Physics 1A: Mechanics
Winter 2016

Discussion: Week 4

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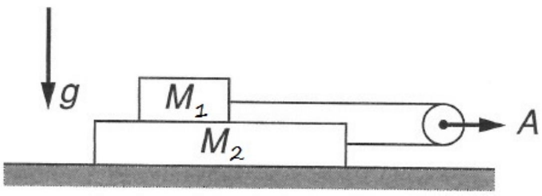
(Alternatively, just e-mail me your questions.)

Problem 1

Mass M_1 slides on top of mass M_2 as shown. Assume $M_2 > M_1$. The two blocks are pulled from rest by a massless rope passing over a massless pulley. The pulley is accelerated at rate A. Block M_2 slides on a table without friction, but there is a constant friction force f between M_1 and M_2 due to their relative motion. Find the tension in the rope.



This is one of those problems in which it's best to just rely on the power of Newton's law. We can immediately write the force equations for both blocks:

$$T - f = M_1 a_1 \ , \tag{1}$$
$$T + f = M_2 a_2 \ . \tag{2}$$

Make sure that you understand why friction works in the direction of tension for the lower mass: if something is experiencing friction, it's a bit as if this thing *dragged* the floor along with it. Of course, usually floor does not move, but in this example block M_2 can move!

Now, we do not know a_1 and a_2, but we do know how the relative accelerations of M_1 and M_2 with respect to each other, a_1' and a_2', satisfy $a_1' = -a_2'$. Now we also know that

$$a_1 = a_1' + A \quad \text{and} \quad a_2 = a_2' + A \quad \Rightarrow \tag{3}$$
$$\Rightarrow \quad a_1 = a_1' + A \quad \text{and} \quad a_2 = -a_1' + A \ , \tag{4}$$

and we can add the last two equations by sides to get

$$a_1 + a_2 = 2A \quad \Rightarrow \quad a_2 = 2A - a_1 \ . \tag{5}$$

This we can plug back into (2) to get

$$T - f = M_1 a_1 \ , \tag{6}$$
$$T + f = 2M_2 A - M_2 a_1 \ . \tag{7}$$

Fist, let us subtract the second equation from the first to obtain

$$-2f = (M_1 + M_2)a_1 - 2M_2A \qquad \Rightarrow \qquad a_1 = \frac{2M_2A - 2f}{M_1 + M_2} \ . \tag{8}$$

Then we add those same two equations by sides to get

$$2T = 2M_2A + a_1(M_1 - M_2) = 2M_2A + \frac{2(M_2A - f)(M_1 - M_2)}{M_1 + M_2} \qquad \Rightarrow \tag{9}$$

$$\Rightarrow \qquad T = M_2A + \frac{(M_2A - f)(M_1 - M_2)}{M_1 + M_2} \ . \tag{10}$$

This can be simplified (admittedly, only a little) to

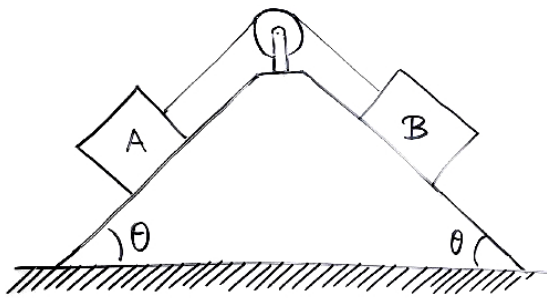
$$T = \frac{2M_1M_2A + f(M_2 - M_1)}{M_1 + M_2} \ . \tag{11}$$

Problem 2

Blocks A and B of mass M_A and M_B are joined by a cable of constant length and negligible mass. They are placed on two sides of a frictionless, triangular block with sides sloping at an angle θ as shown. The cable is threaded over a massless pulley.

a) What is the tension in the cable?

b) Imagine the whole system in question is in an elevator moving upwards with acceleration a_0. What is the tension in the cable now?



a) Since we haven't been told otherwise, we assume $M_A \neq M_B$. This means that the system is not stable and masses will move.

Let us write the equations of motion for the masses:

$$T - m_A g \sin\theta = m_A a_A \ , \tag{12}$$
$$m_B g \sin\theta - T = m_B a_B \ . \tag{13}$$

From the setup it is obvious that $a_A = a_B = a$, so that we are really dealing with

$$T - m_A g \sin\theta = m_A a \ , \tag{14}$$
$$m_B g \sin\theta - T = m_B a \ . \tag{15}$$

Then we have

$$(m_B - m_A) g \sin\theta = (m_A + m_B) a \quad \Rightarrow \quad a = \frac{(m_B - m_A) g \sin\theta}{m_A + m_B} \quad \text{and} \tag{16}$$
$$2T = (m_A + m_B) g \sin\theta + (m_A - m_B) a \quad \Rightarrow \quad T = \frac{2 m_A m_B g \sin\theta}{m_A + m_B} \ . \tag{17}$$

b) We intuitively feel that the only difference here is that we substitute $a \to a + g$ in the final solution. Let's see if that's true!

We can express the upward acceleration a as the sum of its components: the component perpendicular to the slope, $a\cos\theta$, and the component parallel to the slope $a\sin\theta$. The $a\cos\theta$ part will only contribute to the normal force, while $a\sin\theta$ will affect the motion. Specifically, now we have $a'_A = a'_B$ in the frame of the wedge, but for an outside observer

$$a_A = a'_A + a_0 \sin\theta \quad \text{and} \quad a_B = a'_B - a_0 \sin\theta \tag{18}$$
$$\Rightarrow \quad a'_A = a_A - a_0 \sin\theta \quad \text{and} \quad a'_B = a_B + a_0 \sin\theta \ , \tag{19}$$
$$\Rightarrow \quad a_A - a_0 \sin\theta = a_B + a_0 \sin\theta \quad \Rightarrow \quad a_B = a_A - 2a_0 \sin\theta \ . \tag{20}$$

We plug that into (13) to get

$$T - m_A g \sin\theta = m_A a_A \ , \tag{21}$$

$$m_B g \sin\theta - T = m_B (a_A - 2a_0 \sin\theta) \qquad \Rightarrow \qquad a_A = (g + 2a_0)\sin\theta - \frac{T}{m_B} \tag{22}$$

$$\Rightarrow \qquad T - m_A g \sin\theta = m_A (g + 2a_0)\sin\theta - T\frac{m_A}{m_B} \ , \tag{23}$$

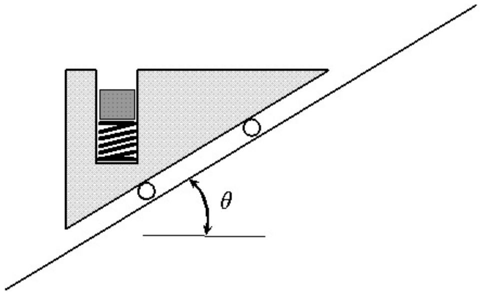
$$\Rightarrow \qquad T\left(1 + \frac{m_A}{m_B}\right) = 2m_A (g + a_0)\sin\theta \qquad \Rightarrow \qquad T = \frac{2m_A m_B (g + a_0)\sin\theta}{m_A + m_B} \ , \tag{24}$$

exactly as we predicted.

Problem 3

Even if you haven't learned about springs yet, do not worry! The only thing you need to know about springs in this problem is that the force that the spring exerts is given by $\vec{F} = -k\vec{x}$, where k is a constant. The minus sign means that the spring acts opposite to the direction of its displacement: if you squeeze the spring, it will act against the squeezing, and if you try to elongate a spring, it will act against that too. You know that from an everyday experience!

A piston is placed on top of a spring inside a wedge-shaped block fitted with small rollers so that it can roll down an inclined plane, as shown. When the block is at rest the spring is compressed x_1 from it's equilibrium position, but when the block is rolling down the plane the spring is compressed only $x_2 < x_1$. What is the inclination angle of the plane? (Neglect friction.)



Hint: Assume some masses both for the piston on the spring and the block, but bear in mind that you don't know them and that in the end the solution should not depend on them. Pssst: it sounds more complicated than it is, so don't panic!

We denote the mass of the piston as m and the mass of the block as M. First, we use the information about the spring when the block is not moving to calculate the spring constant. We can immediately write

$$mg = kx_1 \qquad \Rightarrow \qquad k = \frac{mg}{x_1} \ . \tag{25}$$

When the system composed of piston and the block is moving, its acceleration is given by the Newton's second law:

$$(m+M)g\sin\theta = (m+M)a \qquad \Rightarrow \qquad a = g\sin\theta \ . \tag{26}$$

It should be clear that only the vertical component of this acceleration will affect the spring. From geometry we find that this vertical component is equal to

$$a_v = a\sin\theta = g\sin^2\theta \ . \tag{27}$$

How does the motion down the slope affect the spring? Use your physical intuition: the mass m pushes on the spring due to acceleration g, but the spring "moves away" from it with acceleration a_v. It must be that the overall effect of the mass on the spring is smaller! Hence we write

$$m(g - a_v) = mg\big(1 - \sin^2\theta\big) = mg\cos^2\theta = kx_2 \ . \tag{28}$$

We plug in our expression for k to obtain

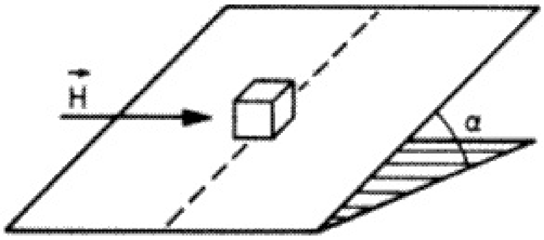
$$mg\cos^2\theta = mg\frac{x_2}{x_1} \qquad \Rightarrow \qquad \cos^2\theta = \frac{x_2}{x_1} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{x_2}{x_1}} \ . \tag{29}$$

Problem 4

A packet of weight W rests on a rough inclined plane that makes an angle α with the horizontal.

a) If the coefficient of static friction $\mu = 2\tan\alpha$, find the least horizontal force H_{min}, acting transverse to the slope of the plane that will cause the particle to move.

b) In what direction will it go?



Hint: note that the friction acts along the plane of the incline, not only along the direction of the force H.

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a) The component of weight acting perpendicular to the incline is $W\cos\alpha$, and the component acting along the incline is $W\sin\alpha$. Therefore the magnitude of the total force acting along the plane of the incline is

$$F = \sqrt{(H)^2 + (W\sin\alpha)^2} = \sqrt{H^2 + W^2\sin^2\alpha}\ . \tag{30}$$

Then, the friction is given by

$$f = \mu W\cos\alpha = 2\tan\alpha\ W\cos\alpha = 2W\sin\alpha\ . \tag{31}$$

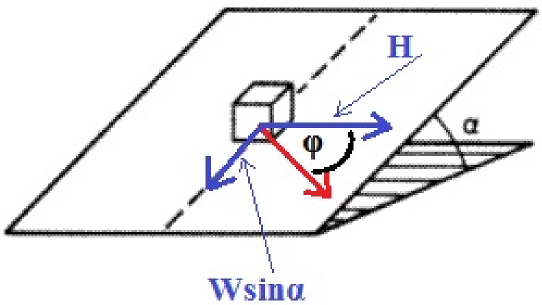
Now then the condition for the packet to start moving is that H_{min} is such that

$$F = \sqrt{H_{\text{min}}^2 + W^2\sin^2\alpha} \geq f = 2W\sin\alpha\ , \tag{32}$$

$$H_{\text{min}}^2 + W^2\sin^2\alpha \geq 4W^2\sin^2\alpha\ , \tag{33}$$

$$H_{\text{min}}^2 \geq 3W^2\sin^2\alpha \qquad \Rightarrow \qquad H_{\text{min}} = \sqrt{3}W\sin\alpha\ . \tag{34}$$

b) If we define an angle φ as in the picture below



then

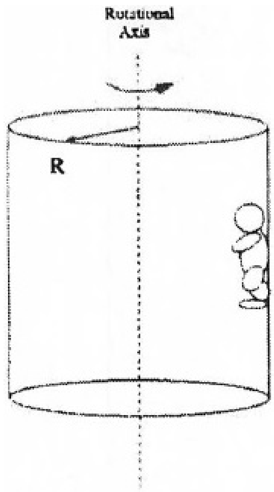
$$\tan\varphi = \frac{W\sin\alpha}{H_{\text{min}}} = \frac{W\sin\alpha}{\sqrt{3}W\sin\alpha} = \frac{1}{\sqrt{3}} \qquad \Rightarrow \qquad \varphi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}\ . \tag{35}$$

Problem 5

This is a Corbin midterm problem.

The Spin-Out (also known informally as the "barf-barrel" [LOL - AW]) was a large, vertically oriented, cylindrical amusement part ride. As riders stood against the curved wall facing inward, the cylinder would begin to rotate. At some point, the floor would drop out and the riders, stuck to the wall, would be suspended over a floor-less void.

For our purposes, take the radius of the chamber to be R, the mass of the rider under consideration to be m, and the static and kinetic coefficients of friction between the rider and the wall of the cylinder to be μ_s and μ_k, respectively.



a) For reasons of liability [not to mention basic human decency - AW], the operators must wait until the passengers are actually "stuck" to the wall before removing the floor. How fast must the walls be moving (tangential speed) before the floor can be pulled out from under the riders?

The force of friction must balance the weight of a passenger. The maximal *static* friction is given by

$$f_s^{(\max)} = \mu_s N \ , \tag{36}$$

where N is the normal force. This normal force is also what makes the passenger spin around, i.e., it plays the role of the centripetal force. This means that we have

$$N = ma_r = \frac{mv^2}{R} \ , \tag{37}$$

so that now

$$f^{(\max)} = \frac{\mu_s mv^2}{R} \ . \tag{38}$$

Then we must have

$$f^{(\max)} \geq mg \qquad \Rightarrow \qquad v \geq \sqrt{\frac{gR}{\mu_s}} \ . \tag{39}$$

b) Evaluate your answer to part a) in the limits $\mu \to 0$ and $\mu \to \infty$ (where μ is whatever the relevant coefficient of friction is) and explain the results.

In the limit $\mu_s \to 0$ the velocity diverges, which is what we expect because the smaller the friction, the bigger the velocity must be in order to keep the passenger in (remember that the normal force is proportional to v^2!). Conversely, in the limit $\mu_s \to \infty$ the minimal velocity becomes infinitesimal, because with infinite friction even the smallest velocity suffices.

c) The floor has been pulled out from our rider and he is stuck to the wall. How large is the force of friction acting on him? Does it depend on his weight? Should he be worried (assuming weight might be an issue)? Explain.

We can see that our final formula in part a) does not depend on mass. Therefore, once a required velocity is obtained, one doesn't have to worry about their weight. The force of friction acting on the passenger is what it has to be to keep the passenger in place: that is $f_s = mg$ (upwards).

d) Now suppose something goes wrong and the cylinder slows to a tangential speed v (less than the threshold value required to keep passengers safe). If our rider is initially at rest with respect to the cylinder wall, how long does it take him to slide down a vertical distance d (with respect to the wall)?

The equation of motion of our rider is now given by (note that we have kinetic friction now!)

$$f_k - mg = -ma \qquad \Rightarrow \qquad \mu_k N - mg = \mu_k \frac{mv^2}{R} - mg = -ma \qquad \Rightarrow \qquad a = g - \frac{\mu_k v^2}{R} = \frac{gR - \mu_k v^2}{R} \ , \tag{40}$$

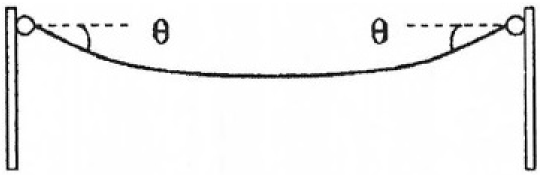
and

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dR}{gR - \mu_k v^2}} \ . \tag{41}$$

Problem 6

This is a Corbin midterm problem - back then he included one textbook problem in his midterm. He NEVER does it now, but this problem still shows you what he's interested in.

Most problems directed at introductory mechanics class assume that ropes, cords or cables have so little mass compared to the other objects in the problem that one can safely ignore their mass. However, if the rope is the *only* object in the problem, then clearly one cannot ignore its mass. Suppose you have a clothesline attached to two poles. The clothesline has a mass M, and each end makes an angle θ with the horizontal.



a) What is the tension at the ends of the clothesline?

The sum of vertical components of tensions at the ends of the rope must balance the weight. I.e., we need

$$2T_{\text{end}} \sin\theta = mg \qquad \Rightarrow \qquad T_{\text{end}} = \frac{mg}{2\sin\theta}\ . \tag{42}$$

b) What is the tension at the lowest point?

The tension at the lowest point is the sum of horizontal tensions at the ends of the rope,

$$T_{\text{middle}} = 2T\cos\theta = \frac{mg}{\tan\theta}\ . \tag{43}$$

c) Why can't we have $\theta = 0$?

Because that would require infinite tension. We can see that indeed, both (42) and (43) diverge in this limit.

d) Discuss your results for parts a) and b) in the limit $\theta \to \frac{\pi}{2}$.

In the limit $\theta \to \frac{\pi}{2}$ the result form part a) becomes

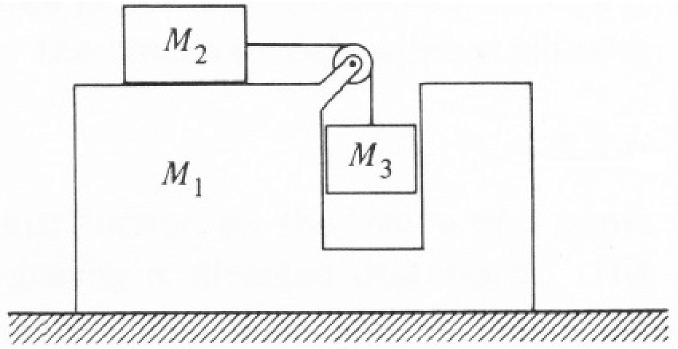
$$T_{\text{end}} = \frac{mg}{2}\ , \tag{44}$$

which totally makes sense as then we just have two vertical ropes (both with tension T) holding mass m.

Similarly, in this limit the tension in the middle becomes zero, which is also what we would expect to happen - the lowest end of a vertically hanging rope experiences no tension.

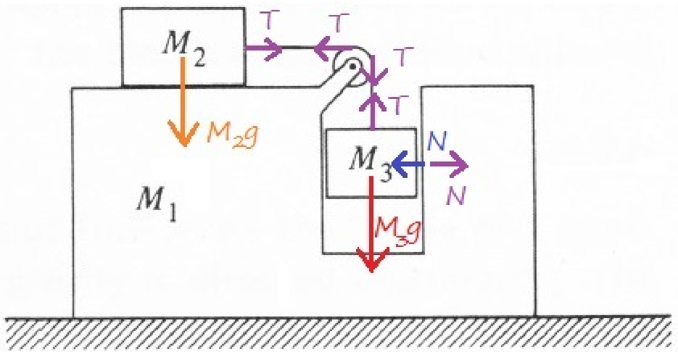
Problem 7

Determine the acceleration of mass M_1 relative to the ground in the following machine. All surfaces are frictionless and the pulley and rope are massless.

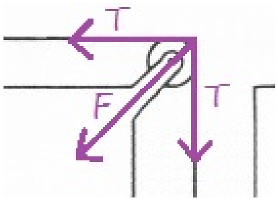


Some of you may look at the picture above and start wondering: why should mass M_1 move at all?

Note that the two tensions at the pulley may be combined into single force that acts on M_1 - this is what makes it move! Then, while moving, block M_1 pushes on M_3 which produces the normal force N. Of course, following the Newton third law, M_3 acts on M_1 with the same force N. The whole situation can be depicted as below (here we assume that M_1 will move to the left, but in the end we will find out which direction it moves from the sign of the acceleration):



Specifically, the pulley detail can be depicted as



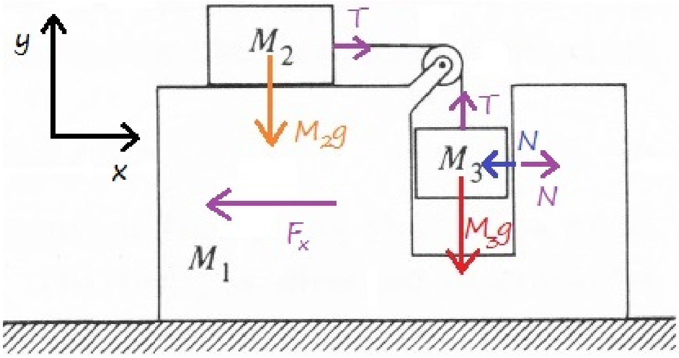
where

$$F = 2\left(T\cos\frac{\pi}{4}\right) = \sqrt{2}T \ . \tag{45}$$

Then for the purposes of this problem we are interested in the horizontal part of F,

$$F_x = F\sin\frac{\pi}{4} = T \ , \tag{46}$$

which is the force that contributes to M_1's acceleration. Then we can redraw the situation considered as



Based on this picture we can write down the equations of motion:

$$T = M_2 a_2 \ , \tag{47}$$
$$T - M_3 g = M_3 a_3 \ , \quad \text{(note we do not specify the direction of } a_3\text{)} \tag{48}$$
$$N = M_3 a \ , \tag{49}$$
$$-T + N = M_1 a \ , \tag{50}$$

where we already used the fact that M_1 and M_3 will move with the same horizontal acceleration a. (Also, based on the picture we may expect that a will be negative.)

We have four equations and five unknowns T, N, a_2, a_3 and a. Clearly, we need another equation. We again use the old rope trick: the length of the rope must be constant, and this length can be expressed by

$$l = l_1 + l_2 = (x_p - x_2) + (y_p - y_3) \ , \tag{51}$$

where x_p, y_p are coordinates of the pulley and x_2 and y_3 are x- and y-coordinates of M_2 and M_3, respectively. Then we can differentiate twice both sides of (51) to obtain

$$0 = a - a_2 - a_3 \ , \tag{52}$$

where we have used the fact that the pulley moves along with M_1 in the horizontal direction. Then we use

$$a_2 = a - a_3 \tag{53}$$

in (47),

$$T = M_2 a - M_2 a_3 \ , \tag{54}$$

plug that into (48) and (50) (plugging (49) into the latter at the same time),

$$M_2 a - M_2 a_3 - M_3 g = M_3 a_3 \quad \text{and} \quad - M_2 a + M_2 a_3 + M_3 a = M_1 a \quad \Rightarrow \tag{55}$$
$$\Rightarrow \quad a_3 = \frac{M_2 a - M_3 g}{M_2 + M_3} \quad \text{and} \quad a = \frac{M_2 a_3}{M_1 + M_2 - M_3} \quad \Rightarrow \tag{56}$$
$$\Rightarrow \quad a = \frac{M_2}{M_1 + M_2 - M_3} \frac{M_2 a - M_3 g}{M_2 + M_3} \ , \tag{57}$$
$$\Rightarrow \quad a\big(M_1 M_2 + M_1 M_3 - M_3^2\big) = -M_2 M_3 g \ , \tag{58}$$
$$\Rightarrow \quad a = -g \frac{M_2 M_3}{M_1 M_2 + M_1 M_3 - M_3^2} \ . \tag{59}$$

Problem 8

This is a Corbin final problem.

On the planet Xergon the force of gravity is not $\vec{F} = -g\hat{j}$, instead it is $\vec{F} = -gy\hat{j}$, where g is a constant.

a) Write and solve the equation of motion for the position (as a function of time) of an object shot upwards with velocity $\vec{v} = v_0\hat{j}$ at $t = 0$ and solve for the motion. The object goes up, reverses direction and hits the ground much as it does in the gravitation field of the Earth, however $y(t)$ is different.

b) How much time will it take for the object to hit the ground?

c) If the initial velocity is $\vec{v} = v_{0x}\hat{x} + v_{0y}\hat{y}$, $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$, what is the equation of the r of the mass as a function of x and the angle θ with respect to the x-axis?

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a) The equation of motion is from definition

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{y}}{dt^2} \ . \tag{60}$$

In our case this becomes

$$-gy = m\frac{d^2y}{dt^2} \quad \Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{g}{m}y \quad \Rightarrow \quad y(t) = A_0 \sin\left(\sqrt{\frac{g}{m}}t\right) + B_0 \cos\left(\sqrt{\frac{g}{m}}t\right) \ . \tag{61}$$

Now we apply the initial condition that $y(t=0) = 0$, which leads us to $B_0 = 0$, and then we used the fact that we know the initial velocity:

$$\left.\frac{dy}{dt}\right|_{t=0} = A_0\sqrt{\frac{g}{m}} \cos\left(\sqrt{\frac{g}{m}}t\right)\Big|_{t=0} = A_0\sqrt{\frac{g}{m}} = v \quad \Rightarrow \quad A_0 = \sqrt{\frac{mv^2}{g}} \ . \tag{62}$$

Finally then the object's trajectory is given by

$$y(t) = \sqrt{\frac{mv^2}{g}} \sin\left(\sqrt{\frac{g}{m}}t\right) \ . \tag{63}$$

b) We want the object to make it to the highest point and back to the ground. This will happen for the argument of the sine function going from 0 to π. That is, at the moment the object hits the ground we have

$$\sqrt{\frac{g}{m}}t = \pi \quad \Rightarrow \quad t = \pi\sqrt{\frac{m}{g}} \ . \tag{64}$$

c) In this case the equation for r is

$$r = \sqrt{v_{0x}^2 t^2 + \frac{mv_{0y}^2}{g}\sin^2\left(\sqrt{\frac{g}{m}}t\right)} \ . \tag{65}$$

We want to express this as a function of x and the angle θ. If the body starts at $x = 0$, the after time t it has traveled

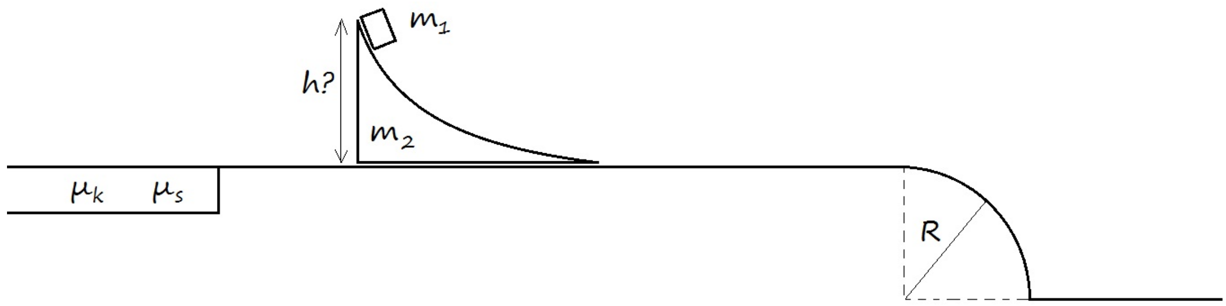
$$x = v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}} \ . \tag{66}$$

This way we obtain

$$r = \sqrt{x^2 + \frac{mv_{0y}^2}{g}\sin^2\left(\sqrt{\frac{g}{m}}\frac{x}{v_{0x}}\right)} = \sqrt{x^2 + \frac{mv^2\sin^2\theta}{g}\sin^2\left(\sqrt{\frac{g}{m}}\frac{x}{v\cos\theta}\right)} \ . \tag{67}$$

Problem 9

This is a Corbin midterm problem.



Consider the picture shown above. A block of mass m_1 sits on a ramp of mass m_2 sits on a ramp of mass m_2; the ramp, in turn, sits on a frictionless horizontal table. The right end of the table joins smoothly to a rounded surface of radius R (frictionless) and the left end of the table meets up with a long horizontal surface described by friction coefficients μ_s and μ_k.

We move this problem to Week 6 worksheet.

Problem 10

This is a homework problem.

The gravitational pull of the earth on an object is inversely proportional to the square of the distance of the object from the center of the earth. At the earth's surface this force is equal to the object's normal weight mg, where $g = 9.8\ \left[\frac{\text{m}}{\text{s}^2}\right]$, and at large distances, the force is zero. If a 20,000-kg asteroid (*use symbols first!*) falls to earth from a very great distance away, what will be its minimum speed as it strikes the earth's surface, and how much kinetic energy will it impart to our planet? You can ignore the effects of the earth's atmosphere. How does this energy compare to the energy of a nuclear bomb in 1945?

We move this problem to Week 5 worksheet.