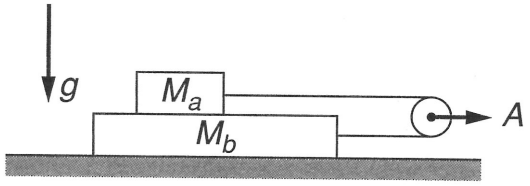
Physics 1A
Spring 2015

Challenge Problem 10

Mass M_a slides on top of mass M_b as shown. Assume $M_b > M_a$. The two blocks are pulled from rest by a massless rope passing over a massless pulley. The pulley is accelerated at rate A. Block M_b slides on a table without friction, but there is a constant friction force f between M_a and M_b due to their relative motion.

Find the tension in the rope.



Solution. Drawing free body diagrams for the two blocks, gives the following Newton's Second Law equations assuming that T is the magnitude of the tension in the rope, and f is the friction force on mass M_b dude to mass M_a:

$$T - f = M_a a_a, \qquad T + f = M_b a_b. \tag{1}$$

We have also implicitly used Newton's Third Law when we write that the friction force on block M_b is equal and opposite that on block M_a. There is also a constraint

$$a_a + a_b = 2A. \tag{2}$$

We therefore have a system of three equations in three unknowns a_a, a_b, T, and we can solve for T to obtain

$$\boxed{T = \frac{2M_a M_b A + (M_b - M_a) f}{M_a + M_b}}. \tag{3}$$