

Final Project Documentation

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1 Recurrence Relations/Dynamic Programming

1.1 Bell numbers

The Bell numbers represent the number of ways to count partitions of (or equivalently equivalence relations on) an n element set. The n -th bell number is given by the recurrence

$$B_n = \sum_{k=1}^n \binom{n-1}{k-1} B_{n-k}$$

for $n \geq 0$.

1.2 Catalan numbers

The Catalan numbers form a sequence of natural numbers that occur in various counting problems, often involving recursively-defined objects. They can be expressed by the recurrence relation

$$C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$$

for $n \geq 0$.

Their closed form is given by

$$\binom{2n}{n} - \binom{2n}{n+1}$$

1.3 Fibonacci numbers

The Fibonacci numbers, commonly denoted F_n , form a sequence such that each number is the sum of the two preceding ones, with $F_0 = 0$, $F_1 = 1$, and the recurrence given by

$$F_n = F_{n-1} + F_{n-2}$$

for $n > 1$.

1.4 Stirling numbers of the first kind

1.5 Stirling numbers of the second kind

2 Permutations and Combinations

2.1 Combinations without repetition

A combination without repetition is a selection of items from a collection, such that the order of selection does not matter. A k -combination of an n element set S is a subset of k distinct elements. The number of k -combinations is equal to the binomial coefficient given by

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

2.2 Permutations without repetition

k -permutations of n are the different ordered arrangements of a k -element subset of an n -set. This number is given by

$$P(n, k) = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-k+1) = \frac{n!}{(n-k)!}$$

2.3 Combinations with repetition

A k -combination with repetitions allowed is a sequence of k not necessarily distinct elements of S , where order is not taken into account, i.e. the number of ways to sample k elements from a set of n elements allowing for duplicates but disregarding different orderings. Using the *stars and bars method*, it can be shown that this number is given by

$$\binom{n+k-1}{k}$$

2.4 Permutations with repetition

Permutations with repetition are ordered arrangements of k elements from a set S with n elements where repetition is allowed. The number of permutations with repetition of size k is simply k^n .

2.5 Generate permutations of a string

2.6 Generate all bit strings of length n

3 Relations

3.1 # of relations

3.2 # of transitive relations

There is no known closed formula for counting the number of transitive relations. The (perhaps inefficient) approach taken in this algorithm is as follows

- (1) Generate all possible relations for an n element set (given by the power set of the cartesian product of the set $\{1, 2, 3, \dots, n\}$)
- (2) For each relation generated in (1), check that for each (a, b) , if there is a point of the form (b, c) , then (a, c) must be in the relation

3.3 # of (ir)reflexive relations

3.4 # of symmetric relations

3.5 # of antisymmetric relations

3.6 # of equivalence relations

4 Sets

4.1 Generate power set

4.2 Generate cartesian product

5 Isomorphisms

maybe total orders?

6 Default

No documentation provided.