Physics 1A: Mechanics

Winter 2016

Discussion: Week 3

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Office hours: Mondays, Tuesdays, Wednesdays 4-7 p.m. *by appointment!*

(Alternatively, just e-mail me your questions.)

Problem 1

Let us attempt to derive kinematics formulas for an object moving along a circle. We'll do it piece by piece so don't worry - you'll be fine ;).

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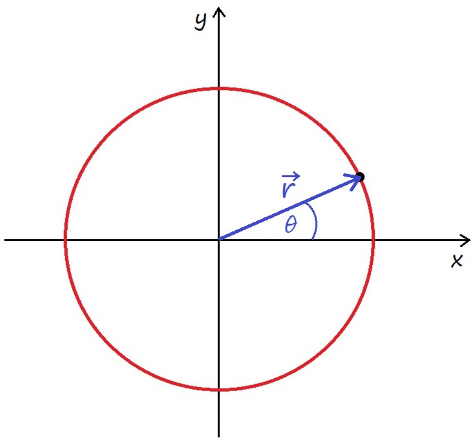
a) We are considering an object moving along a circle of radius r, as depicted below:



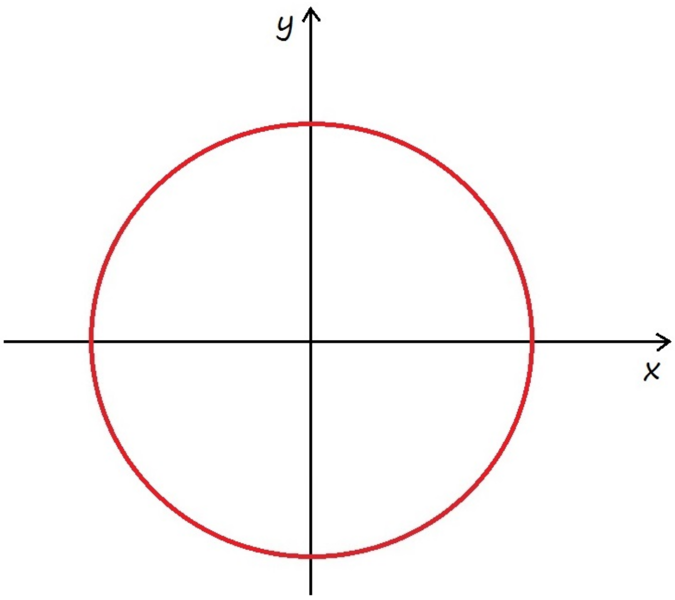
Note that the center of the circle has been aligned with the center of the frame of reference.

The position of an object is described by the position vector $\vec{\boldsymbol{r}}$. Note that $|\vec{\boldsymbol{r}}| = r$, the radius of the circle. For the purpose of derivation it is better to express this position vector by means of other variables. Specifically, express $\vec{\boldsymbol{r}}$ in terms of r, the angle θ, and unit vectors $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$.

Hint: If you're at a loss, read about the so-called polar coordinates.

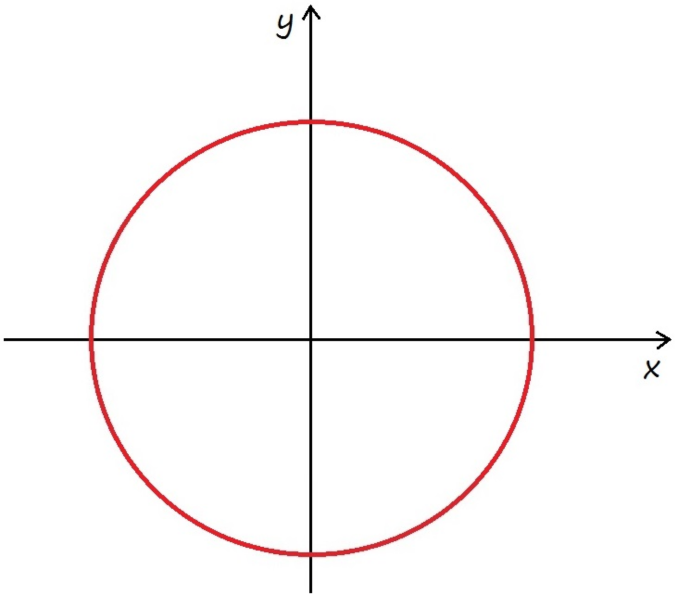
b) That $\vec{v} = \frac{d\vec{r}}{dt}$ is *always* true, also for circular motion. Using your result from a), obtain an expression for $\vec{v}$ in circular motion. Note: for circular motion the angle θ describing the position of the object changes in time, i.e., $\theta = \theta(t)$.
Hint: Use the chain rule when taking the derivative! Remember: $\vec{r}$ is a function of θ, and then θ is a function of t.
Hint 2: To introduce some consistency into our derivations we usually consider the object to be moving counterclockwise along the circle. It is not very important for this problem, but I thought it may make it easier for you to imagine the motion.

c) If you did part b) correctly, you should have obtained an expression with a factor of $\big(-\sin\theta\,\hat{\boldsymbol{x}} + \cos\theta\,\hat{\boldsymbol{y}}\big)$. This factor is a unit vector (check this! One does that by checking whether the vector's magnitude is equal 1.) whose direction will change with θ. Please draw this vector for a few different θs on the diagram below. In what direction does it point? Is it along the radius, or maybe clockwise around the circle, or maybe otherwise? Note whether this direction follows your expectation.



d) Now we move further. As before, it is *always* true that acceleration is given by $\vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt}$. Calculate acceleration in the circular motion.
Hint: the calculation will involve a product rule and a chain rule, and the final expression is a sum of two terms.

e) Once more, if you derived the expression above correctly, you should have obtained two terms. One of them has a factor of $\big(-\sin\theta\ \hat{\boldsymbol{x}} + \cos\theta\ \hat{\boldsymbol{y}}\big)$, which we already know from b) and c), and one has a factor of $\big(-\cos\theta\ \hat{\boldsymbol{x}} - \sin\theta\ \hat{\boldsymbol{y}}\big)$. Please draw this unit vector on the picture below. How can one interpret the two terms in $\vec{\boldsymbol{a}}$?



An important note: there really aren't that many interesting circular motion problems. I will use some of the more interesting homework problems in this worksheet, but make sure you do the rest of the homework too because I will not dwell on circular motion! Honestly, it is just like the linear motion, only instead of meters one uses angles to measure it.

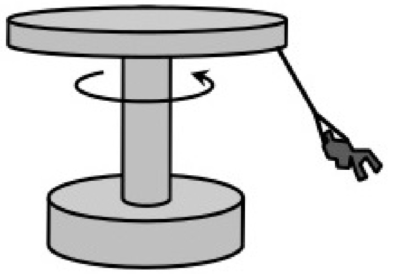
Also, all of the following problems involve the notion of force, so make sure you have reviewed last Friday's lecture!

Problem 2

How is water removed from clothes during the spin cycle of a washer?
Hint: Your answer should involve physics-related terms such as "force", "acceleration" etc.

Problem 3

You take your little cousin Bill to a local amusement park. The kid could ride on a merry-go-round for hours, so you're stuck beside it waiting for him for quite some time. Bored, you're starting to consider Bill's motion around the center of the carousel. Bill's mass is m. The seats of the carousel are hung using lines of approximate length L. You note that when the carousel is not moving, the seats are approximately d_0 away from the center pole. In full motion, the seats are D away from the pole. What is Bill's total acceleration, and what is his linear velocity?



Problem 4

Every human learns pretty early that if they hit something hard, e.g. a wall, how much it will hurt depends on how fast one moves (among other things). Same thing applies to falling: it's pretty safe to jump from a bench or a classroom table, while it's a rather stupid idea to jump from the roof of the Physics and Astronomy Building. Intuitively, after a few experiments (they can be so-called "thought experiments", you don't exactly need to jump off PAB to check this) we may infer that this has something to do both with the speed gained and with how fast this speed is diminished due to the collision (we will really elaborate on that soon). We also realize that when things fall, they gain in speed. Obviously, this is because when free falling, objects accelerate due to unrestricted action of gravity. That is why it hurts little to jump from a bench, while it hurts plenty to jump off of a building: if we jump from a larger height, there is more time to gain speed until we hit the ground.

Having concluded that, let us figure out the following thing. The floor of an average train is about as high as a classroom table. We know it's pretty safe to jump from this height. Why then it is not such a great idea to jump out of a fast moving train? Please try to use physical terms in your reasoning.

Problem 5

This is a homework problem.

A uniform cable of weight w hangs vertically downward, supported by an upward force of magnitude w at its top end. What is the tension in the cable

a) at its top end;

b) at its bottom end;

c) at its middle?

Your answer to each part must include a free-body diagram.

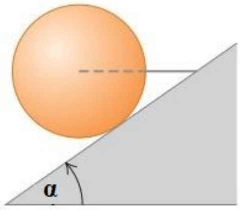
Hint: For each question choose the body to analyze to be a section of the cable or a point along the cable.

d) Graph the tension in the rope versus the distance from its top end.

Problem 6

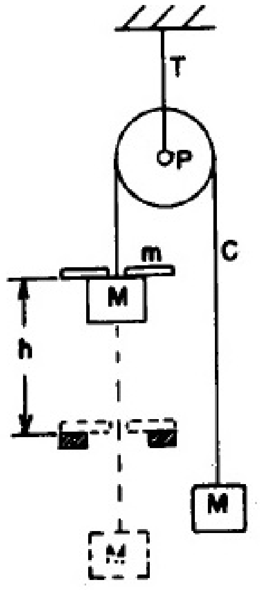
This is another homework problem.

A horizontal wire holds a solid uniform ball of mass m in place on a tilted ramp that rises at an angle α above the horizontal. The surface of this ramp is perfectly smooth, and the wire is directed away from the center of the ball (see figure).
a) Draw a free-body diagram for the ball.
b) How hard does the surface of the ramp push on the ball?
c) What is the tension in the wire?



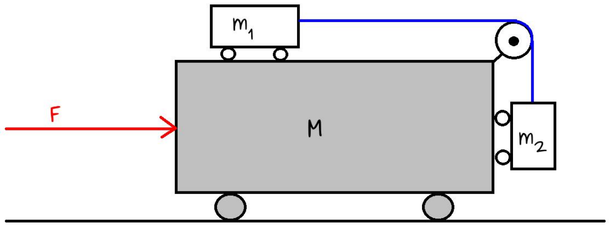
Problem 7

An early arrangement for measuring the acceleration of gravity, called Atwood's machine, is shown in the figure. The pulley and cord have negligible mass and friction. The system is balanced with equal masses M on each side, and then a small C-shaped object of mass m is added to one side (as in the picture). After accelerating through a certain distance h the C-shaped object is caught on a ring and the two equal masses then move on with constant speed v. Find the value of g that corresponds to the measured values of m, M, h, and v.



Problem 8

What horizontal force F must be applied to M so that m_1 and m_2 do not move relative to M? Neglect friction.



Problem 9

And yet another homework problem...

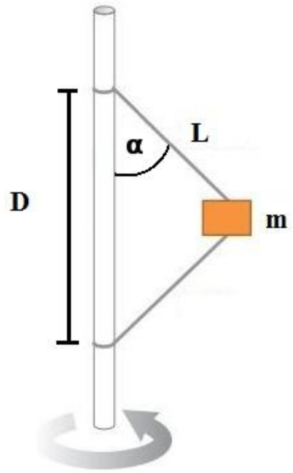
A block of mass m is attached to a vertical rod by means of two strings of same length L. When the system rotates about the axis of the rod, the strings are extended as shown in the diagram and the tension in the upper string is T_1.

a) What is the tension in the lower cord?

b) How many revolutions per minute does the system make?

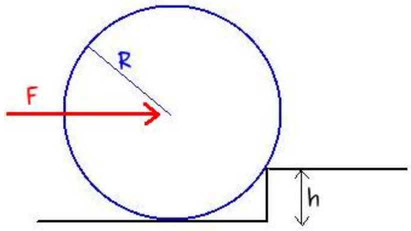
c) Find the number of revolutions per minute at which the lower cord just goes slack.

d) Explain what happens if the number of revolutions per minute is less than in part c).



Problem 10

What horizontal force F is needed to push a wheel mass M and radius R over a block of height h?

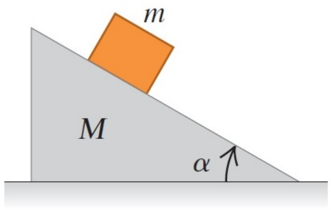


Hint: as usual, start from making a drawing! Is the above drawing completely correct? Specifically, in kinematics, where do we apply forces?

Problem 11

A homework problem once more.

Moving Wedge. A wedge with mass M rests on a frictionless, horizontal tabletop. A block with mass m is placed on the wedge (see figure). There is no friction between the block and the wedge. The system is released from rest.
a) Calculate the acceleration of the wedge and the horizontal and vertical components of the acceleration of the block.
b) Do your answers to part a) reduce to the correct results when M is very large?
c) As seen by a stationary observer, what is the shape of the trajectory of the block?



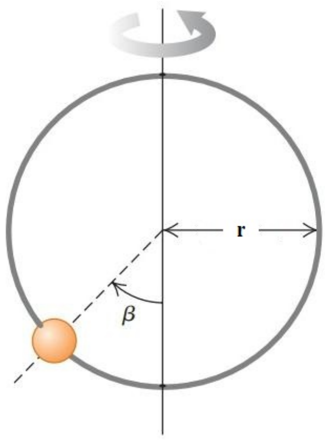
Problem 12

The last homework problem!

A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius r. The hoop rotates at a constant frequency f about a vertical diameter (see figure).

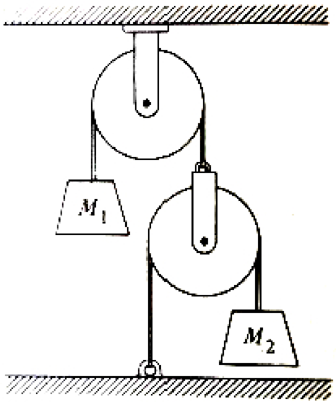
a) Find the angle β at which the bead is in vertical equilibrium. (Of course, it has a radial acceleration toward the axis.)

b) Is there some condition on the frequency f for the bead to move from the bottom of the loop at all?



Problem 13

Masses M_1 and M_2 are connected to a system of strings and pulleys as shown. The strings are massless and of constant length, and the pulleys are massless and frictionless.



a) What do you expect the acceleration of mass M_1 to be in the following limits? Explain your answers.

i) $M_2 \to 0$,

ii) $M_2 \to \infty$.

b) Determine the acceleration of mass M_1 and check that your answer agrees with your expectations from part a).

c) Suppose that the whole apparatus of this problem is in an elevator accelerating relative to the ground with acceleration A.

i) Determine the acceleration of mass M_1 relative to the ground,

ii) Determine the acceleration of mass M_1 relative to the elevator,

iii) Does your expression for the acceleration of mass M_1 relative to the elevator make sense in the limit $A \to 0$ and in the limit $A \to -g$?

Problem 14

This is an original Corbin midterm problem.

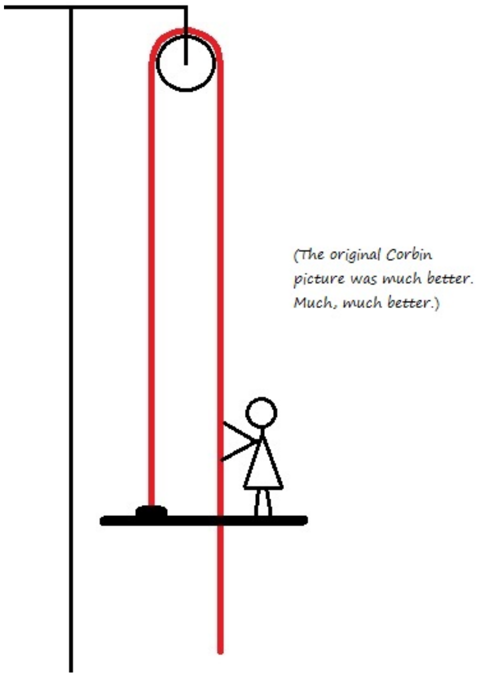
Eager to escape before she is detected, a jewel thief (mass M_1) lowers herself from the roof of the museum in the maintenance lift (mass M_2). The lift is operated manually, by pulling on a rope that is draped over a massless pulley, as shown. For the following questions, assume she is accelerating downward at a rate $|\vec{a}| = a$.

a) How much force is the jewel thief applying to the rope?

b) Strangely enough, someone has left a bathroom scale sitting on the bottom of the lift and the thief is standing on it. How much does it say she weighs?

c) How does the mass of the lift compare to the mass of the thief? How do we know that? Using your answers in part a) and b), explain what would happen if things were the other way around.

d) What is the fastest rate at which the jewel thief can accelerate downwards? Explain.



Problem 15

A small pebble is lodged in the tread of a tire of radius R. If this tire is rolling at speed V without slipping on a horizontal road, find the equations for the x and y coordinates of the pebble as a function of time. Let the pebble touch the road at $t = 0$. Find also the velocity and acceleration components as a function of the time.