Mathematical Morphology

- On completion the students will learn and be able to implement
  - What is Morphology?
  - Morphological image processing techniques
    - Dilation
    - Erosion
    - Opening
    - Closing
  - Applications

Application

Application

- Preprocessing
  - Filtering
  - Shape simplification
- Segmentation using object shape
- Object quantification
  - Area, perimeter etc.
- Enhancing object structure
  - Skeletonization, thinning, thickening, convex hull, object marking etc.

Mathematical Morphology

What is Morphology?

Commonly denotes a branch of biology that deals with the form and structure of animals and plants

Tool for extracting image components that are useful for representation and description of region shape, boundary, skeleton, convex hull etc.

Images as Point Sets

$$X = \{(2,0), (2,1), (1,2), (3,2) \ldots\ldots\}$$

$$X^c =$$

A in Z^2

$a = (a_1, a_2)$

$a \in A$

$b \notin A$

Subset

$A \subseteq B$

Union

$A \cup B$

Intersection.

$A \cap B$

Complement

$A^c = \{b \mid b \notin A\}$

Difference.

$A - B = \{\omega \mid \omega \in A \text{ and } \omega \notin B\}$

Reflection of B

$$\hat{B} = \{ w \mid w = -b \text{ for } b \in B \}$$

Translation $\Rightarrow$ $z = \{ z_1, z_2 \}$

$$A_z = \{ c \mid c = a + z \text{ for } a \in A \}$$

Morphological transformation

Morphological Transformation ψ

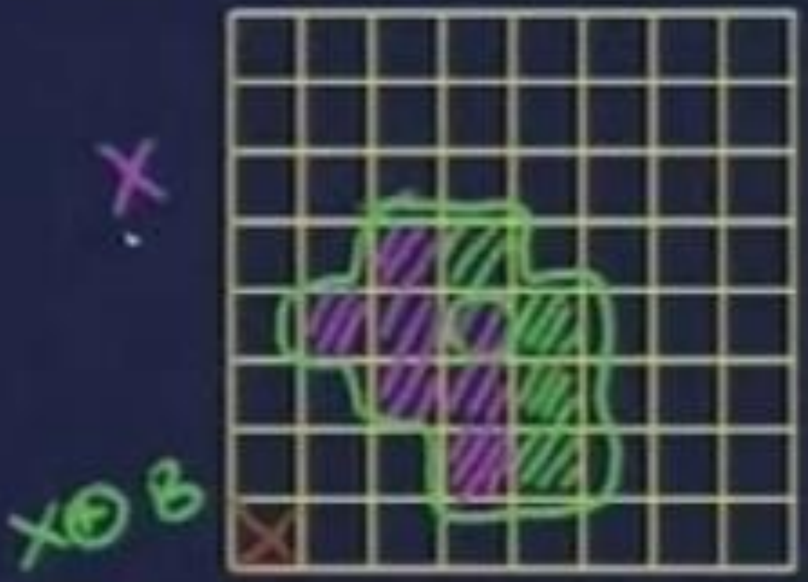
Gives a relation of the image X, with another small point set B, called structuring element

Dilation $\oplus$

$$X \oplus B = \{ p \in Z^2 \mid p = x + b,\ x \in X,\ b \in B \}$$

$$X \oplus B = \{ p \mid (\hat{B})_p \cap X \neq \phi \}$$

Dilation

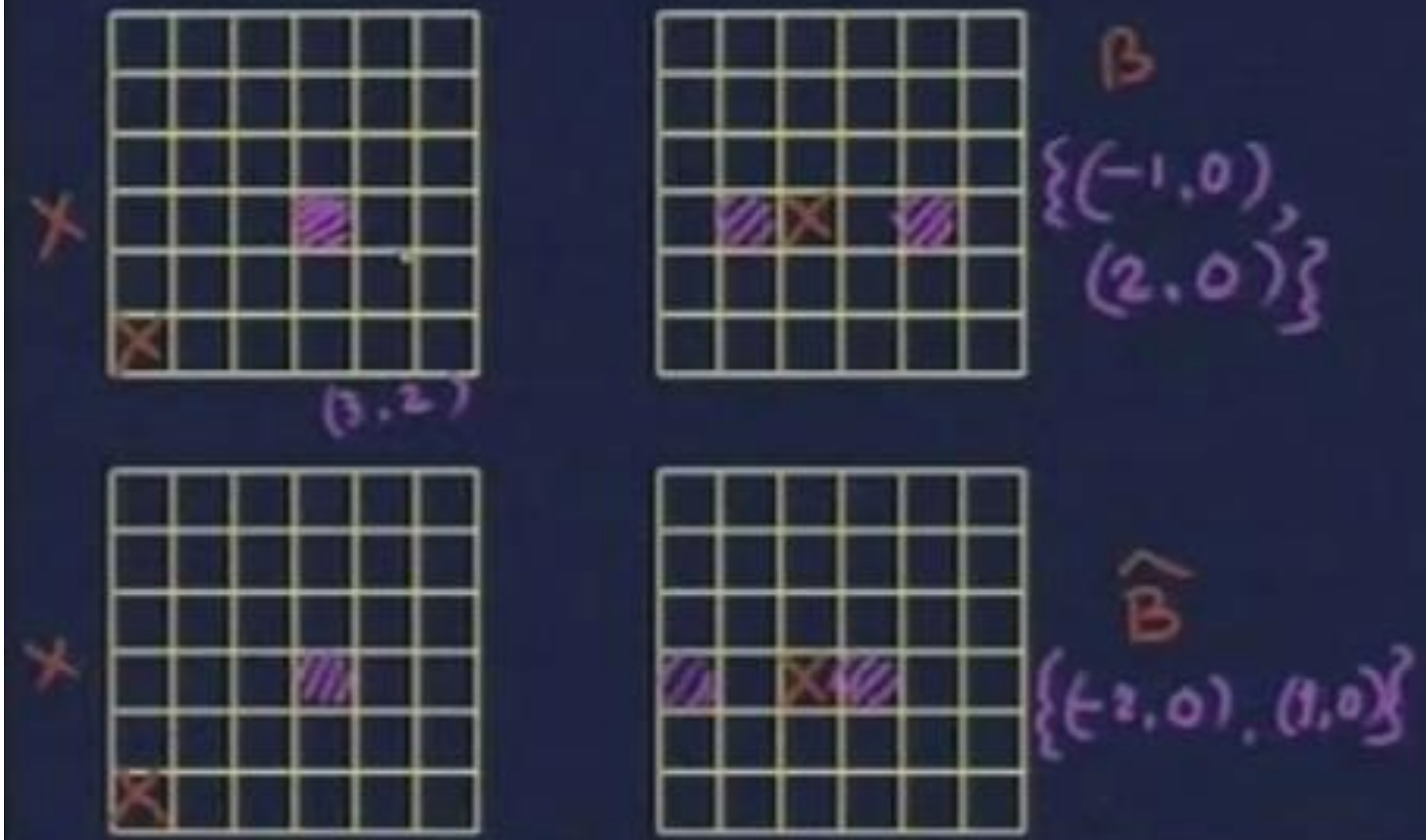


$$B = \{\underline{(0,0)}, (1,0)\}$$

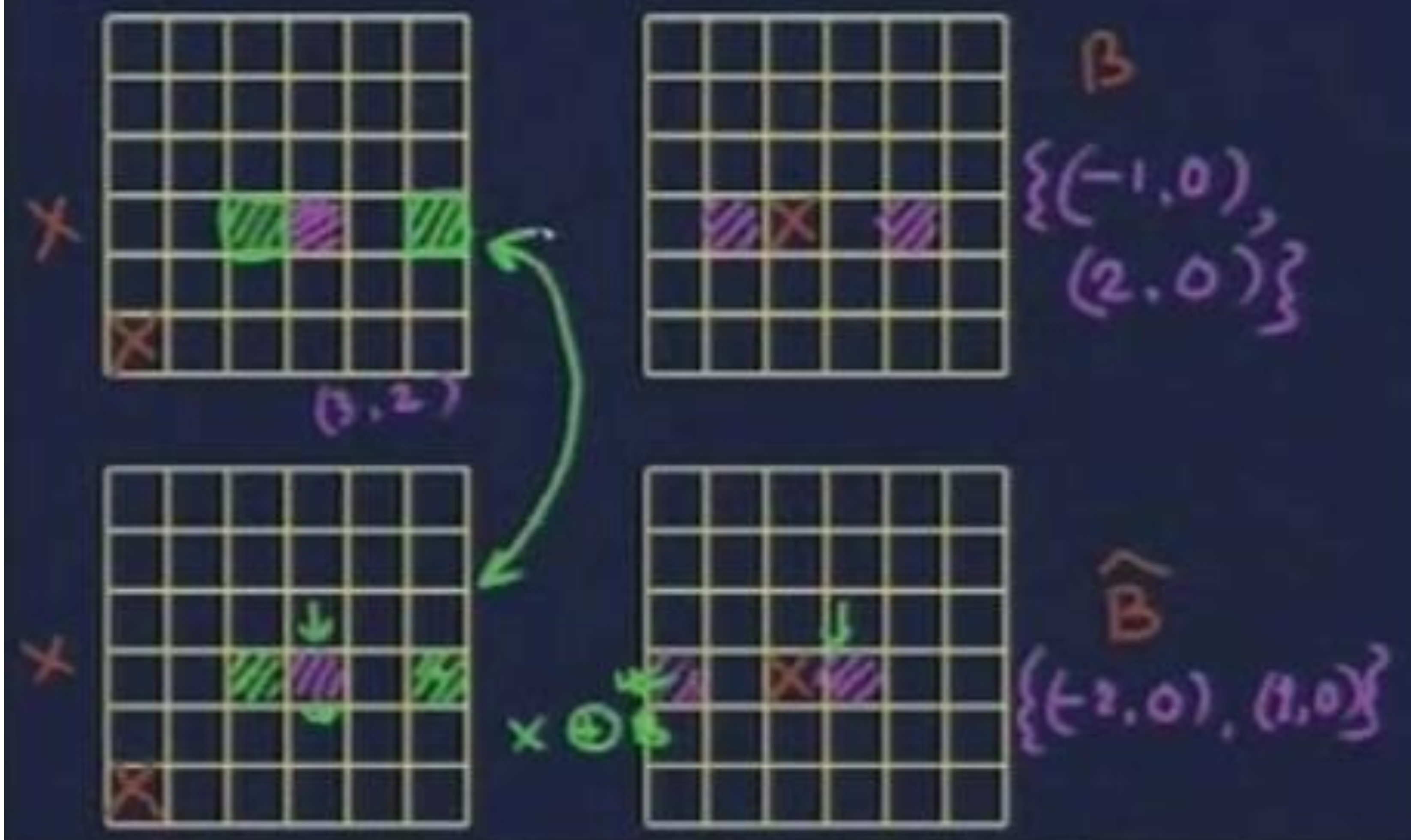
$$X = \{(3,2), (2,2), (3,1) \cdots\cdots\}$$

$$(3,2) + (1,0) = (4,2)$$

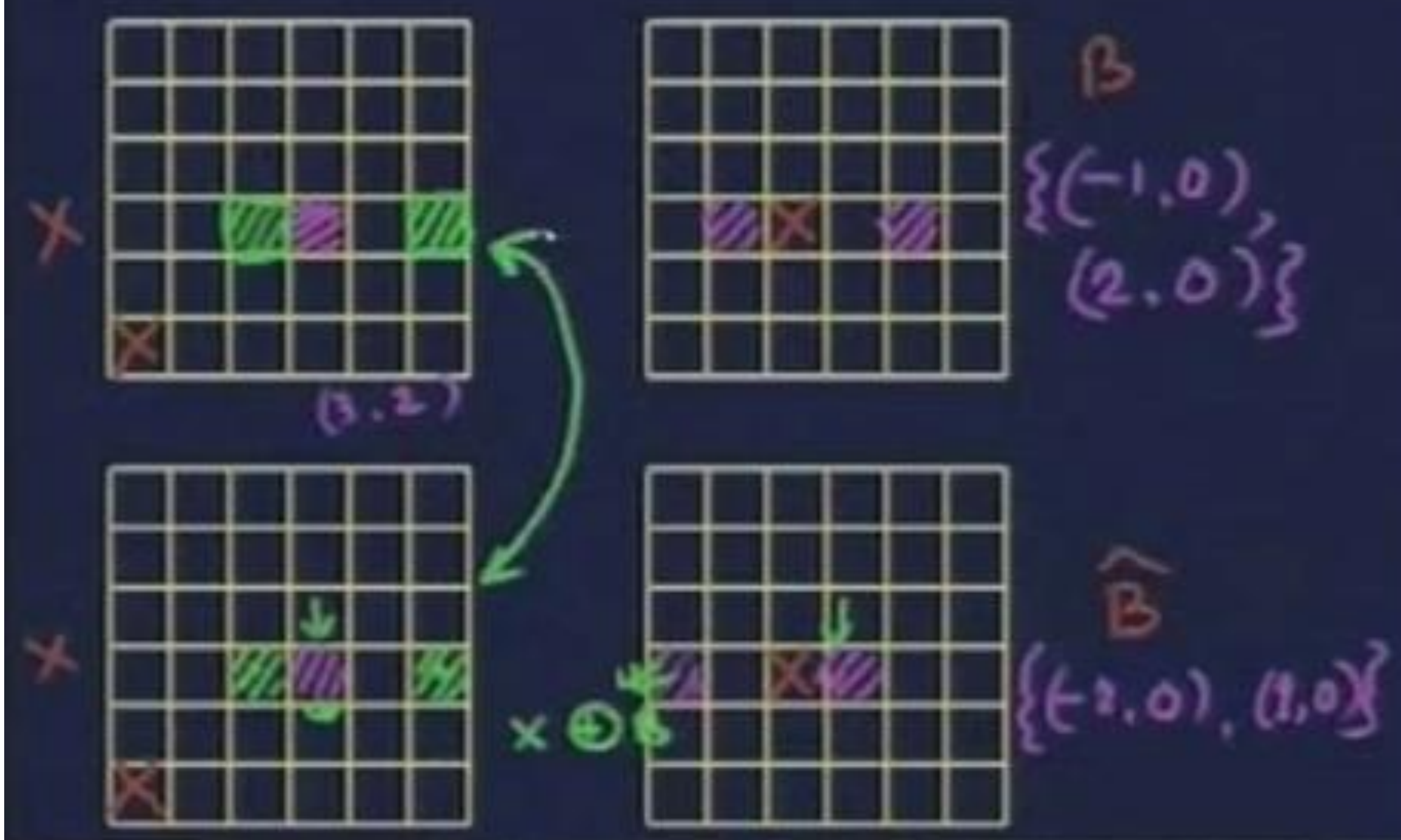
Dilation



Dilation

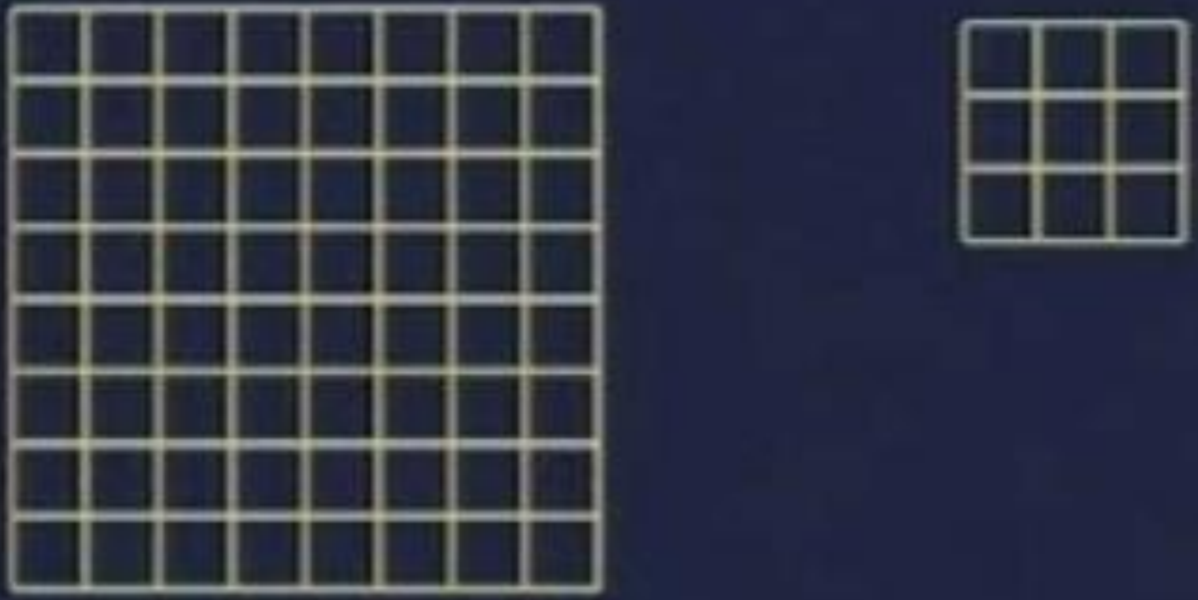


Dilation



Dilation Application

Erosion



$X \qquad\qquad B$

$$X \ominus B = \{ p \in Z^2 \mid p + b \in X \text{ for every } b \in B \}$$

$$X \ominus B = \{ p \mid (B)_p \subseteq X \}$$