600 -g collar C may slide along pole AB. The spring has an undeformed length of 250 mm and a spring constant of 135 N/m. Knowing that the collar is released from rest at A and that

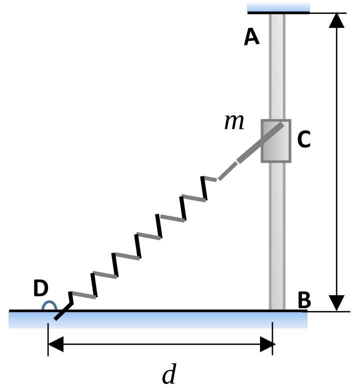
$h =$ 400.0 mm

$d =$ 300.0 mm

determine the speed of collar at B

a ) if there is no friction between collar C and rod AB,

b ) if there is friction between collar and rod with a coefficient of friction $\mu_K = 0.2$




## *SOLUTION*

| Given Data | | | | |
|---|---|---|---|---|
| Friction Coefficient: | $\mu_K =$ | | | 0.20 |
| **Spring constant:** | **k =** | | | **135.00 N/m** |
| Spring unstretched length: | $l_O =$ | 250 | mm = | 0.25 m |
| **Collar Mass:** | **m =** | **600 gr** | | **0.60 kg** |
| Rod's height: | $h =$ | 400.0 | mm = | 0.40 m |
| **See figure:** | **d =** | **300 mm =** | | **0.30 m** |
| See Figure: | AD = | 500.0 | mm = | 0.50 m |

Let us solve the problem in general with friction.

$K_A + U_A + W_{others} = K_B + U_B$

$v_A = 0 \quad ⇨ \quad K_A = 0$

$v_B = v \quad ⇨ \quad K_B = ½\, m\, v^2$

$y_A = h \quad ⇨ \quad (U_g)_A = m\, g\, h$

$y_B = 0 \quad ⇨ \quad (U_g)_B = 0$

$l_A = AD = (h^2 + d^2)^{1/2}$

$\Delta l_A = x_A = (h^2 + d^2)^{1/2} - l_O$

$(U_e)_{lA} = ½ kx^2 \quad ⇨ \quad (U_{el})_A = ½ k\, [\,(h^2 + d^2)^{1/2} - l_O]^2$

$\Delta l_B = x_B = d - l_O \quad ⇨ \quad (U_{el})_B = ½ k\, [d - l_O]^2$

For $W_{others}$:

$\Sigma F_x = 0$

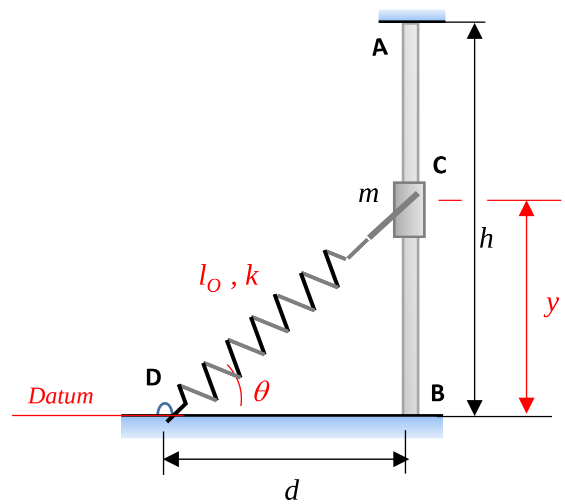
$N - F_S \cos\theta = 0$

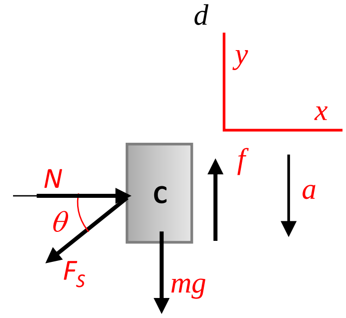
$N = F_S \cos\theta$

$f = \mu_K N = \mu_K F_S \cos\theta$

$f = \mu_K N = \mu_K\, k\, \Delta l\, \cos\theta$

$\Delta l = l - l_O = d/\cos\theta - l_O =$




F.B.D. of C

Therefore:

$f = \mu_K\, k\, (d/\cos\theta - l_O)\cos\theta$

Now

$W_f = -\int f\, dy \qquad y = h \text{ to } 0$

But

$y = d\tan\theta \qquad dy = d\,(1 + \tan^2\theta)\, d\theta = d\,(d\theta/\cos^2\theta)$

Substitute to get:

$W_f = -\int \mu_K\, k(d/\cos\theta - l_O)\cos\theta(\, d/\cos^2\theta\,)d\theta$

$W_f = -\mu_K\, kd \int(1/\cos^2\theta - l_O/\cos\theta\,)d\theta$

***General Solutions***

$\int d\theta/\cos\theta = \ln(1/\cos\theta + \tan\theta)$

$\int d\theta/\cos^2\theta = \tan\theta$

$W_f = -\mu_K kd\,(d\tan\theta - l_O\ln|1/\cos\theta + \tan\theta|)$ $\qquad \theta = 0$ to $\tan^{-1}(h/d)$

Substitute Values:

for $\theta = 0$ $\quad \tan\theta = 0$, and $\cos\theta = 1$

for $\theta = \tan^{-1}(h/d)$ $\quad \tan\theta = h/d = 400/300 =$ 1.33 $\quad$ and $\quad \cos\theta = d/AD = 0.3/0.5 = 0.6$

Therefore:

$W_f = -\mu_K kd\,[d(h/d) - l_O\ln(AD/d + h/d) - (0 - l_O\ln|1/1 + 0|)]$

$W_f = -(0.2) \times (135\ N/m)(0.3m)\ [(0.3(0.4/0.3) - 0.25 \times \ln(0.5/0.3 + 0.4/0.3)] =$ -1.02 j

## WITH Friction

$K_B = ½\,(0.6\ kg)v^2 =$ **$0.3\ v^2$**

$(U_g)_A = (0.6\ kg) \times (9.8\ m/s^2) \times (400.0/1000)\ m =$ **2.35 j**

$(U_{el})_A = ½(135)\,[\,(0.4^2 + 0.3^2)^{1/2} - 0.25]^2 =$ **4.21 j**

$(U_{el})_B = ½k\,[d - l_O]^2 = ½(135)\,(0.3 - 0.25)^2 =$ **0.17 j**

$W_{others} =$ **-1.02 j**

$0 + 2.35 + 2.35 - 1.02 = 0.3v^2 + 0.17$

$0.3v^2 =$ 5.38 j

$v = (1.42/0.3)^{1/2}$

**v = 4.23 m/s**

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## WITHOUT Friction

$K_B = ½\,(0.6\ kg)v^2 = 0.3\ v^2$

$(U_g)_A = (0.6\ kg) \times (9.8\ m/s^2) \times (400.0/1000)\ m =$ 2.35 j

$(U_{el})_A = ½(135)\,[\,(0.4^2 + 0.3^2)^{1/2} - 0.25]^2 =$ 4.21 j

$(U_{el})_B = ½k\,[d - l_O]^2 = ½\,(0.3 - 0.25)^2 =$ 0.17 j

$W_{others} = 0$

$0 + 2.35 + 0.25 + 0 - 0.66 = 0.3v^2 + 0.17$

$0.3v$ 6.39 j

$v = (1.77/0.3)^{1/2}$

**v = 4.62 m/s**