alice has a $5 \,\mathrm{m}$ length of rope, then by using the capstan, approximately what is the maximum load she can hold? Assume that all but $1 \,\mathrm{m}$ of the rope will wrapped around the drum.