Physics 1A
Spring 2015

Challenge Problem 2

An elevator is on the ground at $t = 0$. It ascends from the ground with uniform speed. At time t_d (the "drop time") a boy drops a marble through the floor. The marble falls with uniform acceleration g and hits the ground an amount of time T later.

(a) Find the height of the elevator at the drop time t_d.

(b) On physical grounds, what would you expect your answer to be in the limit $T/t_d \to 0$. Why?

(c) Does your expectation of what would happen from part (b) match your mathematical answer from part (a)?

Solution.

(a) Let the "up" direction be the positive y-direction, and let the elevator ascend from $y = 0$ at $t = 0$. The position of the elevator + ball as a function of time is

$$y(t) = \begin{cases} v_0 t & , 0 \leq t \leq t_d \\ h + v_0(t - t_d) - \frac{1}{2}g(t - t_d)^2 & , t_d \leq t \leq t_d + T \end{cases} \tag{1}$$

Notice that in the expression that applies after t_d, we have used the fact that the ball is in free fall, but when it was dropped, it had velocity v_0 since it had been moving with the elevator at that moment. Using the condition $y(t_d) = h$ in the expression for the interval $[0, t_d]$ and the condition $y(t_d + T) = 0$ in the expression for the interval $[t_d, t_d + T]$, we obtain the following equations:

$$h = v_0 t_d \tag{2}$$

$$0 = h + v_0 T - \frac{1}{2} g T^2. \tag{3}$$

This is a system of two equations in two unknowns h and v_0. Solving for v_0

in the first equation and plugging this into the second equation gives

$$0 = h + h\frac{T}{t_d} - \frac{1}{2}gT^2. \tag{4}$$

Solving for h gives

$$\boxed{h = \frac{\frac{1}{2}gT^2}{1 + \frac{T}{t_d}}} \tag{5}$$

(b) The limit $T/t_d \to 0$ means that T is vastly smaller than t_d. This happens when the elevator ascends extremely slowly because in this case, t_d is rather large, while T is comparatively small because the ball will be in free fall after the drop, and its average velocity will therefore far exceed that during its ascent. Notice that in this physical scenario, the initial velocity of the ball as its dropped would be very small, because its initial velocity equals that of the elevator.

Given these observations, in this limit we expect $h \to \frac{1}{2}gT^2$ because that's what one would get if the ball were dropped from rest from a height T and took a time T to fall. To the ground.

(c) As $T/t_d \to 0$, out answer from part **(a)** approaches $\frac{1}{2}gT^2$, precisely as expected from our physical intuition in part **b**.