Physics 22: Homework 2 Hints

1. Since the field is uniform and the piecewise-smooth surfaces are all planar, then the flux may be determined by a simple dot product
$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot \hat{n} A.$$
All of the relevant face areas may be easily determined using geometry, whereas the directions of the relevant normal vectors can also be written out quite clearly. Note that the net electric flux through this prism (which bounds a finite volume) will be zero.

2. The circular base of the hemisphere is open, and the flux through the actual hemispherical surface is desired. That being said, recall that electric flux may be conceptualized as simply counting electric-field lines piercing a surface. In this respect, it helps to consider closing the hemispherical surface by inserting a disk at its opening. Once this is done, compare the number of piercings through the disk to the piercings through the hemisphere. With a fairly simple argument, one should be able to see that the flux will be $\Phi_E = E\pi R^2$.

3. If one draws the typical field-line profile of such a charge, it should be clear that because the charge is placed at the center of the cube, then any arbitrary rotation of the cube about any axis will not change the way in which the field lines pierce a given face. Indeed, one can always rotate the cube but simultaneously rotate his/her perspective to make the cube look as it was in the original orientation. Although the problem gives pretty much gives it away, it should be clear based on the discussion above that the flux through each of the faces will be the same. Although the brute-force calculation of the flux through a face using the flux integral,
$$\Phi_E = \int_{\mathcal{S}} \vec{E} \cdot d\vec{A},$$
is slightly challenging, it should be clear, via Gauss's Law, what the net flux through the entire cube must be. Then, because all faces are equivalent, then each face should have an equal share of the total flux.

4. This problem poses a situation in which a nonuniform field is piercing a planar surface. Note that $\vec{E}$ and $\hat{n}$ are parallel at all points on the planar surface, so that one need not worry about orientation. That being said, notice that, on the one hand, if one moves along the surface at a fixed value of x (i.e., along a line on the surface that is parallel to the y-axis), then the field changes its magnitude while, obviously, maintaining its direction. On the other hand, if one moves along the surface at a fixed value of y (i.e., along a line on the surface that is parallel to the x-axis), then the field maintains both its magnitude and its direction. Recognizing this, the simplest endeavor would be to exploit this by considering dA to be a strip of area with length, a, and width, dy: $d\vec{A} = \hat{z} a dy$. Setting up in this way, one may then integrate $\vec{E} \cdot d\vec{A}$ to obtain the net electric flux through this surface.