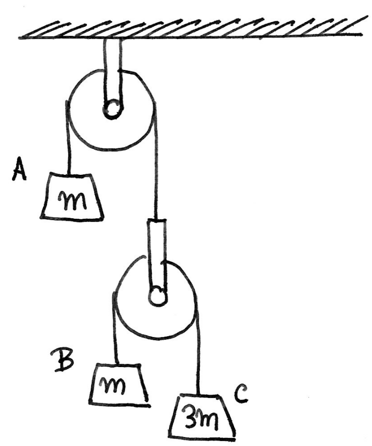
Physics 1A
Spring 2015

Challenge Problem 12

The pulleys in the diagram below are massless and have frictionless axles. The cables are massless and have constant length. The top pulley is attached to the ceiling of a room. What are the accelerations a_A, a_B, a_C of the three hanging masses?



Solution. We first solve this problem in the more general case that m_A, m_B, m_C are arbitrary, then we set $m_A = m, m_B = m, m_C = 3m$ at the end. This is not necessary, but it is a more powerful method since then we can check a lot more limiting cases by independently sending the various masses to many different limits if we so desire.

Suppose that the magnitude if the tension in the rope on the top pulley is denoted T_1, and the magnitude of the tension in the rope on the bottom pulley is denoted T_2. Drawing free body diagrams for the three masses in the y-direction and using Newton's Second Law gives the following equations:

$$T_1 - m_A g = m_A a_A, \qquad T_2 - m_B g = m_B a_B, \qquad T_2 - m_C g = m_C a_C. \quad (1)$$

Drawing a free body diagram for the bottom pulley and noting that the pulley is massless gives the following y-direction Newton's Second Law equation for that pulley:

$$T_1 - 2T_2 = 0. \tag{2}$$

Thus far, Newton's Second Law has given us four equations and five unknowns T_1, T_2, a_A, a_B, a_C. Thus, we need more equations. If a_p is the acceleration of the bottom pulley, then we also have the following constraints, which we already derived in lecture under slightly different circumstances:

$$a_A = -a_p, \qquad 2a_p = a_B + a_C \tag{3}$$

Now we have six equations and six unknowns since we have added one unknown: a_p. Solving these equations for the desired accelerations gives

$$a_A = g\left[-1 + \frac{8m_B m_C}{4m_B m_C + m_A(m_B + m_C)}\right] \tag{4}$$

$$a_B = g\left[-1 + \frac{4m_A m_C}{4m_B m_C + m_A(m_B + m_C)}\right] \tag{5}$$

$$a_C = g\left[-1 + \frac{4m_A m_B}{4m_B m_C + m_A(m_B + m_C)}\right] \tag{6}$$

If we set $m_A = m, m_B = m, m_C = 3m$, then we obtain a massive simplification:

$$\boxed{a_A = \frac{1}{2}g, \qquad a_B = -\frac{1}{4}g, \qquad a_C = -\frac{3}{4}g.} \tag{7}$$