Physics 1A
Spring 2015

Challenge Problem 4

A cylindrical drum of radius R rolls down a slope *without slipping*. Its center has velocity v and acceleration a parallel to the slope. What are the drum's angular velocity ω and angular acceleration α in terms of v and a? Justify your answer mathematically.

Solution. In a time Δt, the angle $\Delta\theta$ for which the drum has rolled equals $\Delta s/R$ where Δs is the amount of its circumference has moved along the slope which gives

$$\Delta s = R\Delta\theta. \tag{1}$$

On the other hand, *because the drum is rolling without slipping*, drawing a picture convinces one that the amount Δs that the circumference has rolled along the ground in time Δt equals the displacement Δx of the center of the drum;

$$\Delta s = \Delta x. \tag{2}$$

If we combine these two facts into one equation and divide both sides by Δt, then we get

$$\frac{\Delta x}{\Delta t} = R\frac{\Delta\theta}{\Delta t}. \tag{3}$$

Taking the limit $\Delta t \to 0$ on both sides therefore gives

$$\boxed{v = R\omega}. \tag{4}$$

Taking the derivative of both sides with respect to time then gives

$$\boxed{a = R\alpha}. \tag{5}$$