Parametric optimal control problem:

$$\min_{x(\cdot),u(\cdot)} \quad \Phi(x(t_f)) \tag{1a}$$

$$\text{s.t.} \quad \dot{x}(t) = f(x(t), u(t), p), \tag{1b}$$

$$x(t_0) = x_0 \tag{1c}$$

$$x^{lo} \leq x(t) \leq x^{up}, \tag{1d}$$

$$u^{lo} \leq u(t) \leq u^{up} \qquad \forall t \in [t_0, t_f] \tag{1e}$$

Discretiezed with multiple shooting variables: $s_0, \cdots, s_N \; q_0, \cdots, q_N$

$$\min_{x(\cdot),u(\cdot)} \quad \Phi(S_N) \tag{2a}$$

$$\text{s.t.} \quad s_{i+1} = x(t_{i+1}; t_i, s_i, q_i, p) \quad i = 0, ..., N-1 \tag{2b}$$

$$s_0 = x_0 \tag{2c}$$

$$x^{lo} \leq s_i \leq x^{up}, \qquad i = 0, ..., N \tag{2d}$$

$$u^{lo} \leq q_i \leq u^{up} \qquad i = 0, ..., N \tag{2e}$$

$x(t; t_0, s, q, p)$ is the solution of

$$\dot{x}(t) = f(x(t), q, p) \tag{3}$$

$$x(t_0) = s \tag{4}$$

$$a(s,q) = \begin{bmatrix} x_0 - s_0 \\ x(t_1; t_0, s_0, q_0, p) - s_1 \\ \vdots \\ x(t_N; t_{N-1}, s_{N-1}, q_{N-1}, p) - s_N \end{bmatrix} \tag{5}$$

$$b(s,q) = \begin{bmatrix} x^{lo} - s \\ s - x^{up} \\ q^{lo} - q \\ q - q^{up} \end{bmatrix} \tag{6}$$

Lagrange function at point (s, q, λ, μ)

$$\mathcal{L}(s,q,\lambda,\mu) = \Phi(s_N) - \lambda^\top a(s,q) - \mu^\top b(s,q) \tag{7}$$

$$\nabla\mathcal{L}(s,q,\lambda,\mu) = \begin{bmatrix} \nabla_x\Phi(s_n) - \nabla_x a(s,q)\lambda - \nabla_x b(s,q)\mu \\ a(s,q) \\ b(s,q) \end{bmatrix} \tag{8}$$

$$\nabla^2\mathcal{L}(s,q,\lambda,\mu) = \begin{bmatrix} \nabla^2\Phi(s,q) - \nabla^2 a(s,q)\lambda & \nabla a(s,q) & \nabla b(s,q) \\ \nabla a(s,q)^\top & & \\ \nabla b(s,q)^\top & & \end{bmatrix} \tag{9}$$

At iteration (x_i, λ_i, μ_i) the following qp needs to be solved:

$$\min_{x(\cdot),u(\cdot)} \quad \Delta x^\top \nabla_x^2 \mathcal{L}(x_i,\lambda_i,\mu_i)\Delta x + \nabla\Phi(x_i)\Delta x \tag{10a}$$

$$\text{s.t.} \quad a(x_i) + \nabla a(x_i)\Delta x = 0 \tag{10b}$$

$$b(x_i) + \nabla b(x_i)\Delta x \leq 0 \tag{10c}$$

The following terms include the evaluation of the dynamical system: $a(x_i), \nabla_x a(x_i)\lambda, \nabla_x^2$

Functions for the following terms needs to be implemented: $x(t;t,s,q,p)$, $\nabla_{(s,q)}x(t;t,s,q,p)\lambda$ $\nabla^2_{(s,q)}x(t;t,s,q,p)\lambda$

Dimensions:

$$t \in \mathbb{R} \tag{11}$$

$$x \in \mathbb{R}^{n_x} \tag{12}$$

$$q \in \mathbb{R}^{n_q} \tag{13}$$

$$p \in \mathbb{R}^{n_p} \tag{14}$$

$$\tag{15}$$

- `inp.thoriz` Integration horizon in the form of a 2×1 matrix. At the beginning this can be assumed to be [0,1]. Later time transformation.
- `inp.sd` Initial value for differential states

- `inp.sa` Initial value for algebraic states -¿ ignore atm
- `inp.q` Control parameter
- `inp.p` Model parameter
- `inp.sensdirs` Matrix of directional derivates of dimension $1+n_x+n_q+n_p \times n_{sens}$, where n_{sens} is the number of derivates. Order of directions: [t, x, q, p]. t can be assumed to be 0.
- `inp.lambda` Adjoint directional derivatives. Matrix of dimension $n_x \times n_{adj}$, where n_{adj} is the number of adjoint derivates.

Output -¿ see readme