Physics 1A
Spring 2015

Challenge Problem 19

A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with 4/5 of its initial kinetic energy. Find the ratio of the mass of the unknown particle to the mass of the proton, assuming that the collision is elastic.

Solution. Let M be the mass of the proton, V_0 its initial velocity, and V its rebound velocity. Let m be the mass of the unknown particle, and let v be its velocity after the collision.

Momentum conservation gives

$$MV_0 = MV + mv \tag{1}$$

The collision is elastic, so kinetic energy is conserved which means that

$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2. \tag{2}$$

The problem tells us that the proton rebounds with 4/5 of its initial kinetic energy. This can be mathematically expressed as

$$\frac{1}{2}MV^2 = \frac{4}{5}\left(\frac{1}{2}MV_0^2\right). \tag{3}$$

If we plug this fact into the energy conservation equation, and do a little algebra, then we get

$$\frac{1}{5}V_0^2 = \frac{m}{M}v^2 \tag{4}$$

The given constraint (**??**) also tells us that $V = -2V_0/\sqrt{5}$ (note the negative sign because the proton rebounds!), and plugging this into the momentum conservation equation (**??**) and doing a little algebra gives

$$V_0\left(1 + \frac{2}{\sqrt{5}}\right) = \frac{m}{M}v \tag{5}$$

If we square both sides of this equation, and then divide it by (**??**), then we find

$$\boxed{5\left(1+\frac{2}{\sqrt{5}}\right)^2=\frac{m}{M}}. \tag{6}$$