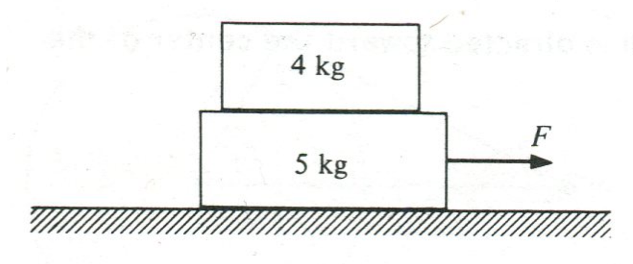
Physics 1A
Spring 2015

Challenge Problem 6

A 4-kg block rests on top of a 5-kg block, which rests on a frictionless table. The coefficient of friction between the two blocks is such that the blocks start to slip when the horizontal force F applied to the lower block is 27 N. Suppose that a horizontal force is now applied only to the upper block, what is the maximum value of this force for the blocks to slide on the table without slipping relative to each other?



Solution. Let the mass of the top block be $m_1 = 4\,\text{kg}$ and the mass of the bottom block be $m_2 = 5\,\text{kg}$. Let $F = 27\,\text{N}$ denote the force that when exerted on the bottom block will just cause the blocks to slip relative to one another, as in the figure above. In this situation, free body diagrams show that the sum of the forces in the x- and y-directions on the top block are

$$\sum F_x = f_s, \qquad \sum F_y = N_{12} - mg. \tag{1}$$

where N_{12} is the normal force of block 2 on block 1, and f_s is the force of static friction on block 1. For the bottom block we have

$$\sum F_x = F - f_s, \qquad \sum F_y = N - N_{12} - mg \tag{2}$$

where we have already used Newton's Third Law twice to equate the magnitudes forces of friction and normal forces of blocks 1 and 2 on each other. Because both blocks are stationary for all times in the y-direction, their accelerations in the y-direction are both zero. Their accelerations in the x-direction are not zero, but they are the same for both blocks since they are just about to slip, but are still staying in contact without slipping. Let's call their accelerations in the x-direction a. Then applying Newton's Second Law gives the following four equations:

$$f_s = m_1 a \tag{3}$$
$$N_{12} - mg = 0 \tag{4}$$
$$F - f_s = m_2 a \tag{5}$$
$$N - N_{12} - mg = 0. \tag{6}$$

This is four equations in four unknowns f_s, a, N_{12}, N. We can solve for all of them, but we'll see in a moment that we only really need f_s;

$$f_s = \frac{m_1}{m_1 + m_2} F. \tag{7}$$

Notice that you don't really even need the y-equations to determine this. Notice that because the blocks are just about to slip relative to each other, the static friction force we just solved for is the maximum possible static friction force between the two blocks.

Now what happens if we apply a force to the top block, call it F', then if we exert the maximum possible force on the top block before it starts to slip on the bottom block, then the static friction force in these case will again be the maximum possible as we determined above, however, the x-equations generated by Newton's Second Law are as follows as you should convince yourself of by drawing free body diagrams:

$$F' - f_s = m_1 a \tag{8}$$
$$f_s = m_2 a \tag{9}$$

Where now the unknowns are F and a. Solving for a in the second equation, and then plugging this into the first equation gives

$$F' = f_s + \frac{m_1}{m_2} f_s = f_s \frac{m_1 + m_2}{m_2} \tag{10}$$

now plugging in the result (7) for f_s in the first part gives

$$F' = \left(\frac{m_1}{m_1 + m_2} F \right) \frac{m_1 + m_2}{m_2} = \frac{m_1}{m_2} F. \tag{11}$$

Plugging in the numbers we get

$$F' = \frac{4\,\mathrm{kg}}{5\,\mathrm{kg}} (27\,\mathrm{N}) = \boxed{21.6\,\mathrm{N}}. \tag{12}$$