Physics 1A: Mechanics
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1st Midterm Review

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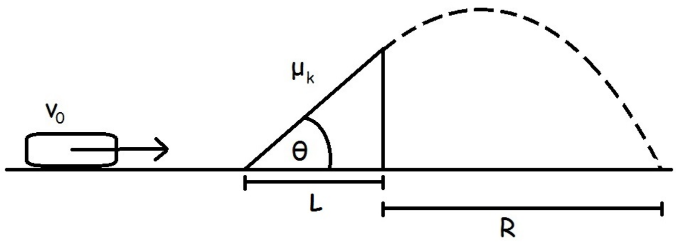
(Alternatively, just e-mail me your questions.)

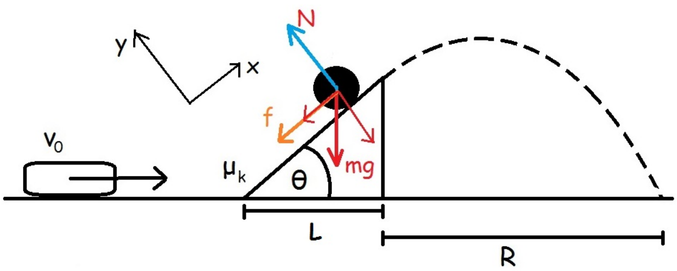
Problem 1

This is an original Corbin problem.

A block slides (with a large speed v_0) over a frictionless horizontal surface towards an immovable ramp. The ramp is inclined at an angle θ, its length is L, and is covered by a material that presents a coefficient of kinetic friction μ_k to the block.
a) What speed will the block have at the top of the ramp?
b) What does it mean if the speed you calculated in part a) is imaginary? What can you deduce from this?
c) What happens to the answer in part a) in the limit $\theta \to 0$? Is this what you'd expect? Explain/resolve any discrepancy.
d) How far from the vertical edge of the ramp will the block land? Evaluate your answer in the limit v_0 is very large (compared to...?) and interpret the result.



You should always start by drawing a picture:



a) Let us use a (parallel-to-the-incline)-(perpendicular-to-the-incline) frame of reference. There is no net force acting along the perpendicular-to-the-incline direction

$$N - mg\cos\theta = 0 \qquad \Rightarrow \qquad N = mg\cos\theta \;, \tag{1}$$

and the force acting along the incline yields

$$-ma_x = -mg\sin\theta - \mu_k N \ , \tag{2}$$

or explicitly

$$-ma_x = -mg\sin\theta - \mu_k mg\cos\theta = -mg\Big(\sin\theta + \mu_k\cos\theta\Big) \qquad \Rightarrow \tag{3}$$

$$\Rightarrow \qquad a_x = g\Big(\sin\theta + \mu_k\cos\theta\Big) \ . \tag{4}$$

Thus we have

$$v(t) = v_0 - a_x t \ . \tag{5}$$

We still do not know t. Now how much time does it take to travel $d = \frac{L}{\cos\theta}$?:

$$\frac{L}{\cos\theta} = v_0 t - \frac{1}{2}a_x t^2 \qquad \Rightarrow \qquad \frac{1}{2}a_x t^2 - v_0 t + \frac{L}{\cos\theta} = 0 \tag{6}$$

$$\Delta = v_0^2 - \frac{2a_x L}{\cos\theta} \ , \qquad \sqrt{\Delta} = \sqrt{v_0^2 - \frac{2a_x L}{\cos\theta}} \ , \tag{7}$$

$$t = \frac{v_0 - \sqrt{v_0^2 - \frac{2a_xL}{\cos\theta}}}{a_x} \ . \tag{8}$$

Then

$$v = v_0 - a_x t \ = \ v_0 - a_x\frac{v_0 - \sqrt{v_0^2 - \frac{2a_xL}{\cos\theta}}}{a_x} = \sqrt{v_0^2 - \frac{2a_xL}{\cos\theta}} = \sqrt{v_0^2 - \frac{2g\Big(\sin\theta + \mu_k\cos\theta\Big)L}{\cos\theta}} = \tag{9}$$

$$= \sqrt{v_0^2 - 2gL\left(\tan\theta + \mu_k\right)} \ . \tag{10}$$

b) This means that the time we've calculated is imaginary, and that means that there is no time such that the block reaches $d = \frac{L}{\cos\theta}$ with the initial velocity v_0. For real solutions we need to have

$$\Delta \geq 0 \qquad \Rightarrow \qquad v_0^2 - \frac{2a_xL}{\cos\theta} \geq 0 \qquad \Rightarrow \qquad v_0^2 \geq \frac{2g\Big(\sin\theta + \mu_k\cos\theta\Big)L}{\cos\theta} = 2gL\left(\tan\theta + \mu_k\right) \ . \tag{11}$$

c) In the limit $\theta \to 0$ we have $\tan\theta = \frac{\sin\theta}{\cos\theta} = 0$. Then (10) becomes

$$v = \sqrt{v_0^2 - 2gL\mu_k} \ . \tag{12}$$

This makes sense, as then we would just have a block with an initial velocity v_0 going from a frictionless surface to a surface of a coefficient of kinetic friction μ_k and length $d = \frac{L}{\cos\theta} \to L$.

d) At the end of the ramp we have

$$v = \sqrt{v_0^2 - 2gL\left(\tan\theta + \mu_k\right)} \ . \tag{13}$$

This can be decomposed into the coefficient parallel and perpendicular to the ground:

$$v_{0,x} = v\cos\theta = \sqrt{v_0^2 - 2gL\left(\tan\theta + \mu_k\right)}\cos\theta \ , \qquad v_{0,y} = v\sin\theta = \sqrt{v_0^2 - 2gL\left(\tan\theta + \mu_k\right)}\sin\theta \ . \tag{14}$$

Let's calculate the time it will take the block to fall to the ground:

$$0 = h + v_{0,y}t - \frac{1}{2}gt^2 = L\tan\theta + v_{0,y}t - \frac{1}{2}gt^2 \quad \Rightarrow \quad \frac{1}{2}gt^2 - v_{0,y}t - L\tan\theta = 0 \tag{15}$$

$$\Delta = v_{0,y}^2 + 2gL\tan\theta \ , \qquad \sqrt{\Delta} = \sqrt{v_{0,y}^2 + 2gL\tan\theta} \ , \tag{16}$$

$$t = \frac{v_{0,y} + \sqrt{v_{0,y}^2 + 2gL\tan\theta}}{g} \ , \tag{17}$$

so that we have

$$R = v_{0,x}t = v_{0,x}\frac{v_{0,y} + \sqrt{v_{0,y}^2 + 2gL\tan\theta}}{g} \ . \tag{18}$$

If v_0 is very large, then also $v_{0,y}$ is very large and then the term $2gL\tan\theta$ does not matter at all, so that we have

$$R = v_{0,x}t = v_{0,x}\frac{2v_{0,y}}{g} = \frac{2v^2\sin\theta\cos\theta}{g} = \frac{v^2\sin(2\theta)}{g} \ . \tag{19}$$

Problem 2

This is a Corbin midterm problem.

Every year aficionados of air-guns, catapults, trebuchets and the like gather on the East Coast to participate in a charity fund-raising event called *Pumpkin Chunkin'*. The goal is simple - launch a pumpkin into the air and try to achieve a greater range than anyone else. The current record appears to be about 4695 ft (or nearly a mile!). I have seen unofficial claims of shots (outside the event) well in excess of a mile.

Let's say a pumpkin is launched from a height h. It achieves a maximum height H and a range R. In terms of these quantities find

a) The time of flight of the pumpkin.
b) The x- and y-components of the pumpkin's initial velocity.
c) The speed the pumpkin was traveling with and the direction it was traveling in when it returned to the ground.
d) Evaluate your answers to part c) in the limit $h \to 0$ and discuss the result.

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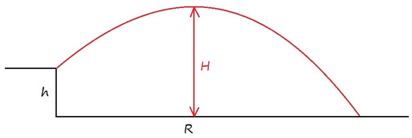
a) b) c) We are considering the following situation:



The position of the pumpkin is described by two following equations of motion:

$$y(t) = h + v_{0y}t - \frac{1}{2}gt^2 \quad \text{and} \quad x(t) = v_{0x}t \ . \tag{20}$$

where we do not know neither v_{0x} nor v_{0y} yet. The total time of the flight $t_{\rm tot}$ is one that satisfies

$$t_{\rm tot} = h + v_{0y}t_{\rm tot} - \frac{1}{2}gt_{\rm tot}^2 = 0 \quad \Rightarrow \quad \frac{1}{2}gt_{\rm tot}^2 - v_{0y}t_{\rm tot} - h = 0 \tag{21}$$

$$\Rightarrow \quad \Delta = v_{0y^2} + 2gh \quad \Rightarrow \quad t_{\rm tot} = \frac{v_{0y} + \sqrt{v_{0y^2} + 2gh}}{g} \ , \tag{22}$$

where we have chosen the bigger of the solutions for obvious reasons.

Now we also know that

$$R = v_{0x}t_{\rm tot} \quad \Rightarrow \quad v_{0x} = \frac{R}{t_{\rm tot}} = \frac{Rg}{v_{0y} + \sqrt{v_{0y^2} + 2gh}} \ . \tag{23}$$

Then for the highest point of the trajectory we can also write

$$0 = \sqrt{v_{0y}^2 - 2g(H-h)} \quad \Rightarrow \quad v_{0y} = \sqrt{2g(H-h)} \ , \tag{24}$$

which we can use to rewrite the total time as

$$t_{\rm tot} = \frac{\sqrt{2g(H-h)} + \sqrt{2gH}}{g} \ , \tag{25}$$

and the horizontal part of the velocity as

$$v_{0x} = \frac{Rg}{\sqrt{2g(H-h)} + \sqrt{2gH}} \ . \tag{26}$$

Finally, the speed that the pumpkin is traveling with as it hits the ground is

$$v \ = \ \sqrt{v_x^2(t_{\rm tot}) + v_y^2(t_{\rm tot})} = \sqrt{v_{0x}^2 + \left(v_{0y}^2 + 2(-g)(-h)\right)} = \sqrt{\frac{R^2g^2}{\left(\sqrt{2g(H-h)} + \sqrt{2gH}\right)^2} + 2gH} \ . \tag{27}$$

The direction of this speed is given by

$$\tan\theta = \frac{v_y(t_{\rm tot})}{v_{0x}} = \frac{-2g\sqrt{H(H-h)} - 2gH}{Rg} \ . \tag{28}$$

d) In the limit $h \to 0$ the last two equations become

$$v = \sqrt{\frac{g\left(R^2 + 16H^2\right)}{8H}} \tag{29}$$

and

$$\tan\theta = \frac{v_y(t_{\text{tot}})}{v_{0x}} = \frac{-4H}{R} \ . \tag{30}$$

It is interesting to look at t_{tot}, which in this limit becomes

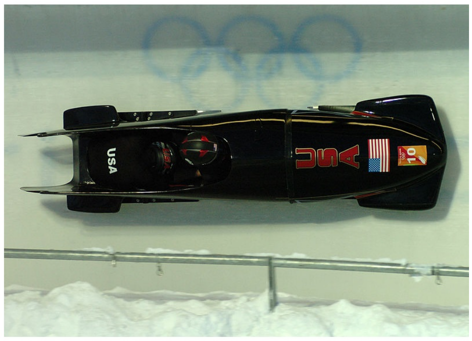
$$t_{\text{tot}}\Big|_{t=0} = 2\sqrt{\frac{2H}{g}} \ , \tag{31}$$

which completely agrees with our intuition: the total time is twice the time it takes the object to go either up or down by height H.

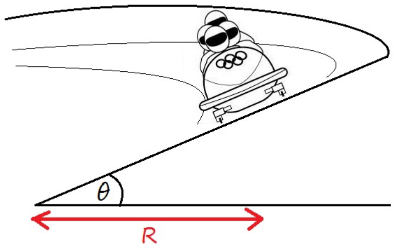
Problem 3

In bobsleigh (a winter sport) teams of two or four teammates make timed runs down narrow, twisting, banked, iced tracks in a gravity-powered sled. It's an Olympic sport of long tradition - below you can see (left to right) the Swiss bobsleigh team in 1910, an East German bobsleigh in 1950 and team USA in 2006:





Let us consider the motion of such a bobsleigh when it's riding along a banked curve of radius R, inclined at an angle θ to the horizontal as shown:



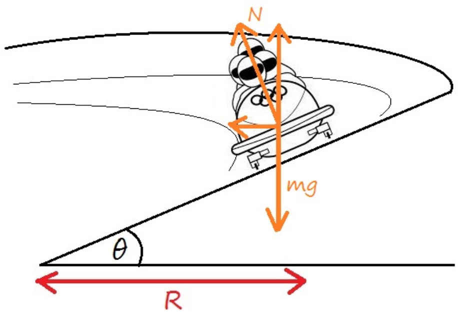
You can assume that the coefficients of friction between the bobsleigh and the track are negligibly small.

a) At what speed should the bobsleigh travel in order to make it safely around the curve (that is, without slipping up or down)?

If the bobsleigh does not move up or down the curve it means that it stays on the path characterized by radius R. For this radius the centripetal acceleration needed to sustain the motion is

$$a_r = \frac{v^2}{R} \qquad \Rightarrow \qquad v = \sqrt{a_r R}\ . \tag{32}$$

Now, this radial acceleration is given by the horizontal part of the normal force, see figure below:



Note: $N \neq mg\cos\theta$! This is because due to the radial friction the situation is not static. Instead, we know for sure that the vertical part of N balances the weight, $N\cos\theta = mg$, and then

$$a_r = \frac{N\sin\theta}{m} = \frac{g\sin\theta}{\cos\theta} \qquad \Rightarrow \qquad v = \sqrt{gR\tan\theta}\ . \tag{33}$$

b) One of the team members has a true weight given by W. What is her apparent weight if the bobsleigh travels safely around the curve?

We know that

$$W = mg \ . \tag{34}$$

The apparent weight of an object is given by the value of the normal force acting on it. Thus

$$W_{\text{apparent}} = N \ . \tag{35}$$

Now, we know that $N = \frac{mg}{\cos\theta}$, so that

$$W_{\text{apparent}} = \frac{mg}{\cos\theta} = \frac{W}{\cos\theta} \ . \tag{36}$$

c) A bobsleigh competition is being organized in high mountains, where one can use naturally occurring embankments along with artificial ones. The organizers are considering the following problem: when a bobsleigh leaves such a naturally shaped track, that bobsleigh is headed straight towards a precipice. Bobsleighs have brakes that will stop them well before the precipice, however, the organizers are considering safety measures in case brakes in one of the bobsleighs break down. At this particular point in track an average speed of a bobsleigh is V, and the precipice is a distance D away. By comparing the amount of friction required to stop the bobsleigh while it's headed straight towards the abyss to the amount of friction required to turn the bobsleigh away (while maintaining a constant speed), determine the safest course of action for bobsleigh teams.

If the bobsleigh is to go straight, then it needs to loose all of its speed before the precipice. That is we need (in the limiting case when the bobsleigh stops just before the precipice)

$$V^2 - 2aD = V^2 - \frac{2fD}{m} = 0 \ , \tag{37}$$

where f is the friction. Then

$$f = \frac{mV^2}{2D} \ . \tag{38}$$

On the other hand if the bobsleigh is to turn we need the friction to act as a centripetal force that will force the bobsleigh to move in a circle of radius D, that is we need

$$f = \frac{mV^2}{D} \ . \tag{39}$$

This means that in order to just stop the bobsleigh one needs half the friction needed for the bobsleigh to turn. This in turn means that, paradoxically, it's safest for the bobsleigh with faulty brakes to just continue straight towards the precipice.

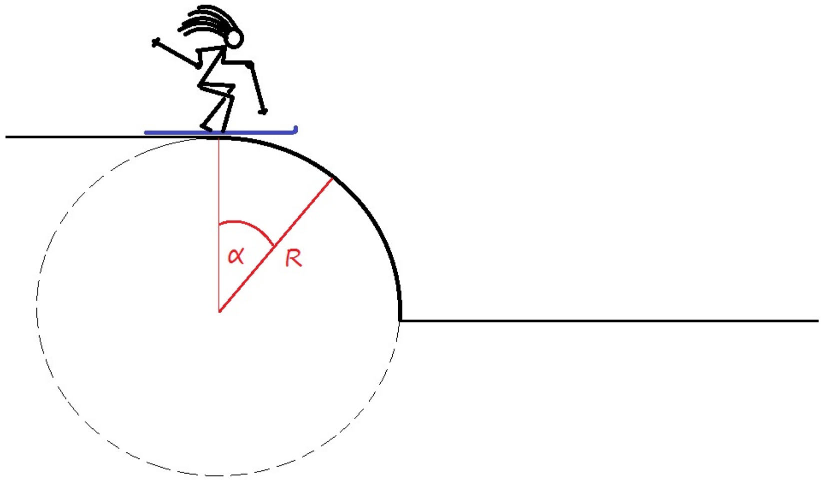
Problem 4

A group of students gets bored on a skiing trip and decides to try something new. They plan to ski down a perfectly spherical part of the hill (see figure).

a) First, a coy student starts at the top of the "sphere" with very small initial speed - barely a push to get him going (meaning you can assume the initial velocity is zero). He then skids down the side of the ball. At what angle α does he lose contact with the hill and fly off at a tangent?

b) Another student is much braver: she approaches the spherical part of the hill with an initial speed v_0. And what angle does she lose the contact with the ground?

c) What initial speed must the next student have to shoot off the top of the hill without ever touching the spherical surface?



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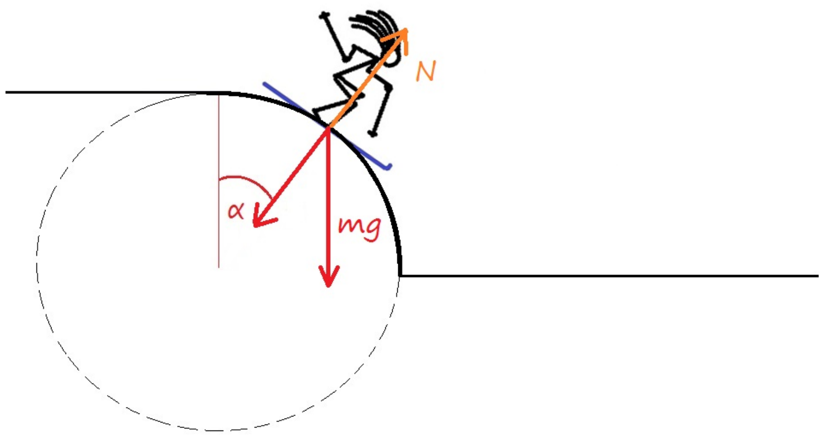
a) Let us consider the motion of the student somewhere along the hill, as pictured below:



The situation is not static so that the normal force does not balance gravity. In fact, the forces along the radius of the sphere must combine to provide the radial acceleration. I.e.,

$$mg\cos\alpha - N = ma_r = \frac{mv^2}{R}\ . \tag{40}$$

Now, when the skier loses contact with the hill *the tension N goes to zero!* This if we put $N = 0$ in the above equation we will get a condition for the point where the students flies off:

$$g\cos\alpha = \frac{v^2}{R}\ . \tag{41}$$

In this equation we do not know both the angle α and the velocity v the students has at this point, so that we clearly need another equation.

We obtain this equation in the following way: ultimately it's gravity that is responsible for the student's gain in speed. At an angle α the student has traveled a distance

$$h = R(1 \cos\alpha) \tag{42}$$

downwards. We can then use $v^2 - v_0^2 = 2ax$ and write

$$v^2 = 2gh = 2gR(1 - \cos\alpha) \ . \tag{43}$$

I know some of you were skeptical about this point during the review. Let us solve for v^2 in a yet another way using kinematics. We know that the translational acceleration along the sphere is

$$a_t = g\sin\theta \ . \tag{44}$$

Then

$$dv_t = a_t \ dt \ . \tag{45}$$

We can express the distance along the arc traveled in time dt as

$$ds = v_t \ dt = R \ d\theta \qquad \Rightarrow \qquad dt = \frac{R}{v_t} \ d\theta \ , \tag{46}$$

so that now we are dealing with

$$dv_t = a_t \frac{R}{v_t} \ d\theta \qquad \Rightarrow \qquad v_t \ dv_t = a_t R \ d\theta = gR\sin\theta \ d\theta \ . \tag{47}$$

Now we are ready to integrate:

$$\int_{v_0}^{v_f} v_t \ dv_t = gR\int_{\theta_0}^{\theta_f} d\theta \ \sin\theta \qquad \Rightarrow \tag{48}$$

$$\Rightarrow \qquad \frac{1}{2}\big[v_t^2\big]_{v_0}^{v_f} = gR\big[-\cos\theta\big]_{\theta_0}^{\theta_f} \ , \tag{49}$$

$$\frac{1}{2}\big(v_f^2 - v_0^2\big) = gR\Big(\cos\theta_0 - \cos\theta_f\Big) \ , \tag{50}$$

$$v_f^2 - v_0^2 = 2gR\Big(\cos\theta_0 - \cos\theta_f\Big) \ . \tag{51}$$

In our case $\theta_0 = 0$, so that this becomes

$$v_f^2 - v_0^2 = 2gR\big(1 - \cos\theta\big) \ . \tag{52}$$

See? I told you! ;)

Now then we can plug (43) into (41) to obtain

$$g\cos\alpha = 2g(1\cos\alpha) \qquad \Rightarrow \qquad \cos\alpha = \frac{2}{3} \qquad \Rightarrow \qquad \alpha = \cos^{-1}\left(\frac{2}{3}\right) = 48.19^\circ \ . \tag{53}$$

Note that this angle does not depend on mass! Every object sliding down a spherical surface falls off at this angle.

b) Here we need to adjust equation (43) to

$$v^2 - v_0^2 = 2gR(1 - \cos\alpha) \qquad \Rightarrow \qquad v^2 = v_0^2 + 2gR(1 - \cos\alpha) \ . \tag{54}$$

Then (41) becomes

$$g\cos\alpha = \frac{v_0^2}{R} + 2g(1 - \cos\alpha) \qquad \Rightarrow \qquad \cos\alpha = \frac{v_0^2}{3gR} + \frac{2}{3} \qquad \Rightarrow \qquad \alpha = \cos^{-1}\left(\frac{v_0^2}{3gR} + \frac{2}{3}\right) \ . \tag{55}$$

Note that this result nicely converges to (53) for $v_0 = 0$, as it should.

c) If the skier shoots off without ever touching the spherical surface it means that she detaches at α equal zero, or, equivalently, for the argument of $\cos^{-1}$ equal to 1. This sets the condition for th minimal initial shoot off velocity (for all higher velocities the argument of $\cos^{-1}$ will be bigger than 1 and thus the formula will become meaningless)

$$\frac{v_0^2}{3gR} + \frac{2}{3} \geq 1 \qquad \Rightarrow \qquad v_0^{(\text{shoot off})} \geq \sqrt{gR} \, . \tag{56}$$

Problem 5

Victor is on the 100th floor of an office building. At each of times $t_1 = 0$ and $t_2 > 0$ he drops a metal ball from a window. While the balls are falling, will the distance between them increase, decrease, or stay the same? Explain your answer using kinematics. You may neglect air resistance.

We need to write down equations of motion for both objects. For the first ball the equation of motion is

$$y_1(t) = H - \frac{1}{2}g(t - t_1)^2 = H - \frac{1}{2}gt^2 \ , \tag{57}$$

where H is the height of the building (which we don't know). For the second ball the equation of motion is

$$y_2(t) = H - \frac{1}{2}g(t - t_2)^2 \ . \tag{58}$$

The distance between the two falling balls is at any time $t > t_2$ given by

$$D = y_2(t) - y_1(t) \ . \tag{59}$$

Let us plug in the equations of motion:

$$D = H - \frac{1}{2}g(t - t_2)^2 - \big(H - \frac{1}{2}gt^2\big) = -\frac{1}{2}g\big(t^2 - 2tt_2 + t_2^2\big) + \frac{1}{2}gt^2 = \frac{1}{2}gt_2\big(2t - t_2\big) > 0 \ . \tag{60}$$

We can easily see that for any t and t_2 such that $t > t_2 > 0$ the expression above is always positive and is also a linear function of time - meaning that the distance between the two objects will always increase.

Problem 6

This is a Corbin midterm problem.

A small hot-air balloon slowly rises from the surface of the Earth at a constant speed v_b. Nearby a young child holds a loaded slingshot above his head, pointed straight up. When the balloon reaches a height H above the slingshot, the child fires a marble with a large velocity v_m along a vertical path adjacent to that of the balloon.

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a) How fast is the marble moving (relative to the child) when it first overtakes the balloon?

We write down the equations of motion for both the marble and the balloon:

$$y_b(t) = H + v_b t \quad \text{and} \quad y_m(t) = v_m t - \frac{1}{2}gt^2 \ . \tag{61}$$

When the balloon and marble are at the same height, we have

$$y_b(t) = y_m(t) \quad \Rightarrow \quad H + v_b t = v_m t - \frac{1}{2}gt^2 \quad \Rightarrow \quad \frac{1}{2}gt^2 + (v_b - v_m)t + H = 0 \ , \tag{62}$$

$$\Rightarrow \quad \Delta = \left(v_b - v_m\right)^2 - 2gH \ , \quad \sqrt{\Delta} = \sqrt{\left(v_b - v_m\right)^2 - 2gH} \quad \Rightarrow \tag{63}$$

$$\Rightarrow \quad t_\pm = \frac{v_m - v_b \pm \sqrt{\left(v_b - v_m\right)^2 - 2gH}}{g} \ . \tag{64}$$

We choose the smaller of the times (the other one is when the marble passes the balloon on its way back down to the ground), so that the marble's velocity at that point comes out to be

$$v(t_-) = v_m - gt_- = v_b + \sqrt{\left(v_b - v_m\right)^2 - 2gH} \ . \tag{65}$$

b) How far above the balloon will the marble appear to go?

We maximize the distance between the balloon and the marble:

$$D = y_m(t) - y_b(t) = -\frac{1}{2}gt^2 + (v_m - v_b)t - H \ , \tag{66}$$

$$\frac{dD(t)}{dt} = -gt + v_m - v_b = 0 \quad \Rightarrow \quad t_{\max} = \frac{v_m - v_b}{g} \ , \tag{67}$$

so that

$$D_{\max} = D(t_{\max}) = -\frac{1}{2}g\left(\frac{v_m - v_b}{g}\right)^2 + (v_m - v_b)\frac{v_m - v_b}{g} = \frac{1}{2}\frac{(v_m - v_b)^2}{g} \ . \tag{68}$$

c) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon? Explain.

At that point in time

$$v(t_{\max}) = v_m - gt_{\max} = v_b \ . \tag{69}$$

This makes sense because as soon as the marble's velocity is smaller than v_b, the distance between the marble and the balloon starts to diminish.

d) How much time will elapse between the marble's first encounter with the balloon and its last?

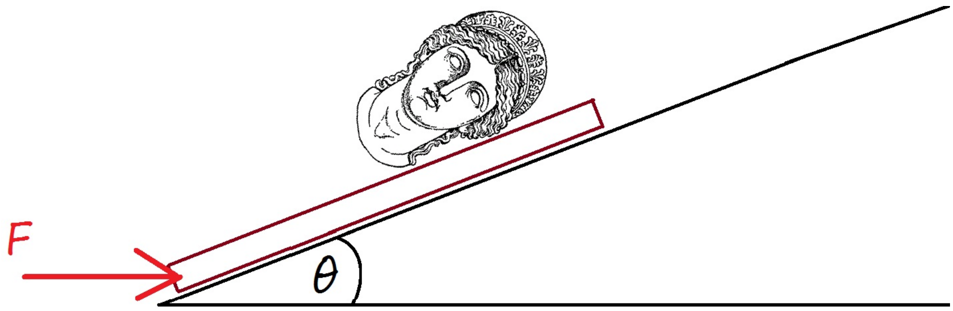
We already solved for that in part a)! This time is

$$\Delta t = t_+ - t_- = \frac{2\sqrt{\left(v_b - v_m\right)^2 - 2gH}}{g} \ . \tag{70}$$

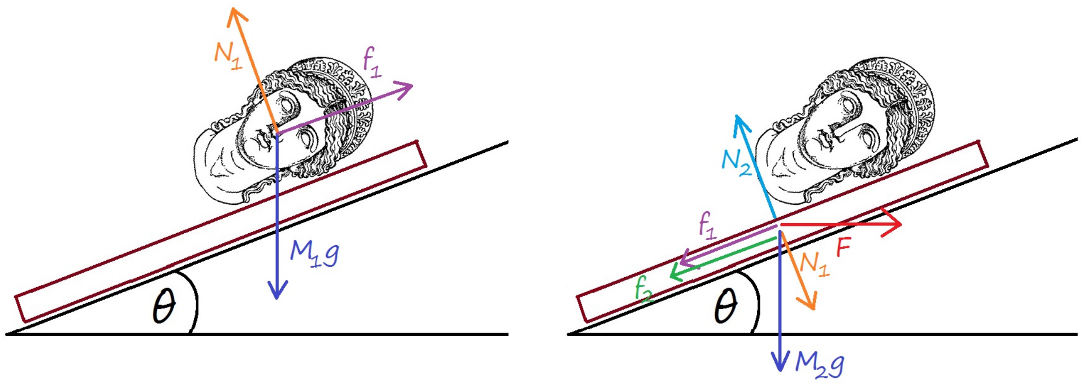
Problem 7

This is a Corbin midterm problem.

After robbing the museum, the thieves need to carry a very unwieldy art piece (of mass M_1) up a hill to their hideout. Instead of risking tripping and falling down the hill with their hands full, they place the sculpture on a wooden plank (of mass M_2) and lay the plank on the hill. Then they apply a horizontal force F to the bottom of the plank, as shown below. The coefficients of friction between the art piece and the plank are given as μ_{s1} and μ_{k1}, while the coefficients of friction between the plank and the ground are given as μ_{s2} and μ_{k2}.



a) Assume the art piece stays at rest relative to the plank as the plank slowly rises up the plane. Sketch the coordinate system you intend to use on the figure above, then below draw free-body diagrams for the sculpture and the plank. For full credit you need to label each diagram and clearly identify (correctly and unambiguously) each of the forces that act on the objects and the directions in which they act.



b) What is the magnitude and direction of the sculpture's acceleration?

Since the sculpture and the plank do not move with respect to each other, we can treat them as one body of mass $M_1 + M_2$. Then we can write

$$F\cos\theta - f_{k2} - (M_1 + M_2)g\sin\theta = (M_1 + M_2)a\ . \tag{71}$$

Since

$$f_{k2} = \mu_{k2}N_{\text{total}} = \mu_{k2}\big[(M_1 + M_2)g\cos\theta + F\sin\theta\big]\ , \tag{72}$$

we have

$$F\cos\theta - \mu_{k2}\big[(M_1 + M_2)g\cos\theta + F\sin\theta\big] - (M_1 + M_2)g\sin\theta = (M_1 + M_2)a \qquad \Rightarrow \tag{73}$$

$$\Rightarrow \qquad a = \frac{F(\cos\theta - \mu_{k2}\sin\theta) - (M_1 + M_2)g(\sin\theta + \mu_{k2}\cos\theta)}{M_1 + M_2} \tag{74}$$

c) How large is the force of friction acting on the art piece?

Since the sculpture's equation of motion is

$$f_1 - M_1 g \sin\theta = M_1 a \ , \tag{75}$$

the force of friction acting on it is

$$\begin{aligned} f_1 &= M_1 g \sin\theta + M_1 a = M_1 \frac{F(\cos\theta - \mu_{k2}\sin\theta) - (M_1+M_2)\mu_{k2} g\cos\theta}{M_1+M_2} = & (76)\\ &= F\frac{M_1(\cos\theta - \mu_{k2}\sin\theta)}{M_1+M_2} - \mu_{k2} M_1 g\cos\theta \ . & (77) \end{aligned}$$

d) How large would the horizontal force have to be in order to cause the sculpture to begin to slide?

The maximum static friction that can act on the sculpture is

$$f_1^{(\mathrm{max})} = \mu_{s1} N_1 = \mu_{s1} M_1 g \cos\theta \ . \tag{78}$$

Thus the maximum force F that can be applied without the sculpture sliding down is given by

$$\begin{aligned} \mu_{s1} M_1 g\cos\theta &= F_{\mathrm{max}} \frac{M_1(\cos\theta - \mu_{k2}\sin\theta)}{M_1+M_2} - \mu_{k2} M_1 g\cos\theta \qquad \Rightarrow & (79)\\ F_{\mathrm{max}} &= \frac{(\mu_{s1}+\mu_{k2})(M_1+M_2) g\cos\theta}{\cos\theta - \mu_{k2}\sin\theta} & (80) \end{aligned}$$

Any force F bigger than F_{max} will cause the sculpture to slide.

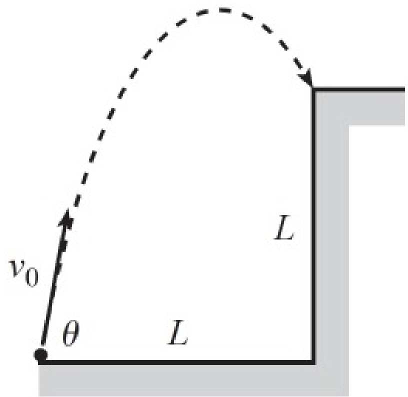
Problem 8

A small ball is launched from the ground onto the corner of a cliff as shown.

a) For which launch angles θ in the range $[0, \pi]$ is it possible for the ball to hit the corner? Explain.

b) For a given angle θ belonging to the above range, what launch speed v_0 is necessary for the ball to hit the corner?

c) Examine the limiting cases $\theta \to \frac{\pi}{2}$ and $\theta \to \frac{\pi}{4}$. What happens to the required launch speed in each of these cases according to the formula you derived in part b)? Does the behavior of your formula make sense in these limiting cases? *(Hint: if this behavior makes little sense then examine the formula carefully - is it correct? [Probably not - either that or you have rather poor physical intuition ;).])*



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a) The launch angle must be in the open interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. For $\theta \geq \frac{\pi}{2}$ the ball will go either vertically or to the left, so it clearly can't hit the corner. If $\theta \leq \frac{\pi}{4}$, notice that even "infinite" speed would just make the ball hit the vertical wall by traveling on a near-straight line. Any finite speed will just hit even lower.

b) We are keeping the angle θ fixed. We want

$$y(t_{\text{hit}}) = v_0 \sin\theta \; t_{\text{hit}} - \frac{1}{2} g t_{\text{hit}}^2 = L \quad \text{and} \quad x(t_{\text{hit}}) = v_0 \cos\theta \; t_{\text{hit}} = L \; . \tag{81}$$

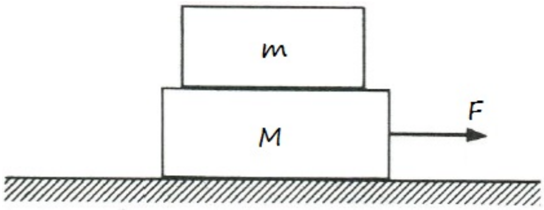
We use the second of equations to solve for t_{hit} and plug that into the first equation to get

$$v_0 = \sqrt{\frac{gL}{2\cos^2\theta(\tan\theta - 1)}} \; . \tag{82}$$

c) In both limits $v_0 \to \infty$. This makes sense: we discussed the case $\theta = \frac{\pi}{4}$ in part a), and the explanation for $\theta = \frac{\pi}{2}$ is the same: one needs an infinite speed to hit L if one launches the ball almost vertically.

Problem 9

A block of mass m rests on top of a block of mass M, which rests on a frictionless table. The coefficient of friction between the two blocks is such that the blocks start to slip when the horizontal force applied to the lower block is equal F. Suppose that now a horizontal force is applied only to the upper block, what is the maximum value of this force for the blocks to slide on the table without slipping relative to each other?



In the first situation, in the moment just before the blocks slip (so static friction is maximal), we have the following equations of motion:

$$mg = N_1 \ , \tag{83}$$
$$f = \mu_s mg = ma \ , \tag{84}$$
$$F - f = Ma \ . \tag{85}$$

Then we easily obtain

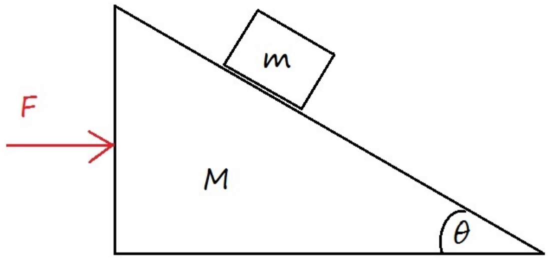
$$F - m_a = Ma \qquad \Rightarrow \qquad a = \frac{F}{M+m} \ , \tag{86}$$
$$f = F\frac{m}{M+m} \ . \tag{87}$$

Now we apply force F' to the upper block and want to find the maximal force under which the blocks move together without slipping. The equations of motion are now

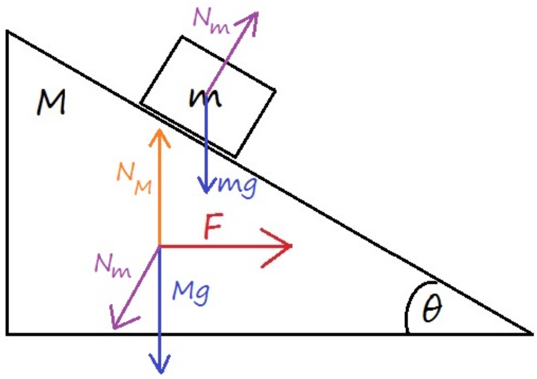
$$F' - f = ma \ , \tag{88}$$
$$f = Ma \qquad \Rightarrow \qquad a = \frac{f}{M} \qquad \Rightarrow \qquad F' - f = f\frac{m}{M} \tag{89}$$
$$\Rightarrow \qquad F' = f\left(1 + \frac{m}{M}\right) = f\frac{m+M}{M} = F\frac{m}{M} \ . \tag{90}$$

Problem 10

A block of mass m rests on a frictionless wedge of mas M, which itself rests on a frictionless floor. What external force must be applied to prevent block from sliding down?



The free body diagrams will look the following way:



We are mostly interested in forces acting in the horizontal direction. For the wedge the equation of motion is

$$F - N_m \sin\theta = Ma \ , \tag{91}$$

while for the block on the wedge the equation of motion is

$$N_m \sin\theta = ma \ . \tag{92}$$

Note that we have used the fact that in the situation described both objects have the same acceleration a.

We find N_m from the condition

$$N_m \cos\theta = mg \qquad \Rightarrow \qquad N_m = \frac{mg}{\cos\theta} \ , \tag{93}$$

and plug it into (92) to solve for a,

$$a = g\tan\theta \ , \tag{94}$$

which we then plug into (91) and solve for F:

$$F = (M+m)g\tan\theta \ . \tag{95}$$