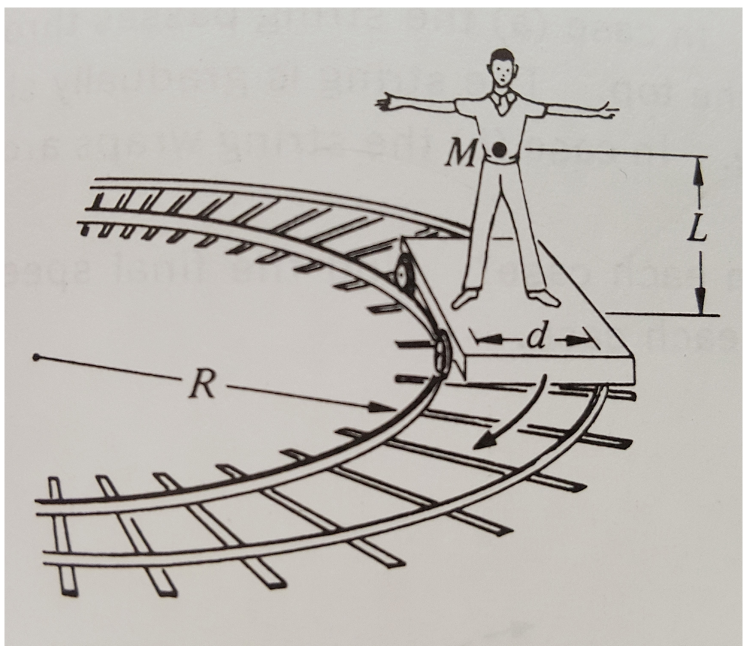
Physics 1A
Spring 2015

Challenge Problem 25

A man of mass M stands on a railroad car which is rounding an unbanked turn of radius R at speed v. His center of mass is a height L above the car, and his feet are a distance d apart.

How much weight is on each of his feet?



Solution.

We first note that the rate of change of $\mathbf{L}'$, the angular momentum of the man relative to his center of mass, is zero[1], and this implies that the net

[1] This is not actually as obvious as it might seem at first. It's clear that he's not rotating about his center of mass in the plane containing his arms and legs, but if you were to watch him from above from a frame of reference fixed relative to the track, you would see that he is spinning around his center of mass along an axis passing vertically through his center of mass. Nonetheless, this spinning occurs at a constant rate, so the rate of change of the spinning, and hence the associated angular momentum, is zero

external torque about this center of mass is zero:

$$\boldsymbol{\tau}'_{\text{ext}} = 0. \tag{1}$$

The external forces on the man are the normal forces on his feet, his weight, and the friction on his feet keeping his body moving in a circle. Let N_1 be the magnitude of the normal force on his inner foot, let N_2 be the magnitude of the normal force on his outer foot, and let f denote the magnitude of the friction force, Newton's Second Law in the x- and y-directions reveals the following relationships between these various forces.

$$N_1 + N_2 = Mg \tag{2}$$

$$f = M\frac{v^2}{R} \tag{3}$$

Here we have assume that d is much smaller than R, so we can treat the radius at which the force of friction acts as R for both of his feet, despite the fact that his inner foot is at radius R and his outer foot is at radius $R + d$.

Setting the sum of the external torques about this center of mass to zero gives

$$-fL - N_1\frac{d}{2} + N_2\frac{d}{2} = 0. \tag{4}$$

Note that gravity can be treated as acting at his center of mass, so it exerts no torque about his center of mass. We thus have three equations in three unknowns, N_1, N_2, and f. We are after N_1 and N_2 since these are the contact forces between each of his feet and the platform on which he stands. Some algebra gives

$$\boxed{N_1 = \frac{Mg}{2} - \frac{Mv^2}{R}\frac{L}{d}, \qquad N_2 = \frac{Mg}{2} + \frac{Mv^2}{R}\frac{L}{d}} \tag{5}$$

Does this result make sense? Are there limiting cases you can check? What about when $v \to 0$?