

DISCRETE MATHEMATICAL STRUCTURES

MAT 203	DISCRETE MATHEMATICAL STRUCTURES	CATEGORY	L	T	P	CREDITS
		BSC	3	1	0	4

Preamble:

The purpose of this course is to create awareness in students about the basic terminologies used in advanced courses in Computer Science and develop rigorous logical thinking for solving different kinds of problems in Computer Science. This course helps the learner to apply the theory and applications of elementary Counting Principles, Propositional Logic, Predicate Logic, Lattices, Generating Functions, Recurrence Relations and Algebraic Structures eventually in practical applications.

Prerequisite: A sound background in higher secondary school Mathematics

Course Outcomes: After the completion of the course the student will be able to

CO#	CO
CO1	Check the validity of predicates in Propositional and Quantified Propositional Logic using truth tables, deductive reasoning and inference theory on Propositional Logic (Cognitive Knowledge Level: Apply)
CO2	Solve counting problems by applying the elementary counting techniques - Rule of Sum, Rule of Product, Permutation, Combination, Binomial Theorem, Pigeonhole Principle and Principle of Inclusion and Exclusion (Cognitive Knowledge Level: Apply)
CO3	Classify binary relations into various types and illustrate an application for each type of binary relation, in Computer Science (Cognitive Knowledge Level: Understand)
CO4	Illustrate an application for Partially Ordered Sets and Complete Lattices, in Computer Science (Cognitive Knowledge Level: Apply)
CO5	Explain Generating Functions and solve First Order and Second Order Linear Recurrence Relations with Constant Coefficients (Cognitive Knowledge Level: Apply)
CO6	Illustrate the abstract algebraic systems - Semigroups, Monoids, Groups, Homomorphism and Isomorphism of Monoids and Groups (Cognitive Knowledge Level: Understand)

Mapping of course outcomes with program outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	✓	✓	✓	✓								✓
CO2	✓	✓	✓	✓								✓
CO3	✓	✓	✓	✓		✓						✓
CO4	✓	✓	✓	✓		✓						✓
CO5	✓	✓	✓	✓								✓
CO6	✓	✓	✓	✓								✓

Abstract POs defined by National Board of Accreditation			
PO#	Broad PO	PO#	Broad PO
PO1	Engineering Knowledge	PO7	Environment and Sustainability
PO2	Problem Analysis	PO8	Ethics
PO3	Design/Development of solutions	PO9	Individual and team work
PO4	Conduct investigations of complex problems	PO10	Communication
PO5	Modern tool usage	PO11	Project Management and Finance
PO6	The Engineer and Society	PO12	Life long learning

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination Marks (%)
	Test 1 (%)	Test 2 (%)	
Remember	30	30	30
Understand	30	30	30
Apply	40	40	40
Analyze			
Evaluate			
Create			

Mark Distribution

Total Marks	CIE Marks	ESE Marks	ESE Duration
150	50	100	3

Continuous Internal Evaluation Pattern:

Attendance	10 marks
Continuous Assessment Tests (Average of Series Tests 1 & 2)	25 marks
Continuous Assessment Assignment	15 marks

Internal Examination Pattern:

Each of the two internal examinations has to be conducted out of 50 marks. First series test shall be preferably conducted after completing the first half of the syllabus and the second series test shall be preferably conducted after completing remaining part of the syllabus. There will be two parts: Part A and Part B. Part A contains 5 questions (preferably, 2 questions each from the completed modules and 1 question from the partly completed module), having 3 marks for each question adding up to 15 marks for part A. Students should answer all questions from Part A. Part B contains 7 questions (preferably, 3 questions each from the completed modules and 1 question from the partly completed module), each with 7 marks. Out of the 7 questions, a student should answer any 5.

End Semester Examination Pattern:

There will be two parts; Part A and Part B. Part A contains 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 full questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carries 14 marks.

Syllabus

Module – 1 (Fundamentals of Logic)

Mathematical logic - Basic connectives and truth table, Statements, Logical Connectives, Tautology, Contradiction. Logical Equivalence - The Laws of Logic, The Principle of duality, Substitution Rules . The implication - The Contrapositive, The Converse, The Inverse.

Logical Implication - Rules of Inference. The use of Quantifiers - Open Statement, Quantifier. Logically Equivalent – Contrapositive, Converse , Inverse , Logical equivalences and implications for quantified statement, Implications , Negation .

Module - 2 (Fundamentals of Counting Theory)

The Rule of Sum – Extension of Sum Rule . The Rule of Product - Extension of Product Rule . Permutations. Combinations. The Binomial Theorem (without proof). Combination with Repetition. The Pigeon hole Principle. The Principle of Inclusion and Exclusion Theorem (Without Proof) - Generalization of the Principle. Derangements.

Module - 3 (Relations and Functions)

Cartesian Product - Binary Relation. Function – domain , range-one to one function, Image-restriction. Properties of Relations- Reachability Relations, Reflexive Relations, Symmetric Relations, Transitive relations, Anti-symmetric Relations, Partial Order relations, Equivalence Relations, Irreflexive relations.

Partially ordered Set – Hasse Diagram, Maximal-Minimal Element, Least upper bound (lub), Greatest Lower bound(glb) (Topological sorting Algorithm- excluded). Equivalence Relations and Partitions - Equivalence Class.

Lattice - Dual Lattice , Sub lattice , Properties of glb and lub , Properties of Lattice , Special Lattice , Complete Lattice, Bounded Lattice, Completed Lattice , Distributive Lattice.

Module - 4 (Generating Functions and Recurrence Relations)

Generating Function - Definition and Examples , Calculation techniques, Exponential generating function. First order linear recurrence relations with constant coefficients – homogeneous, non-homogeneous Solution. Second order linear recurrence relations with constant coefficients, homogeneous, non-homogeneous Solution.

Module - 5 (Algebraic Structures)

Algebraic system-properties- Homomorphism and Isomorphism. Semi group and monoid – cyclic monoid , sub semi group and sub monoid, Homomorphism and Isomorphism of Semi group and monoids. Group- Elementary properties, subgroup, symmetric group on three symbols ,The direct product of two groups, Group Homomorphism, Isomorphism of groups, Cyclic group. Rightcosets - Leftcosets. Lagrange's Theorem

Text Book

1. Discrete and Combinatorial Mathematics (An Applied Introduction), Ralph P Grimaldi, B V Ramana , 5th Edition, Pearson

Reference Books

- 1) Kenneth H. Rosen, Discrete Mathematics and Its Applications with Combinatorics and Graph Theory, Seventh Edition, MGH, 2011
- 2) Trembly J.P and Manohar R, “Discrete Mathematical Structures with Applications to Computer Science”, Tata Mc Graw Hill Pub. Co. Ltd., New Delhi, 2003.
- 3) Bernard Kolman, Robert C. Busby, Sharan Cutler Ross, “Discrete Mathematical Structures”, Pearson Education Pvt Ltd., New Delhi, 2003
- 4) Kenneth H. Rosen, “Discrete Mathematics and its Applications”, 5/e, Tata Mc Graw Hill Pub. Co. Ltd, New Delhi 2003
- 5) Richard Johnsonbaugh, “Discrete Mathematics”, 5/e, Pearson Education Asia, New Delhi, 2002.
- 6) Joe L Mott, Abraham Kandel, Theodore P Baker, “Discrete Mathematics for Computer Scientists and Mathematicians”, 2/e, Prentice-Hall India, 2009.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Show that $R \vee M$, $\neg R \vee S$, $\neg M$, $\neg S$ cannot exist simultaneously (without using truth table)
2. Represent the following statement in symbolic form “Not every city in Canada is clean”.

Course Outcome 2 (CO2):

1. How many possible arrangements are there for the letters in MASSASAUGA in which 4 A's are together?
2. Find the number of integers between 1 and 1000 inclusive, which are not divisible by 5, 6 or 8

Course Outcome 3 (CO3):

1. If $A = \{1, 2, 3, 4\}$, give an example of a relation R that is reflexive and symmetric but not transitive.
2. Let Z be the set of integers. R is a relation called “Congruence Modulo 3” defined by $R = \{ (x, y) / x \in Z, y \in Z, x - y \text{ is divisible by } 3 \}$. Show that R is an equivalence relation.

Course Outcome 4 (CO4):

1. Assume $A = \{ a, b, c \}$. Let $P(A)$ be its power set and ‘ \leq ’ be the subset relation on the power set. Draw the Hasse diagram of $(P(A), \leq)$.
2. What is meant by Bounded Lattice? Give an example.

Course Outcome 5 (CO5):

1. Solve $a_r - 3a_{r-1} - 4a_{r-2} = 3^r$ using Generating function method; Given $a_0 = 1, a_1 = 2$.
2. Find the generating function for the sequence $1, 3, 3^2, 3^3, \dots$

Course Outcome 6 (CO6):

1. Prove that the group $\{ 1, -1, i, -i \}$ is cyclic with generators i and $-i$.
2. State and prove Lagrange's Theorem.

Model Question Paper

QP CODE:

Reg No: _____

Name : _____

PAGES : 3

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, MONTH & YEAR

Course Code: MAT 203

Course Name: Discrete Mathematical Structures

Max.Marks :100

Duration: 3 Hrs

PART A

Answer all Questions. Each question carries 3 Marks

1. Show the following implication without constructing the truth table: $(P \wedge Q) \Rightarrow P \rightarrow Q$
2. Write the negation of the following statement. "If I drive, then I will not walk"
3. What is pigeon hole principle? Explain. If you select any five numbers from 1 to 8 then prove that at least two of them will add up to 9 .
4. In how many ways can the letters of the word ALLAHABAD be arranged ?
5. Show that the divisibility relation $'/'$ is a partial ordering on the set \mathbb{Z}^+ .
6. Consider the functions given by $f(x) = 2x+3$ and $g(x) = x^2$. Find $(g \circ f)$ and $(f \circ g)$.
7. What is meant by exponential generating function? Explain.
8. Provide one example of linear homogeneous recurrence relation. Mention the degree also.
9. What is a monoid ? Explain.
10. Let $(A, .)$ be a group. Show that $(ab)^{-1} = b^{-1}a^{-1}$

(10 x 3 = 30 Marks)

PART B

(Answer any one Question from each Module. Each question carries 14 Marks)

- 11.
- (a) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

(6 marks)

(b) Show that from

(ii) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$.

(iii) $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows.

(8 marks)

OR

12.

(a) Show that $(x)(P(x) \vee Q(x)) \Rightarrow ((x)P(x) \vee (\exists x)Q(x))$ using indirect method of proof.

(6 marks)

(b) Discuss indirect method of proof. Show that the following premises are inconsistent

(i) If Jack misses many classes through illness, then he fails high school.

(ii) If Jack fails high school, then he is uneducated.

(iii) If Jack reads a lot of books, then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books.

(8 marks)

13.

(a) Explain binomial theorem. Determine the coefficient of x^9y^3 in the expansion of $(x+y)^{12}$, $(x+2y)^{12}$ and $(2x-3y)^{12}$ using binomial theorem.

(6 marks)

(b) How many 5 digit numbers can be formed from the digits 1,2,3,4,5 using the digits without repetition?

(i) How many of them are even?

(ii) How many are even and greater than 30,000?

(8 marks)

OR

14.

(a) There are 8 guests in a party. Each guest brings a gift and receives another gift in return. No one is allowed to receive the gift they bought. How many ways are there to distribute the gifts?

(6 marks)

(b) Six papers are set in an examination of which two are mathematical. Only one examination will be conducted in a day. In how many different orders can the papers be arranged so that

(i) Two mathematical papers are consecutive?

(ii) Two mathematical papers are not consecutive?

(8 marks)

15.

- (a) Let $A = \{1, 2, 3, 4, \dots, 11, 12\}$ and let R be the equivalence relation on $A \times A$ defined by $(a, b) R (c, d)$ iff $a+d = b+c$. Prove that R is an equivalence relation and find the equivalence class of $(2, 5)$

(8 marks)

- (b) What is a chain lattice? Explain. Also show that every chain is a distributive lattice.

(6 marks)

OR

16.

- (a) Suppose $f(x) = x+2$, $g(x) = x-2$, and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $(g \circ f)$, $(f \circ g)$, $(f \circ f)$ and $(g \circ g)$

(8 marks)

- (b) Let R and S be two relations on a set A . If R and S are symmetric, Prove that $(R \cap S)$ is also symmetric.

(6 marks)

17.

- (a) Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ for $r \geq 2$; Given $a_0 = 0$; $a_1 = 41$ using generating functions

(8 marks)

- (b) Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2$ using generating function.

(6 marks)

OR

18.

- (a) Solve $a_n - 3a_{n-1} + 2 = 0$; $a_0 = 1$ $n \geq 1$, using generating functions.

(8 marks)

- (b) Use generating function to solve the following recurrence relation $a_n = 2a_{n-1} + 2^n$; with $a_0 = 2$.

(6 marks)

19.

- (a) Prove that the set ' \mathbb{Q} ' of rational numbers other than 1 forms an abelian group with respect to the operation ' $*$ ' defined by $a * b = a+b-ab$.

(8 Marks)

- (b) Show that the direct product of two group is a group.

(6 Marks)

OR

20.

- (a) Show that the subgroup of a cyclic group is cyclic.

(8 Marks)

- (b) Let $(A, *)$ be a group. Show that $(A, *)$ is an abelian group if and only if $a^2 * b^2 = (a * b)^2$ for all ' a ' and ' b ' in A

(6 Marks)

TEACHING PLAN

No	Contents	No of Lecture Hrs
Module – 1 (Fundamentals of Logic) (9 hrs)		
1.1	Mathematical logic, Basic Connectives and Truth Table	1
1.2	Statements, Logical Connectives, Tautology, Contradiction	1
1.3	Logical Equivalence, The Laws of Logic	1
1.4	The Principle of duality, Substitution Rules	1
1.5	The implication, The Contrapositive, the Converse , the Inverse	1
1.6	Logical Implication, Rules of Inference, Logical Implication	1
1.7	The use of Quantifiers, Open Statement, Quantifier, Negation	1
1.8	Logically Equivalent, Contrapositive, The Converse, The Inverse	1
1.9	Logical Implications	1
Module - 2 (Fundamentals of Counting Theory) (9 hrs)		
2.1	The Pigeon-hole Principle	1
2.2	The Rule of Sum	1
2.3	Extension of Sum Rule	1
2.4	The Rule of Product	1
2.5	Extension of Product Rule , Permutations	1
2.6	Combinations, Combination with repetition	1
2.7	The Binomial Theorem	1
2.8	The Principle of Inclusion and Exclusion Theorem (Without Proof) Generalization of the Principle	1
2.9	Derangements	1
Module - 3 (Relations and Functions) (9 hrs)		
3.1	Cartesian Product, Binary Relation, Function, Domain, Range , One to One Function Image - Restriction	1
3.2	Properties, Reachability Relations, Reflexive Relations, Symmetric Relations, Transitive relations, Antisymmetric Relations.	1

3.3	Partial Order relations	1
3.4	Equivalence Relation, Irreflexive Relations.	1
3.5	Partially ordered Set, Hasse Diagram.	1
3.6	Maximal-Minimal Element, Least Upper bound, Greatest Lower Bound	1
3.7	Equivalence Relations and Partitions, Equivalence Class	1
3.8	Lattice- Dual Lattice, sub lattice, Properties of glb and lub	1
3.9	Properties of Lattice, Special Lattice, Complete Lattice, Bounded Lattice, Completed Lattice, Distributive Lattice	1
Module - 4 (Generating Functions and Recurrence Relations) (9 hrs)		
4.1	Generating Function, Definition and Examples	1
4.2	Exponential Generating Function.	1
4.3	First Order Linear Recurrence Relations with Constant Coefficients (Lecture I)	1
4.4	First Order Linear Recurrence Relations with Constant Coefficients (Lecture II)	1
4.5	Homogeneous Solution	1
4.6	Non homogeneous Solution	1
4.7	Second order linear recurrence relations with constant coefficients	1
4.8	Homogeneous Solution	1
4.9	Non homogeneous Solution	1
Module - 5 (Algebraic Structures) (9 hrs)		
5.1	Algebraic System-Properties, Homomorphism and Isomorphism	1
5.2	Semi group, Monoid, Cyclic monoid	1

5.3	Sub semigroup and sub monoid	1
5.4	Homomorphism and Isomorphism of Semigroup, Monoids and Groups	1
5.5	Elementary Properties, Subgroup, Symmetric group on three symbols	1
5.6	The direct Product of two Groups	1
5.7	Group Homomorphism, Isomorphism, Cyclic group	1
5.8	Right coset, Left coset	1
5.9	Lagrange's Theorem	1

