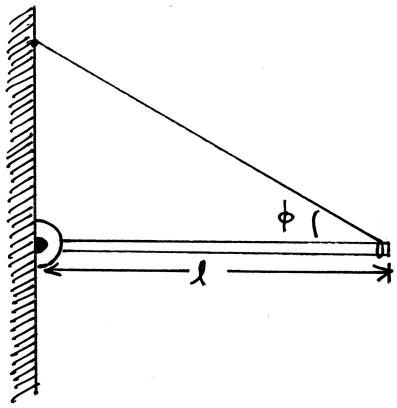
Physics 1A
Spring 2015

Challenge Problem 21

A thin rod of mass M and length ℓ is connected to a wall by a hinge. The rod is horizontal, and it is held stationary by a massless cable running from its free end to the wall. The cable makes an angle ϕ with the horizontal.

(a) What is the tension in the cable?
(b) What are the components of the force of the rod on the hinge?



Solution.
(a) The tension in the cable can be determine by examining only the torques acting on the rod. If we pick the origin of our coordinates to be at the hinge, and if we pick the positive x-direction point to the write in the figure and let the positive y-direction point upward, then the net torque on the rod in the z-direction is

$$\sum \tau_z = T \sin\phi \ell - \frac{Mg\ell}{2} \tag{1}$$

On the other hand, the angular acceleration of the rod is zero, so we get

$$T \sin\phi \ell - \frac{Mg\ell}{2} = 0, \tag{2}$$

and therefore

$$\boxed{T = \frac{Mg}{2\sin\phi}} \tag{3}$$

(b) To determine the force of the rod on the hinge, we analyze the forces on the rod to first determine the force of the hinge on the rod, and then we note that the force of the rod on the hinge has the same components but with opposite sign because of Newton's Third Law. Drawing a free body diagram for the rod gives

$$\sum F_x = -T\cos\phi + N_x = 0 \tag{4}$$

$$\sum F_y = T\sin\phi - Mg + N_y = 0 \tag{5}$$

where N_x, N_y are the components of the force of the hinge on the rod, both of which can be either positive or negative (we don't know their directions quite yet). This is two equations in two unknowns N_x, N_y since we already solved for the tension in part (a). Solving these equations and plugging in our previous result for the tension gives

$$\boxed{N_x = \frac{Mg}{2\tan\phi}, \qquad N_y = \frac{Mg}{2}}. \tag{6}$$