Partridges in Pear Trees – LaTeX

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Motivation

The preparation of scientific documents with mathematical content has always given their authors and printers difficulty in rendering the material to paper or screen. This article is part of an exercise to illustrate how different systems for preparing such documents compare in capability and ease of use.

The text preparation systems to be compared are LaTeX ([1]), Typst ([2]) and Rmarkdown ([3]). We note that bibliographic support is also of interest to users.

1 A useful example

The Christmas counting song "The Twelve Days of Christmas" suggest the singer receives one gift – we shall unabashedly simply declare that the Partridge in a Pear Tree is a single gift – on the first of twelve days, then that gift plus two more on the second, and so forth. For our needs, we want to have a general formula for the total number of gifts received after n days. While the commonly known song uses 12 days, there are variants with other values of n. The Faroe Islands use the inflationary value $n = 15$. In our exposition, we will rely on the Wikipedia reference [4] as our authority.

Thus we seek a formula for $T(n)$, the total number of presents received after the n'th day.

Single day number of presents

On a single day, the number of presents $S(n)$ is clearly the sum of an arithmetic progression (Equation 1).

$$S(k) = \sum_{i=1}^{k} i \tag{1}$$

Clearly we now want to compute

$$T(n) = \sum_{j=1}^{n} S(k)$$

The formula for $S(k)$ is well known, and derived by noting that writing the sequence forwards and then backwards illustrates that twice the sum is $k * (k + 1)$, so we have

$$S(k) = \frac{k(k+1)}{2}$$

We can use this so that we find

$$2T(n) = \sum_{k=1}^{n} (k^2 + k)$$

We therefore need

$$Q(n) = \sum_{k=1}^{n} k^2$$

To give our provisional expression as

$$T(n) = \frac{Q(n) + S(n)}{2}$$

$Q(n)$ is a well-known summation, often proved by mathematical induction,

$$Q(n) = \frac{n(2n+1)(n+1)}{6}$$

Thus we want:

$$\begin{aligned} T(n) &= \frac{1}{2}\left(\frac{n(2n+1)(n+1)}{6} + \frac{n(n+1)}{2}\right) \\ &= \frac{2n^3 + 3n^2 + n + 3n^2 + 3n}{12} \\ &= \frac{2n^3 + 6n^2 + 4n}{12} \\ &= \frac{n^3 + 3n^2 + 2n}{6} \end{aligned}$$

References

[1] L. Lamport. *LATEX: A Document Preparation System.* Addison-Wesley Publishing Company, 1986.

[2] Laurenz Mädje. *A Programmable Markup Language for Typesetting.* PhD thesis, 2022, 2022.

[3] Yihui Xie, J.J. Allaire, and Garrett Grolemund. *R Markdown: The Definitive Guide.* Chapman and Hall/CRC, Boca Raton, Florida, 2018.

[4] The Twelve Days of Christmas (song), December 2025. Page Version ID: 1326578965.