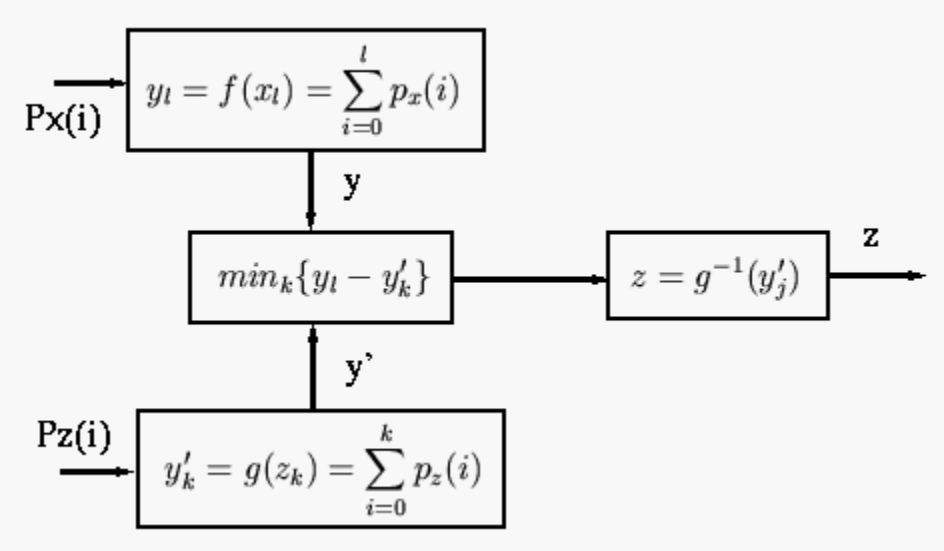
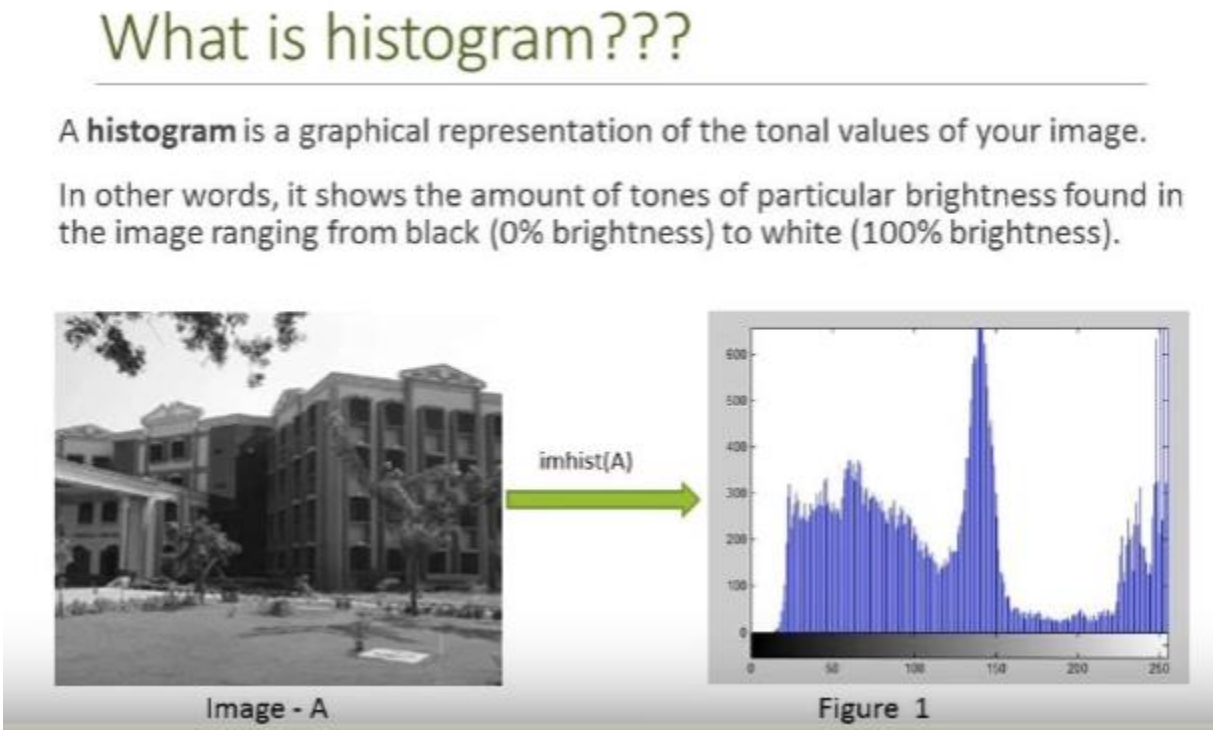
Histogram Specification

Here we want to convert the image so that it has a particular histogram that can be arbitrarily specified. Such a mapping function can be found in three steps:

- Equalize the histogram of the input image
- Equalize the specified histogram
- Relate the two equalized histograms





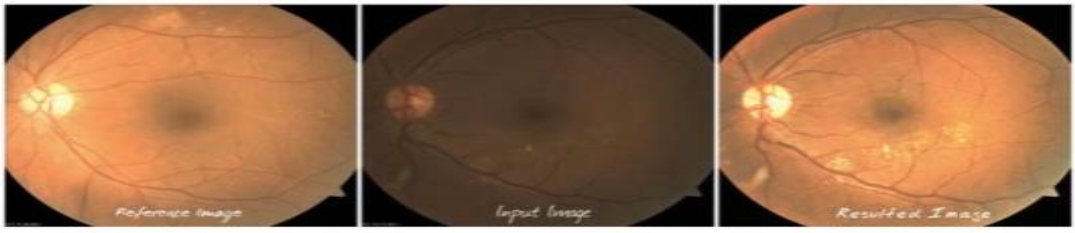
Histogram Specification

Histogram specification is the transformation of an image so that its histogram matches a specified histogram.

Reference image **Input image** **Resultant image**

Reference image **Input image** **Resultant image**

Application



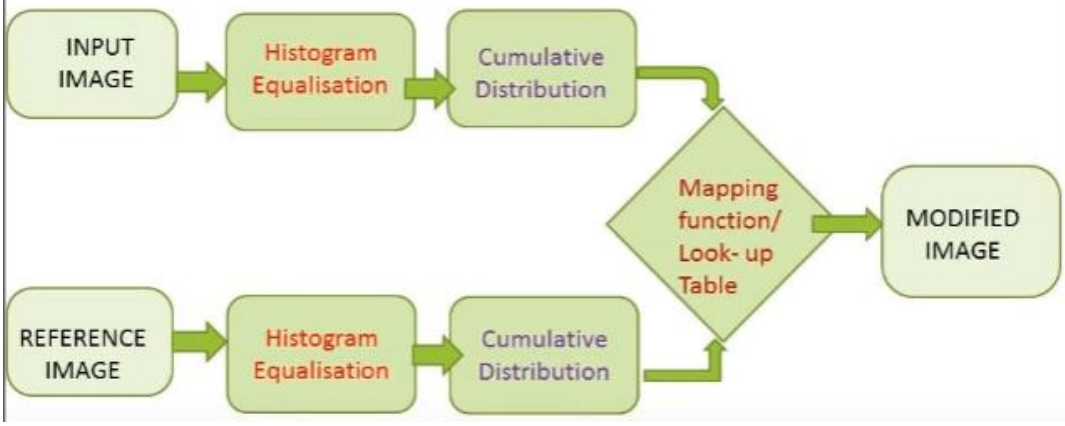
Retinal image enhanced by using proper 'histogram specified' reference image.

Objective:

Here we want to convert the image so that it has a particular histogram that can be arbitrarily specified. Such a mapping function can be found in three steps:

- Equalize the histogram of the input image.
- Equalize the specified histogram.
- Relate the two equalized histograms.

Histogram specification in a nutshell



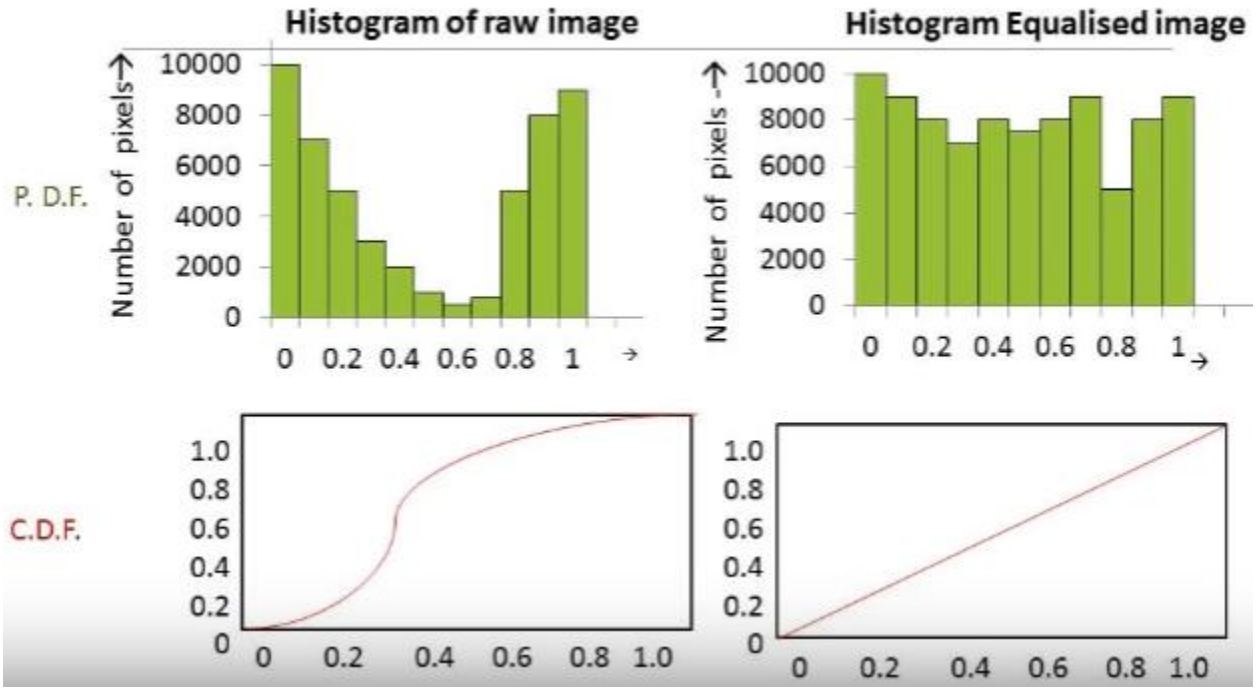
Histogram Equalisation

- It is a technique for adjusting the image intensities to enhance the contrast.
- Let ' f ' be an image represented by a matrix ' M ' of 'r' rows and 'c' columns, the integer pixel intensities ranging from 0 to L-1 where 'L' is the possible intensity values (often 256). Then normalised histogram of ' f ' with a bin for each possible intensity.

$$p_n = \frac{\text{number of pixels with intensity n}}{\text{total number of pixels}} \qquad n = 0,1,2,\ldots.. L-1$$

Image - A

Figure 2



Math behind histogram specification

- We first equalize the histogram P_x of the input image X :

$$y = f(x) = \int_0^x p_x(u)\,du$$

- We then equalize the desired histogram P_z of the output image :

$$y' = g(z) = \int_0^x p_z(u)\,du$$

- The inverse of the above transform is $z = g^{-1}(y')$.

- As the two intermediate images y and y' both have the same equalized histogram, they are actually the same image, ie., $y = y'$.

- Overall transform from the given image to the desired image can be found to be:

$$z = g^{-1}(y') = g^{-1}(y) = g^{-1}(f(x)).$$

- However, as the image gray levels are discrete, the continuous mapping obtained above can only be approximated.

- The discrete histograms $h_y[i]$ and $h_{y'}[i]$ are not necessarily identical.

- We therefore need to relate each gray level in x with $h_x[i] \neq 0$ to a gray level in z with $h_z[i] \neq 0$, so that the mapping from y to y' can be established.

Steps of the algorithm.

Step 1: Find histogram of input image h_x, and find its cumulative H_x, the histogram equalization mapping function.

$$H_x[j] = \sum_{i=0}^{j} h_x[i]$$

Step 2: Specify the desired histogram h_z, and find its cumulative distribution H_z, the histogram equalization mapping function is given by:

$$H_z[j] = \sum_{i=0}^{j} h_z[i]$$

Step 3: Relate the two mapping to build a lookup table for the over all mapping. Specifically, for each input level i, find an output level j so that $H_z(j)$ best matches $H_x(i)$:

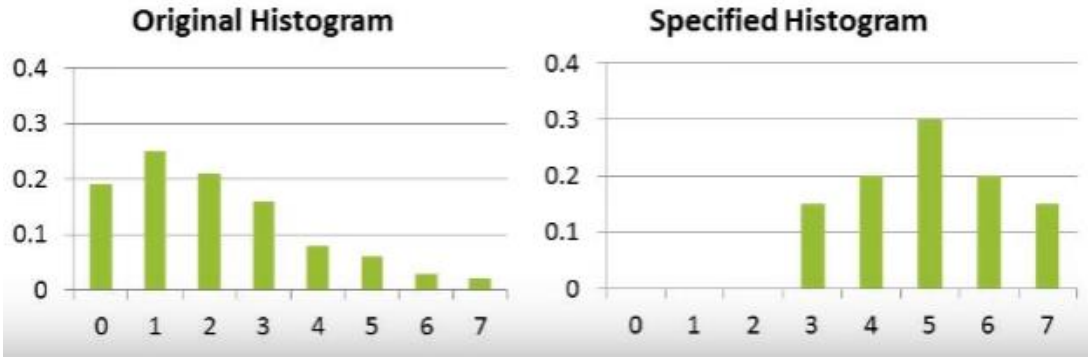
$$| H_x[i] - H_z[j] | = \min_k | H_x[i] - H_z[k] |.$$

and then we setup a lookup entry-

$$lookup[i] = j.$$

Example:

The histogram of the given image and the histogram desired are shown below-



Step 1 : Equalise p_x to get mapping y= f(x) ; Step 2 : Equalise p_z to get mapping y' = g(z)

X_i (L)	n_j No. of pixels	h_x equalisation	$Y=H_x$ C.D.F
0	790	0.19	0.19
1	1023	0.25	0.44
2	850	0.21	0.65
3	656	0.16	0.81
4	329	0.08	0.89
5	245	0.06	0.95
6	122	0.03	0.98
7	81	0.02	1.00

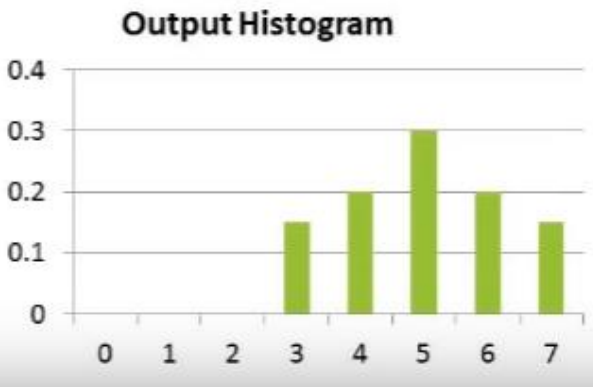
Z_i (L)	p_z equalisation	$y'=H_z$ C.D.F.
0	0.00	0.00
1	0.00	0.00
2	0.00	0.00
3	0.15	0.15
4	0.20	0.35
5	0.30	0.65
6	0.20	0.85
7	0.15	1.00

Step 3 : Obtain overall mapping , x --> y --> y' --> z

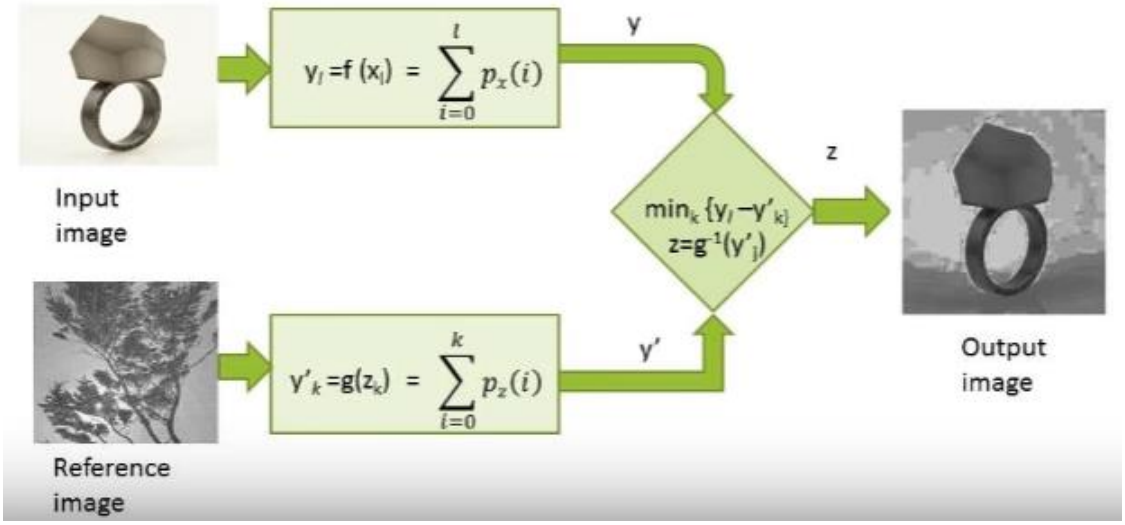
$X_i = i$	$Y_j = H_x$	$Y'_j = H_z$	$Z_j = j$
0	0.19	0.00	3
1	0.44	0.00	4
2	0.65	0.00	5
3	0.81	0.15	6
4	0.89	0.35	6
5	0.95	0.65	7
6	0.98	0.85	7
7	1.00	1.00	7

Look up table

i	0	1	2	3	4	5	6	7
j	3	4	5	6	6	7	7	7



Summary


Input image
$y_l = f(x_l) = \sum_{i=0}^{l} p_x(i)$
y
$\min_k \{y_l - y'_k\}$
$z = g^{-1}(y'_j)$
z
Output image
Reference image
$y'_k = g(z_k) = \sum_{i=0}^{k} p_z(i)$
y'