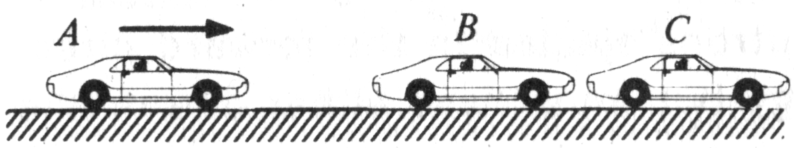
Physics 1A
Spring 2015

Challenge Problem 20

Cars B and C are at rest with their breaks off. Car A plows into car B at high speed, pushing B into C. If the collisions are completely inelastic (i.e. the cars stick together) what fraction of the initial energy is dissipated in the second collision (with car C)? The cars are of identical construction.



Solution. Let's first derive a general result that we can apply not only to this problem (twice), but to any inelastic collision. Suppose that an object of mass m and velocity v collides completely inelastically with an object of mass M at rest. What is the ratio of the final kinetic energy to the initial kinetic energy? Well conservation of momentum tells us that if V is the velocity of the system consisting of both objects stuck together after the collision, then

$$mv = (m + M)V \tag{1}$$

so

$$V = \frac{m}{m+M}v \tag{2}$$

therefore, the ratio of the final to initial kinetic energy is

$$\frac{K}{K_0} = \frac{\frac{1}{2}(m+M)V^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2}{mv^2} = \frac{m}{m+M} \tag{3}$$

Now, let's apply this result to the problem at hand. Let m_c be the mass of each of the cars (which are identical). In the first collision between A and B, we set $m = m_c$ and $M = m_c$, so if K_1 is the kinetic energy after this collision and K_0 is the kinetic energy before, our general result above tells us that

$$\frac{K_1}{K_0} = \frac{m_c}{2mc} = \frac{1}{2}. \tag{4}$$

Applying our general result above again to the second collision with $m = 2m_c$ and $M = m_c$ gives

$$\frac{K_2}{K_1} = \frac{2m_c}{3m_c} = \frac{2}{3}. \tag{5}$$

The energy change in the second collision is

$$K_2 - K_1 \frac{2}{3} K_1 - K_1 = -\frac{1}{3} K_1 = -\frac{1}{3}\frac{1}{2} K_0 = -\frac{1}{6} K_0. \tag{6}$$

So the fraction of the initial energy dissipated in the second collision is therefore

$$\boxed{\left| \frac{K_2 - K_1}{K_0} \right| = \frac{1}{6}}. \tag{7}$$