

# Partridges in Pear Trees – Rmarkdown

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## Motivation

The preparation of scientific documents with mathematical content has always given their authors and printers difficulty in rendering the material to paper or screen. This article is part of an exercise to illustrate how different systems for preparing such documents compare in capability and ease of use.

The text preparation systems to be compared are LaTeX ([1]), Typst ([2]) and R Markdown ([3]). We note that bibliographic support is also of interest to users.

## A useful example

The Christmas counting song “The Twelve Days of Christmas” suggest the singer receives one gift – we shall unabashedly simply declare that the Partridge in a Pear Tree is a single gift – on the first of twelve days, then that gift plus two more on the second, and so forth. For our needs, we want to have a general formula for the total number of gifts received after  $n$  days. While the commonly known song uses 12 days, there are variants with other values of  $n$ . The Faroe Islands use the inflationary value  $n = 15$ . In our exposition, we will rely on the Wikipedia reference [4] as our authority.

Thus we seek a formula for  $T(n)$ , the total number of presents received after the  $n$ 'th day.

## Single day number of presents

On a single day, the number of presents  $S(n)$  is clearly the sum of an arithmetic progression (Equation Arith-progrn).

Arith-progrn

$$S(k) = \sum_{i=1}^k i$$

Once more, but with numbering:

Arith-progrn 2

$$S(k) = \sum_{i=1}^k i$$

Once more, but with the global equation counter reset:

Arith-progrn 1

$$S(k) = \sum_{i=1}^k i$$

One final time, but we are back to using standard numbering:

$$S(k) = \sum_{i=1}^k i \tag{1}$$

Clearly we now want to compute

$$T(n) = \sum_{\{j=1\}}^n S(k)$$

The formula for  $S(k)$  is well known, and derived by noting that writing the sequence forwards and then backwards illustrates that twice the sum is  $k * (k + 1)$ , so we have

$$S(k) = \frac{k(k+1)}{2}$$

We can use this so that we find

$$2T(n) = \sum_{k=1}^n (k^2 + k)$$

We therefore need

$$Q(n) = \sum_{k=1}^n k^2$$

To give our provisional expression as

$$T(n) = \frac{Q(n) + S(n)}{2}$$

$Q(n)$  is a well-known summation, often proved by mathematical induction,

$$Q(n) = \frac{n(2n+1)(n+1)}{6}$$

Thus we want:

$$\begin{aligned} T(n) &= \frac{1}{2} \left( \frac{n(2n+1)(n+1)}{6} + \frac{n(n+1)}{2} \right) \\ &= \frac{2n^3 + 3n^2 + n + 3n^2 + 3n}{12} \\ &= \frac{2n^3 + 6n^2 + 4n}{12} \\ &= \frac{n^3 + 3n^2 + 2n}{6} \end{aligned}$$

We can calculate the summation from within typst using its built-in scripting capabilities:

```
#let n = 12
#((calc.pow(n, 3) + 3*n*n + 2*n)/6)
```

to get the result:

364

Table 1 shows the total number of gifts received as of each day.

<b>Day</b>	<b>Gifts</b>
1	1
2	4
3	10
4	20
5	35
6	56
7	84
8	120
9	165
10	220
11	286
12	364

Table 1: Total gifts received as of each of the 12 days.

## References

- [1] L. Lamport, *LATEX: A Document Preparation System*. Addison-Wesley Publishing Company, 1986. [Online]. Available: <https://books.google.ca/books?id=IgJUAAAAMAAJ>
- [2] L. Mädje, “A Programmable Markup Language for Typesetting,” Doctoral dissertation, 2022.
- [3] Y. Xie, J. Allaire, and G. Grolemund, *R Markdown: The Definitive Guide*. Boca Raton, Florida: Chapman, Hall/CRC, 2018. [Online]. Available: <https://bookdown.org/yihui/rmarkdown>
- [4] “The Twelve Days of Christmas (song).” Accessed: Dec. 09, 2025. [Online]. Available: [https://en.wikipedia.org/w/index.php?title=The\\_Twelve\\_Days\\_of\\_Christmas\\_\(song\)&oldid=1326578965](https://en.wikipedia.org/w/index.php?title=The_Twelve_Days_of_Christmas_(song)&oldid=1326578965)