Physics 22: Homework 9

The following problems deal with electromagnetic induction and motional EMF.

1. Figure 1 shows two concentric conducting loops, the outer connected to a battery and a switch. The switch is initially open. It is then closed, left closed for a while, and then reopened. While explaining your reasoning, fully describe the induced current in the inner loop, if any, during the entire procedure.

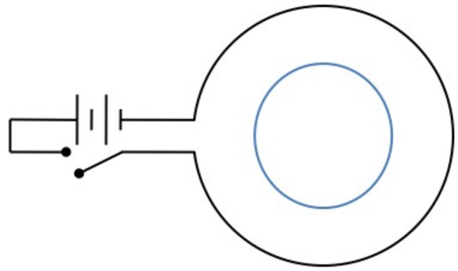

Figure 1: A conducting loop has the ability to run a current when the switch attaching it to the battery is closed. The concentric conducting inner loop will have a magnetic flux linkage with the outer loop when the outer loop runs a current.

2. A bar magnet is moved steadily through a conducting ring, as shown in Figure 2.

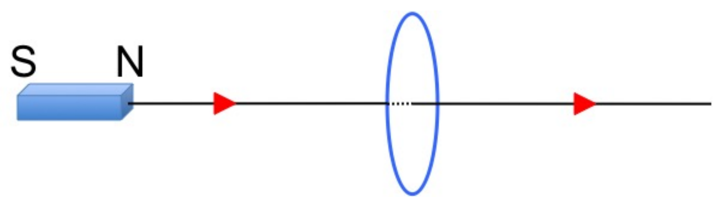

Figure 2: A bar magnet is moved steadily into and out of a conducting loop. The North pole of the magnet is facing the loop as it approaches the loop, while its South pole will be facing the loop as the magnet exits the loop.

   (a) Upon choosing an appropriate direction as the positive sense of current circulation, qualitatively sketch the current and power dissipation in the ring as functions of time. Be sure to indicate the position of a reference point on the bar magnet on your time axis.

   (b) In order for the bar magnet to be moved steadily through the conducting loop, work must be done by, say, the person orchestrating the movement. While explaining your reasoning, qualitatively discuss the sign of the work done by the person throughout the process of moving this magnet steadily into and out of the loop.

3. A square wire loop, of side length $\ell = 3.0$ m, is in a plane perpendicular to a uniform magnetic field, with this field pointing into the page. A 6-Volt light bulb is in series with the loop, as shown in Figure 3. From an initial strength of $B_0 = 2.0$ T, the field is reduced steadily to zero over some time.

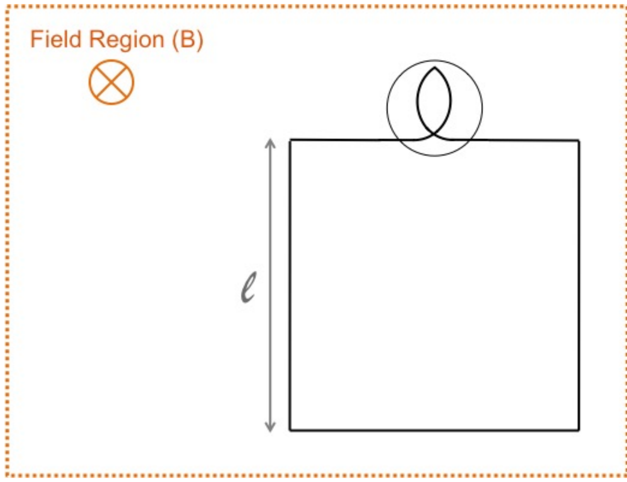

Figure 3: A square wire loop has a 6-V light bulb attached to it. The loop is immersed in a uniform magnetic-field region that points consistently into the page. The field undergoes a steady reduction.

(a) Find the time, Δt, over which the field must be reduced so that the bulb will shine at full brightness during this process.

(b) In which way will the induced current flow in the square wire loop?

4. A solenoid—of length $\ell = 2.0$ m, diameter $D = 15$ cm, and total turns $N = 2000$—has a current running through its coils that is increasing at the rate of $\gamma = 1.0$ kA/s.

(a) Consider a wire loop, of diameter $d = 10$ cm and resistance $R = 5.0\ \Omega$, that is concentric to the solenoid axis and, thus, in a plane perpendicular to the field generated by the solenoid. Determine the magnitude of the current that is induced in this wire loop.

(b) Repeat Part (a) for a similarly oriented wire loop of the same resistance but with a diameter of $d' = 25$ cm.

5. As shown in Figure 4, a solenoid—of length $\ell = 2.0$ m, diameter $D = 30$ cm, and $N = 5000$ turns—has a concentric coil, with $\mathcal{N} = 5$ turns, wrapped around the solenoid with roughly the same diameter and length. This concentric coil has a total resistance $R = 180\ \Omega$. The solenoid runs a current unidirectionally, specifically such that the magnetic field generated by the solenoid points downward relative to Figure 4.

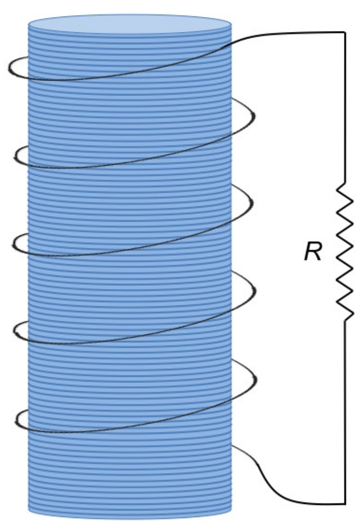

Figure 4: A long, tight-coiled solenoid has a concentric coil wrapped around it that is hooked up to a resistor. The solenoid generates a magnetic field in the downward direction that will eventually decay due to the current running through the coils of the solenoid exponentially decaying.

(a) At $t = 0$, the solenoid current begins to decay exponentially, with a profile

$$I(t) = I_0 e^{-\alpha t},$$

with $I_0 = 85$ A and $\alpha = 0.40\ \mathrm{s}^{-1}$.

i. What is the direction of the current running through the resistor as the current in the solenoid decays?

ii. What is the value of the resistor current at $t = 1.0$ s and at $t = 5.0$ s?

iii. Sketch a rough graph of the resistor current as a function of the time.

(b) Now, assume that the current in the solenoid in Figure 4 has the form shown in the graph in Figure 5.

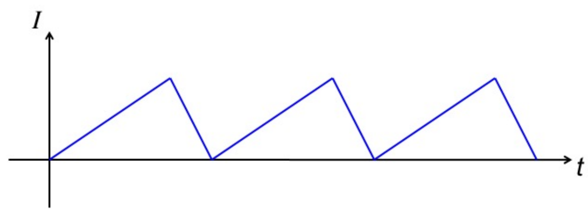

Figure 5: The current profile of the solenoid, behaving as a "saw-tooth" in its time-dependence.

i. Sketch a qualitative plot of the current running through the resistor as a function of the time.

ii. Determine the basic mathematical relationship between the graph in Figure 5 and the result of Part (ci).

(c) Now, assume that the current in the solenoid is given by

$$I(t) = I_0 \sin(\omega t),$$

with $I_0 = 85$ A and $\omega = 210$ rad/s.

i. Find an expression for the current running through the resistor.

ii. Determine the peak current in the resistor.

iii. What is the resistor current when the solenoid's current is at a maximum?

6. A square conducting loop—of side length $\ell = 0.50$ m and resistance $R = 5.0\ \Omega$—moves to the right with constant speed $v = 0.25$ m/s. As shown in Figure 6, at $t = 0$ the rightmost edge of the wire loop enters a region, of width $w = 0.75$ m, of uniform magnetic field with strength $B = 1.0$ T that points into the plane of the page.

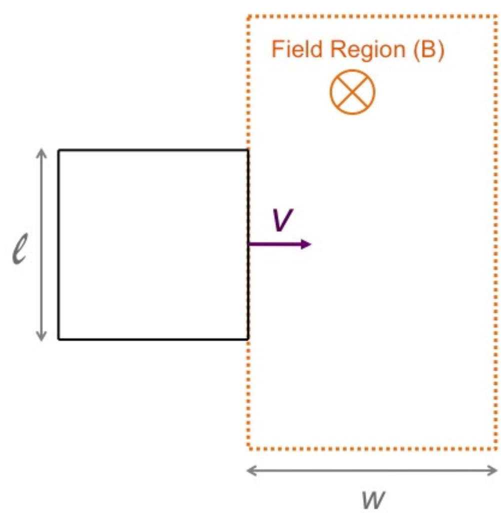

Figure 6: A square conducting loop is traveling at constant velocity entering a uniform magnetic-field region. The loop is small enough that it will be fully immersed within the field region over a range of times, and will completely exit the field region in a time, T.

(a) Determine the time, T, that it takes the loop to completely exit the field region from when its right edge first enters (i.e., referencing $t = 0$).

(b) Taking a Clock-Wise (CW) current through the square loop (relative to Figure 6) as positive, plot the current running through this loop as a function of the time, t, for $t \in [0, T]$.

(c) Plot the power dissipated in the loop as a function of the time, t, for $t \in [0, T]$.

(d) Repeat Parts (a) - (c) in the case where the loop has side length $\ell' = 0.75$ m and the field region has width $w' = 0.50$ m.

(e) Qualitatively describe the changes that would occur in Parts (a) - (c) above if the conducting loop had a right-triangular shape in the orientation shown in Figure 7 as it entered, moved through, and exited the field region.

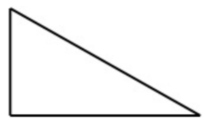

Figure 7: A right-triangular conducting loop that is supposed to enter in place of the square loop of Figure 6, with the exact orientation shown here.

7. A rectangular wire loop—of width $w = 75$ cm and length $\ell = 1.6$ m—is to be turned about an axis that is in the plane of the loop and that cuts the loop in half along its width. This wire loop is immersed in a uniform magnetic field of field strength $B = 0.25$ T. The loop is to produce an alternating emf of frequency $f = 60$ Hz with peak value $\mathcal{E}_{\text{max}} = 6.7$ kV.

   (a) How many turns must this loop have?

   (b) What is the maximum flux through the coils?

   (c) If the resistance of the wires is $R = 50\ \Omega$, sketch the current running through this loop and the power dissipated by this loop as functions of the time.

8. Figure 8 shows a pair of parallel, perfectly conducting rails a distance ℓ apart in a uniform magnetic field, of strength B, that points into the page. A resistor, of resistance R, is connected across the rails on one end, while a conducting bar, of negligible resistance, is being pulled along the rails with a constant speed, v, to the right.

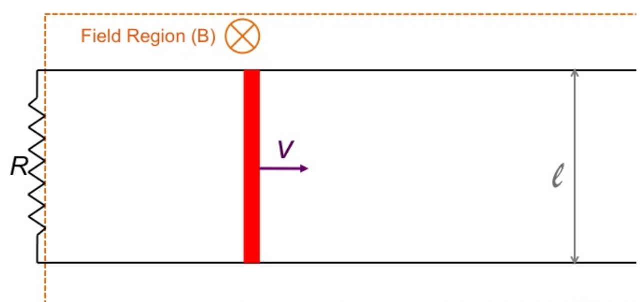

Figure 8: A conducting bar makes electrical contact with a pair of long, parallel, perfectly conducting rails that are attached via a resistor on one end. The bar is moving at a constant velocity, and the system is immersed in a uniform magnetic-field that points into the page.

   (a) What is the direction of the induced current in the resistor?

   (b) Why must positive work be done by an external agent to ensure the movement of the conducting bar at constant velocity?

   (c) Determine the rate at which work must be done by the external agent pulling the bar.

   (d) Suppose the resistor in Figure 8 is replaced by an ideal voltmeter.

      i. To which rail should the positive terminal of the voltmeter be connected to ensure that it reads a positive voltage?

      ii. At what rate must work be done by the external agent pulling the conducting bar at constant velocity in this case?

9. Consider the exact same setup as Problem 8 with the exact same figure (Figure 8). Further suppose that the rails are frictionless and that the bar has a mass m and is initially at rest (i.e., $v(t=0)=0$). A constant force, F, is applied to the bar in the rightward direction (relative to Figure 8).

   (a) Describe the motion of the bar as the force continues to push it, and provide a rough sketch of the velocity of the bar as a function of the time, t.

   (b) Using Newton's 2$^{\text{nd}}$ Law and the method of separation of variables, obtain a quantitative expression for the velocity of the bar as a function of the time, t, in terms of the given parameters.

   (c) Determine at what time the bar's acceleration becomes zero and the speed of the bar at that point in time.

10. Consider a similar setup to Problem 8, now consisting of a resistor, of resistance R, in series with a battery, of emf $\mathcal{E}$, that is in series with a switch that is initially open, and again with perfectly conducting, frictionless rails and a perfectly conducting sliding bar. Initially the bar is at rest, and now no external agent pushes it. The setup is shown in Figure 9.

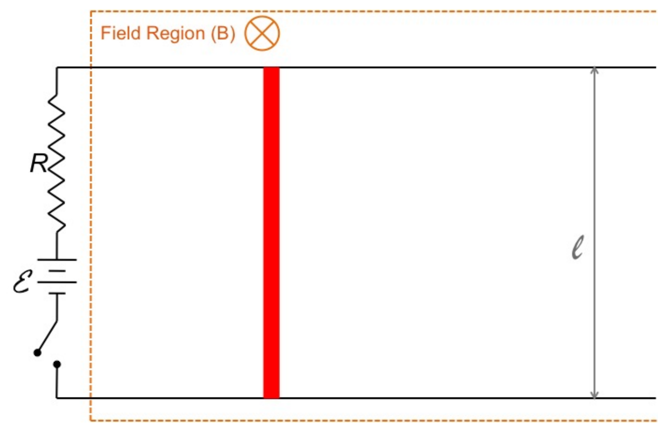

Figure 9: A conducting bar makes electrical contact with a pair of long, parallel, perfectly conducting, frictionless rails that are attached to a resistor, battery, and switch (all in series) on one end. The bar is initially at rest, and the system is immersed in a uniform magnetic-field that points into the page. The switch is closed at $t=0$ after being open for a very long time.

   (a) Describe the bar's subsequent motion.

   (b) The bar eventually reaches a constant velocity. Why?

   (c) Determine the value of this terminal speed in terms of the given quantities.

   (d) What role does the resistance in the circuit play on the system's dynamics? Discuss.

11. A copper disk—of diameter $D = 90$ cm and of negligible thickness—is spinning at a frequency $f = 3600$ rpm about a thin, conducting axle running through its center, as shown in Figure 10. A uniform magnetic field, of strength $B = 1.5$ T, is directed perpendicular to the disk (i.e., along the axle spinning the disk); specifically, it is directed upward relative to Figure 10. A stationary conducting brush maintains contact with the disk's rim, and a voltmeter is placed with one lead on the brush and the other lead on the conducting axle. Determine the voltmeter's reading and polarity.

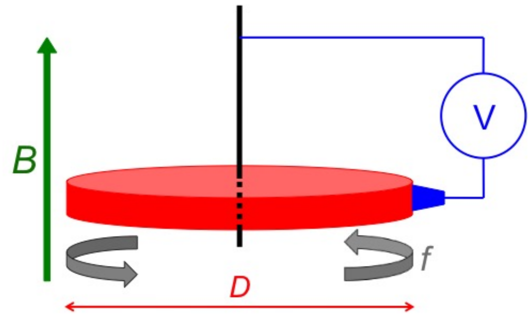

Figure 10: A copper disk is rotating at a fixed frequency, f, while immersed in a uniform magnetic field that points perpendicular to the plane of the disk (specifically, upward for a horizontally oriented disk). The conducting axle about which it rotates has an ideal voltmeter connecting it to the rim of the disk, with the connection to the perimeter of the disk being made by a conducting brush that consistently maintains electrical contact with the perimeter.

12. A circular wire loop—of resistance R and radius a—lies with its plane perpendicular to a uniform magnetic field. The field strength changes from an initial value, B_1, to a final value, B_2. By integrating the loop current over time, show that the total charge that moves around the loop is independent of how the field changes with time, and obtain an expression for this total charge in terms of the given quantities.