30 April 2020 Friday BC Calc

9.5 Alternating Series Test, Abs/Conditional Convergence

Homework

Day 17

Absolute Convergence

If a series $\sum_{n=0}^{\infty} |a_n|$

(the sum of the absolute values)

converges, then the series

$$\sum_{n=0}^{\infty} a_n$$

converges absolutley

Alternating series

Definition.
A series in which the terms are alternating + and − is called an alternating series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots$$

alternating harmonic series

$a_n = \frac{1}{n}$

$a_n > 0$

$1-2+3-4+5 - - -$

alternating

LEIBNITZ'S THEOREM

(the alternating series test)

The series $\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 - - -$

converges if all three conditions are satisfied

1) each u_n is positive

2) $u_n \geqslant u_{n+1}$ for all n starting with a rank N

3) $\lim_{n \to \infty} u_n = 0$

Alternating Series (Leibniz's Theorem):

*** this section deals with series that have positive an negative terms.**

ex. $\sum_{\square}(-1)^n\left(\frac{1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

Alternating Series Test:

Let $a_n>0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ **and** $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ **converge if the following two conditions are met:**

1. $\lim_{n\to\infty} a_n = 0$ $\left(n^{\text{th}} \text{ test}\right)$

2. $a_{n+1} \le a_n$ for all n $\left(\text{ratio test } \frac{a_{n+1}}{a_n} \le 1\right)$

Alternating harmonic series

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+------.$

1) $u_n=\frac{1}{n}>0$ **because $n\geq 1$**

n positive integer

2) $u_{n+1}\leq u_n$

$\frac{1}{n+1}<\frac{1}{n}$ yes because $\frac{1}{n+1}-\frac{1}{n}<0$

(you must justify this step)

$\frac{n-n-1}{n(n+1)}<0$

True $\frac{-1}{n(n+1)}<0$

3) $u_n=\frac{1}{n}$ $\lim_{n\to\infty}\frac{1}{n}=0$

[illegible] $n\geq 1$

Determine if the following sereis converge abs , converge cond or diverge.

$$1)\ \frac{1}{2} - \frac{2}{2^2+1} + \frac{3}{3^2+1} - \frac{4}{4^2+1} + \dots$$

$$2)\ 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \dots$$

$1 - \frac{1}{2!} + \frac{1}{3!} - - - -$

1) $u_n = \frac{1}{n!} > 0 \quad n \geq 1$

2) $u_n > u_{n+1}$

$\frac{1}{n!} > \frac{1}{(n+1)!}$

$\frac{1}{n!} - \frac{1}{(n+1)!} > 0$

$\frac{n+1-1}{(n+1)!} > 0 \quad \frac{n}{(n+1)!} > 0 \quad n \geq 1$

3) $\lim_{n \to \infty} \frac{1}{n!} = 0$

This series is convergent (alternating series test)

$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n!} \right| = \sum_{n=1}^{\infty} \frac{1}{n!} = \underset{\text{conv}}{e-1}$$

Absolute convergent

1) $u_n = \frac{n}{n^2+1}$

1) $\frac{n}{n^2+1} > 0$, $n \geq 1$ n positive integer n^2+1 sum of squares

2) $u_n > u_{n+1}$

$\frac{n}{n^2+1} > \frac{n+1}{(n+1)^2+1}$ See if is true for $n \geq 1$

$\frac{n}{n^2+1} - \frac{n+1}{(n+1)^2+1} > 0$

$\frac{n[n^2+2n+2] - (n+1)(n^2+1)}{(n^2+1)((n+1)^2+1)} > 0$

$\frac{\cancel{n^3}+2n^2+2n-\cancel{n^3}-n-n^2-1}{(n^2+1)((n+1)^2+1)} > 0$

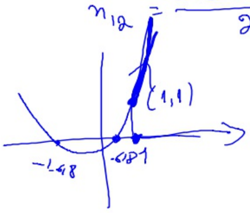
$\frac{n^2+n-1}{(n^2+1)((n+1)^2+1)} > 0$ for $n \geq 1$

$n \geq 1$

$n_{12} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$\frac{-1+\sqrt{5}}{2}$, $\frac{-1-\sqrt{5}}{2}$

≈ 0.618 ≈ -1.618



$f(n) = n^2+n-1$ $f(1) = 1+1-1$

$f'(n) = 2n+1$

$n \geq 1$

n	1	
$f(n)$	1	↗
$f'(n)$	+	+ +

$\lim\limits_{n\to\infty} \frac{n}{n^2+1} = \lim\limits_{n\to\infty} \frac{1}{2n} = 0$

3) $\lim\limits_{n\to\infty} \frac{n}{n^2+1} = 0$

Convergent according Leibnitz's test

$\sum\limits_{n=1}^{\infty} \left| (-1)^{n+1} \frac{n}{n^2+1} \right| = \sum\limits_{n=1}^{\infty} \frac{n}{n^2+1}$

$a_n = \frac{n}{n^2+1}$ $\quad b_n = \frac{1}{n}$ $\quad \lim\limits_{n\to\infty} \frac{a_n}{b_n} = \lim\limits_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{n^2}{n^2+1} = 1$

$\int_1^{\infty} \frac{dx}{x} = \cancel{\lim}$ div

$\sum\limits_{n=1}^{\infty} \frac{1}{n}$ $\quad p = 1$ harmonic series div

$\int^{\infty} \frac{x}{x^2+1} dx \qquad x^2+1 = u \qquad 2x\,dx = du$

$\lim\limits_{a\to\infty} \int_1^a \frac{x}{x^2+1} dx = \lim\limits_{a\to\infty} \frac{1}{2} \ln|x^2+1| \Big|_1^a$

$\frac{1}{2} \left[\lim\limits_{a\to\infty} \ln|a^2+1| - \ln 2 \right]$

∞

The absolute values is not conv

$\frac{1}{2} - \frac{2}{2^2+1} + \cdots$ is $\boxed{\text{conditional conv}}$

Do the series converge or diverge?

$$ex.\ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

$$ex\ \sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}}$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2}$$

$$ex.\ \sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n}$$

$$ex\ \frac{2}{1}-\frac{1}{1}+\frac{2}{2}-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\dots$$

Absolute and Conditional Convergence

- Used for series that are not alternating but contain both (+) and (-) terms. We check to see whether the abs. value case converges, and, if so, we say it converges absolutely.
- Not everything converges absolutely. Some series converge, but their abs value case does not. This is called conditional convergence.

Def:

1. Σa_n has absolute convergence if $\Sigma|a_n|$ converges.
2. Σa_n has conditional convergence if Σa_n converges and $\Sigma|a_n|$ diverges.

(note: easier to check $|a_n|$ case first. If $|a_n|$ converges, then a_n converges absolutely. If $|a_n|$ diverges, need to check a_n by alternate series or ratio test)

Determine if conv or diverge, if conv. determine if absolute or conditional.

ex. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

ex. $\sum_{n=0}^{\infty} \frac{(-1)^{n} n!}{2^{n}}$

ex. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

ex. $\sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}}{3^n}$

① $\sum |a_n| = \sum_{n=0}^{\infty} \frac{1}{3^n}$ $\sum \left(\frac{1}{3}\right)^n$

$|r| < 1$

$\frac{1}{3}$ ∴ conv.

Converges absolutely.

$ex. \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

Remainder Formula/Error form for Alt. Series

- if summing up n terms, the remainder(error) is less than the (n+1) term.

ex. approx the sum of the first six terms of $ex. \sum_{n=1}^{\infty} (-1)^{n+1} (\frac{1}{n!})$

error/remainder: $< a_7$